12/10-2021—14/10-
-2026

Campus

Microperforado ■ Bandas de color

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12/I 2022 г. **План работы (от 21/III 2021)**

1) AF DW dynamics paper – subm. to PRA – Referees' reports 14/II 22
2) DW scattering at film edge Pawel – subm. PRA – Referees
3) Vortex gyromodes thick dots / nanowires – subm. Dieß, anal. Nina
4) Benjamin – bone shaped dots theory – subm. Sci. Rep. – POLS Svenogr.
5) Skyrmion vs ferrimagnet u Jagg (Ni) Зухра, Анализ
6) Bloch point MS – accepted Sci. Rep. – Oberste APC
7) Proposal GV, groups consolidation – Jekyll. – Subm. 19/I 22
8) Cgeiach azim. DW bowing circular hole Mateusz, $\theta(\varphi)$ DW
9) Nagra travel report Poznan
10) Reviewing APL Wu-Kang
11) Nyumi invited Speakers Sol-SkyMag '2022
12) Cnet Santander.pl – (10 kPLN – nanowire)
13) Nagra u Ryerech 8 lecture no 45 min – Poznan, go 31/I 2022
14) Bneem 6 CV Poznan conf – July u Oct. 11 u, 20/II 2021 г. 24/I u.u. 31/I 2022
15) New members "Dole" – 3 mo
16) Von 7 kE RISE – MSCA project go 15/XII 2021
17) Arneng Hopfian – dot Simul by Gracezyk
18) Hiring Investigador (Ricardo) – bunch mo us my project
19) BP domain wall – c Papxagen
20) Rev. JKEP / Apaob u MMM / Angelova, Mansell / Lee
21) Our Conf (ex MMM – Intermag) 2022 u nagra. gpaeag. Oberwolfach
22) Conf Seville, June 5-10, 2020 Abs. go Jan. 15, 2022. Hotel booked u Hotel booked
23) Abs. JEMS '2022 July 24-29, Warsaw go March 3, 2022 – Hotel Booked
24) Abs NAP-2022, Sept 11-16, Krakow (invited) go Apr. 25.
25) Intermediate report „Ministerio "PID 2019 project go March 1, 2022 – res. Rep. go 15 Feb
26) Bloch point DW dynamics in nanowires
27) Oberl na cnet Joany POLS „3D magnetism" project – subm. Feb. 9, 2022

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(12/X) Подготовка proposal - GV с Якуковым.
Организовал свой talk (BP) на Spintronics Conf. Poznan, Oct. 20-23, 2021.

(13/X) В Poznan downtown - получил номер PESEL. Беседа с Mateusz, Maciej, Mathieu - Skyrmion dot on top of Co-Soft mag. film. Skyrmion  Sk. core вытесняет свой core вдоль M of soft film и становится более стабильным, при меньшем DMI

(14/X) Перевел свою часть по 3D magnetism to Spanish и послал Якукову. Беседа с Klos и Тартаковская GV in stripe array. Расчет $I_0(P)$, $I_1(P)$ isolated stripe для J. Klos. Пена Отжига FMR с M_{dp} - Aharoni, без учета реальных N_{xx}, N_{yy} для неоднородной low-f mode но как в Gus' PRB'2002.

(15/X) Беседа с Jaroslaw о вкладе dip. pinning, Gus' PRB'2002. Mateusz и Mathieu представили dynamics of patterned film with ○ △ □ holes в Co-film with PMA. Области вблизи holes близки к 5) vortex-ring ch-plane state и могут быть описаны через vortex ⇒ ring spectra in finite H_z Завершил GV project external Col. letters.

(16/X) Наладил с помощью A. получение SMS.
→ (17) GV-project - оформлено certificates на всех моих 4 projects
b) remind с contracts by Arantxa. c) Обновл. мои publications to On-line app
d) послал Якукову ext. collaborators letters.
(18) активировал my Polish-Santander card by SMS и пров. счет
(19) Contact Roadmap magnonics (имя, address grants) p-35, 36
(20) подпис. BP доклад на Poznan conference Sat. Oct. 23
(21) подпис. 2 темы для Master, PhD students на 25-26/X.
(22) Организовать опл. APC (50% на 50%) of Sci Reports paper. Couc.
→ 7) IVA и Cristanto" за III кв. 2021.
→ 8) print Ikerbasque contract.

(17/X) Работа над GV-proposal вчера consolidados с Якуковым.

(18/X) Активировал Santander Pol. card. Выбрал PIN и password к счету. Kaufland.

(19/X) Meeting с Krawczyk group. Доклад Katarzina о взаимодействии SW - stripe с Skyrmion at the top of stripe. Рассеяние SW Gus' PRB'02 ω_n, $\cos(x_n y) e^{ik_x x}$, $\vec{M}_0 \| \hat{x}$, $\omega_n \to \omega_{n'}$ на skyrmion за счет $\omega_n(k_x)$. Якуков подал Kiva GV project, подписав его за меня. Л. West Un. в Kiev name - 220 €. Посылка Л. Дубо - Kiev Inv. Proofs "BP" paper.

(20-29/X) Работа в Poznan - HoBS - project с Леней и Piotr. Inv. talk Symp. on Spintronics and Quantum information" Poznan Oct 20-23 2021 - "Bloch points" Oct. 23, 2021. 2 лекции в UAM "Mag. vortices" М. Skyrm. Подписал contracts POIS Hopf. Project (60,000 PLN, 6 выплат по 10 k PLN до 30 June 2023), выставил счет 10 k PLN на Task 1 - "Bloch points: statics" подписал contract на 10 лекций в UAM (можно online) до 31.05.2022 - 4 k PLN.

(30/X) Полет домой POZ → Munich → BSO. Л. с Антоном - встретили в аэропорту Бургоса (по маршруту Burgos - La Guardia), ужин, камин.

(31/X) Осень дома с Л. (по маршруту Burgos - La Guardia), ужин, камин.

(1/XI) Бумаги. Срочные e-mails.

(2/XI) Рецензир. APL paper Wu-Kong, Vortex core reversal. E-mails J. Julian, Cristina о открытой Investigador position. Бумаги.

(3/XI) С Л. в "Alcampo" Oiartzun. E-mail Ricardo о position. Бумаги.

(4/XI) Решение 3D Vortex equations for thick dot on Aug. 2014. Vortex GM - thick dot
метод local magnetostatics Fegel и eq. Mathieu или eq. Parabolic Cylinder
в Давиденко от способа аппроксимации $J(z) = (dz \cdot \sigma(z) z)$ или $J(z) = J(0)\cos pz$ или $J(z) = J(0)(1 - p^2 z^2)$, $p^2 = -\frac{1}{2} J''(0)/J(0) > 0$, $p(\beta)$ - вычисл. аналитически.

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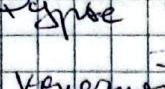
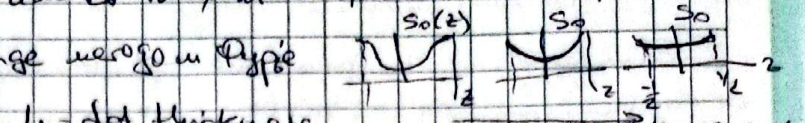
In Aug. 2014 установлено что СЗ/ФП ур-я Thiele:

(1) дает $n=0$ гуромоде $S_0(z)$ - profile н/м чего напоминает. решение

1D методом Фурье  имеет max at $z=0$, middle dot ферромагнетик.

н/м чтобы изменить exchange методом Фурье 

с ростом $y \sim 1/L^2$, т.е. с уменьшением L - dot thickness $S_0(z)$ имеет max н/м $z = \pm 1/2$ dot top/bottom, превращаясь в $S_0(z) = const$ н/м $L \to 0$.

Т.е. численное решение 1D методом Фурье говорит что exchange (finite y) обеспечивает $S_0(z)$ top/bottom localization.

(2) Local magnetostatics, reducing to Mathieu eq. $\frac{d^2S}{d\xi^2} + [a - 2q\cos 2\xi]S(\xi) = 0$ См. 30/VIII 2014

$a = \left(\frac{2}{p}\right)^2 \frac{\lambda}{y}$, $q = \frac{1}{2}\left(\frac{2}{p}\right)^2 \frac{J(0)}{y}$, $\xi = \frac{p}{2}z$, $z \in [-\frac{1}{2}, \frac{1}{2}]$. Нужно $a_2, b_2 = 0$ для $q \gg 1$

$J(z) = \int dz' \chi(zz') = \int \frac{dk}{k} I^2(k)\left[1 + e^{-\beta k/2}\operatorname{ch}\beta k z\right] dk$, $J(0) = 2\int \frac{dk}{k} I^2(k)\left[1 - e^{-\beta k/2}\right]$

in units $(8\pi M_s^2)$. $p = p(\beta)$ растет с $\beta\uparrow$. $p(\beta) = 1 - 1.5$ для $\beta = 0.2 \div 0.7$

$q(\beta, L) = \frac{2}{p^2(\beta)} \frac{J(0,\beta)}{y}$, h (in units $8\pi M_s^2$) $= \frac{1}{8\pi M_s^2} M_s^2 \left(\frac{l_e}{L}\right)^2 \left[\ln\frac{R}{R_c} + \frac{5}{4}\right] =$

$= \frac{[\dots]}{8\pi}\left(\frac{l_e}{L}\right)^2$. $q(\beta, L) = \frac{2}{p^2(\beta)} 2\int \frac{dk}{k} I^2(k)\left(1 - e^{-\beta k/2}\right) \frac{8\pi}{[\dots]} \frac{L^2}{L_e^2} \sim L^2$.

$q \sim L^2$. Чтоб иметь малое q надо обеспечить малое L - dot thickness, через Mathie eq. может справиться, дает СФ $S_n(z)$ и ω_n н/м малых L.

(3) Local magnetostatics, reducing to parabolic cylinder equation

$yS'' + [\lambda - J(z)]S(z) = 0$. $J(z) = J(0)(1 - p^2z^2)$, $y'' + (\dots) = 0$.

Standard form $y''(x) - \left(\frac{1}{4}x^2 + a\right)y(x) = 0$, $y'' + \left(\frac{1}{4}x^2 - a\right)y(x) = 0$.

или NIST $\frac{d^2S}{dt^2} = \mu^4(1 - t^2)S(t)$.

$\frac{d^2S}{dz^2} + \left[\frac{\lambda - J_0}{y} + \frac{1}{y}J_0p^2z^2\right]S(z) = 0$. $\frac{1}{y}J_0p^2z^2 = \frac{1}{4}x^2$, $\left(\frac{1}{y}J_0p^2\right)^{1/2} z = \frac{1}{2}x$.

$z = \frac{1}{2}x \cdot \frac{1}{\left(\frac{1}{y}J_0p^2\right)^{1/2}}$

$\frac{d^2}{dz^2} = \frac{d}{dz}\frac{d}{dz} = \frac{d}{d\left(\frac{x}{2}\frac{1}{\sqrt{\dots}}\right)} \frac{d}{d\left(\frac{x}{2}\frac{1}{\sqrt{\dots}}\right)} = 4(\sqrt{\dots})^2 \frac{d^2}{dx^2}$.

Немного, см. 11/VIII 2014 г. Надо $\left(\frac{1}{y}J_0p^2\right)^{1/2} z^2 = \frac{1}{2}x^2$, $\frac{J_0p^2}{y} = d$

$d^{1/2}z^2 = \frac{1}{2}x^2$, $z = \frac{1}{\sqrt{2}}\frac{1}{d^{1/4}}x$

$\frac{d^2S}{dz^2} + \left[\frac{\lambda - J_0}{y} + dz^2\right]S(z) = 0$.

$\frac{d^2}{dz^2} = \frac{d^2}{dx^2}\left(\frac{1}{\sqrt{2}d^{1/2}}\right)^{-1} = \frac{d^2}{dx^2}\sqrt{2}d^{1/2}$.

$\frac{d^2S(x)}{dx^2}\sqrt{2}d^{1/2} + \left[\frac{\lambda - J_0}{y} + d\frac{1}{2d^{1/2}}x^2\right]S(x) = 0$, $\frac{d^2S(x)}{dx^2} + \left[\frac{\lambda - J_0}{y\sqrt{2}d^{1/2}} + \frac{x^2}{4}\right]S(x) = 0$

Def. $a = -\frac{(\lambda - J_0)}{2yd^{1/2}}$, $\frac{d^2S(x)}{dx^2} + \left(\frac{x^2}{4} - a\right)S(x) = 0$, $x = \sqrt{2}d^{1/4}z$.

$\lambda = \frac{g\omega}{\gamma} = \frac{\omega}{\gamma}\cdot\frac{2\pi M_s}{\gamma} = 2\omega\frac{M_s}{\gamma}$. $J_0 = J(0) = 2\int \frac{dk}{k} I^2(k)\left(1 - e^{-\beta k/2}\right)$, $I(k) = \int_0^1 d\rho \frac{J_1(k\rho)}{\rho}$

$\Rightarrow a = \frac{(J_0 - \lambda)}{2yd^{1/2}}$. Вид СФ зависит от знака a.

$a = \frac{1}{2yd^{1/2}}\left[J_0 - \frac{1}{8\pi M_s^2} 2\omega\frac{M_s}{\gamma}\right] = \frac{1}{2yd^{1/2}}\left[8\pi M_s^2 J_0 - 2\omega\frac{M_s}{\gamma}\right] = \frac{1}{2yd^{1/2}}\frac{2M_s}{\gamma}\left[4\pi M_s\gamma J_0 - \omega_0\right]$

$a = \frac{M_s}{\gamma y d^{1/2}}\left[\omega_M J_0 - \omega\right]$. $\omega_0 = \omega_M J(0,\beta)$. $J(0,\beta \ll 1) = 2\int \frac{dk}{k} I^2(k)\left(1 - e^{-\beta k/2}\right) \approx$

$\approx 2\int \frac{dk}{k} I^2(k)\frac{\beta k}{2} = \beta\int dk\, I^2(k)$. ω_G (JAP '2002) $= \frac{1}{2}\gamma M_s \frac{(2/\beta)^2}{\chi(0)} = \frac{1}{2}\gamma M_s \frac{4}{9}\cdot 4\pi F(\beta) =$

$= 8\pi\gamma M_s F(\beta)$. $F(\beta) = \int \frac{dt}{t} f(\beta t) I^2(t) \approx \frac{1}{2}\int \frac{dt}{t}\beta t I^2(t) = \frac{1}{2}\beta\int dt\, I^2(t)$.

ω_G (JAP '02) $= 2\omega_M F(\beta) = \beta\omega_M \int dt\, I^2(t)$. $a(\beta \ll 1) = \frac{M_s}{\gamma y d^{1/2}}\left[\omega_M\beta\int dk\, I^2(k) - \omega\right]$

$a(\beta \ll 1) = \frac{M_s}{\gamma y d^{1/2}}\left[\omega_G(\beta) - \omega\right]$, $a > 0$ для $n=0$ GM, $a < 0$ для $n = 1, 2, \dots$ GM.

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Если $a>0$ для ω_0 - GM и $a<0$ для $\omega_n(n>0)$, то ω_0 - GM есть особая, localized mode at top/bottom chr surfaces.
Числ. расчет $M/\overline{M}_s$ подтвердил что $a>0$ есть корень собственных. тонкой локализованной моды.

Если $y(x)$ есть решение $y''(x) - (\frac{1}{4}x^2 + a)y(x) = 0$, то $y(xe^{i\frac{\pi}{4}}, -ia)$ есть решение eq. Thiele $s''(x) + (\frac{1}{4}x^2 - a)s(x) = 0$. $y(x,a)$ выражается через confluent hypergeometric function $F_1(a,b,x) = M(a,b,x)$.

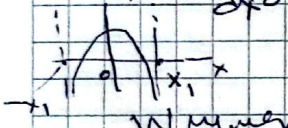
In Aug. 2014 я предположил $\frac{\partial s}{\partial z}\Big|_{z=\pm\frac{1}{2}} \sim \frac{\partial s}{\partial x} = 0$ и используя $\frac{dM}{dx} \sim M(a, f(a), x)$ решал $M(a, f(a), x)\Big|_{z=\pm 1/2} = 0$, что дало спектр $a_n \to \omega_n$.
т.е. метод parabol. cylinder equation предполагал использование bound. conditions at $z = \pm 1/2$, в отличие от метода Mathieu.

$a_n = a_n(L)$, т.к. $z = \pm 1/2$, соотв. $x = \sqrt{2}\alpha^{1/4}(\pm\frac{1}{2})$, $\alpha = \frac{J_0\rho^2}{\gamma} = \alpha(\beta, L)$.
$a = \frac{M_s}{\gamma\gamma\alpha^{1/2}}(\omega_M J_0 - \omega)$. Спектр $a_n = \frac{M_s}{\gamma\gamma\alpha^{1/2}}(\omega_M J_0 - \omega_n)$, $\omega_n = \omega_M J_0 - \frac{M_s}{\gamma\gamma\alpha^{1/2}} a_n(L)$.
Есть один корень $a_n > 0$, $n=0$ localized mode и набор корней $a_n < 0$ — (delocalized modes).
Можно $\omega_n = \omega_n^m + \omega_n^{ex} = \gamma_0\omega_M - \frac{M_s}{\gamma}\frac{1}{\gamma\alpha^{1/2}} a_n$.

$\gamma\alpha^{1/2} = \gamma\frac{\rho\sqrt{J_0}}{\sqrt{\gamma}} = \rho\sqrt{J_0\gamma}$, $\omega_n = \omega_n^m - \frac{M_s}{\gamma}\frac{1}{\rho\sqrt{J_0\gamma}} a_n(L)$, $\omega_{ex} = -\frac{M_s}{\gamma}\frac{1}{\rho\sqrt{J_0\gamma}} a_n(L) \sim \frac{a_n(L)}{L}$
$\omega_{ex}^n \sim \frac{a_n(L)}{L^2}$, т.к. $\gamma \sim 1/L^2$, $\omega_n^{ex}(L) \sim L a_n(L)$.

(5/XI) E-mails to Eli (new secretary), Arthur (student program), Ruben, Ricardo.
Eq. Thiele in local magnetostatics approximation можно записать как ур.:

$u(x) - \frac{d^2s}{dx^2} - \frac{x^2}{4}s(x) = -as(x)$, $-a = E$ имеет смысл энергии.
$u(x) = -\frac{1}{4}x^2$ - потенциал.



В выход. переменных ур. $\lambda s(z) = -\gamma s'' + J(z)s(z)$, см. 9,8 Aug. 2014.
применить метод WKB.

Задуре во Лена. Сказал ей, чтоб делала Mathieu - расчет CD/CS, для малых L, малых q ($q \le 7$).

(6-7/XI) Отдых дома. Б-ница. За публикации с Л.

(8/XI) подготовка Travel report Poznan. E-mails Ruben/Eli (monitors) и Оксана (sci. Rep. MS) Berganza.

(9/XI) Ездил в ЦИРУ/ЕНИ. 2 monitors доставлены в мой office. Отдал Eli Travel reports Poznan вкл. CV, Tests и Invoice 1690€ APC. Sci. Rep. и Block point.
С Л. на andarreta и „Lidl" Hermani.

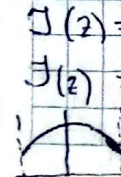
(10/XI) Переписка с Eli и Окс. Тел. с Окс. - обещала сделать draft ответов Referees Berganza - Sci. Rep. Рец. работ ЖТФ/Орлов (major rev.).

(11/XI) Отдых дома - Б-ница.

(12/XI) Составил approx. список invited SolSkyMag-2022 и начал публиковать.

(13/XI) Vortex gyrotropes thick dot. Вернулся к WKB-расчету 8/VIII 2014 г. ω_c - WKB

Eq. Thiele $-\gamma s'' + J(z)s(z) = \lambda s(z)$, $\lambda/(8\pi M_s^2) = \frac{\omega}{\omega_M}$, $\frac{\gamma}{8\pi M_s^2} = \frac{1}{8\pi}\left(\frac{l_e}{L}\right)^2\left[\ln\frac{R}{a_0} + s/4\right]$
$J(z) = \int_{-1/2}^{1/2} dz' \chi(zz'), \quad (8\pi M_s^2)\beta\int dk I^2(k) e^{-k|z-z'|} = \chi(zz')$, $I(k) = \int_0^1 dg\, g\, J_1(kg)$
$J(z) = (8\pi M_s^2)\, 2\int\frac{dk}{k} I^2(k)\left[1 - e^{-\beta k/2}\operatorname{ch}\beta k z\right]$. (*) $s'' + f(z)s(z) = 0$, $f(z) = \frac{1}{\gamma}[\lambda - J(z)]$



Решение ур. (*) я получил осенью в виде $s_0(z) = u(z)\operatorname{ch}P(z)$ где $n=0$ мода

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$u(z) \sim \frac{1}{f^{1/4}(z)}$, $\Phi(z) = \int_0^z dz\, f^{1/2}(z)$, $\frac{\partial S_0(z)}{\partial z} = 0 \left(z = \pm \frac{1}{2}\right)$ далее ед. на ω $\int_0^{1/2} dz f^{1/2}(z) = -\frac{1}{4}\frac{f'(1/2)}{f^{3/2}(1/2)}$

Имп $\omega_{0/2\pi} = 1{,}30$ ГГц $n_{fm} L = 100$ nm, $R = 150$ nm.

Mathews - Walker, Ch. 1 далее $S(z) = e^{i\Phi(z)}$ - WKB-анзац и если $|\Phi''| = \frac{1}{2}\frac{f'}{\sqrt{f}} << |f(z)|$, то $\Phi(z) = \pm \int dz\, f^{1/2}(z)$. Сначала в Φ' аппроксимация далее

$u(z) = \pm \int dz f^{1/2}(z) + \frac{i}{4} \ln f$ или $S(z) \approx \frac{1}{f^{1/4}(z)}\left[C_+ e^{i\int dz f^{1/2}(z)} + C_- e^{-i\int dz f^{1/2}(z)}\right]$

$f(z) = \frac{1}{y}[\lambda - U(z)]$, $f'(z) = -\frac{1}{y}U'(z)$. Чтобы выполнялось $\frac{1}{2}\left|\frac{f'}{\sqrt{f}}\right| << |f(z)|$

Def. $\Phi_0(z) = \int_{z_0}^z dz\, f^{1/2}(z)$, $S(z) = \frac{1}{f^{1/4}(z)}\left[C_+ e^{i\Phi_0(z)} + C_- e^{-i\Phi_0(z)}\right]$ - несколько далее $f(z_0) = 0$, z_0 точка поворота

WKB functions $W_\pm(z) = \frac{1}{f^{1/4}(z)} e^{\pm i\Phi_0(z)}$, $\Phi_0(z) = \int_{z_0}^z dz\, f^{1/2}(z)$.

Вычисление: через $S(z) = \frac{1}{f^{1/4}(z)}\left[C_+ e^{i\Phi(z)} + C_- e^{-i\Phi(z)}\right]$, $\Phi(z) = \int dz f^{1/2}(z)$

Условие $\frac{\partial S}{\partial z}|_{z=\pm 1/2} = 0$, $S(z) = u(z)\left[C_+ e^{i\Phi(z)} + C_- e^{-i\Phi(z)}\right]$, $u(z) = f^{-1/4}(z)$.

$\frac{\partial S}{\partial z} = u'\left[C_+ e^{i\Phi} + C_- e^{-i\Phi}\right] + u\Phi' i\left[C_+ e^{i\Phi} - C_- e^{-i\Phi}\right] = 0$

$\Phi(z) = \int dz f^{1/2}(z) = \int_0^z dz f^{1/2}(z) + \Phi(0) = \int_0^z dz f^{1/2}(z) + a$, $a = \Phi(0)$

$\Phi'(z) = f^{1/2}(z)$ - нечет. по z. $u'(z) = \frac{\partial}{\partial z} f^{-1/4}(z) = -\frac{1}{4} f^{-5/4}(z)\frac{\partial f}{\partial z}$ - нечет. по z.

$\Phi(z)$ - нечет. по z? $\Phi(-z) = \int_0^{-z} dz f^{1/2}(z) + a = |z = -z'| = -\int_0^{z'} dz' f^{1/2}(-z') + a =$

$= -\int_0^{z'} dz f^{1/2}(z) + a = [-\Phi(z)^0 + a] + a = -\Phi(z) + 2a$, $\Phi(-z) \neq -\Phi(z)$

$\Phi(-z) = -\Phi(z) + 2a$

$z = -\frac{1}{2}$, $u'(-\frac{1}{2})\left[C_+ e^{i\Phi(-1/2)} + C_- e^{-i\Phi(-\frac{1}{2})}\right] + u(-\frac{1}{2})\Phi'(-\frac{1}{2}) i\left[C_+ e^{i\Phi(-\frac{1}{2})} - C_- e^{-i\Phi(-\frac{1}{2})}\right] = 0$

$z = +\frac{1}{2}$, $u'(\frac{1}{2})\left[C_+ e^{i\Phi(1/2)} + C_- e^{-i\Phi(1/2)}\right] + u(\frac{1}{2})\Phi'(1/2) i\left[C_+ e^{i\Phi(1/2)} - C_- e^{-i\Phi(1/2)}\right] = 0$

- система 2-х уравнений, чтобы найти C_+, C_-. $u'(\pm\frac{1}{2}) = u'_\pm$, $u(\pm\frac{1}{2}) = u_\pm$, $\Phi(\pm\frac{1}{2}) = \Phi_\pm$, $\Phi'(\pm 1/2) = \Phi'_\pm$

$$\begin{cases} C_+\left[u'_- e^{i\Phi_-} + i u_- \Phi'_- e^{i\Phi_-}\right] + C_-\left[u'_- e^{-i\Phi_-} - u_- i\Phi'_- e^{-i\Phi_-}\right] = 0 \\ C_+\left[u'_+ e^{i\Phi_+} + i u_+ \Phi'_+ e^{i\Phi_+}\right] + C_-\left[u'_+ e^{-i\Phi_+} - i u_+ \Phi'_+ e^{-i\Phi_+}\right] = 0 \end{cases}$$

$\det \hat{A} = \left[u'_- + i u_- \Phi'_-\right] e^{i\Phi_-}\left[u'_+ - i u_+ \Phi'_+\right] e^{-i\Phi_+} - \left[u'_+ + i u_+ \Phi'_+\right] e^{i\Phi_+}\left[u'_- - i u_- \Phi'_-\right] e^{-i\Phi_-} = 0$

$\Phi(-1/2) = \Phi_- = -\Phi_+ + 2a$. $\Phi_- - \Phi_+ = \ldots$ $\Phi_+ - \Phi_- = \int_0^{1/2} dz f^{1/2} - \int_0^{-1/2} dz f^{1/2} =$

$= \int_0^{1/2} dz f^{1/2}(z) - \int_0^{1/2} dz f^{1/2}(z) = |z = -z'| = \int_0^{1/2} dz f^{1/2} - \int_0^{1/2}(-1) dz' f^{1/2}(-z') =$

$= \int_0^{1/2} dz f^{1/2}(z) + \int_0^{1/2} dz' f^{1/2}(z') = 2\int_0^{1/2} dz f^{1/2}(z) = 2\left[\Phi_+ - a\right]$, $\Phi_+ - \Phi_- = 2\Phi_+ - 2a$

$\Phi_+ - \Phi_- = \psi$

$\left[-u'_+ + i u_+ \Phi'_+\right]\left[u'_+ - i u_+ \Phi'_+\right] e^{-i\psi} - \left[u'_+ + i u_+ \Phi'_+\right]\left[-u'_+ - i u_+ \Phi'_+\right] e^{i\psi} = 0$

$-\left[u'_+ - i u_+ \Phi'_+\right]^2 e^{-i\psi} + \left[u'_+ + i u_+ \Phi'_+\right]^2 e^{i\psi} = 0$, $u_+ = u(\frac{1}{2})$, $u'_+ = u'(\frac{1}{2})$

Def. $u'_+ = v'$, $u_+ = v$, $\Phi'_+ = \Phi'(\frac{1}{2})$, $\Phi(z) = \int_0^z dz f^{1/2}(z) + \Phi(0)$

$\Phi'(z) = f^{1/2}(z) = \frac{1}{u^2(z)}$, $\Phi'_+ = \frac{1}{u_+^2} = \frac{1}{v^2}$

$\left(v' + i v \frac{1}{v^2}\right)^2 e^{i\psi} = \left(v' - i v \frac{1}{v^2}\right)^2 e^{-i\psi}$, $\left(v' + \frac{i}{v}\right)^2 e^{i\psi} = \left(v' - \frac{i}{v}\right)^2 e^{-i\psi}$

$y = \left(v' + \frac{i}{v}\right)^2 e^{i\psi}$, $y = \bar{y}$, y is real, $\mathrm{Im}(y) = 0$, $\mathrm{Im}\left(v' + \frac{i}{v}\right)^2 e^{i\psi} = 0$

$\left(v' + \frac{i}{v}\right)^2 = v'^2 - \frac{1}{v^2} + 2i\frac{v'}{v}$, $y = \left(v' + \frac{i}{v}\right)^2(\cos\psi + i\sin\psi) = \left[v'^2 - \frac{1}{v^2} + 2i\frac{v'}{v}\right] *$

$(\cos\psi + i\sin\psi) = i\left[\left(v'^2 - \frac{1}{v^2}\right)\sin\psi + 2\frac{v'}{v}\cos\psi\right] + (\text{real})$, $\mathrm{Im}(y) = 0$ далее

$\left(v'^2 - \frac{1}{v^2}\right)\sin\psi + 2\frac{v'}{v}\cos\psi = 0$ - ед. на собств. λ_n и w_n. $v = u(\frac{1}{2})$, $v' = u'(1/2)$.

Escaneado con CamScanner

$u(z) = f^{-1/4}(z)$ $\quad u'(z) = -\frac{1}{4} f^{-5/4}(z) f'(z) = -\frac{1}{4} f^{-1/4} \frac{f'}{f} = -\frac{1}{4} u \frac{f'}{f}$.

$v = u$, $u' = -\frac{1}{4} u \frac{f'}{f} = -\frac{1}{4} \frac{f'}{f} v = v \frac{f'}{f}$, $\frac{v'}{v} = -\frac{1}{4} \frac{f'}{f}$

$(v')^2 - \frac{1}{v^2} = \frac{1}{16}\left(\frac{f'}{f}\right)^2 v^2 - \frac{1}{v^2}$, $v = u(+1/2)$, $v^2 = f^{-1/2}(1/2)$

$(v')^2 - \frac{1}{v^2} = \frac{1}{16}\left(\frac{f'}{f}\right)^2 f^{-1/2} - f^{1/2}$ $\quad \frac{1}{v^2} = f^{1/2}(1/2)$.

$tg\psi = -2\frac{v'}{v} \frac{1}{v'^2 - \frac{1}{v^2}} = (-2)\left(-\frac{1}{4}\right)\frac{f'}{f} \frac{1}{\frac{1}{16}\frac{f'^2}{f^2} f^{-\frac{1}{2}} - f^{1/2}}$

$tg\psi = \frac{1}{2}\frac{f'}{f} \frac{1}{f^{1/2}} \frac{1}{\frac{1}{16}\frac{f'^2}{f^2}\frac{1}{f} - 1}$. Def. $g(z) = \frac{1}{4}\frac{f'}{f} f^{-1/2}(z)$

ω_n - freq.

$tg\psi = 2g \cdot \frac{1}{g^2 - 1}$. $\quad g(z) = \frac{1}{4}\frac{f'(z)}{f^{3/2}(z)}$, $z = +1/2$, $\psi = \Phi_+ - \Phi_- =$

$= \Phi(+\frac{1}{2}) - \Phi(-\frac{1}{2})$

$\psi = \Phi_+ - \Phi_- = 2\Phi_+ - 2a = 2\Phi(+\frac{1}{2}) - 2\Phi(0)$, $\Phi(z) = \int_0^z dz\, f^{1/2}(z) + \Phi(0)$

$\psi = 2\int_0^{1/2} dz\, f^{1/2} + 2\Phi(0) - 2\Phi(0) = 2\cdot\int_0^{1/2} dz\, f^{1/2}(z)$.

Eq. $tg\psi = \frac{2g}{g^2 - 1}$, $z = +\frac{1}{2}$ есть eq. на спектр λ_n и ω_n.

$f(z) = \frac{1}{\gamma}[\lambda - J(z)]$, $f'(z) = -\frac{1}{\gamma} J'(z)$. При $\beta \to 0$, $J'(z) \to 0$ flat pot.

и $g(z) \to 0$, $tg\psi \approx 0$, $\psi \approx n\pi$, $\int_0^{1/2} dz\, f^{1/2}(z) = n\pi/2$. $f(z) \approx const > 0$

$f^{1/2}(0)\frac{1}{2} = n\pi/2$, $f^{1/2}(0) = n\pi$, $f(0) = \frac{1}{\gamma}[\lambda - J(0)] = (n\pi)^2$

$\lambda_n = \gamma(n\pi)^2 + J(0)$. $J(\alpha, \beta << 1) \approx \beta \int dk\, I^2(k) = 2F(\beta)$ – из Gus' JAP'02

ω_G^m (JAP'02) $= \omega_M \cdot 2F(\beta) = \beta\omega_M \int dt\, I^2(t)$. $\lambda_n = 2\omega_n \frac{M_s}{\gamma}$, $\frac{\lambda_n}{(8\pi M_s)^2} = \frac{\omega_n}{\omega_M}$

$2\frac{M_s}{\gamma}\omega_n = \gamma(n\pi)^2 + (8\pi M_s^2) J(0) = \frac{\omega_n}{\omega_M}(8\pi M_s^2)$, см. 4/XI.

$\frac{\omega_n}{\omega_M} = J(0) + \frac{\gamma}{8\pi M_s^2}(n\pi)^2 = 2F(\beta) + \frac{\gamma}{8\pi M_s^2}(n\pi)^2$, $2F(\beta) = \frac{\omega_G^m}{\omega_M}$ (JAP'02)

$<<1$ $\left(\frac{\omega_n}{\omega_M} = \frac{\omega_G^m}{\omega_M} + \frac{\gamma}{(8\pi M_s^2)}(n\pi)^2\right)$, $\frac{\gamma}{8\pi M_s^2} = \frac{1}{8\pi}\left(\frac{Le}{L}\right)^2\left[\ln\frac{R}{R_c} + \frac{5}{4}\right]$

Это сходится к Eq. (4) для $\omega_n(L)$ из Ding, Sci Rep. 2014. $f(z) > 0$.

Если $\beta \to 0$, $g(z) \neq 0$ и eq $tg\psi = \frac{2g}{g^2 - 1}$ надо решать.

$\psi = \Phi(+\frac{1}{2}) - \Phi(-\frac{1}{2}) = 0$ когда $\beta \to 0$ и будет ω_{min} dot. V. motion uniform along oz

Находим C_-/C_+ ratio: $C_+[u'_+ + iu_+\Phi'_+]e^{i\Phi_+} + C_-[u'_+ - iu_+\Phi'_+]e^{-i\Phi_+} = 0$

$C_-/C_+ = -\frac{[u'_+ + iu_+\Phi'_+]}{[u'_+ - iu_+\Phi'_+]} e^{2i\Phi_+}$, $\frac{C_-}{C_+} = -\frac{[v' + iv\Phi'_+]}{[v' - iv\Phi'_+]} e^{2i\Phi_+}$

$\Phi'_+ = \Phi'(+1/2) = f^{1/2}(1/2) = \frac{1}{u^2(1/2)} = \frac{1}{v^2}$ $\quad \Phi_+ = \Phi(1/2) = \int_0^{1/2} dz\, f^{1/2}(z) + \Phi(0)$

$\frac{C_-}{C_+} = -\frac{[v' + i\frac{1}{v}]}{[v' - i\frac{1}{v}]} e^{2i\Phi_+} = -\frac{[v' + \frac{i}{v}]^2}{[v'^2 + \frac{1}{v^2}]} e^{2i\Phi_+}$, $(v' + \frac{i}{v})^2 = v'^2 - \frac{1}{v^2} + 2i\frac{v'}{v} =$

$= (v'^2 - \frac{1}{v^2})(1 - itg\psi)$.

Сам. ур. $tg\psi = -2\frac{v'}{v}\frac{1}{[v'^2 - \frac{1}{v^2}]}$. $1 - itg\psi = \frac{1}{\cos\psi}[\cos\psi - i\sin\psi] = \frac{1}{\cos\psi} e^{-i\psi}$

$\frac{C_-}{C_+} = -\frac{(v'^2 - \frac{1}{v^2})}{(v'^2 + \frac{1}{v^2})}(1 - itg\psi)e^{2i\Phi_+} = -\frac{(v'^2 - 1/v^2)}{(v'^2 + 1/v^2)}\frac{1}{\cos\psi} e^{-i\psi} e^{2i\Phi_+}$

$2\Phi_+ - \psi = 2\Phi_+ - (\Phi_+ - \Phi_-) = \Phi_+ + \Phi_-$. (!)

$v'^2 - \frac{1}{v^2} = f^{1/2}\left[\frac{1}{16}\frac{f'^2}{f^2}\frac{1}{f} - 1\right] = f^{1/2}[g^2 - 1]$

$v'^2 + \frac{1}{v^2} = f^{1/2}\left[\frac{1}{16}\frac{f'^2}{f^2}\frac{1}{f} + 1\right] = f^{1/2}[g^2 + 1]$, $\frac{C_-}{C_+} = -\frac{(v'^2 - 1/v^2)}{(v'^2 + 1/v^2)} \frac{e^{i(\Phi_+ + \Phi_-)}}{\cos\psi}$

$\frac{C_-}{C_+} = -\frac{(g^2 - 1)}{(g^2 + 1)} \frac{e^{i(\Phi_+ + \Phi_-)}}{\cos\psi}$, $z = \pm\frac{1}{2}$

Ф $S(z) = \frac{C_+}{f^{1/4}(z)}\left[e^{i\Phi(z)} - \frac{(g^2 - 1)}{(g^2 + 1)} \frac{e^{i(\Phi_+ + \Phi_-)}}{\cos\psi} e^{-i\Phi(z)}\right]$, $\Phi(z) = \int_0^z dz\, f^{1/2}(z) + \Phi(0)$

Escaneado con CamScanner

При $\beta \to 0$, $g \to 0$, $\psi \to \pi n$, $c_-/c_+ \approx e^{i(\Phi_+ + \Phi_-)} \frac{1}{\cos(n\pi)} = (-1)^n e^{i(\Phi_+ + \Phi_-)}$

$S(z) \sim \left[e^{i\Phi(z)} + (-1)^n e^{i(\Phi_+ + \Phi_-)} e^{-i\Phi(z)} \right]$, $\Phi_+ + \Phi_- = \int_0^{1/2} dz f^{1/2}(z) + \int_{-1/2}^{0} dz f^{1/2}(z) + 2\Phi_0 =$

$= \int_0^{1/2}(\dots) - \int_0^{1/2} dz f^{1/2}(-z) + 2\Phi(0) = 2\Phi(0)$. $\Phi(z) = \int_0^z dz f^{1/2}(z) + \Phi(0)$

$S(z) \sim \left[e^{i\left(\int_0^z dz f^{1/2} + \Phi_0\right)} + (-1)^n e^{i2\Phi_0} e^{-i\left(\int_0^z dz f^{1/2} + \Phi_0\right)} \right]$

$S(z) \sim e^{i\Phi_0} \left[e^{i\tilde{\Phi}} + (-1)^n e^{-i\tilde{\Phi}} \right]$, $\tilde{\Phi}(z) = \int_0^z dz f^{1/2}(z)$.

$S(z) \sim e^{i\Phi_0} \cos\tilde{\Phi}(z)$ при n - even, $S(z) \sim e^{i\Phi_0} i \sin\tilde{\Phi}(z)$ при $n = 1, 3, 3 \dots$ odd.

$\tilde{\Phi}(z) = \int_0^z dz f^{1/2}(z) \approx \int_0^z dz f^{1/2}(0) = f^{1/2}(0)\, z = n\pi z = q_n z$ в этом и предыдущем.

$S(z) \sim e^{i\Phi_0} \cos(n\pi z)$, n - even, $S_n(z) \sim i e^{i\Phi_0} \sin(n\pi z)$, n - odd.

И в том и другом случае $\partial S_n / \partial z = 0$, $z = \pm 1/2$ для both even and odd "n".

Вид $\Phi(z) = \int_0^z dz \sqrt{\lambda_n - J(z)} \frac{1}{\sqrt{J}} + \Phi(0)$ видимо обобщает $\Phi_n(z)$ из Ding'APL'14.

$g(z) = g(1/2) = \text{const}$. $S(\pm z) \sim \left[e^{i\Phi(z)} - \frac{(g^2-1)}{(g^2+1)} \frac{e^{i(\Phi_+ + \Phi_-)}}{\cos\psi} e^{-i\Phi(z)} \right]$ $\Phi_+ + \Phi_- = 2\Phi_0$

$S(-z) \sim \left[e^{i\Phi(-z)} - \frac{(g^2-1)}{(g^2+1)} \frac{e^{i2\Phi_0}}{\cos\psi} e^{-i\Phi(-z)} \right]$, $\Phi(-z) = -\Phi(z) + 2\Phi_0$

$S(-z) \sim \left[e^{-i\Phi(z)} e^{2i\Phi_0} - \frac{(g^2-1)}{(g^2+1)} \frac{e^{2i\Phi_0}}{\cos\psi} e^{i\Phi(z)} e^{-2i\Phi_0} \right]$ – изобразить рез. / нерез.

$S(z) \sim 1/f^{1/4}(z)$ сингулярно при $f(z) = 0$, но на периоде $\beta \to 0$

Это не сказалось, ибо $f(z) \to f(0) = \text{const} > 0$. Но, видимо, the lowest mode ψ_0, соответствует области $f(z) < 0$, $\lambda_0 < J(z)$ при z в центре зоны.

Пока WKB $S(z)$ игнорирует точки поворота $f(z) = 0$.

Также WKB ломается когда точек поворота нет, $f(z) > 0$, $\lambda_n > J(z)$, т.е. для гигамодов с $n = 1, 2, \dots$. Для the lowest ψ_0, видимо, надо сшивать решения $S(z)$ слева и справа точки поворота. Сшивка и есть WKB-method.

При $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + [V(x) - E]\psi(x) = 0$ $V(x) - E = (a - x) F_0$, $F_0 = -V'(a)$

вблизи turning points $x = a$, $x = b$. $V(x) - E = (x - b) F_1$, $F_1 = V'(b)$ –

$V(x) - E \sim (x - a)$ или $\sim (x - b)$ приводит к Airy equation $-\frac{\hbar^2}{2m}\psi'' + (a - x) F_0 \psi = 0$

near a (и b) точек. Airy eq. $\psi'' - u\psi(u) = 0$

Ассимптотический вид $Ai(u) \underset{u \to \infty}{\approx} \frac{1}{\sqrt{\pi}} \frac{1}{(-u)^{1/4}} \cos\left[\frac{2}{3}(-u)^{3/2} - \pi/4 \right]$ дает

сшивку решений $\psi_1(x) \approx \frac{1}{2\sqrt{q(x)}} \exp\left\{ -\frac{1}{\hbar} \int_x^a q(y)\, dy \right\}$, $q(x) = i p(x) = \sqrt{2m[V(x) - E]}$

при $x < a$ и $\psi_2(x) = \frac{1}{\sqrt{p(x)}} \cos\left\{ \frac{1}{\hbar} \int_a^x p(y)\, dy - \frac{\pi}{4} \right\}$ при $x > a$.

$J(z) = \frac{1}{\gamma}[\lambda - J(z)] \sim [E - V(z)]$. Turning point z^*, $J(z^*) = 0$, $J(z^*) = \lambda$.

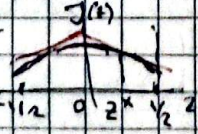
Разложить $J(z)$ в ряд в окрестности $z = z^*$. $J(z) = J(z^*) + J'(z^*)(z - z^*) + \frac{1}{2} J''(z^*)(z - z^*)^2 + \dots$

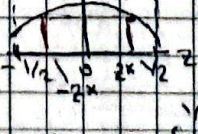
$J(z) = 2\int \frac{dk}{k} I^2(k)\left[1 - e^{-\beta k/2} \operatorname{ch}(\beta k z)\right]$, $J'(z) = -2\beta \int dk\, I^2(k) e^{-\beta k/2} \operatorname{sh}(\beta k z)$

$z^* \in [0, +\frac{1}{2}]$ или $[-\frac{1}{2}, 0]$, $J'(z^*)$ конечно.

Сшивка by Airy func

не эффективна, она зависит- ся Δ potential $J(z)$ far $z \to \pm\infty$.





Эвристика (доказывается в WKB при $n \to \pm\infty$): $\int_{z^*(\omega)}^{1/2} dz \frac{\omega}{\hbar} \tilde{\Phi}(z) = \int_{z^*}^{1/2} dz f^{1/2}(z) = \pi(n + \frac{1}{2})$

– интеграл между turning points z^* и $1/2$.

Но для ψ_0 ($n = 0$)-mode можно Maslov-method, когда $a_0(q)$ не меняет знак.

Escaneado con CamScanner

(15/xi) Исследовать ход для Mathieu method для ω_0 - гармоника? Mathieu eq. Lowest GM

Аб за инвайт-Steigen: $a_0(q) \approx -\frac{1}{2}q^2$ при $q \ll 1$

$\frac{d^2s}{dz^2} + [a - 2q\cos 2z]\,s(z) = 0$, $z = \frac{\pi}{2}z$. $J(z) = J(0)\exp$.

$\chi(zz') = 8\pi M_s^2 \beta \int dk\, e^{-\beta k|z-z'|} I^2(k)$ см. Apr.c 2020.

$\chi(zz') = 8\pi M_s^2 \beta\, K(\beta|z-z'|)$, $K(x) = \int dk\, e^{-xk} I^2(k)$, $I(k) = \int_0^1 dz\, J_1(kz)$

$F(z) = \int_{-1/2}^{1/2} dz'\, K(\beta|z-z'|) = \frac{2}{\beta}\int \frac{dk}{k} I^2(k)\left[1 - e^{-\beta k/2}\operatorname{ch}\beta k z\right]$

$F(z) = \frac{1}{\beta} J(z)$, $J(z) = 2\int \frac{dk}{k} I^2(k)\left[1 - e^{-\beta k/2}\operatorname{ch}\beta k z\right]$

$J(z) = \int L\cdot\chi(zz')$ в единицах $(8\pi M_s^2)$. S, g, β в нормировочном $F(z)$.

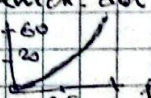
$q = \frac{2}{\beta^2}\frac{J(0)}{\gamma} = \frac{2}{\beta^2(\beta)} J(0,\beta) \frac{8\pi}{\left[\ln\left(\frac{R}{R_c}\right) + \frac{5}{4}\right]} \frac{L^2}{L_e^2}$. $c(\beta) = \ln\frac{R}{R_c} + \frac{5}{4}$

$q(\beta, L) = \frac{2J(0,\beta)}{\beta^2(\beta)} 8\pi \frac{L^2}{L_e^2}\frac{1}{c(\beta)}$. Def. $z = R/L_e$.

$R/R_c = \frac{1}{0.68}\left(\frac{R}{L_e}\right)^{2/3}\frac{1}{\beta^{1/3}}$

$\frac{L^2}{L_e^2} = \left(\frac{\beta R}{L_e}\right)^2 = \beta^2 z^2$. $q(\beta) = 16\pi \frac{J(0,\beta)}{\beta^2(\beta)}\beta^2 z^2 \frac{1}{c(\beta)}$, $c(\beta) = \ln\frac{R}{R_c} + 5/4$

$J(0,\beta) = 2\int_0^\infty \frac{dk}{k} I^2(k)\left[1 - e^{-\beta k/2}\right]$, $\beta(\beta)$ из $\cos\frac{1}{2}\beta(\beta) = \frac{J(1/2,\beta)}{J(0,\beta)}$.

MathCad „vortex_TM_thick.dot - Mathias.mcd" даёт $J(\beta)$ быстрее расчёт с β для цилиндра $n=10$. q 

, $q(\beta=1) \approx 60$, но $q(\beta) \ll 1$ уже при $\beta \ll 1$.

$a_0(q) \approx -2q + 2\sqrt{q} - \frac{1}{4} + O(1/\sqrt{q})$ при $q \gg 1$.

Частота $\omega = a(q)\cdot\omega_0$, $\omega_0 = \frac{1}{2}\gamma M_s \frac{\beta^2}{4\beta z^2}\frac{1}{z^2} c(\beta) = \frac{1}{\gamma 2\pi}\omega_M \frac{\beta^2(\beta)}{\beta E}\frac{1}{z^2} c(\beta, z)$ scale.

Ибо $a_0(q) \approx -2q$, тогда $\frac{\omega_0}{\omega_M} = |a_0(q)|\,\omega_y = 2q\omega_y = 16\pi z^2 \frac{J(0,\beta)}{\beta^2}\frac{3^2\beta^2 z^2}{c(\beta)} \times 2\times$

$\times \frac{1}{32\pi}\frac{\beta^2}{\beta^2 z^2} c(\beta) = \pi J(0,\beta)$. $\frac{\omega_0(\beta)}{\omega_M} \approx \pi^2 J(0,\beta)$ будет близко к точному

Тем $\omega(\beta, z) = a(q)\,\omega_y(\beta, z)$ из-за зависимости $|a_0(q)|$.

Дано $\beta = \frac{100\text{ nm}}{150\text{ nm}} = 2/3$, $z = \frac{150\text{ nm}}{20\text{ nm}} = 7.5$. $\frac{\omega_0}{2\pi} \approx 1.76$ GHz ($\omega_M/2\pi = 30$ GHz).

Это в 8 раз больше ω_0 - эксп. Ошибка 15% в Ding, Sci Rep 14

ω_0 можно немного (3-5%) повышать из-за in-plane exchange $\sim (L_e/R)^2$.

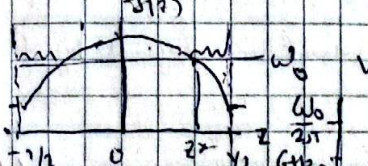
Alternative: Где $\int_{z^*(\omega)}^{1/2} dz\, f^{1/2}(z) = 1/2$? $f(z) = \frac{1}{\gamma}\left[\lambda - J(z)\right]$. z^* из $J(z^*) = \lambda$.

$\frac{\lambda}{\gamma} = \frac{g\omega}{\pi}\frac{1}{\left(M_s^2\frac{c}{\beta^2 z^2}\right)}$, $\frac{\lambda}{\gamma} = \frac{2\pi M_s}{\gamma}\cdot\omega\frac{\beta^2 z^2}{c M_s^2} = \frac{\beta^2 z^2}{c}\frac{2\omega}{\gamma M_s} = 8\pi\frac{\omega}{\omega_M}\frac{\beta^2 z^2}{c(\beta)}$

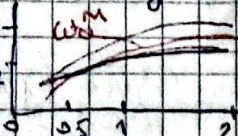
$\frac{J(z)}{\gamma} = \frac{8\pi M_s^2}{M_s^2 c}\frac{1}{\beta^2 z^2} J(z) = 8\pi\frac{\beta^2 z^2}{c} J$, $f(z) = \frac{\lambda}{\gamma} - \frac{1}{\gamma}J(z) = 8\pi\frac{\omega}{\omega_M}\frac{\beta^2 z^2}{c} - 8\pi\frac{\beta^2 z^2}{c}J(z)$

$f(z) = 8\pi\frac{\beta^2 z^2}{c}\left[\frac{\omega}{\omega_M} - J(z)\right] = f_0\left[\omega - J(z)\right]$, $f_0 = 8\pi\frac{\beta^2 z^2}{c}$, ω в ω_M units.

$f(z^*) = 0$, $\omega = J(z^*)$, $z^* = J^{-1}(\omega)$. $\int_{z^*(\omega)}^{1/2} dz\, f_0^{1/2}\sqrt{\omega - J(z)} = \pi/2$, $n=0$.



Из вида $\Phi(z) \sim \sqrt{f(z)}$ ясно что $J(\frac{1}{2}\beta) < \omega_0 < J(0,\beta)$.



$\omega_1 < \omega_0 < \omega_2(\beta)$

Удивительно, но решение Mathieu $\omega_0^M(\beta)$ можно искать между $\omega_1(\beta)$ и $\omega_2(\beta)$ исходя turning point при $q > 18$ ($\beta > 0.5$). когда $a_0(q)$ и $\omega_0^M(q)$ уже почти не отличаются, если интересует $q \gg 1$.

Escaneado con CamScanner

$f(z) = f_0\left[\omega - J(z)\right] = f_0\left[\omega - J(0)\cos pz\right]$. $\int_{z^*}^{1/2} dz\, f^{1/2}(z) = f_0^{1/2}\int_{z^*}^{1/2} dz\left[\omega - J_0\cos pz\right]^{1/2}$

можно взять. $J_M = \int dz\sqrt{1 - b\cos pz}$ | Гр.-Рыжик | 2.576

$J_M = \int dz\sqrt{\omega - J_0\cos pz} = 2\sqrt{a+b}\,(..) = 2\sqrt{\omega + J_0}\,E(\delta,\tau) - \frac{2J_0\sin(..)}{\sqrt{\omega - J_0\cos pz}}$.

$pz = x$, $J_M = \frac{1}{p}\int^x dx\sqrt{\omega - J_0\cos x} = \frac{2}{p}\sqrt{\omega + J_0}\,E(\delta,\tau) - \frac{2J_0\sin x}{\sqrt{\omega_0 - J_0\cos x}}\frac{1}{p}$

$\sin\delta = \sqrt{\frac{(\omega+J_0)(1-\cos x)}{2(\omega - J_0\cos x)}}$, $\tau^2 = \frac{2J_0}{(\omega + J_0)}$. $p = p(\beta) \in \left[0, \frac{2}{3}\pi\right]$, $x_{max} = \frac{2}{3}\pi\cdot\frac{1}{2} = \frac{\pi}{3}$

$\int_{z^*}^{1/2} dz\sqrt{\omega - J_0\cos pz} = \frac{1}{p}\int_{pz^*}^{p/2} dx\sqrt{(...)} = \frac{2}{p}\sqrt{\omega + J_0}\,E(\delta,\tau) - \frac{2}{p}\frac{J_0\sin pz}{\sqrt{\omega - J_0\cos pz}}\Big|_{z^*}^{1/2} =$

$= F(z)\Big|_{z^*}^{1/2} = F(\nu_2) - F(z^*)$, $F(z^*) = \Big|\, f(z^*) = 0,\ \omega = J(z^*)\,\Big|$ и J_M отдаст

Сингулярен $\sim \frac{1}{\sqrt{f(z^*)}}$ если $\sin pz^* \neq 0$. Если $\sin pz^* = 0$, $pz^* = 0, \pi, \ldots$

Min $z^* = \frac{\pi}{p}$. Даже при $p(\beta \gg 1) = p_{max} = \frac{2}{3}\pi$, $z^* = \frac{1}{2}$ но и тогда сингулярность, $|k| \le \frac{1}{2}$.

Может сингулярность "умнет" в $E(\delta,\tau)$ и $\sin pz$-члене сократилось?

$E(\varphi,k) = \int_0^{\varphi} d\alpha\sqrt{1 - k^2\sin^2\alpha}$. Неочевидно, почему J_M сингулярен.

$1 - a\cos x = 1 - a\left(1 - 2\sin^2\frac{x}{2}\right) = 1 - a + 2a\sin^2\frac{x}{2}$, $a = \frac{J(0)}{\omega_0} > 1$.

$J_M = (..)\int^{x^*} dx\,(...)$, а надо $\int^{p/2} dx\,(...)$, $x^* = pz^*$

$\int_0^{1/2} dz(..) = \int_0^{z^*} dz(..) + \int_{z^*}^{1/2} dz(..)$, $\int_{z^*}^{1/2} dz(..) = \int_0^{1/2} dz(..) - \int_0^{z^*} dz(..)$

Альтернатива не нужна, т.к. теор. $\int_{z^*}^{1/2} dz$ table integral. (интегрируем $f(z) < 0$ при $z < z^*$.

J_M быстрее, чем $\int_0^{z^*} dz\,(..)$ при $\omega > J_0$, но при $z \in [0, z^*]$ $\omega < J_0$.

Пусть $J(z) = J_0(1 - p^2z^2)$, $\omega - J(z) = \omega - J_0 + p^2z^2J_0$

$J_p = \int_{z^*}^{1/2} dz\sqrt{\omega - J_0 + J_0p^2z^2} = \left|\frac{\omega - J_0}{J_0p^2} = -\nu^2 < 0\right| = p\sqrt{J_0}\int_{z^*}^{1/2} dz\sqrt{z^2 - \nu^2} =$

$= p\sqrt{J_0}\left[\frac{1}{2z\sqrt{z^2-\nu^2}} - \frac{1}{2}\nu^2\ln\left(z + \sqrt{z^2-\nu^2}\right)\right]_{z^*}^{1/2}$, $\frac{1}{\sqrt{z^2-\nu^2}}$ — расходится при z^*, точка $(z^*)^2 = \nu^2$ — полюса.

$\omega - J_0 + p^2J_0z^{*2} = 0$, $z^{*2} = \frac{J_0 - \omega}{p^2J_0}$.

С $J(z) = J_0(1 - p^2z^2)$ альтернатива $\int_{z^*}^{1/2} dz\sqrt{f(z)} = \pi i/2$ — также не нужна.

→ Phase integral $\Phi(z \to z^*)$ сингулярен. в точке поворота $z \to z^*$, $f(z^*) = 0$.

(16/XI) Im. speakers SolSkyMag. Rozz. Бинц. Sept toEli signed factures Dell' Monitors,

(17/XI) перемена SolSkyMag. Hopfion-gyrovector. High order EM

Получим B/XI eq. на спектр vortex gyrohodes $tg\psi = \frac{2g}{(g^2-1)}$, $g(z) = \frac{1}{4}\frac{f'(z)}{f^{3/2}(z)}$,

при $z = \pm 1/2$. $\psi = \Phi\left(+\frac{1}{2}\right) - \Phi\left(-\frac{1}{2}\right) = 2\int_{-1/2}^{1/2} dz\, f^{1/2}(z)$, ω_n при $n = 1, 2, 3, \ldots$

$f(z) = f_0\left[\frac{\omega}{\omega_M} - J(z)\right]$, $f_0 = 8\pi\frac{\beta^2z^2}{c}$, $z = R/\lambda_e$.

$f'(z) = -f_0J'(z) = 2\beta f_0\int_0^{\infty} dk\, I^2(k)\,e^{-\beta k/2}\,sh(\beta kz)$, $g\left(\frac{1}{2}\right) = \frac{1}{4}\cdot\frac{f'(1/2)}{f^{3/2}(1/2)}$

$e^{-\beta k/2}\,sh\left(\beta k\frac{1}{2}\right) = e^{-x}\frac{1}{2}\left(e^x - e^{-x}\right) = \frac{1}{2}\left(1 - e^{-2x}\right) = \frac{1}{2}\left(1 - e^{-\beta k}\right)$

$f'\left(\frac{1}{2}\right) = \beta f_0\int dk\, I^2(k)\left(1 - e^{-\beta k}\right)$. $f\left(\frac{1}{2}\right) = f_0\left[\omega - J\left(\frac{1}{2}\right)\right]$, $f\left(\frac{1}{2}\right)$ нашли.

$J\left(\frac{1}{2}\right) = 2\int\frac{dk}{k}I^2(k)\left[1 - e^{-\beta k/2}\,ch\,\beta k/2\right] = \int\frac{dk}{k}I^2(k)\left(1 - e^{-\beta k}\right)$

Mathcad "Vortex_EM_thick_dot_Mathieu.mcd" нарисуем $\psi(\omega)$ монотонно растет, $tg\psi(\omega)$ с разрывами, $g(\omega)$ быстро падает с ω и ω/ω_M

$G(\omega) = \frac{2g(\omega)}{g^2(\omega) - 1} < 0$ + быстро падает до "0" с ростом ω. Т.е. для больших ω/ω_M, $G(\omega) \approx 0$, и равенство есть $tg\psi(\omega) = 0$, $\psi(\omega) = n\pi$, как для $\beta \to \infty$ предела.

Escaneado con CamScanner

Т.к. $\psi(\omega) \approx n\pi$ для ω_n, то $tg\psi \approx tg(n\pi) + \frac{1}{\cos^2\psi|_{n\pi}} \cdot (\psi - n\pi) = \psi - n\pi$.

решаем $\psi(\omega) - n\pi = \frac{2g(\omega)}{g^2(\omega)-1}$, чтоб получить ω_n.

Plotted $\omega_1(L)$, $\omega_2(L)$ - разумные. $\omega_1(L)$ linear min при $\beta \approx 0.7$, $R = 150$ nm.

(18/XI) Переписывался любезно в новом UPV/EHU-office. Ricardo's office. Сегодня другому сотруднику, переписывался с Ar. Ding. Angel: в мой office могут подселить new investigator. Приглашение inv. Speakers.

(19/XI) Рец. Ansalone et al. „Gauge field DMI" MMM - proceedings.

(20/XI) Сл. в „Lidl", „Carefour" Обаввіга. Бинт.

(21/XI) Отдых 50 на „Бинт".

(22/XI) Анализ list of invited Speakers SolSkyMag-2022. Now invitations. Ramirdor Krawczyk от ann. 10 kPLN „Umowa o dzieło", 1 этап.

(23/XI) Анализ сделанного по FM thick dots и nanowires (с утолщением core и $R \sim R_0$, $\beta \gg 1$). Еще в папке „Vortex-TM-Glob-2.mcd" с частотами $\omega_n(L)$ по eq. Ding'2014: $\omega_1(L)$ linear min, the lowest mode есть $\omega_1(L)$ при $L > 200$ nm, $R = 250$ nm. В file „Vortex_TM_thick_dot_11.mcd" (2014) результаты расчет нелинейной parabolic cylinder of $\omega_n(L)$ и eff $S_n(z)$.

→ $\omega_1(L)$ linear min как в Ding'14 расчете, так и в new WKB расчете!

(24-25/XI) Переписка с Julian Alexander (Ricardo appl.), Cristina, Oksana (MMM'2022 talk), Nona (Inspions), Arkady (Inspis GV project), MMM'2022 (see), El (Sanchez Sci. Reports). Бинт.

(26/XI) Ikerbasque (Aitor) извещение о promotion standard 3k€ с Jan.2022. Получил 2 Nanoscale papers (Dobrovolskiy et al., Berganza dal.) 2020 в списке high impact publications (IF2020 = 7.79 Nanoscale) и есть сотрудничество с local experimentalists (M. Goiriena-Goikoetxea и A. Garcia-Arribas из Bilbao). Meeting с Julian - согласование и подпись комиссии по Ricardo appl. Investigator position UPV/EHU из моего Ministerio project.

(27-28/XI) Запомнил docs на Ricardo и послал to Julian. Бинт.

(29/XI) Получил 9150 PLN на Poznań-Santander conf. по Inspion project.

(30/XI) Отправлены docs на Ricardo's position by Correo interno.

Работа над Berganza paper. Бинт. С Л. и А. в „Lidl" Hernani.

(X-I) Berganza Sci. Rep. manuscript и обвеян Referees. Послан files Oksana.

(2/XII) Berganza Suppl. Mat, corrected, послан Oksana. Рец. ЖЭТФ/обзор.

(3/XII) E-mails Iñigo (Ikerbasque) о вкл. меня в top 2% по citations в 1996-2020 и п.4 по интегрированию в UPV/EHU по всем спец. Sk.-reading BP model для Josè (Berganza MS). R. Stone Book proposal.

→ UAM-Poznań - lecture, Monday Dec. 20, 11:30 (Maciej). Л. поспала кнеб Лена 12 leg-23€

Sent corrected Suppl. Mat. Oksana.

(4/XII) Invitations speakers Sol-Sky Mag'2022. 7/XII 205€ West. Union Лена, кнеб.

(5-8/XII) Работа над переделкой Berganza Sci. Rep. и ответ to Referees. Мужев ожидался Ond. Artem (Porto) из GV group project переписка с Arcady Julian, Oksana по поводу отказа и GV и ministerio projects.

(9/XII) Запрос Convocatoria UPV/EHU с анкетой Ricardo (напрасно Ruben to call them). Cancelled MMM talk on Bloch points. Бинт. С Л. „Lidl" Hernani. Berganza sent last MS version.

(10/XII) Berganza с Артемом Бондаренко: Vortex dynamics, программа to write a brief paper, обед в Eng. Faculty. Confirmed что Ruben is co-Supervisor of Ricardo.

(11/XII) Revision Berganza Skyrm. manuscript. 12/XII послан его Eider и Oksana.

(13/XII) Meeting с Artem, он умеет расч. $\omega_n(L)$ cylinder dot $R = 250$ nm.

Dobrolubov: это он тоже находил energy of moving deformed vortex как $\delta E = E_{dyn}(\vec{x}) - E_{stat}(\vec{x})$ и искал зависимость $\delta E(\vec{x})$, где $\vec{x} = \omega \cdot |\vec{x}|$

Vortex $\mathcal{L} = \frac{1}{2} M_v (\dot{\vec{X}})^2 + \frac{1}{2} (\vec{G} \times \vec{X}) \dot{\vec{X}} - W(\vec{X})$. Если считать, что $W(\vec{X})$ это kinetic energy с $v = \omega |\vec{X}|$, то $W(\vec{X}) = \frac{1}{2} \varkappa |\vec{X}|^2 = \frac{1}{2} \frac{\varkappa}{\omega^2} |\dot{\vec{X}}|^2 =$

$= \frac{1}{2} M_v |\dot{\vec{X}}|^2 \to M_v = \frac{\varkappa}{\omega^2}$, ω - GMO ω_0 frequency, $\omega_0 = \varkappa / G$, то

$M_v = \frac{\varkappa}{\omega_0^2} = \frac{\varkappa}{(\varkappa/G)^2} = \frac{G^2}{\varkappa} = \frac{(2\pi M_s L/\gamma)^2}{2\pi M_s^2 \frac{L^2}{R}} = \frac{9}{20} \frac{R}{\gamma^2} 2\pi = \frac{9\pi}{10} \frac{R^2}{\gamma}$ (w/sin)

$M_v = \frac{9\pi}{10} \frac{R}{\gamma^2}$ - giant vortex mass. Domino stab. $\varkappa \sim M_s^2 \frac{L^2}{R}$ coll. Gus PRB'10 $M_v \sim \frac{L^2}{\gamma^2}$.

Ivanov JETP Lett '2010: $G_3 (\hat{z} \times \ddot{\vec{X}}) + M_v \ddot{\vec{X}} + G (\hat{z} \times \dot{\vec{X}}) = \vec{F}$ eq. Thiele

использ. $\omega_0, \omega_{\pm 1}$ отсюда $G_3 = \frac{0.63}{\omega_M} \frac{R}{\gamma^2}$, $M_v = 0.58 \frac{L}{\gamma^2}$.

G_3 важен если $\lambda = \frac{G_3 \omega^2}{G}$ не мало. Для G-M-O ω_0 vortex mode

$\lambda = \frac{G_3 \omega_0^2}{G} = 0.63 \frac{1}{\omega_M} \frac{R}{\gamma^2} (0.18 \beta \omega_M)^2 \frac{1}{2\pi M_s \frac{L}{\gamma}} = 0.63 (0.18)^2 \frac{R}{\omega_M \gamma^2} \frac{L^2}{R^2} \omega_M^2 \frac{\gamma^2}{\omega_M L} =$

$= 0.63 \cdot (0.18)^2 \cdot 2 \frac{L}{R} = 0.04 \frac{L}{R}$, $\lambda = \frac{G_3 \omega_0^2}{G} = 0.04 \frac{L}{R} \ll 1$, т.к. $\frac{L}{R} \ll 1$

для thin dots. // Вклад G_3-гироскопа в vortex dynamics очень мал $\lambda \ll 1$,

(14/XII) переписка с Bergama Sci. Rep. MS (resubmitted (3 me thin dot) Dec. 13). Отказ.

(15/XII) С п. в Банк La Caixa" оформили Debit/Credit Cards Confirmo" new passw / El. firma" привязка к п. mobile phone. Личный счёт п. к mob. 635233325 привязать не смогли. Счёт А. привяз. к его mobile phone не успели. Буд. попробуем. В Киев посылки 28€ т. Ане, Катюше

(16/XII) Сдан travel report Artemenko.

(17/XII) Ели сообщ. что на my project не хватает суммы для оплаты Artem. из-за reservation 22k€ на 3/m Ricardo на 2022.

(18/XII -19) подготовка On-line lecture Poznan „Spin waves". Посадили елку.

(20/XII) On-line lecture #3, Poznan. Complain letter PRA на Ricardo's manuscript Mn_2Au. посылка Киев, духи и Лене, 12kg, 23€.

(21-26/XII) Уборка дома, отдых, б-ные. Рец. Non-papers, 3D vortex, Nano Lett. С п. в „Lidl", „Carrefour" Olaberria.

(27/XII) Replies to referees. Pawel - PRA, SW scan. manuscript.

(28/XII) Умер п. папа. Ответ Pawel - PRA. Посылка п. Дуб, 12kg, 23€.

Ricardo: все еще не прислали contract, change of PRA - Mn_2Au paper.

(29/XII) Update Ikerbasque CV. Уборка, б-ные.

(30/XII) Послал Replies to PRA - Pawel MS с всеми Co-authors.

(31/XII) С п. в „AlCampo" Diarbizu. Оплата Santander deb. card 3-й транш ~450€ за А. участку. Оплачен 392,40€ (32,70 x 12 months) „Arrendburu" за Jan. - Dec. 2021. Update CV. Отпр. „bi" с Ант.

(1/I) **2022**. Update Ikerbasque info + uploaded new CV.
С п. в San Sebastian с мальчиками. Б-ные.

(2/I) POLS „3D mag. textures" project.

Task 2. Study of stabilization of 3D solitons - hopfions (4-12 month) (Oct. 21 - June 22)
Cyl. geometry - QH, роль DMI, magnetostatics, высших уравн. сохран.

Task 3. Mag. soliton dynamics from 2D to 3D (8-14 month, Feb. - Aug. 2022)
3D vortex / skyrmion string oscillations gyroscopic, BP - SW, BP dynamics.
нахождение 3D - vortex gyrotropic modes in cyl. dots/nanowires
сделать WKB / Manton calc. of $\omega_n(\beta)$ - vortex gyromodes in cyl. dots.

Escaneado con CamScanner

(3/I) Vortex GM thick dots/wires – какие записать ещё?

Две papers: 1) Vortex GM – WKB/Mathieu approx. $w_n(l)$
2) Vortex GM – wires & graphene v. core + non-linear terms

Осе симм. инфракрасная расш. 3D vortex gyrotropic dynamics $\vec{X}(z,t)$ без gyrov. force(t) или с gyrov. core (Φ). Учёт v. core нужен лишь для R сравнимых с ξ vortex, т.е. dots/wires с малым R. Vortex mass M_0 неинтересна. Ее можно добавить в $\dot{X}$ term. Сначала в конце расчёта $w_n(l)$.

Написание paper (1) Vortex GM – Mathieu/WKB local W_{nm} approx.

(4/I) Заявка Burgundy + Grand Prize Innovat (2 нед), НИВ или dissert.

(5/I) Написание WKB/Mathieu approx. paper.

(6/I) Рождество дома. Бонус. Vortex GM thick dot paper.

(7/I) Vortex GM thick dot paper. 13/XII/II общее направление $\psi = \frac{2\gamma}{(g^2-1)}$ где γ eq.

$S(z) \sim \left[e^{i\Phi(z)} - \frac{(g^2-1)}{(g^2+1)} \frac{1}{\cos\psi} e^{2i\Phi_0} e^{-i\Phi(z)} \right]$, $\frac{(g^2-1)}{(g^2+1)}$ где ψ eq.

$tg^2\psi = \frac{1}{\cos^2\psi} - 1$. $\cos^2\psi = \frac{1}{tg^2\psi+1} = \frac{1}{\frac{4g^2}{(g^2-1)^2}+1} = \frac{(g^2-1)^2}{(g^2+1)^2}$, $\cos\psi = \frac{g^2-1}{g^2+1}$

– странно до этого.

$\frac{g^2-1}{g^2+1} \frac{1}{\cos\psi} = \frac{(g^2-1)}{(g^2+1)} \frac{(g^2+1)}{(g^2-1)} = 1$, $S(z) \sim \left[e^{i\Phi(z)} - e^{2i\Phi_0} e^{-i\Phi(z)} \right]$ $S(z)$ – вещ.

$\Phi(-z) = -\Phi(z) + 2\Phi_0$, $S(-z) \sim \left[e^{-i\Phi(z)} e^{2i\Phi_0} - e^{2i\Phi_0} e^{-i\{-\Phi(z)+\Phi_0\}} \right] \sim \left[e^{-i\Phi(z)} e^{2i\Phi_0} - e^{i\Phi(z)} \right]$

$S(-z) \sim \left[e^{i\Phi(z)} - e^{2i\Phi_0} e^{-i\Phi(z)} \right] = S(z)$. + $S(z)$ is even. А где odd $\chi(z)$?

Надо $S(z) \sim \left[e^{i\Phi(z)} \pm e^{2i\Phi_0} e^{-i\Phi(z)} \right]$, $S(-z) \sim \left[\pm e^{i\Phi(z)} e^{2i\Phi_0} \pm e^{2i\Phi_0} e^{-i\{-\Phi(z)+\Phi_0\}} \right]$

$S(-z) \sim \left[\pm e^{i\Phi(z)} + e^{2i\Phi_0} e^{-i\Phi(z)} \right] \sim \left[e^{i\Phi(z)} \pm e^{2i\Phi_0} e^{-i\Phi(z)} \right]$ – опять то же. $S(-z) = S(z)$?

13/XII или при $p \to 0$, $g \to 0$ было $\psi = n\pi$, $\cos\psi = (-1)^n$, $S(z) \sim \cos(\pi n z)$, n – even

Будучи надо $S(z) \sim \left[e^{i\Phi(z)} + \mathrm{sign}(\cos\psi)\, e^{2i\Phi_0} e^{-i\Phi(z)} \right]$ $S(z) \sim \sin(\pi n z)$, n – odd.

$\psi = \Phi_+ - \Phi_-$, $\Phi_+ + \Phi_- = 2\Phi_0$. $\Phi(-z) = -\Phi(z) + 2\Phi_0$. $\Phi(z) = \int_0^z dz \sqrt{V_2} + \Phi_0$

WKB functions содержат $\Phi(z) = \int^z dz \sqrt{V_2(z)}$ – indefinite integral, т.к.

точно $\partial\Phi/\partial z = \sqrt{V_2(z)}$ выбирается. $\Phi(z) = \int_{z_0}^{z} dz \sqrt{V_2(z)} = \int_{z_0}^{0} + \int_{0}^{z}$

$\int_{z_0}^{0} dz \sqrt{V_2(z)} = |z = -t| = -\int_{z_0}^{0} \sqrt{V_2(t)}$. $\Phi(z) = \int_{z_0}^{z} = F(z) + C$, $\frac{\partial F}{\partial z} = \Phi'(z) = \sqrt{V_2(z)}$

$\Phi(-z) = \int_0^{-z} dz \sqrt{V_2} + \Phi_0 = |z=-t| = -\int_0^{t} dt \sqrt{V_2(-t)} + \Phi_0 = -\int_0^{t} dz \sqrt{V_2(z)} + \Phi_0$

$\Phi(-z) = \int_0^{-z} + \Phi_0 = \int_0^{z} + \int_z^{-z} + \Phi_0 = \int_0^{z} - \int_{-z}^{z} + \Phi_0 = \Phi(z) + \int_z^{-z} (\ldots)$

$\int_{-z}^{z} dz \sqrt{V_2} = 2\int_0^{z} dz \sqrt{V_2}$, $\Phi(-z) = \int_0^{z} + \Phi_0 - 2\int_0^{z} + \Phi_0 = -\int_0^{z} dz \sqrt{V_2} + \Phi_0 =$

$= -\left[\Phi(z) - \Phi_0 \right] + \Phi_0 = -\Phi(z) + 2\Phi_0$, $\Phi(-z) = -\Phi(z) + 2\Phi_0$, $\Phi(z)$ – нечет.

если $\Phi_0 = \Phi(0) = 0$.

$S(-z) = \left[e^{i\Phi(-z)} + \mathrm{sign}(\cos\psi)\, e^{2i\Phi_0} e^{-i\Phi(-z)} \right] = \left[e^{i\{-\Phi(z)+2\Phi_0\}} + \mathrm{sign}(.)\, e^{2i\Phi_0} \times \right.$

$\left. \times e^{-i\{-\Phi(z)+2\Phi_0\}} \right] = \left[e^{2i\Phi_0} e^{-i\Phi(z)} + \mathrm{sign}(.)\, e^{i\Phi(z)} \right] \sim \left[e^{i\Phi(z)} + \mathrm{sign}(.) e^{2i\Phi_0 - i\Phi} \right]$

$S(z) = \left[e^{i\Phi(z)} + c e^{2i\Phi_0} e^{-i\Phi(z)} \right] = e^{i\Phi_0} \left[e^{i(\Phi(z)-\Phi_0)} + c e^{-i(\Phi(z)-\Phi_0)} \right]$ имеет смысл.

Def. $\tilde{\Phi}(z) = \Phi(z) - \Phi_0$, тогда $S(z) = e^{i\Phi_0} \left[e^{i\tilde{\Phi}(z)} + c e^{-i\tilde{\Phi}(z)} \right]$, $\tilde{\Phi}(z) = \int_0^z dz \sqrt{V_2}$ – odd

При $c = \mathrm{sign}(\cos\psi) = +1$, $S_n(z) = e^{i\Phi_0} 2\cos\tilde{\Phi}(z)$ – вещ. с Φ.

При $c = \mathrm{sign}(\cos\psi) = -1$, $S_n(z) = 2i e^{i\Phi_0} \sin\tilde{\Phi}(z)$ – чисто с Φ.

Вообще $S(z) = C_+ \frac{e^{i\Phi_0}}{\sqrt{V_2(z)}} \left[e^{i\tilde{\Phi}(z)} + c e^{-i\tilde{\Phi}(z)} \right]$, $c = \mathrm{sign}(\cos\psi)$, $S = S_x + iS_y$

с Φ – или чисто $\frac{1}{\sqrt{V_2(z)}}$

(8/I) подрост. + Submitted abstract Nano 2022, Sevilla, June 6-10, 2022.
Booked Hotel in Sevilla с отменой до 25го June 1.

Escaneado con CamScanner

9/I Лекция 2 ч ит 31/1(?), IVA декларация до 24/I, Ricardo Mn2An MS, GM-thick dot paper, перепечатка и Site SolSkyMag '2022.

10/I с П. в „tide" Hermani GM-thick dot paper, Бишь.

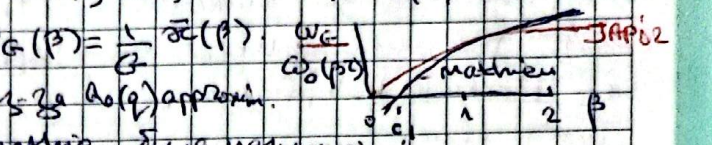
11/I Бишь. Update SolSkyMag '22 website.

12/I с П. в „Carrefour Market", Тополя. Бишь.

13/I Method Mathieu g.e. small $q \ll 1$, $a_0(q) = -\frac{1}{2}q^2 + \frac{7}{128}q^4 - \dots$ дает $\omega_0(\beta)$ при $\beta < 0,2$ которая значительно ниже 2ой $\omega_G(\beta)$ JAP'02, где бралось просто $S(z) = \text{const}$ и $\omega_G(\beta) \sim \omega_M \beta \cdot 0,18$.

Вывод: method Mathieu для the lowest GM (thick dot) с поправкой $S(z)$ работает лишь для thick dots при $q \gg 1$ (при $\beta = 0,5$, $q = 12,8$)

GuS JAP'02: $S(z) = \text{const}$, $\frac{\partial^2 S}{\partial z^2} + \frac{1}{y}[\lambda - J(z)]S(z) = 0 \to \lambda - J(z) = 0$, $\lambda_0 = \int dz\, J(z) = \int dz\, dz' \gamma(zz') = \bar{\varkappa}$, $\omega_G(\beta) = \frac{1}{G}\bar{\varkappa}(\beta)$. $\frac{\omega_G}{\omega_0(\beta,\pi)}$ Mathieu $= 0$ при $\beta = 0,13$, $\pi = \frac{R}{L} = 10$, что дает $a_0(q)$ аппрокс. Надо брать $\beta > 0,4 \div 0,5$, чтоб $\omega_0(\beta,\pi)$ Mathieu была разумной.



WKB Согласно Olver, WKB approximation хорошее, если $\varphi(x) \ll 1$, где

$\varphi(x) = \frac{4f(x)f''(x) - 5(f')^2}{16 f^3(x)}$, $\frac{d^2w}{dx^2} = f(x)w(x)$, $f(x)$ осцилл. – $f(z)$ убывает.

$f(z) = \frac{1}{y}[\lambda - J(z)]$, $f'(z) = -\frac{1}{y}J'(z)$, $f''(z) = -\frac{1}{y}J''(z)$

$\varphi(z) = -\frac{4f(z)f'' - 5f'^2}{16 f^3(z)}$, $\varphi(z) = -\frac{1}{16}\frac{1}{\frac{1}{y^3}(\lambda - J)^3}\left[4(\lambda - J)\frac{J''}{y}(-1) - 5\cdot\frac{1}{y^2}(J')^2\right] = \frac{1}{16}\frac{y}{(\lambda - J)^3}\left[4(\lambda - J)J'' + 5(J')^2\right]$, $\lambda = \omega/\omega_M$

$J(z) = 2\int_0^\infty \frac{dk}{k} I^2(k)\left[1 - e^{-\beta k/2}\operatorname{ch}\beta k z\right]$, $I(k) = \int_0^1 ds\, s\, J_1(ks)$.

Когда $\varphi(z) = \frac{1}{16}\frac{y}{[\lambda - J(z)]^3}\left[4(\lambda - J)J'' + 5(J')^2\right]$ будет $\ll 1$?

$f(z) = \frac{1}{y}[\omega - J(z)]$ и $f(z) = f_0(b,z)[\omega - J(z)]$, $y = \frac{1}{f_0(b,z)}$, $\lambda = \frac{\omega}{\omega_M}$, $\lambda \to \omega$.

Подсчет $\varphi(x) \to \varphi(z)$ численно дает неопределенные результаты. Нельзя сказать что $|\varphi(z)| \ll 1$. Оставим $\varphi(z)$, ведь $w_n(L)$ правдоподобны.

14/I Написание GM-thick dot manuscript. $\tilde{\Phi}(z) = \int_0^z dz\, f^{1/2}(z)$

Even eigenfunctions $S_n(z) \sim \cos\tilde{\Phi}(z)$, odd eigenfunctions $\sin\tilde{\Phi}(z)$.

$f(z) = \frac{1}{y}[\lambda - J(z)]$, $\lambda = \omega/\omega_M$. $\tilde{\Phi}(z,\lambda) = \int_0^z \frac{dz}{\sqrt{y}}\sqrt{\lambda - J(z)}$. Для $S_n(z)$ надо брать $\lambda = \lambda_n$, где λ_n из $\operatorname{tg}\psi(\lambda) = \frac{2g(\lambda)}{g^2(\lambda) - 1}$

$g(\lambda) = \frac{1}{4}\frac{S'(1/2,\lambda)}{f^{3/2}(\frac{1}{2},\lambda)}$, y (in units $8\pi M^2$) $= \frac{1}{8\pi}\left(\frac{\omega}{L}\right)^2\left[\ln\frac{R}{R_0} + \frac{\Sigma}{4}\right]$, $\psi(\lambda) = \tilde{\Phi}(\frac{1}{2},\lambda) - \tilde{\Phi}(-\frac{1}{2},\lambda) = 2\tilde{\Phi}(\frac{1}{2},\lambda)$, $\tilde{\Phi}(z)$ нечет.

$\cos\psi = (\pm)\frac{(g^2-1)}{(g^2+1)}$, $\sin^2\psi = 1 - \cos^2\psi = 1 - \frac{(g^2-1)^2}{(g^2+1)^2} = \frac{(g^2+1)^2 - (g^2-1)^2}{(g^2+1)^2} = \frac{4g^2}{(g^2+1)^2}$

$\sin\psi = (\pm)\frac{(g^2+1)}{2g}$. Насколько сильно $\tilde{\Phi}_n(z)$ зависит от z? Сильно ли $\tilde{\Phi}_n(z)$ отличается от $\tilde{\Phi}_n(z) = kz$ с целым $k = k_z$ эффект. wave vector $\parallel Oz$?

15/I С П. и Ю. в Загребе, кафе. Бишь.

16,17/I Отдых дома. Бишь. „Crystal Liga"

18/I Приезд Бердыча/Sa'Rep. Skytimbu MS as is. Публикация на NAP-2022. Cracow

Escaneado con CamScanner

(18,20/I) Среща с Jaroslaw Klos „Pinning DW in stripe array", Адам Мицк.
Conf. Solsztyniag - 2022 неделната.
(21-24/I) Модул IVA „Конгресите" за 2021 г. С П. в „Carrefour market".
Работа над GM-thick doc paper. Среща с Пеб(Covid). Вливане в Liga „Crystal" to Liga „Elita".
(25/I) Среща с Оксана по Nanowires project. Мне материал DW с BP nanowires.
Intermediate report по PID-2019 project - до March 1. Но e-mail а не получен и на сите министерия (Кунстантия/San Sebastian) никакви писмени.
(26/I) E-mails to/from Мирков (Quantum Center), Ricardo (Mn2Au paper), Мирков (Заявление GV proposal до 16,I), Ricardo (PhD certificate) Ricardo (Mn2Au paper, rebuttal letter). Модул. lecture # 4 Познан.
(27/I) E-mails Мирков (PhD Ricardo), Gonzalo Muga (my affiliation)
(28/I) Модул. lecture #4, Познан. Бълга. Quantum Center.
(29/I) В Carrefour "Оливерия с П. Бълга. Адам Дома. Skype с Ruben.
(30/I) "Rebuttal letter on Edge-DW" paper Lucian Iren incl. Pawel,
(31/I) Lecture # 4, Poznan. Vortex dyn. in circular, Tri-layer, elliptical dots.
(1/II) Кореспонд с Ненси по POLS - project array. Бълга.
(2-3/II) Subsidiacion GV - grant application (таблици според grants участниците, N - reduction grant amounts с цветом членове Групата или всех UPV - участниците?).
(4/II) Ministerio grant „Nanowires" meeting on-line (3 часа).
New results по Bloch point DW dynamics by Felipe и Jose - - мне сделан бълга. Заплата маса в „тогата" (257 €).
3D magnetism" presentation в С. Donelly (Finocchio Conf.
(5-6/II) Борба за выход в „Лигу чемпионов" chess.com. Лечим g.p. дома.
С П. за обеда и в Оливерия „Carrefour" „Lidl".
(7/II) E-mails to Ruben (Javier), Ricardo (Mn2Au), Maciej (Lect.), Akkady (GV project)
(8/II) Meeting с Julian, Ruben, Ricardo Javier, в „Lukas" на вино/jamon/queso.
Обсъжд. с Ruben нового грантов to support PhD или postdoc new.
Skype с Нена: progress POLS project. Оформил и послал Лене и Maciej docs на след. конгрес за 2nd task „3D magn. textures" of POLS project.
Кореспонд с Ricardo, Ruben, P. Fischer.
(9/II) Кореспонд с Ненси, Ruben, Ricardo, Javier. послал to Maciej Signed docs на след. POLS конгрес (10 k€ LN). С П. в „Eroski" Urbil.
(10/II) Intermedio report „Nanowires" project. Запис. Оксана. GV-project Akkady info.
(11/II) Nanowires" report. Предложи. to Ricardo, Ruben. Арантха сделал бълга в „Nanowire" project
(12-13/II) С П. в „Lidl" Оливерия. „Nanowires" project report.
(14/II) Послал Nanowire report to "Cristina UPV/EHU с comments, Кто part D by co-ordinators не запомнил. Тел. разговор с Оксана: 1) сказал кто я отдал Арантха своим предлож. сделал бълга в „Nanowire" project; 2) Felipe прислал BP-DW simulations - обсудим; 3) Nanowire sec. D сделать не удалось (нет времени ещё пока, ещё 10/II) 4) Ruben сделал бълга в BP-DW dyn. используя gyrofield (?) Ricardo re-submitted Mn2Au-MS to PRA, e-mails Ruben, Ricardo.

BP mass
→ Dynamics of Bloch point. BP mass. Y.A. Izyaev, E.B. Sonin, JETP'1989.
BP motion along Bloch line, v-arc string. DW in xz plane.
Field gradient ∥ Oy, film normal $H_z = H_z y$ - potential well for DW
∞-film vs bulk с гистерез. Wm и uniaxial anisotropy xz - easy plane"
Oy ∥ движения DW xz. BP движется ∥ Oz, в DW plane. Bloch DW.
$\Theta_0(y) = 2\arctg \exp\left[\frac{y-A}{D}\right]$. DW energy $\Sigma_{DW}(q,\varphi) = \ldots + MH_z^2 q^2$. m=cosΘ, φ-azim.
Bloch line ∥ Oz, $\varphi(x) = \pm 2\arctg \exp(x/\Lambda_0)$

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$z^2 = x^2+y^2+z^2$ $r \ll \Delta_0 = \sqrt{A/K}$ – Bloch point = hedgehog (exchange dominates)

$V = \frac{\partial z}{\partial t}$ – BP velocity $\parallel Oz$, Bloch line. $\Lambda_0^2 = \frac{A}{2\pi M_s^2}$,

$\ell^2 = \frac{z_0}{2\pi H_z'}$, $z_0 = \sqrt{AK}$. If $\Lambda_0 \gg \ell$ (large $\frac{\partial H_z}{\partial y}$) $\Rightarrow$ $m_{BP} = \frac{\pi}{2}\frac{M}{\gamma^2 H_z'}\ln\frac{\Lambda_0}{r_m}$,

$r_m = \max(\ell, \Delta)$ – cutoff radius. Exp m_{BP} u ucheniya Ivan Iosifa in presentation on Feb. 4. // Iz eya BP mass give gluvsemma $\parallel Oz$ v ∞-film (bulk) w/m gradient Bonnero gradient $H_z' = \frac{\partial H_z}{\partial y}$ ($\Lambda_0 \gg \ell$, ℓ – small).

→ A nego m_{BP} gna gluvsemma BP $\parallel Oz$ (wire length) in finite radius nanowire nog gelstvuem uniform mag. field H_z.

→→ A. Shevchenko, M. Barabash. Nanoscale Res. Lett. 10, 159 (2015).

A General formalism for determination of the effective mass of nanoscale structural inhomogeneities of th DW in uniaxial ferromagnet.

Oglenka $m_{BP} \approx \frac{\Delta}{\gamma^2}$, zoo sovnagaet c Oglenkoi Kafaba '89 w/m $\Lambda_0 \ll \ell$ (small H_z').

$\Delta = \sqrt{A/K}$ 1D DW width in bulk FM.

// Joze abbeun, zoo on simulated BP motion (the BP-DW) nog gelstvuem 6 xy-plane, as oglero nanowire edge k gHyzong.

(15/II) Thiaville ansatz (2003) for BP (BP=DW) in cylinder, cm. 9-10/II 2021

$\vec{m} = (m_\varphi, m_\rho, m_z)$, $\vec{m}(\vec{r}) = \vec{m}(\beta, z)$ – axial symmetry, $Q_H(topol) = 0$.

Thiaville-BP: $m_\varphi = 0$, $m_\rho = \sqrt{1-m_z^2}$, $m_z = \frac{z}{r}\frac{\beta^2}{\beta^2+\rho^2}$, $r^2 = x^2+y^2+z^2$, $\rho^2 = x^2+y^2$.

Sph-BP ansatz (Tejo '2021) w/m helicity $\gamma = \pi/2$: $m_\varphi = 0$, $m_\rho = \sin\theta$, $m_z = p\cos\theta$,

$\vec{r} = (r, \theta, \varphi)$ – spherical coordinates, $\cos\theta = z/r$, $m_z = p\frac{z}{r}$ – Bunyak

v Thiaville ansatz c $m_z = \frac{z^2}{r}\frac{\beta^2}{\beta^2+\rho^2}$ – Ohm obnagaem w/m $\rho \to 0$, 6 BP center.

Est u gyrovector $\vec{G}$ gna Thiaville – BP?

Gyrovector BP-DW nanowire

Conseau Gui EPL '2016: emergent el.-m. field tensor $F^e_{\mu\nu} = \vec{m}\cdot[\partial_\mu\vec{m}\times\partial_\nu\vec{m}]$

gyrotensor $G_{ab} = J\int d^3\vec{r}\ \vec{m}\cdot\left[\frac{\partial\vec{m}}{\partial x_a}\times\frac{\partial\vec{m}}{\partial x_b}\right]$, $J = \frac{M_s}{\gamma}$, a,b – spatial coordinates,

$\vec{m}(\vec{r},t) = \vec{m}[\vec{r}, \vec{X}(t)]$, $\vec{X}$ – soliton center. If rigid soliton motion $\vec{m}[\vec{r},\vec{X}] = \vec{m}(\vec{r}-\vec{X})$

to $G_{ab} = J\int d^3\vec{r}\ F^e_{ab}$, $F^e_{ab} = \vec{m}\cdot\left[\frac{\partial\vec{m}}{\partial x_a}\times\frac{\partial\vec{m}}{\partial x_b}\right]$, $a,b = (x,y,z)$ u.u. (ρ,φ,z) u.u. (r,θ,φ) – spherical coord.

Dve cylindrical nanowires gbogstvo $\vec{r} = (\rho,\varphi,z)$ cyl. coordinates. $\vec{m} = m_\rho\hat{\rho} + m_\varphi\hat{\varphi} + m_z\hat{z}$

w/m $m_\varphi\hat{\varphi}$: $\left(\frac{\partial\vec{m}}{\partial z}\times\frac{\partial\vec{m}}{\partial\rho}\right) = \left[\frac{\partial m_\varphi}{\partial z}\frac{\partial m_z}{\partial\rho} - \frac{\partial m_z}{\partial z}\frac{\partial m_\varphi}{\partial\rho}\right]\hat{\rho} = [\dots]\hat{\rho}$.

$m_\varphi\hat{\varphi}\cdot\left(\frac{\partial\vec{m}}{\partial z}\times\frac{\partial\vec{m}}{\partial\rho}\right) = m_\varphi\hat{\varphi}\cdot[\dots]\hat{\rho} = (\dots)\hat{\varphi}\cdot\hat{\rho} = 0$.

w/m $m_z\hat{z}$: $\left(\frac{\partial\vec{m}}{\partial\rho}\times\frac{1}{\rho}\frac{\partial\vec{m}}{\partial\varphi}\right) = 0$, т.к. $\frac{\partial\vec{m}}{\partial\varphi} = 0$, $\vec{m} = \vec{m}(\rho,z)$.

$G_{ab} = J\int d^3\vec{r}\ \vec{m}\cdot\left[\frac{\partial\vec{m}}{\partial x_a}\times\frac{\partial\vec{m}}{\partial x_b}\right] = J\int d^3\vec{r}\ g_{ab}$, $g_{ab} = \vec{m}\cdot\left[\frac{\partial\vec{m}}{\partial x_a}\times\frac{\partial\vec{m}}{\partial x_b}\right]$

Bonpoc gna Thiaville – BP $g_{ab} \equiv 0$ gyrovector density $\equiv 0$.

$\frac{\partial\vec{m}}{\partial\rho} = \frac{\partial}{\partial\rho}\left[m_\varphi\hat{\varphi} + m_z\hat{z}\right] = \frac{\partial m_\varphi}{\partial\rho}\hat{\varphi} + \frac{\partial m_z}{\partial\rho}\hat{z}$, ecm $\frac{\partial\varphi}{\partial\rho} = 0$.

$\frac{\partial\vec{m}}{\partial\varphi} = \frac{\partial}{\partial\varphi}\left[m_\varphi\hat{\varphi} + m_z\hat{z}\right] = \frac{\partial m_\varphi}{\partial\varphi}\hat{\varphi} + \frac{\partial m_z}{\partial\varphi}\hat{z} = 0$, ecm $\frac{\partial\varphi}{\partial\varphi} = 0$ → Her. $J_{ab} \neq 0$.

→ Note: Arsken $\partial\hat{\rho}/\partial\varphi = \hat{\varphi}$, $\partial\hat{\varphi}/\partial\varphi = -\hat{\rho}$, $\partial\hat{\rho}/\partial\rho = 0$, $\partial\hat{\varphi}/\partial\rho = 0$ [2.2.3]

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→ Т.е. чтоб $G_{ab} \neq 0$, $g_{ab} \neq 0$ для BP надо либо отказаться от Thiaville-ansatz, либо признать, что BP motion is not rigid, и G_{ab} нелокально ($\forall a,b \; F^e_{ab} = 0$). for Thiaville ans.

$g_{ab} = F^e_{ab}$ вырожден. $\frac{\partial}{\partial x_a}$. G_{ab} вырожден. $\frac{\partial}{\partial X_a}$ как следует из Thiele eq

$\vec{m}(\vec{r},t) = \vec{m}(s,z,t)$. BP-DW motion can rigid along Oz, т.к. характерная длина $\|Oz$ может ∞. $\vec{m}(s,z,t) = \vec{m}[s, X_s, z - Z(t)]$. $X_s(t)$

Зависимость от X_s не знаем.

$G_{ab} = \int\int dx\,dz\, \vec{m} \cdot \left[\frac{\partial \vec{m}}{\partial X_a} \times \frac{\partial \vec{m}}{\partial X_b}\right] = \int f dV\, g_{ab}$, $g_{ab} = \vec{m} \cdot \left[\frac{\partial \vec{m}}{\partial X_a} \times \frac{\partial \vec{m}}{\partial X_b}\right]$

Чтобы $g_{ab} \neq 0$, $\vec{X} \neq 0$, $m_s = 0$ - нарушаем dyn. ansatz.

$\vec{m} = (0, m_\varphi, m_z)$, $g_{ab} = m_\varphi[\cdot\cdot] + m_z[\cdots]$. $g_{ab} = \begin{pmatrix} 0 & g_{s\varphi} & g_{sz} \\ g_{\varphi s} & 0 & g_{\varphi z} \\ g_{zs} & g_{z\varphi} & 0 \end{pmatrix}$, $Z = X_z$

$g_{s\varphi} = \vec{m} \cdot \left[\frac{\partial \vec{m}}{\partial X_s} \times \frac{\partial \vec{m}}{\partial X_\varphi}\right] = 0$, т.к. $\vec{m}$ не зависит от X_φ

$g_{sz} = \vec{m} \cdot \left[\frac{\partial \vec{m}}{\partial X_s} \times \frac{\partial \vec{m}}{\partial X_z}\right] = (m_\varphi \hat{\varphi} + m_z \hat{z}) \cdot \left[\frac{\partial \vec{m}}{\partial X_s} \times \frac{\partial \vec{m}}{\partial X_z}\right]$

$\frac{\partial \vec{m}}{\partial X_s} \times \frac{\partial \vec{m}}{\partial X_z} = -\frac{\partial \vec{m}}{\partial X_s} \times \frac{\partial \vec{m}}{\partial z} = -\frac{\partial \vec{m}}{\partial X_s} \times \left[\frac{\partial m_\varphi}{\partial z}\hat{\varphi} + \frac{\partial m_z}{\partial z}\hat{z}\right]$

$m_z = \frac{z}{r}\frac{b^2}{b^2+s^2}$, $m_\varphi = \sqrt{1-m_z^2}$. $\frac{\partial m_\varphi}{\partial z} = \frac{1}{2\sqrt{1-m_z^2}}(-)2 m_z \frac{\partial m_z}{\partial z} = -\frac{m_z}{m_\varphi}\frac{\partial m_z}{\partial z}$

$\frac{\partial}{\partial z}(m_\varphi^2 + m_z^2) = \frac{\partial}{\partial z}(1) = 0$. $\frac{\partial \vec{m}}{\partial z} = -\frac{m_z}{m_\varphi}\frac{\partial m_z}{\partial z}\hat{\varphi} + \frac{\partial m_z}{\partial z}\hat{z} = \left[-\frac{m_z}{m_\varphi}\hat{\varphi} + \hat{z}\right]\frac{\partial m_z}{\partial z}$

$\vec{m}[\vec{s}, z, t] = \vec{m}[\vec{s}, \vec{X}, z - Z(t)]$ - надо. Или $\vec{X} = (X_s, y) = 0$, $\vec{m}(\vec{s}, \vec{X}, z - Z) = \vec{m}(s,z)$ static

Для $\vec{m} = m_\varphi \hat{\varphi} + m_z \hat{z} = (0, m_\varphi, m_z)$ - cylindr. и $X_z = 0$

Для $\vec{X} = (X, y, z)$ - декартовы компоненты. components.

$g_{xy} = \vec{m} \cdot \left[\frac{\partial \vec{m}}{\partial x} \times \frac{\partial \vec{m}}{\partial y}\right]$, $\vec{m} = \vec{m}[\vec{s}, X, y, z - Z]$. $g_{ab} = \begin{pmatrix} 0 & g_{xy} & g_{xz} \\ g_{yx} & 0 & g_{yz} \\ g_{zx} & g_{zy} & 0 \end{pmatrix}$

$\frac{\partial \vec{m}}{\partial X_a} \times \frac{\partial \vec{m}}{\partial X_b} = \left[\hat{\varphi}\frac{\partial m_\varphi}{\partial X_a} + \hat{z}\frac{\partial m_z}{\partial X_a}\right] \times \left[\hat{\varphi}\frac{\partial m_\varphi}{\partial X_b} + \hat{z}\frac{\partial m_z}{\partial X_b}\right] = \hat{\varphi}\times\hat{z}\frac{\partial m_\varphi}{\partial X_a}\frac{\partial m_z}{\partial X_b} +$

$+ \hat{z}\times\hat{\varphi}\frac{\partial m_z}{\partial X_a}\frac{\partial m_\varphi}{\partial X_b} = \left|\hat{\varphi}\times\hat{z} = \hat{s}\right| = \hat{s}\left[\frac{\partial m_\varphi}{\partial X_a}\frac{\partial m_z}{\partial X_b} - \frac{\partial m_z}{\partial X_a}\frac{\partial m_\varphi}{\partial X_b}\right]$, $X_a = X, y, z$

$g_{ab} = \vec{m} \cdot \left[\frac{\partial \vec{m}}{\partial X_a} \times \frac{\partial \vec{m}}{\partial X_b}\right] = (m_\varphi \hat{\varphi} + m_z \hat{z}) \cdot \hat{s}\left[\frac{\partial m_\varphi}{\partial X_a}\frac{\partial m_z}{\partial X_b} - \frac{\partial m_z}{\partial X_a}\frac{\partial m_\varphi}{\partial X_b}\right] = 0$,

т.к. $\hat{\varphi}\cdot\hat{s} = 0$ и $\hat{z}\cdot\hat{s} = 0$.

Т.е. сами предположили dyn. $m_s = 0$, $\vec{m} = m_\varphi \hat{\varphi} + m_z \hat{z}$, автоматом $g_{ab} = 0$ Все зависимости от вида $\vec{m} = \vec{m}(\vec{s}, X, y, z - Z) = \vec{m}(\vec{r}, \vec{X})$. И rigid BP-DW приближение тут не учтем (!). Thiaville ansatz - $m_s = 0$ (static).

Даже если static $\vec{m}$ (Thiaville, например) $\vec{m} = (0, m_\varphi, m_z)$, надо dynamic ($\vec{X} \neq 0$) $\vec{m}(\vec{r}, \vec{X}(t))$ с $m_s \neq 0$. $m_s = 0$ - pure dynamic, $m_s \sim |\dot{\vec{X}}|$.

Но G_{ab}, g_{ab} - density m_s-ya $G_{ab} \dot{X}_a \dot{X}_b$ dynamic term in Thiele eq. должны сливаться при $\vec{X} = 0$, $G_{ab}(\vec{X} = 0)$. тогда $G_{ab}(\vec{X} = 0) = 0$.

→ Вывод: любой ansatz $\vec{m} = (m_s, m_\varphi, m_z)$ с m_s(static) $= 0$ дает $G_{ab} = 0$ в linear Thiele eq, $G_{ab}(\vec{X} = 0) = 0$.

А как же 2D case $\vec{m}(0, m_\varphi, m_z)$ у него $G_{xy}, G_{yx} \neq 0$ и $G_z \neq 0$?

→ Вывод не верен. Это фокус сфер. coordinates?

$g_{xy} = \vec{m} \cdot \left[\frac{\partial \vec{m}}{\partial x} \times \frac{\partial \vec{m}}{\partial y}\right]$, $\frac{\partial \vec{m}}{\partial x} \times \frac{\partial \vec{m}}{\partial y} = \left[\frac{\partial m_x}{\partial x}\hat{x} + \frac{\partial m_y}{\partial x}\hat{y} + \frac{\partial m_z}{\partial x}\hat{z}\right] \times \left[\frac{\partial m_x}{\partial y}\hat{x} + \frac{\partial m_y}{\partial y}\hat{y} + \frac{\partial m_z}{\partial y}\hat{z}\right]$

$= (\hat{x}\times\hat{y})\left[\frac{\partial m_x}{\partial x}\frac{\partial m_y}{\partial y} - \frac{\partial m_y}{\partial x}\frac{\partial m_x}{\partial y}\right] + (\hat{x}\times\hat{z})\left[\frac{\partial m_x}{\partial x}\frac{\partial m_z}{\partial y} - \frac{\partial m_z}{\partial x}\frac{\partial m_x}{\partial y}\right] + (\hat{y}\times\hat{z})\left[\frac{\partial m_y}{\partial x}\frac{\partial m_z}{\partial y} - \frac{\partial m_z}{\partial x}\frac{\partial m_y}{\partial y}\right]$

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$\hat{x}\times\hat{y}=\hat{z}$, $\hat{x}\times\hat{z}=-\hat{y}$, $\hat{y}\times\hat{z}=\hat{x}$

$g_{xy}=(m_x\hat{x}+m_y\hat{y}+m_z\hat{z})\left[\hat{z}[\dots]+\hat{y}[\dots]+\hat{x}[\dots]\right]\neq 0$ by Stokes' Theorem.

Hago Gab$(\vec{X}=0)$, haciendo como Gab cumpla en sfera $\vec{m}(\vec{r})$, y que $\frac{\partial\varphi}{\partial x}\to\frac{\partial\varphi}{\partial x^a}$, "rigid" Solitón approximation.

$g_{xy}=\vec{m}\cdot\left[\frac{\partial\vec{u}}{\partial x}\times\frac{\partial\vec{u}}{\partial y}\right]$, $\frac{\partial\vec{u}}{\partial x}=\frac{\partial\varphi}{\partial x}\left[\dots\right]$

$\hat{s}=\frac{x}{s}\hat{x}+\frac{y}{s}\hat{y}$, $\hat{\varphi}=\frac{1}{s}(-y\hat{x}+x\hat{y})$, $\hat{z}=\hat{z}$, $s=\sqrt{x^2+y^2}$

$\frac{\partial}{\partial x}(s\hat{s})=\frac{\partial}{\partial x}\left[\frac{1}{s}(x\hat{x}+y\hat{y})\right]=\frac{\partial}{\partial x}\frac{1}{s}\cdot[x\hat{x}+y\hat{y}]+$

$+\frac{1}{s}\hat{x}+\frac{1}{s}(x\hat{x}+y\hat{y})=-\frac{1}{s^2}\frac{\partial s}{\partial x}(x\hat{x}+y\hat{y})+\frac{1}{s}\hat{x}=$

$=\hat{x}\frac{s}{s^2}-\frac{x}{s^2}\hat{s}+\frac{1}{s}(\hat{s}\cos\varphi-\hat{\varphi}\sin\varphi)=-\frac{1}{s}\sin\varphi\,\hat{\varphi}$

$\frac{\partial\hat{s}}{\partial x}=\frac{1}{s}(-y\hat{x}+x\hat{y})=-\frac{1}{s^2}\frac{s}{x}(-y\hat{x}+x\hat{y})+\frac{1}{s}\hat{y}=-\frac{1}{s^3}x\hat{\varphi}+\frac{1}{s}(\sin\varphi\,\hat{s}+\cos\varphi\,\hat{\varphi})=\frac{\sin\varphi}{s}\hat{\varphi}$

$\frac{\partial\hat{s}}{\partial y}=\frac{1}{s}\sin\varphi\,\hat{s}$, $\frac{\partial\hat{\varphi}}{\partial x}=-\frac{1}{s}\sin\varphi\,\hat{s}$.

$\frac{\partial\vec{u}}{\partial x}=\frac{\partial\vec{u}}{\partial s}\hat{s}+\frac{\partial\vec{u}}{\partial\varphi}\hat{\varphi}+\frac{\partial\vec{u}}{\partial z}\hat{z}+m_s\left(-\frac{1}{s}\right)\sin\varphi\,\hat{\varphi}+m_\varphi\frac{1}{s}\sin\varphi\,\hat{s}=$

$=\hat{s}\left[\frac{\partial\vec{u}}{\partial s}+\frac{1}{s}\sin\varphi\,m_\varphi\right]+\hat{\varphi}\left[\frac{\partial\vec{u}}{\partial\varphi}-\frac{1}{s}\sin\varphi\,m_s\right]+\frac{\partial\vec{u}}{\partial z}\hat{z}$

$\frac{\partial\hat{s}}{\partial y}=\frac{1}{s}\frac{\partial}{\partial y}(x\hat{x}+y\hat{y})=-\frac{1}{s^2}\frac{s}{y}(x\hat{x}+y\hat{y})+\frac{1}{s}\hat{y}=\frac{1}{s}(-x\hat{x}+y\hat{y})=\frac{\cos\varphi}{s}\hat{\varphi}$

$\frac{\partial\hat{\varphi}}{\partial y}=\frac{1}{s}\frac{\partial}{\partial y}(-y\hat{x}+x\hat{y})=-\frac{1}{s^2}\frac{s}{y}(-y\hat{x}+x\hat{y})+\frac{1}{s}(-\hat{x})=-\frac{1}{s}(\cos\varphi\,\hat{s}-\sin\varphi\,\hat{\varphi})=-\frac{\cos\varphi}{s}\hat{s}$

$\frac{\partial\vec{u}}{\partial y}=\frac{\partial\vec{u}}{\partial s}\hat{s}+\frac{\partial\vec{u}}{\partial\varphi}\hat{\varphi}+\frac{\partial\vec{u}}{\partial z}\hat{z}+m_s\frac{\cos\varphi}{s}\hat{\varphi}+m_\varphi\left(-\frac{\cos\varphi}{s}\right)\hat{s}=$

$+\hat{s}\left[\frac{\partial\vec{u}}{\partial s}-\frac{1}{s}\cos\varphi\,m_\varphi\right]+\hat{\varphi}\left[\frac{\partial\vec{u}}{\partial\varphi}+\frac{1}{s}\cos\varphi\,m_s\right]+\frac{\partial\vec{u}}{\partial z}\hat{z}$.

$\frac{\partial\vec{u}}{\partial x}\times\frac{\partial\vec{u}}{\partial y}=\left[\hat{s}\left(\frac{\partial\vec{u}}{\partial x}\right)_s+\hat{\varphi}\left(\frac{\partial\vec{u}}{\partial x}\right)_\varphi+\hat{z}\left(\frac{\partial\vec{u}}{\partial x}\right)_z\right]\times\left[\hat{s}\left(\frac{\partial\vec{u}}{\partial y}\right)_s+\hat{\varphi}\left(\frac{\partial\vec{u}}{\partial y}\right)_\varphi+\hat{z}\left(\frac{\partial\vec{u}}{\partial y}\right)_z\right]=$

$=\hat{s}\times\hat{\varphi}\left[\left(\frac{\partial\vec{u}}{\partial x}\right)_s\left(\frac{\partial\vec{u}}{\partial y}\right)_\varphi-\left(\frac{\partial\vec{u}}{\partial y}\right)_s\left(\frac{\partial\vec{u}}{\partial x}\right)_\varphi\right]_z+\hat{s}\times\hat{z}\left[\left(\frac{\partial\vec{u}}{\partial x}\right)_s\left(\frac{\partial\vec{u}}{\partial y}\right)_z-\left(\frac{\partial\vec{u}}{\partial y}\right)_s\left(\frac{\partial\vec{u}}{\partial x}\right)_z\right]_\varphi+$

$+\hat{z}\times\hat{\varphi}\left[\left(\frac{\partial\vec{u}}{\partial x}\right)_\varphi\left(\frac{\partial\vec{u}}{\partial y}\right)_z-\left(\frac{\partial\vec{u}}{\partial y}\right)_\varphi\left(\frac{\partial\vec{u}}{\partial x}\right)_z\right]_s$,

$\hat{s}\times\hat{\varphi}=\hat{z}$, $\hat{\varphi}\times\hat{z}=\hat{s}$, $\hat{z}\times\hat{s}=\hat{\varphi}$

$\frac{\partial\vec{u}}{\partial x}\times\frac{\partial\vec{u}}{\partial y}=\hat{z}[\dots]_z+\hat{\varphi}[\dots]_\varphi+\hat{s}[\dots]_s$

$\vec{m}\cdot\left[\frac{\partial\vec{u}}{\partial x}\times\frac{\partial\vec{u}}{\partial y}\right]=\{m_z[\dots]_z+m_\varphi[\dots]_\varphi+m_s[\dots]_s\}$. Tomemos vacío $m_s=0$.

$\vec{m}\cdot\left[\frac{\partial\vec{u}}{\partial x}\times\frac{\partial\vec{u}}{\partial y}\right]=m_z\left[\frac{1}{s}\sin\varphi\,m_\varphi\frac{\partial\vec{u}}{\partial\varphi}-\frac{\partial\vec{u}}{\partial\varphi}\left(-\frac{1}{s}\cos\varphi\,m_\varphi\right)\right]+m_\varphi\left[\frac{\partial\vec{u}}{\partial s}\left(-\frac{1}{s}\cos\varphi\,m_s\right)-\frac{\partial\vec{u}}{\partial z}\sin\varphi\,m_s\right]+$

$+m_z\frac{1}{s}\left[\sin\varphi\frac{\partial\vec{u}}{\partial x}+\cos\varphi\frac{\partial\vec{u}}{\partial y}\right]\cdot\left[\cos\varphi\frac{\partial\vec{u}}{\partial x}-\sin\varphi\frac{\partial\vec{u}}{\partial y}\right]=m_z m_\varphi\frac{1}{s}\left[\sin\varphi\frac{\partial\vec{u}}{\partial x}+\cos\varphi\frac{\partial\vec{u}}{\partial y}\right]+$

$\frac{\partial\varphi}{\partial x}=\frac{\partial\varphi}{\partial x}\sin^2\varphi+\cos^2\varphi\frac{\partial\varphi}{\partial x}+\sin\varphi\cos\varphi\frac{\partial\varphi}{\partial y}-\sin\varphi\cos\varphi\frac{\partial\varphi}{\partial y}$

$g_{xy}=\vec{m}\cdot\left[\frac{\partial\vec{u}}{\partial x}\times\frac{\partial\vec{u}}{\partial y}\right]=m_z m_\varphi\frac{1}{s}=\frac{\partial\varphi}{\partial z}m_\varphi\frac{1}{s}=\frac{1}{s}m_\varphi\left[m_z\frac{\partial\varphi}{\partial z}+m_\varphi\frac{\partial\varphi}{\partial z}\right]$

$\frac{\partial\varphi}{\partial z}=\frac{\partial\varphi}{\partial z}\sqrt{1-m_z^2}=\frac{1}{2\sqrt{1-m_z^2}}(-2m_z)\frac{\partial\varphi}{\partial z}=-\frac{m_z}{m_\varphi}\frac{\partial\varphi}{\partial z}$, $g_{xy}=\frac{1}{s}m_\varphi\left[m_z\frac{\partial\varphi}{\partial z}+m_\varphi\frac{\partial\varphi}{\partial z}\right]=\frac{1}{s}\left[-m_z^2+m_\varphi^2\right]\frac{\partial\varphi}{\partial z}$. Hablaremos grado con m_z^2. Con $m_z+m_\varphi^2$ (cuan m_φ^2).

Escaneado con CamScanner

то $g_{xy} = \frac{1}{s}\left[m_x^2 + m_y^2\right]\frac{\partial m_z}{\partial s} = \frac{1}{s}\frac{\partial m_z}{\partial s}$. $G_{xy} = \int dV g_{xy} = \frac{M_s}{J}\int d\varphi \int ds \int dz \frac{1}{s}\frac{\partial m_z}{\partial s} \cdot$

$\cdot \frac{1}{s}\frac{\partial m_z}{\partial s}$. Если $m_z \neq f(z)$ winding vortex, то $G_{xy} = \frac{M_s}{J} L \int d\varphi \int ds \frac{1}{s}\frac{\partial m_z}{\partial s} =$

$= 2\pi \frac{M_s}{J} L\, m_z(s)\Big|_0^R = 2\pi \frac{M_s}{J} L\left[m_z(R) - m_z(0)\right]$ — Bloch point, для G_z — gyrovector.

$G_{xy} = -2\pi \frac{M_s}{J} L p = -G_z$. Надо $G_z = -2\pi \frac{M_s}{J} L p$, т.е. сохранить

sign m_z^2, изменив sign m_z^2 in G_{xy} definition (atoms at $\hat{s} \times \hat{\varphi} = -\hat{\varphi}$).

$g_{xy} = \frac{1}{s}\left[-m_z^2 - m_\varphi^2\right]\frac{\partial m_z}{\partial s} = -\frac{1}{s}\frac{\partial m_z}{\partial s}$ или $m_s = 0$.

$G_{xy} = \int dV g_{xy} = -\int dz \int d\varphi s\, ds \frac{\partial m_z}{\partial s}\frac{1}{s} = -2\pi \int dz \int ds \frac{\partial m_z}{\partial s}$, $\vec{m} = \vec{m}(s,z)$

(BP) Thiaville BP: $m_z(s,z) = \frac{z}{4}\frac{R^2}{R^2+s^2}$ — нечет по z. $\int_{-1/2}^{1/2} dz\, m_z(s,z) = 0$ BP

и $G_{xy} = 0$ для thiaville ansatz.

Расчет выше $\vec{m} = (m_s, m_\varphi, m_z)$ — cylindrical comp. с $m_s = 0$, $\vec{r} = (x,y,z)$ декартовы компоненты. Оставим цилиндрические. Надо сделать одинаковые компоненты $\vec{m}$ и $\vec{r}$. Если $\vec{m} = (m_s, m_\varphi, m_z)$, то и $\vec{r} = (s,\varphi,z)$ cylindrical comp

Проверим Skyr index BP ($\frac{9}{4\pi} 2\pi$.) основываясь на Wang-Dzialoshinskii 2020, верно ли $Q_H = 0$? для thiaville BP.

Аналогично для расчета Gab Bloch point $\nabla w = \vec{G} = \sin\theta \nabla\theta \times \nabla\Phi$

с $\vec{m} = \vec{m}(\theta,\Phi)$, θ, Φ — spherical angles.

$m_x = \sin\theta\cos\Phi$, $m_y = \sin\theta\sin\Phi$, $m_z = \cos\theta$, $\operatorname{tg}\Phi = \frac{m_y}{m_x}$.

Для thiaville BP, $\operatorname{tg}\Phi = \frac{x/s\sqrt{1-m_z^2}}{-y/s\sqrt{1-m_z^2}} = -\frac{x}{y}$. $\Phi = \operatorname{arctg}\left(\frac{x}{y}\right)(-1)$

$\frac{\partial\Phi}{\partial x} = \frac{\partial}{\partial x}(-1)\operatorname{arctg}\left(\frac{x}{y}\right) = (-1)\frac{1}{1+(x/y)^2}\frac{1}{y} = \frac{-y}{x^2+y^2} = -\frac{y}{s^2} = -\frac{1}{s}\sin\varphi$

$\frac{\partial\Phi}{\partial y} = -\frac{\partial}{\partial y}\operatorname{arctg}\left(\frac{x}{y}\right) = -\frac{1}{1+(x/y)^2}\times\left(+\frac{1}{y^2}\right) = \frac{x}{x^2+y^2} = \frac{1}{s}\cos\varphi$. $\frac{\partial\Phi}{\partial z} = 0$.

$-\sin\theta\nabla\theta = \nabla\cos\theta = \nabla m_z$.

$\vec{G} = -\nabla m_z \times \nabla\Phi$ $\nabla\Phi = (\Phi'_x, \Phi'_y, 0)$, $\nabla m_z = \vec{a}$, $\vec{G} = -\vec{a}\times\nabla\Phi$

$\nabla\Phi = (b_s, b_\varphi, 0)$. $\vec{G} = -\vec{a}\times\nabla\Phi = -\begin{vmatrix} a_s & a_\varphi & a_z \\ b_s & b_\varphi & 0 \end{vmatrix} = \ldots$

$\vec{a} = \nabla m_z(s,z) = \hat{s}\frac{\partial m_z}{\partial s} + \hat{z}\frac{\partial m_z}{\partial z}$ $\vec{b} = \nabla\Phi$

$\vec{a} = (a_s, 0, a_z)$, $a_\varphi = 0$

$\vec{G} = -\vec{a}\times\nabla\Phi = -\hat{\varphi}(a_z b_s) - \hat{z}(a_s b_\varphi) + \hat{s}(a_z b_\varphi)$..

$b_s = b_x\cos\varphi + b_y\sin\varphi = b_x\cos\varphi + \ldots = -\frac{1}{s}\sin\varphi\cos\varphi + \frac{1}{s}\cos\varphi\sin\varphi = 0$.

$b_\varphi = -b_x\sin\varphi + b_y\cos\varphi = -\left(-\frac{1}{s}\sin\varphi\right)\sin\varphi + \frac{1}{s}\cos\varphi\cos\varphi = \frac{1}{s}$

$\vec{g} = -\vec{a}\times\nabla\Phi = -\hat{z}\,a_s b_\varphi + \hat{s}\,a_z b_\varphi = \hat{s}\frac{1}{s}a_z - \hat{z}\frac{1}{s}a_s$, $\vec{a} = \nabla m_z$

$a_z = \frac{\partial m_z}{\partial z}$, $a_s = \frac{\partial m_z}{\partial s}$. $a_z = \frac{\partial}{\partial z}\frac{z}{2}\frac{R^2}{R^2+s^2}$, $a_s = \frac{\partial m_z}{\partial s}$.

$g_s = \frac{1}{s}a_z = \frac{1}{s}\frac{\partial}{\partial z}m_z$, $g_z = -\frac{1}{s}a_s = -\frac{1}{s}\frac{\partial m_z}{\partial s}$. компоненты $\vec{g}$.

$(-g_{xy})$

Global $\vec{G}$ - gyrovector, $\vec{G} = \int dV\vec{g}$, $G_z = \int dz\int d^2s\,\frac{1}{s}\frac{\partial m_z}{\partial s} = 0$, т.к. $m_z(s)$ - нечет.

$G_s = \int dV g_s = \int dz \int d^2s \frac{1}{s}\frac{\partial m_z}{\partial z} = 2\pi \int dz \int ds \frac{\partial m_z}{\partial z} = 2\pi \int ds\, m_z(z,s)\Big|_{z=-1/2}^{z=1/2}$

$G_s = 4\pi \int ds\, m_z(1/2, s)$, $I = \int_0^R ds\, m_z(z,s) = \int_0^R ds \frac{z}{\sqrt{z^2+s^2}}\frac{R^2}{(R^2+s^2)} = R^2 z\int_0^R ds \frac{1}{\sqrt{z^2+s^2}(R^2+s^2)}$

$I(z,R) = R^2 z\int_0^R ds \frac{1}{\sqrt{z^2+s^2}(s^2+R^2)} = R^2 z\frac{1}{R^2}\int_0^1 dr \frac{1}{\sqrt{(z/R)^2+r^2}}\frac{1}{r^2+(R/R)^2}$

$r = s/R$, $z = 1/2$ здесь.

$I(z,t) = z t^2 \int_0^1 dr \frac{1}{\sqrt{(z/R)^2 + r^2}} \frac{1}{(t^2+1)^2}$, $t = \frac{b}{R} \sim 5 \div 10$ nos, $b \sim 10$ nm, $R \sim 50$ nm, $b/R \sim 1$

$G_s = 4\pi J \cdot I(z,t) = 2\pi L \left(\frac{M_s}{\gamma}\right) I_s\left(\frac{z}{R}, t\right)$, $I_s\left(\frac{z}{R}, t\right) = t^2 \int_0^1 dr \frac{1}{\sqrt{(z/R)^2 + r^2}(t^2+1)}$

$z/R = \frac{1}{2R} \gg 1$ Gyr. long nanowire. $I_s \approx t^2 \frac{R}{z} \int_0^1 dr \frac{1}{t^2 + r^2} =$

$= t^2 \frac{R}{z} \frac{1}{t} \arctg \frac{r}{t} \Big|_0^1 = t \frac{R}{z} \arctg\left(\frac{1}{t}\right)$.

$G_s = 2\pi L \left(\frac{M_s}{\gamma}\right) t \frac{2R}{L} \arctg\left(\frac{1}{t}\right) = 4\pi R \left(\frac{M_s}{\gamma}\right) t \arctg\left(\frac{1}{t}\right)$, $G_s \approx 4\pi R \left(\frac{M_s}{\gamma}\right)$, when $t \gg 1$

Nanowire c BP-DW in thinwall. (gyr. vector $\left(\begin{smallmatrix} \text{vort} \\ t \ll 1 \end{smallmatrix}\right)$

global $\vec{G} = (G_s, 0, 0)$, $G_s = 4\pi R \left(\frac{M_s}{\gamma}\right)$ when $t \gg 1$.

Gyroforce $\vec{F}_g = \vec{G} \times \vec{V} = \hat{z} G_s V_e - \hat{\varphi} G_s V_z$. Each DW motion c $\vec{V}$ along nanowire length, the gyroforce $\vec{F}_g = -\hat{\varphi} G_s V_z$ $\vec{V}$ tends to incline BP k nanowire edge, $s = R$.

BP-DW gyro vector $G_s = 4\pi R \left(\frac{M_s}{\gamma}\right) \cdot \arctg\left(\frac{1}{t}\right)$, $t = \frac{b}{R}$, b - BP size

Suppose $t \ll 1$ (when b - equilibrium eq $\min W(b)$), $\arctg\left(\frac{1}{t}\right) \approx \frac{\pi}{2} - t$

$G_s (t \ll 1) \approx 4\pi R \left(M_s/\gamma\right) t \cdot \frac{\pi}{2} = 2\pi^2 b \left(M_s/\gamma\right)$

(16/ii) Revised PISA9 „Nanowires reverse in vortex and the Gyrotropic UPV/EHU.

Implementation of Orec. no inhomog BP in cyl. nanowire in spherical.

BP-DW in cyl. nanowire: 1) Vortex b_{eq} eq $\min W(b)$, 2) Vortex χ_z, χ_s dyn.

Gyr. BP&DW when $W(\vec{x}) = W(0) + \frac{1}{2}\chi_z z^2 + \frac{1}{2}\chi_s x_s^2$, two gaps eq.

Thiaville ansatz: $\vec{m} = \vec{m}(s,z)$, $m_s = 0$, $m_\varphi = \sqrt{1-m_z^2}$, $m_z = \frac{z}{\sqrt{z^2+s^2}} \frac{b^2}{(b^2+s^2)}$, $z^2 = z^2 + s^2$, $s^2 = x^2 + y^2$

$\operatorname{div}\vec{m} = \frac{1}{s}\frac{\partial}{\partial s}(s m_s) + \frac{1}{s}\frac{\partial m_\varphi}{\partial \varphi}^{=0} + \frac{\partial m_z}{\partial z} = \frac{\partial m_z}{\partial z}$. $m_z = \frac{z}{\sqrt{z^2+s^2}} \frac{b^2}{(b^2+s^2)}$

$\frac{\partial m_z}{\partial z} = \frac{b^2}{(b^2+s^2)} \left[\frac{1}{\sqrt{z^2+s^2}} + z\left(-\frac{1}{2}\right)(z^2+s^2)^{-3/2} 2z \right] =$ BP size b - proces

$= \frac{b^2}{(b^2+s^2)} \frac{1}{(z^2+s^2)^{3/2}} \left[z^2 + s^2 - z^2 \right] = \frac{b^2 s^2}{(b^2+s^2)} \frac{1}{(z^2+s^2)^{3/2}}$. $\operatorname{div}\vec{m} = \frac{\partial m_z}{\partial z} = f(z,s)$

$W_m^{vol} = \frac{1}{2} M_s^2 \int dV \int dV' \frac{\operatorname{div}\vec{m}(\vec{r}) \operatorname{div}\vec{m}(\vec{r}')}{|\vec{r} - \vec{r}'|}$, $\frac{1}{\sqrt{z^2+s^2}} = \int dk e^{-k|z|} J_0(ks)$

$J_0(k|\vec{s} - \vec{s}'|) = \sum_m e^{im(\varphi - \varphi')} J_m(ks) J_m(ks')$

$\int_0^{2\pi} d\varphi' J_0(k|\vec{s} - \vec{s}'|) = 2\pi J_0(ks) J_0(ks')$

$\int d\varphi d\varphi' \frac{1}{|\vec{r} - \vec{r}'|} = \int d\varphi \int d\varphi' \int_0^\infty dk e^{-k|z-z'|} \sum_m e^{im(\varphi - \varphi')} J_m J_m(\cdot) = (2\pi)^2 \int dk e^{-k|z-z'|} J_0(ks) \times J_0(ks')$

$W_m^{vol} = \frac{1}{2} M_s^2 (2\pi)^2 \int ds_s \int dz \int ds' s' dz' \operatorname{div}\vec{m}(s,z) \operatorname{div}\vec{m}(s',z') \int dk e^{-k|z-z'|} J_0(ks) J_0(ks')$

$W_m^{vol} = 2\pi^2 M_s^2 \int ds_s s \int dz \int ds' s' \int dz' \int dk e^{-k|z-z'|} J_0(ks) J_0(ks') \frac{b^2 s^2}{(b^2+s^2)} \frac{1}{(z^2+s^2)^{3/2}} \frac{b^2 s'^2}{(b^2+s'^2)(z'^2+s'^2)^{3/2}}$

$I(k,z) = \int_0^R ds_s J_0(ks) \frac{s^2}{(b^2+s^2)} \frac{1}{(z^2+s^2)^{3/2}}$.

$W_m^{vol} = 2\pi^2 M_s^2 b^4 \int dz dz' \int dk e^{-k|z-z'|} I(k,z) I(k,z')$, Moreover no $\int dz dz'$ moreno Bessel? Hen

$J(k,s) = \int dz dz' e^{-k|z-z'|} \frac{1}{(z^2+s^2)^{3/2}} \frac{1}{(z'^2+s'^2)^{3/2}} \cdot \int dz dz' e^{-k|z-z'|} f(z) f(z') =$

$= \int_{z>z'} dz \int_{z'} dz' e^{-k(z-z')} f(z) f(z') + \int_{z<z'} dz \int dz' e^{-k(z'-z)} f(z) f(z') = \int dz e^{-kz} f(z) \times$

$\times \int_{z'} dz' e^{kz'} f(z') + \int dz e^{kz} f(z) \int_{z'} dz' e^{-kz'} f(z')$. Integrando hago $\int_0^z dz' \ldots + \int_z^L dz' \ldots$ una no vengas.

$I_z(k,s) = \int dz \int dz' e^{-k|z-z'|} f(z,s) f(z',s') = \int dz \left[\int_0^z dz' e^{-k(z-z')} f(z,s) f(z',s') + \int_z^L dz' e^{-k(z'-z)} f(z,s) f(z',s') \right] =$

$= \int dz \left[e^{-kz} f(z) \int_0^z dz' e^{kz'} f(z') + \right.$

$e^{kz}\left[f(z)\int_z^L dz' e^{-kz'} f(z')\right]$. Нашли $F_1(z)=\int_0^z dz' e^{+kz'} f(z')$, $F_2(z)=\int_z^L dz' e^{-kz'} f(z')$

$F_1(z)=\int_0^z dz' e^{kz'} \frac{1}{(z'^2+\rho'^2)^{3/2}}$, $F_2(z)=\int_z^L dz' e^{-kz'} \frac{1}{(z'^2+\rho'^2)^{3/2}}$.

В Гр. Ряжских (3.387) есть формула $\int_0^\infty \frac{dx}{(x^2+a^2)^{\nu+1}} e^{-\beta x}$ с ∞-пределами, а не z_0 и z. Есть ли такая формула?

Использовали ли Вольмир и MathCad $W_m^{vol}(B)$ as it is без упрощений?

$$I(k,z)=\left|\rho/R=r,\ kR=x\right|=R^2\int_0^1 dr\, r\, J_0(kRr)\frac{r^2}{\left[(t/R)^2+r^2\right]}\frac{1}{R^3}\frac{1}{\left[(t/R)^2+r^2\right]^{3/2}}$$

$$I(k,z)=\frac{1}{R}\int_0^1 dr\, r\, J_0(rx)\frac{r^2}{(t^2+r^2)}\frac{1}{\left[\left(\frac{t}{R}\right)^2+r^2\right]^{3/2}},\quad z/R=\frac{z}{L}\frac{L}{R}=\beta\frac{z}{L}+\frac{z}{L}\to z.\quad \beta=L/R$$

$$W_m^{vol}(B)=2\pi^2\mu_s^2 B^4 L^2\int dz\int dz'\frac{1}{R}\int dx\, e^{-\frac{x}{R}L|z-z'|}\, I(kz)\,I(kz')$$

$$W_m^{vol}(B)=2\pi^2\mu_s^2 B^4 L^2\frac{1}{R^3}\int_0^1 dz\int_0^1 dz'\int_0^\infty dk\, e^{-\beta k|z-z'|}\, I(kz)\,I(kz'),\ \text{где размерные переменные}$$

$$I(k,t)=\int_0^1 dr\, r\, J_0(kr)\frac{r^2}{(r^2+t^2)}\frac{1}{\left[\beta^2z^2+r^2\right]^{3/2}},\quad t=b/R.$$

$$W_m^{vol}(b)=2\pi\mu_s^2 V\beta t^4\int_0^1 dz\int_0^1 dz'\int_0^\infty dk\, e^{-\beta k|z-z'|}\, I(k,t)\,I(k,z')$$ Volume W_m of Theoville BP

Вид расчета W_m^{face} — см. 25/10 2020.

$$W_m^s=\frac{1}{2}\mu_s^2\int d_1S\int d_2S'\frac{(\vec m\cdot\vec n)(\vec m'\cdot\vec n')}{|\vec r-\vec r'|},\quad \vec m\cdot\vec n=\begin{cases}m_z, & z=L/2\\ -m_z, & z=-L/2\end{cases}$$

$$(\vec m\cdot\vec n)(\vec m'\cdot\vec n')=1\,m_z m_z' \underset{(\Delta z=0)}{} -1\,m_z m_z' \underset{(\Delta z=L)}{},\quad W_m^s=\frac{1}{2}\mu_s^2\int d^2\rho\int d^2\rho'\int dk\sum_\mu J_\mu J_\mu(\ldots)$$

$$\cdot e^{ik(z-z')}2\left[m_z m_z' \underset{\Delta z=0}{} - e^{-kL} m_z m_z' \underset{(\Delta z=L)}{}\right]=\left|m_z\neq f(\varphi)\right|=\frac{1}{2}\mu_s^2(2\pi)^2\int dk\int d\rho\,\rho\int d\rho'\,\rho'\times$$

$$\times J_0(k\rho)J_0(k\rho')2\left[m_z m_z' \underset{\Delta z=0}{} - m_z m_z' \underset{\Delta z=L}{} e^{-kL}\right],\quad 2[\ldots]=m_z\left(\tfrac{L}{2}\right)m_z'\left(+\tfrac{L}{2}\right)+$$

$$+m_z\left(-\tfrac{L}{2}\right)m_z'\left(-\tfrac{L}{2}\right)-m_z\left(\tfrac{L}{2}\right)m_z'\left(-\tfrac{L}{2}\right)-m_z\left(-\tfrac{L}{2}\right)m_z'\left(+\tfrac{L}{2}\right),\quad m_z(z)-\text{odd function of } z$$

$$W_m^s=4\pi^2\mu_s^2\int dk\int d\rho\,\rho\int d\rho'\,\rho' J_0(k\rho)J_0(k\rho')\left[m_z(\rho)\underset{(\Delta z=0)}{}m_z(\rho')-m_z(\rho)\underset{(\Delta z=L)}{}m_z(\rho)e^{-kL}\right]$$

$$K_1(k)=\int d\rho\,\rho\, J_0(k\rho)\, m_z\left(\rho, z=\tfrac{L}{2}\right),\quad K_\pm(k)=\int d\rho\,\rho\, m_z\left(\rho, z=\pm\tfrac{L}{2}\right)J_0(k\rho)$$

$$W_m^s=4\pi^2\mu_s^2\int dk\left[K_+^2(k)-e^{-kL}K_+(k)K_-(k)\right],\quad K_-(k)=\int d\rho\,\rho J_0\, m_z\left(\rho, z=-\tfrac{L}{2}\right)=$$

$$=-\int d\rho\,\rho J_0\, m_z\left(\rho, z=+\tfrac{L}{2}\right)\ \text{so odd } m_z(z).\quad K_-(k)=-K_+(k)$$

$$W_m^s=4\pi^2\mu_s^2\int dk\, K_+^2(k)\left(1+e^{-kL}\right)$$

$$K_+(k)=\int d\rho\,\rho\, J_0(k\rho)\, m_z\left(\rho, z=+\tfrac{L}{2}\right)=\int_0^R d\rho\,\rho\, J_0(k\rho)\frac{z}{\sqrt{z^2+\rho^2}}\frac{\beta^2}{\left(\beta^2+\rho^2\right)}\Bigg|_{z=+L/2}=\left|r=\rho/R\right|=$$

$$=R^2\int_0^1 dr\, r\, J_0(kRr)\frac{z/R}{\sqrt{z^2/R^2+r^2}}\frac{(b/R)^2}{\left[(b/R)^2+r^2\right]}=R^2\int dr\, r\, J_0(kRr)\frac{\beta z}{\sqrt{\beta^2z^2+r^2}}\frac{t^2}{\left[t^2+r^2\right]}\Bigg|_{z=t/2}$$

$$W_m^s(b)=4\pi^2\mu_s^2\frac{1}{R}\int dx\, R^4\left[\int_0^1 dr\, r\, J_0\left(\frac{x}{R}r\right)\frac{\beta z}{\sqrt{\beta^2z^2+r^2}}\frac{t^2}{(t^2+r^2)}\right]^2\left[1+e^{-x\beta}\right]$$

$\pi\frac{1}{R}R^4\beta^2=\pi R^3\frac{L}{R}\beta=V\beta$. $W_m^s(t)=4\pi\mu_s^2V\beta\int dk\left[1+e^{-\beta k}\right]\tilde K_+^2(k)$

$$\tilde K_+(k)=\int_0^1 dr\, r\, J_0(kr)\frac{z}{\sqrt{\beta^2z^2+r^2}}\frac{t^2}{(t^2+r^2)^2}.$$ Вычислить ещё t^2 из $\tilde K_+(k)$.

$$W_m^s(B)=4\pi\mu_s^2V\beta t^4\int_0^\infty\left(1+e^{-\beta k}\right)\tilde K_+^2(k)dk,\quad \tilde K_+(k)=\int_0^1 dr\, r\, J_0(kr)\frac{z}{\sqrt{\beta^2z^2+r^2}}\frac{1}{(t^2+r^2)^2}$$

$t=b/R$ $\qquad z=+\frac{1}{2}$.

$W_{ex}=A\int dV\left[(\nabla\Theta)^2+\sin^2\Theta(\nabla\Phi)^2\right]$, $\sin^2\Theta=1-\cos^2\Theta=1-m_z^2$, $\Phi=-\operatorname{arctg}\left(\frac{x}{y}\right)$ даст Вихрь ansatz

$$(\nabla\Phi)^2=\left(\frac{\partial\Phi}{\partial x}\right)^2+\left(\frac{\partial\Phi}{\partial y}\right)^2+\underbrace{\left(\frac{\partial\Phi}{\partial z}\right)^2}_{=0}=\frac{1}{\rho^2}\sin^2\varphi+\frac{1}{\rho^2}\cos^2\varphi=\frac{1}{\rho^2}.$$

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$m_z = \cos\theta$, $\theta(s,z) = \arccos m_z$, $\frac{\partial\theta}{\partial s} = -\frac{1}{\sqrt{1-m_z^2}}\frac{\partial m_z}{\partial s}$, $\frac{\partial\theta}{\partial z} = \frac{1}{\sqrt{1-m_z^2}}\frac{\partial m_z}{\partial z}$

$(\nabla\theta)^2 = \left(\frac{\partial\theta}{\partial s}\right)^2 + \left(\frac{\partial\theta}{\partial z}\right)^2 = \frac{1}{(1-m_z^2)}\left(\frac{\partial m_z}{\partial s}\right)^2 + \frac{1}{(1-m_z^2)}\left(\frac{\partial m_z}{\partial z}\right)^2 = \frac{1}{(1-m_z^2)}\left[\left(\frac{\partial m_z}{\partial s}\right)^2 + \left(\frac{\partial m_z}{\partial z}\right)^2\right]$

$W_{ex}(B) = A\int dV\left[\frac{1}{(1-m_z^2)}(\nabla m_z)^2 + (1-m_z^2)\frac{1}{s^2}\right]$, $\int dV = 2\pi\int dz\int ds\, s$.

$\frac{\partial m_z}{\partial z} = \frac{B^2 s^2}{(B^2+s^2)}\frac{1}{(z^2+s^2)^{3/2}}$, $\frac{\partial m_z}{\partial s} = \frac{2}{\partial s}\frac{1}{\sqrt{z^2+s^2}}\frac{B^2}{(B^2+s^2)} = 2\left(-\frac{1}{2}\right)(z^2+s^2)^{-\frac{3}{2}}\cdot 2s\frac{B^2}{(B^2+s^2)^2}$

$+\frac{zB^2}{\sqrt{z^2+s^2}(B^2+s^2)^2}(-1)2s = -2B^2 s\frac{1}{(z^2+s^2)^{3/2}(B^2+s^2)} - 2zB^2 s\frac{1}{(B^2+s^2)^2\sqrt{z^2+s^2}} =$

$= -\frac{2B^2 s}{(z^2+s^2)^{3/2}(B^2+s^2)^2}\left[B^2+s^2-2(z^2+s^2)\right] = -\frac{2B^2 s}{(z^2+s^2)^{3/2}(B^2+s^2)^2}\left[B^2-s^2-2z^2\right]$

$W_{ex}(B) = 2\pi A\int dz\int ds\, s[\ldots] = 2\pi AL\int_{-1/2}^{1/2} dz\int_0^{R/L} dv\, v\left[\frac{1}{(1-m_z^2)}R^2(\nabla m_z)^2 + \frac{1}{v^2}(1-m_z^2)\right]$

$R^2(\nabla m_z)^2 = R^2\left[\left(\frac{\partial m_z}{\partial s}\right)^2 + \left(\frac{\partial m_z}{\partial z}\right)^2\right] = \left(\frac{\partial m_z}{\partial v}\right)^2 + \frac{1}{\beta^2}\left(\frac{\partial m_z}{\partial z}\right)^2$. $\frac{2\pi AL}{4\pi M_s^2 V} = \frac{2\pi AL}{4\pi M_s^2\pi R^2 L} =$

$= \left(\frac{l_e}{R}\right)^2$, $l_e^2 = \frac{A}{2\pi M_s^2}$. $\frac{1}{4\pi M_s^2 V}W_{ex}(B) = \left(\frac{l_e}{R}\right)^2\int_{-1/2}^{1/2} dz\int_0^1 dv\, v\left[\frac{1}{(1-m_z^2)^2}\left\{\left(\frac{\partial m_z}{\partial v}\right)^2 + \frac{1}{\beta^2}\left(\frac{\partial m_z}{\partial z}\right)^2\right\} + \right.$

$\left. + \frac{1}{v^2}(1-m_z^2)\right]$. $m_z(s,z) \to m_z(v,z)$, $m_z = \frac{(z/L)L}{\sqrt{L^2\left(\frac{z}{L}\right)^2 + z^2R^2}}\frac{(B/R)^2}{\left[(B/R)^2+v^2\right]}$

$m_z = \beta\frac{z}{\sqrt{\beta^2 z^2 + v^2}}\frac{v^2}{(v^2+v^2)}$.

$\frac{1}{4\pi M_s^2 V}W_{ex}(B) = \left(\frac{l_e}{R}\right)^2\int_{-1/2}^{1/2} dz\int_0^1 dv\, v\left[\frac{1}{(1-m_z^2)}(\nabla m_z)^2 + \frac{1}{v^2}(1-m_z^2)\right]$ BP exchange

// Mathcad для $z=0$ $W_m^{vol}(B)$, $W_m^{face}(B)$ растут с $B\uparrow$, $W_{ex}(B)$ достигает насыщения с ростом B, поэтому $W(B) = W_{ex}(B) + W_m^{vol}(B) + W_m^{s}(B)$ имеет min. !!

Mathcad „Bloch_point_energy-wire.mcdx" построен, $W(B)$ имеет min при радиусах $B/R < 1$. $W(B)$ в Thiaville ansatz (BP).

(17-18/II) Расчет $W(B)$ Bloch point в cylinder. Переписка с Сергеем Маслем, BP Seng Koog (slides), Ruben. Анализ Refs по Bloch points. The maximum.

① R. Hertel, J. Kirschner. JMMM 278, L291-297 (2004). Magnetic droplets in soft magnetic cylinder. Moving BP in cylinder hysteresis loops, coexisting BP и anti-BP, которые быстро аннигилируют, а сами DW ускоряются

② R. Hertel. J. Phys. Cond. Mat. 28 (2016) 483002. Ultrafast DW dynamics in magnetic nanotubes and nanowires. Moving BP exceeds or moving DW в зависимости на „spiral" orbit $\in \omega_+, \omega_-$. ω_+ - CCW (high amplitude), ω_- (low amplitude motion) - CW. $\omega_1 \neq \omega_{1,c}$ (SW) in front of and behind moving DW.

(19/II) $\subset$ A. B. „Carrefour" Occasion и „hide" Hermann.

(20/II) Бонус. Изменен Refs на A. и мой g. doing. (подать до 50+50€)

(21/II) Meeting с Julian и (Ricardo) Ruben. Ruben показал MS draft (Co-авторы Оксана, Felipe, Ruben) по BP-DW dynamics, 6 pages, где он использовал мою concept of „gyrofield". Ruben сказал, что MS file мне пришлет не позже, т.к. Оксана is away.

S.K. Kim прислал slides по DW nanostripes и „VC reversal" Оксана отправила прислал „Overleaf" link to MS. Но не прислал, сославшись на Felipe, что он не готов. Sent e-mail to wait

(22/II) Brownian proofs MS-Bergana и ждем-с.

(23/II) E-mail to A. Fert о B-sphere. Gyromodes. Thick dots - Simulat. By Arsen $W_n(L)$ для $R = 75, 150, 250$ nm. Неудачно. comments Occ. - BP design.

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(24/II) Нужен начал не управ. Новосел. тел. с Фархад о Vortex coring dyn. и BP dynamics in cylindrical nanowires. Запросил Фархада stiffness χ_z no BP-DW displacements. Фархад прислал slides по single E(single vortex) с E(BP-DW) E(SV) < E(BP-DW). Labor reform: Ricardo сообщ. что его контракт на 1Я года разорвался.

(25/II) Мой g-pomp., дома. С Л. в "Hide" обменялись.

(26/II) С Л. и Глеб в Donostia, Igaldo Cafe. Slides DW nanostripes.

(27/II) Slides DW dyn. in nanostripes" Уехал Глеб.

(28/II) • Lecture #5 (Poznan): "DW dynamics in magnetic nanostripes".

Сроочно: 1) Contract Ricardo; ~~2) meeting с A. Fert~~; 3) abstracts SolStyMag'22 4) Refs. по BP и подготовка к meeting online; 5) Информ. Ромен-ев 6) активировать „Caixa" Bank-mobile; 7) Travel report - Caleb

(1/III) Meeting with Albert Fert. Его plenary talk (40+5') at June 27, 28 or 29 at ~10 AM. Title/Abstract нужны. Book hotel for Albert for 3 nights June 26-29 (2 persons) „Londres". Обсужд. hopfions. Он в skyrmion papers учит. $q_z = \frac{1}{4\pi}\int dxdy\, m\left[\partial_x \vec{m} \times \partial_y \vec{m}\right]$, q_x and q_y - 2D обобщение of skyrmion number, 2D topol. charge.

Запросил у Jon, CAM аккаунты 2 „Dell" laptops. Jon сделал мне connection to new printer.

Booked „Londres" Hotel June 26-29, 2022, for Albert Fert for my name ~~my~~ Credit Card. Free Cancelation до 23:59, June 23, 2022.

(2/III) Jon/CAM активировал new „Dell" laptops и установил VPN, MS office etc. Secret words Brandon/Kiev/Kiev, Passw. „kostya25". On-line Zoom meeting с Оксана, Ruben, Felipe, Jose - BP-DW dynamics in nanowires. Мои comments о stiffness, SW, gyrofield, обсужд. papers Fischer APL'2020, Nat. Com. 2019, $v_{DW}(H)$.

(3/III) Gyrofield (kinetic field), Gus. PRL'2008.

$$\mathcal{J}_{kin} = \frac{M_s}{\gamma}\frac{\vec{n}\cdot(\vec{m}\times\dot{\vec{m}})}{(1+\vec{m}\cdot\vec{n})},\quad \vec{n}=\hat{z}\ (\text{Dirac string}) \to \mathcal{J}_{kin} = J\frac{(\vec{m}\times\dot{\vec{m}})\cdot\hat{z}}{p+m_z}$$

$$\vec{H}_g = \frac{1}{M_s}\frac{\partial \mathcal{J}_{kin}}{\partial \vec{m}},\quad \frac{\partial}{\partial \vec{m}}\mathcal{J}_{kin} = \frac{\partial}{\partial \vec{m}} J\frac{(\vec{m}\times\dot{\vec{m}})\cdot\hat{z}}{(p+\vec{m}\cdot\hat{z})} = J\frac{\partial}{\partial \vec{m}}\frac{\vec{m}\cdot(\dot{\vec{m}}\times\hat{z})}{(1+\vec{m}\cdot\hat{z})} =$$

$$= J\frac{(\dot{\vec{m}}\times\hat{z})}{(p+\vec{m}\cdot\hat{z})} + J\cdot\frac{\vec{m}\cdot(\dot{\vec{m}}\times\hat{z})}{(p+\vec{m}\cdot\hat{z})^2}(-1)\hat{z} = J\frac{(\dot{\vec{m}}\times\hat{z})}{(p+m_z)} - \hat{z}J\frac{\vec{m}\cdot(\dot{\vec{m}}\times\hat{z})}{(p+m_z)^2}$$

$\vec{H}_g$ имеет 2 компонента $\perp\hat{z}$ и компонент $\parallel\hat{z}$. $J = M_s/\gamma$.

$$H_g^z = h_z = \frac{1}{M_s}\left[\frac{\partial}{\partial \vec{m}}\mathcal{J}_{kin}\right]\cdot\hat{z} = -\frac{1}{\gamma}\frac{\vec{m}\cdot(\dot{\vec{m}}\times\hat{z})}{(p+m_z)^2} = -\frac{1}{\gamma}\frac{\hat{z}\cdot(\vec{m}\times\dot{\vec{m}})}{(p+m_z)^2}$$

Затем $W_z = -M_s h_z m_z$ - Zeeman energy as driving force of VC reversal

Но $\mathcal{J}_{kin} = \frac{M_s}{\gamma}\frac{\vec{m}\cdot(\dot{\vec{m}}\times\vec{n})}{(1+\vec{m}\cdot\vec{n})} = M_s\vec{m}\cdot\vec{h}_g'$, где $\vec{h}_g' = \frac{1}{\gamma}\frac{(\vec{m}\times\dot{\vec{m}})}{(1+\vec{m}\cdot\vec{n})}$ связано с $\mathcal{J}_{kin}$ (- W_z). Тогда $\vec{h}_g^{\perp} = \frac{1}{\gamma}\frac{(\dot{\vec{m}}\times\hat{z})}{(1+m_z)}$ не имеет z-компонент. $h_g^z = 0$, $\vec{h}_g \perp \hat{z}$.

Осталось $h_z = -\frac{1}{\gamma}\frac{(\vec{m}\times\dot{\vec{m}})_z}{(p+m_z)^2}$ as in Gus. APL'2008. $h_z = -\left[\frac{\partial}{\partial}\frac{\Theta}{2}\right]^{2p}\frac{1}{\gamma}\dot{\Phi}$

Давай definition $\vec{h}_g = M_s^{-1}\frac{\partial \mathcal{J}_{kin}}{\partial \vec{m}}$, $W_z = -M_s h_z m_z$ Запишем через q.

Real $W_z = -\mathcal{J}_{kin} = -M_s\vec{m}\cdot\vec{h}_g'$, $\vec{h}_g' = \frac{1}{\gamma}\frac{(\vec{m}\times\hat{z})}{(1+m_z)}$ не имеет z-компоненты и не имеет вида $W_z = -M_s h_z m_z$.

Оценка h_z для vortex с $\dot{\vec{x}} = \vec{v} = \vec{\omega}\times\vec{x}$ есть $h_z \approx \frac{\omega}{\gamma}\frac{|x|}{R_a} \sim \frac{\omega}{\gamma} \sim 1$ kOe для $\omega \sim 36$ GHz

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Gus'EPL вводит наведенное gyrofield $\vec{H}_g = -\frac{c}{\gamma}\vec{A}_0$, если
Замена $\mathcal{L}_{int}(\text{vortex-SW}) = M_s \vec{m}_s \cdot \vec{H}_g$. $\vec{H}_g$ это поле от soliton (vortex) сконцентрированное на SW. Bilinear soliton – SW interaction term

$$\mathcal{L}_{int} = -\frac{M_s}{\gamma} c\vec{m}_s \cdot \vec{A}_0 = \frac{M_s}{\gamma}\sin\theta(\dot{\varphi}e\ell - \psi\dot{\theta}) \cdot \vec{m}(\theta,\varphi)$$ soliton

$\vec{m}_s(\varphi,\psi)$ SW magn.

In moving frame $(Oz' \parallel \vec{m})$ $\vec{m}_s' = (m_{sx}, m_{sy}, m_{zs}) = (e\ell, \sin\theta\psi, 0)$

$\vec{A}_0 = \frac{1}{c}\frac{\partial \vec{m}}{\partial t} \times \vec{m} + (\vec{A}_0 \cdot \vec{m})\vec{m}$, $\vec{m} \perp \vec{m}_s$ и только $\frac{1}{c}\dot{\vec{m}} \times \vec{m}$ дает вклад

в $\mathcal{L}_{int} = -\frac{M_s}{\gamma} c\vec{m}_s \cdot \vec{A}_0$, $\vec{A}_0 = \frac{1}{c}\dot{\vec{m}} \times \vec{m}$, $\vec{H}_g = -\frac{c}{\gamma}\vec{A}_0 = -\frac{1}{\gamma}\dot{\vec{m}} \times \vec{m}$

Gyrofield from soliton–SW interaction ($\vec{H}_g$ acts on SW) is (Gus'EPL'16)

$$\vec{H}_g = -\frac{1}{\gamma}\dot{\vec{m}} \times \vec{m}, \quad H_g^z = -\frac{1}{\gamma}(\dot{\vec{m}} \times \vec{m})_z$$

Тогда как gyrofield из $\mathcal{L}_{kin}$ обусловлено $\mathcal{L}_{kin} = M_s \vec{m} \vec{h}_g'$,

$\vec{h}_g' = \frac{1}{\gamma}\frac{(\dot{\vec{m}} \times \hat{z})}{(1+m_z)}$. Gyrofield $\vec{h}_g = \frac{1}{M_s}\frac{\partial \mathcal{L}_{kin}}{\partial \vec{m}} = J\frac{(\dot{\vec{m}} \times \hat{z})}{(p+m_z)} - \hat{z}J\frac{(\dot{\vec{m}} \times \hat{z})\cdot\vec{m}}{(p+m_z)^2}$.

$$h_g^z = -\frac{1}{\gamma}\frac{(\vec{m} \times \dot{\vec{m}})_z}{(p+m_z)^2}$$ (Gus'PRL'2008).

Видно что interaction gyrofield $\vec{H}_g$ (Gus'EPL'16) и artificial gyrofield $\vec{h}_g$ и $\mathcal{L}_{kin}$ (Gus'PRL'08) отличаются множителем $-\frac{1}{\gamma}\frac{1}{(p+m_z)^2}$. (!)

т.к. $m_z = \pm p$ соответ. VC reversal $\frac{1}{(p+m_z)^2} \gg 1$, $h_g^z \gg H_g^z$ near VC reversal take moments.

Emergent mag. field $\vec{B} = \sin\theta\nabla\theta \times \nabla\Phi$, $\vec{B} = \text{rot}\vec{A}_e$, $\vec{A}_e = (1-\cos\theta)\nabla\Phi$ не содержит $d\vec{m}/dt$ производных. $\frac{d\vec{m}}{dt}$ входит в emergent-el. field $\vec{E}_e$.

Skype meeting с Оксаной и Ruben.

Оксана представила presentation Felipe и обсудили $v(H)$ для BP-SW. При $H > H_1$ для некоторых R. BP не движется BP-aBP pairs, а BP убегает из наночипа и возбуждается back, что снижает v_{SW}. При $H > H_2$ generation BP-pairs, что резко увеличивает v_{SW}.

H_g по мысли Gus'PRL'2008 is giant ~ 20 T. А физически какой gyrofield? Обратим что все ед. для $h_g(\vec{r},t)$ PRL'2008 не соответствуют to real field, это какие-то фиктивные. Введенная чтоб обеспечить VC-reversal. Напомним $\vec{H}_g = \frac{1}{\gamma}\dot{\vec{m}} \times \vec{m}$ gyrofield из GusEPL'16 только что это real mag. field of soliton–SW interaction.

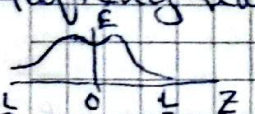
Новое definition of emergent el. and mag. fields [Nagaosa'2013]

$v_m \approx 1.8$ km/s. Видно это max v_{ph} of SW в наночипе. Обсудили какой имеет расчет $\omega(k)$-SW и найти $v_{ph} = \omega/k$ и max v_{ph}.

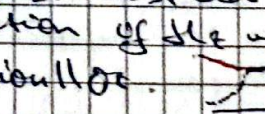
Запросил chess.com насчет 200$ моего приза to Ukrainian army.

4/III Solsky mag парочка и Abstracts. Встреча в Днепр с нтр и Ukrainian families (6 семей). Послал BP+abstract to JEMS'22.

Разбираю новый change E_{BP} as function of H_z и H_x small < 10Oe



$E(z)$ · BP energy as func. BP position $\frac{1}{2}$ or $-\frac{1}{2}$.



т.е. local minimum at $\vec{X}=0$ в наночипе, global max $E(z)$ и global min $E(X_0)$. $E(X_0) = \frac{1}{2}\alpha_0 X_0^2$, с $\alpha_0 > 0$ где d_0 для BP в наночипе.

Запросил нормальный $Z(H_z)$ и $X_0(H_x)$ для нахождения α_0 и α_z.

Escaneado con CamScanner

5/III $\vec{G}$ - gyrovector gyro BP - Thiaville ansatz cylindrical nanowire.

Conv. 15/II $\vec{G}$ (global) $= (G_s, 0, 0)$ uniform towards $G_s \neq 0$, $G_s = 4\pi R\left(\frac{M_s}{\gamma}\right)$ tanθ_{BP} $\left(\frac{1}{t}\right)$ $t = b/R$.

BP - ω_G BP position $\vec{X} = (X, Y, Z)$ or (ρ, φ, z) in cyl. coordinates. $\vec{G} \times \dot{\vec{X}} = \begin{vmatrix} \hat{\rho} & \hat{\varphi} & \hat{z} \\ G_s & 0 & 0 \\ \dot{X}_\rho & \dot{X}_\varphi & \dot{z} \end{vmatrix} = G_s \dot{X}_\varphi \hat{z} - G_s \dot{z} \hat{\varphi}$

Thiele eq. $\vec{G} \times \dot{\vec{X}} = \frac{\partial W}{\partial \vec{X}}$, $G_s \dot{X}_\varphi \hat{z} - G_s \dot{z} \hat{\varphi} = \frac{\partial W}{\partial \vec{X}}$. $W(\vec{X}) = \frac{1}{2}\varkappa(X^2 + Y^2) + \frac{1}{2}\varkappa_z Z^2$.

$\vec{X} = X\hat{x} + Y\hat{y} + Z\hat{z} = X(\hat{\rho}\cos\varphi - \hat{\varphi}\sin\varphi) + Y(\hat{\rho}\sin\varphi + \hat{\varphi}\cos\varphi) + Z\hat{z}$

$\vec{r} = (\rho, \varphi, z)$. $\vec{X} = X_\rho \hat{\rho} + X_\varphi \hat{\varphi} + Z\hat{z}$, $X_\rho = X\cos\varphi + Y\sin\varphi$, $X_\varphi = -X\sin\varphi + Y\cos\varphi$

$\vec{X} = X_\rho \hat{\rho} + X_\varphi \hat{\varphi} + \varkappa_z \hat{z} = X_\rho(\hat{x}\cos\varphi + \hat{y}\sin\varphi) + X_\varphi(-\sin\varphi\hat{x} + \cos\varphi\hat{y}) + \hat{z}z$

$X = X_\rho\cos\varphi - X_\varphi\sin\varphi$, $Y = X_\rho\sin\varphi + X_\varphi\cos\varphi$.

$X^2 + Y^2 = (X_\rho\cos\varphi - X_\varphi\sin\varphi)^2 + (X_\rho\sin\varphi + X_\varphi\cos\varphi)^2 = X_\rho^2 + X_\varphi^2$

$W(\vec{X}) = \frac{\varkappa}{2}(X_\rho^2 + X_\varphi^2) + \frac{1}{2}\varkappa_z Z^2$. $\frac{\partial W}{\partial \vec{X}} = \frac{\partial W}{\partial X_\rho}\hat{\rho} + \frac{\partial W}{\partial X_\varphi}\hat{\varphi} + \frac{\partial W}{\partial Z}\hat{z}$ Da. 6/VIII (так ли?)

$\hat{\rho}$: $\frac{\partial W}{\partial X_\rho} = 0$ $\hat{\varphi}$: $-G_s\dot{z} = \frac{\partial W}{\partial X_\varphi}$, $\hat{z}$: $G_s\dot{X}_\varphi = \frac{\partial W}{\partial Z}$.

$\frac{\partial W}{\partial X_\rho} = \frac{1}{2}\varkappa X_\rho \cdot 2 = 0 \Rightarrow X_\rho = 0$ (?), $-G_s\dot{z} = \varkappa X_\varphi$, $G_s\dot{X}_\varphi = \varkappa_z Z$. $\ddot{X}_\varphi = \frac{\varkappa_z}{G_s}\dot{z} = -\omega_G^2 X_\varphi$

$\dot{X}_\varphi = \frac{\varkappa_z}{G_s}Z$, $-G_s\ddot{z} = \varkappa\dot{X}_\varphi = \varkappa\frac{\varkappa_z}{G_s}Z$, $\ddot{z} = -\frac{\varkappa\varkappa_z}{G_s^2}Z = -\omega_G^2 z$.

Gyrofrequency $\omega_G = \frac{\sqrt{\varkappa\varkappa_z}}{G_s}$.

Проверка $\vec{G} \times \dot{\vec{X}} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ G_x & G_y & 0 \\ \dot{X} & \dot{Y} & \dot{Z} \end{vmatrix} = \hat{x}G_y\dot{Z} + G_x\dot{Y}\hat{z} - \dot{X}G_y\hat{z} - G_x\dot{Z}\hat{y}$

$\vec{G} \times \dot{\vec{X}} = \hat{x}G_y\dot{z} - G_x\dot{z}\hat{y} + G_x\dot{y}\hat{z} + \dot{x}G_y\hat{z} = \frac{\partial W}{\partial X}\hat{x} + \frac{\partial W}{\partial Y}\hat{y} + \frac{\partial W}{\partial Z}\hat{z}$

$\hat{x}$: $G_y\dot{z} = \frac{\partial W}{\partial X}$, $\hat{y}$: $-G_x\dot{z} = \frac{\partial W}{\partial Y}$; $\hat{z}$: $G_x\dot{y} - G_y\dot{x} = \frac{\partial W}{\partial Z}$

$G_x = G_s\cos\varphi$, $Y \to G_y = G_s\sin\varphi$, $G_x\dot{y} - G_y\dot{x} = G_s\cos\varphi\dot{y} - G_s\sin\varphi\dot{x} =$

$= G_s(\dot{y}\cos\varphi - \dot{x}\sin\varphi) = \dot{X}_\varphi G_s = \frac{\partial W}{\partial Z}$, $G_s\dot{z} = \frac{1}{\sin\varphi}\frac{\partial W}{\partial X} = -\frac{1}{\cos\varphi}\frac{\partial W}{\partial Y}$.

$X_\varphi = -X\sin\varphi + Y\cos\varphi$. $\frac{\partial W}{\partial X} = \frac{\partial W}{\partial X_\varphi}\frac{\partial X_\varphi}{\partial X} = \frac{\partial W}{\partial X_\varphi}(-\sin\varphi)$, $\frac{\partial W}{\partial Y} = \frac{\partial W}{\partial X_\varphi}\cos\varphi$

$G_s\dot{z} = -\frac{\partial W}{\partial X_\varphi}$ - верно.

Вывод, если только $G_s \neq 0$, $W(\vec{X}) = \frac{1}{2}\varkappa(X^2 + Y^2) + \frac{1}{2}\varkappa_z Z^2$, то $\omega_G = \frac{\sqrt{\varkappa\varkappa_z}}{G_s}$.

Оба stiffness coefficients $\varkappa$ и $\varkappa_z$ должны быть чтобы BP в нанопроволоке.

$\vec{X}(t) = X_\varphi(t)\hat{\varphi} + Z(t)\hat{z}$. пусть $X_\varphi(t) = X_0\cos\omega_G t$, $Z(t) = X_0\sin\omega_G t$

$(-G_s X_0\sin\omega t =) -G_s\dot{z} = -G_s\omega_G X_0\cos\omega_G t = \varkappa X_0\cos\omega_G t$, $-G_s\omega_G = \varkappa$ - много.

Надо $X_\varphi(t) = X_\varphi^0\sin\omega_G t$, $Z(t) = Z_0\cos\omega_G t$

$-G_s\dot{z} = -G_s Z_0(-\omega_G)\sin\omega_G t = \varkappa X_\varphi^0\sin\omega_G t$, $G_s Z_0\omega_G = \varkappa X_\varphi^0$

$\sqrt{\varkappa\varkappa_z}Z_0 = \varkappa X_\varphi^0$, $X_\varphi^0 = \sqrt{\frac{\varkappa_z}{\varkappa}}Z_0$ - elliptic motion in $\hat{\varphi}$-$\hat{z}$ plane.

$G_s\dot{X}_\varphi = G_s X_\varphi^0\omega_G\cos\omega_G t = \varkappa_z Z_0\cos\omega_G t$, $G_s X_\varphi^0\omega_G = \varkappa_z Z_0$

$\sqrt{\varkappa\varkappa_z}X_\varphi^0 = \varkappa_z Z_0$, $\sqrt{\varkappa\varkappa_z}\sqrt{\frac{\varkappa_z}{\varkappa}}Z_0 = \varkappa_z Z_0$ - тождество.

Вернемся $\vec{X} = (X, Y, Z)$ неподвижная система где BP-dynamics in BP DW nego $Z(t) \to Z_0 - vt$ или $\to Z(t) - vt$ для rigid DW motion $\parallel Oz$.

Или перейти в moving coordinate frame.

Escaneado con CamScanner

(6/III) SW in cylindrical nanowires. Min $v_{ph}(k_z)$ для BP-DW

① R. Arias, D.L. Mills. Phys. Rev. B 63, 134439 (2001). $\frac{\partial m}{\partial \rho}=0$

Метод magnetostatic potential. Расчет $\omega_{nm}(k_z)$ для разных $k_z \cdot R$. n – номер корня сек. ур., m, $e^{im\varphi}$ – azimuthal index SW

Численно $\omega_{mn} = f(kR)$ для $m = 1, 2, 3$. $\omega(k_z)$ похожа на BVMSW.

Мин $\omega(k_z)$ при $\log(k_z R) \to \min v_{ph}$, использован для оценки, пересчитав ω_M Ni (480 G) to Fe. $kR \approx 10 \to k \approx 2\cdot 10^8 \frac{1}{m}$, $\lambda = \frac{2\pi}{k} \approx 30$ nm.

Min $v_{ph} \approx \frac{0.4 \cdot \gamma 4\pi M_s}{2\cdot 10^8 \frac{1}{m}} = 0.2 \frac{60\ \text{GHz}}{10^8 \frac{1}{m}} = 120 \frac{m}{s}$ (Fe), min $\frac{\omega(k)}{\omega_M} \approx 0.4$

Min $\omega(k) \approx 0.4 \cdot 60$ GHz ≈ 24 GHz.

② J. Rychly et al. J. Phys. D: Appl. Phys. (2018).

Повторение Arias-Mills, но есть явные аналитические ур-я для Bound conditions $\frac{\partial m}{\partial \rho}=0$ и секул. уравнение. SW mode profiles.

Оценка v_{ph} – наш диапазон (по их plots) $v_{ph} \approx 40$-$50 \frac{m}{s}$ для Ni.

$M_s(Fe) \approx 3.5 M_s(Ni)$, $v_{ph}(Fe) \approx 140$-180 m/s. Но это средн всех SW modes с разными "m". Для azim. symmetric waves $<m=0$ $\omega_{0n}(k_z)$ – выше и $v_{ph} = \omega_{0n}(k_z)/k_z$ больше. SW

Оксана подтверждает generation of SW $<m=0$, но ни ω_c, ни k_z в окрестности $\min v_{ph}(k_z)$ не знает. (BP-DW motion).

Послал запрос на SW-nanowires to B. Ivanenko и M. Krawczyk.

(7/III) Вера не может восстановить Sim. Code in Mapple (Mathematica). Maciej сказал, что они могут сделать SW Simulations in COMSOL. Послал ему Fe параметры, $R = 30, 40, 50$ nm и $H_z = 10$-100 mT.

(7-8/III) переписка о моих оценках $\omega(k_z)$, $\min v_{ph}$ in Fe nanowires с Оксаной. Боже, Лига чемпионов. Chess.com не может поспеть в Украину 200 $ prize. Нашумел prize на чай сам и поспеть самому.

(9/III) Maciej подтвердил, что SW Simulations Fe nanowires проведены – полный Summary. Registration для FCT-proposal Гриба as consultant.

Гур.-Мелков. Magnetostatic SW in long cylinder с металлическими стенками ($\tilde{B}_n|_S = 0$) из magn. potential $\psi = J_m(x_\rho) e^{i(m\varphi - kz)}$

$k = k_z$ wave vector ∥ оси wire z, x – wave number in xy-plane

$k^2 = -\mu_{xx} x^2$, $X \frac{J_m'(X)}{J_m(X)} = m \frac{\mu_a}{\mu}$, μ, μ_a components $\hat{\mu}(\omega)$, $\hat{\chi}(\omega)$.

$X = xR$.

$\omega^2 = \omega_H \left[\omega_H + \frac{\omega_M}{1 + k^2R^2/X_{mn}^2(\omega)} \right]$ Eq. (6.70).

n/m $m=0$, нет зависимости $\vec{m}(\rho, \varphi, z)$ от φ, $X_{0n} = k_n R$, из $\frac{\partial J_0(X)}{\partial X} = 0$ или $J_1(X) = 0$, $X = $ n корни $J_1(X) = 0$. Это соотв. $\frac{\partial \vec{m}}{\partial \rho}\Big|_R = 0$ б.с.

$\omega_{0n}^2 = \omega_H \left[\omega_H + \frac{\omega_M}{1 + \frac{k^2}{x_n^2}} \right] = \omega_H \left[\omega_H + \omega_M \frac{x_n^2}{(x_n^2 + k^2)} \right]$, $x_n = \frac{d_n}{R}$.

Учет exchange. $\vec{h}_{ex} = q \nabla^2 \vec{m}$, $\gamma = \gamma M_s q$, $\omega_H \to \omega_H + \gamma k^2$.

Это дает eq. Herring-Kittel $\omega_{\vec{k}}^2 = [\omega_H + \gamma k^2][\omega_H + \gamma k^2 + \omega_M \sin^2 \theta_{\vec{k}}]$

$\vec{k} = (k_x, k_y, k_z)$. $\sin^2\theta_{\vec{k}} = \frac{k_x^2 + k_y^2}{k^2}$.

Для cylindrical wire с $m=0$, $k_x^2 + k_y^2 = k_\rho^2 = x^2$, $x \to x_n$ из-за б.с. $\rho = R$

$K^2 = k_z^2 + x^2 = k^2 + \alpha^2$. $x_n = d_n/R$, $J_1(d_n) = 0$

Escaneado con CamScanner

m=0 SW $\omega_n^2(k) = \left[\omega_H + \gamma k_n^2\right]\left[\omega_H + \gamma k_n^2 + \omega_M \frac{\alpha_n^2}{k^2 + \alpha_n^2}\right]$, $k = k_e$.

$k_n^2 = k^2 + \alpha_n^2$. $\omega_H = \gamma k^2 - \gamma k_e^2 + \omega_H l_e^2 k^2 = \omega_M \frac{A}{2\pi M_s^2} k^2$ =

$= \gamma \cdot 4\pi M_s \frac{A}{2\pi M_s^2} k^2 = \gamma \frac{2A}{M_s}$; $\gamma = \gamma \frac{\mu_0}{M_s}$. $\omega_H = \gamma H_0$. $l_e^2 = \frac{2A}{M_s^2}$

$\frac{1}{\gamma^2}\omega_n^2(k) = \left[H + l_e^2(k^2 + \alpha_n^2)\right]\left[H + l_e^2(k^2 + \alpha_n^2) + 4\pi M_s \frac{k^2}{(k^2 + \alpha_n^2)}\right]$

Удобно $h = H_0/(4\pi M_s)$, $l_e^2 \frac{1}{4\pi M_s}$ — безразм. длина корреляционная

при k^2 на l_e^2, а $\gamma \frac{2A}{M_s} = \omega_M l_e^2 k^2$, где $l_e^2 = 1/(4\pi M_s)^2 \cdot \frac{A}{2\pi M_s^2}$.

$\frac{\omega_n^2(k)}{\omega_M^2} = \left[h + l_e^2 k_n^2\right]\left[h + l_e^2 k_n^2 + \frac{\alpha_n^2}{k_n^2}\right]$, $l_e^2 = H/(4\pi M_s^2)$, $\alpha_n = d_n$

$k \to kR$, $\alpha_n \to \alpha_n R = d_n$, $l_e^2 k_n^2 = l_e^2\left[k^2 + \alpha_n^2\right] = \left(\frac{l_e}{R}\right)^2\left[(kR)^2 + d_n^2\right]$

$\frac{\alpha_n^2}{k_n^2} = \frac{\alpha_n^2}{k^2 + \alpha_n^2} + \frac{d_n^2}{(kR)^2 + d_n^2}$; $kR \to k$.

$\left(\frac{\omega_n^2(k)}{\omega_M^2} = \left[h + \frac{l_e^2}{R^2}(k^2 + d_n^2)\right]\left[h + \frac{l_e^2}{R^2}(k^2 + d_n^2) + \frac{d_n^2}{k^2 + d_n^2}\right]\right)$ SW nanowires

Выражаем через $h = H_0/(4\pi M_s)$, $(l_e/R)^2$ и d_n — dimensionless parameters

Для Fe. $A = 2.1 \cdot 10^{-6} \frac{erg}{cm}$, $M_s = 1.700$ G, Fe

$l_e = \sqrt{\frac{2.1 \cdot 10^{-6} erg/cm}{2\pi (1.7)^2 10^6 erg^2/cm^6}} = \sqrt{\frac{2.1}{2\pi (1.7)^2}} \cdot 10^{-6} cm = 0.340 \cdot 10^{-6} cm = 34$ nm

$4\pi M_s = 4\pi \cdot 1700$ G $= 21.36 \cdot 10^3$ G. $\gamma(Fe)$? $\gamma = 2.9 \frac{GHz}{kOe}$

$\frac{\omega_M(Fe)}{2\pi} = \left(\frac{\gamma}{2\pi}\right) \cdot 4\pi M_s = 2.9 \frac{GHz}{kOe} \cdot 21.36$ kOe $= 61.94$ GHz.

Roots of $J_1(x) = 0$ $\alpha_1 = 3.832$, $\alpha_2 = 7.016$, $\alpha_3 = 10.174$, $\alpha_4 = 13.324$, $\alpha_5 = 16.471$

$4\pi M_s = 21.36$ kOe, $h = 0.01$ соответ. $H_0 = 0.214$ kOe $= 214$ Oe $= 21.4$ mT

при $R = 50$ nm kR соответ. $k = \frac{1}{50\,nm} = 0.02 \frac{1}{nm} = 2 \cdot 10^7 \frac{1}{m} = 2 \cdot 10^5 \frac{1}{cm}$

$v_{ph} = \frac{\omega(k)}{k} \Rightarrow \omega_M \frac{\omega(k) R}{(kR)} = \omega_M R \frac{\omega(k)}{k}$ in absolute units (1/s),

где $\omega(k)$ и k in dimensionless units.

$\omega_M R = \frac{\omega_M}{2\pi} 2\pi R$, $\omega_M R = 61.94$ GHz $\cdot 50$ nm $\cdot 2\pi = 2\pi \cdot 61.94 \cdot 50$.

$10^9 \frac{1}{s}\, 10^{-9} m$ $\left(\omega_M R = 50\,nm\right)$ $= 2\pi \cdot 50 \cdot 61.94 \frac{m}{s} = 19.46 \cdot 10^3 \frac{m}{s}$

Math Cad и SW_cylindr_nanowires.mcd" даст $\min v_{ph} \approx 2 \cdot 10^{-3}$ при $kR = 28$

для $R = 50$ nm, $h = 0.01$. In abs. units $\min v_{ph} = 2 \cdot 10^{-3} \cdot 19.5 \cdot 10^3 \frac{m}{s} \approx 40 \frac{m}{s}$ (!)

$\min v_{ph}(k)$ достигается при $k \approx \frac{28}{50\,nm} = 0.56 \frac{1}{nm} = 0.56 \cdot 10^9 \frac{1}{m} = 56 \cdot 10^6 \frac{1}{cm}$

У Акиз-Миллс $\min \omega(k)$ был при $\log(kR) \approx 1$, $kR \approx 10$, что $v_{ph} = \frac{\omega(k)}{k}$

и $\min v_{ph}(k)$ сдвигается к большим k.

$\omega_{0n}(k)$ ($m = 0$) растут с ростом d_n. Оставим только d_1 для $\min v_{ph}(k)$, $n = 1$.

С ростом H_0 ω_{01} растет, и v_{ph} растет. Зависимость $v_{ph}(k)$ от R нужна.

// Отнесем на время отпуска поиски Украины & Книга 10 Одним с подписями, одеждой, костюмом, национальной одеждой, чтобы определить личность.

→ Вывод: для Fe-nanowires min SW phase velocity $v_{ph}(h, R)$ изменяется в пределах 40–70 m/s для $H_e = 20$–100 mT и $R = 20$–60 nm. Нужен e-mail о phase velocity min v_{ph} Океана. (Это контакт-хер SW $\omega(k)$)

Escaneado con CamScanner

(10/III) Reminders some SolSkyMag invited speakers to send titles/abs.

Arias-Mills '2001 спред $\omega(k)$ нанонити на конечна дебелина маг. material (Fe or Ni). $m=1$, $h=0,19$ (4 kOe grnd Fe), $p=0,05; 0,2; 2$

$p=\frac{k_e}{R}=\frac{3.4\,nm}{R}$ - $p=2$ ($R=1.7$ nm) - перпендикулярно. grnd Fe.

$p=0.2$ $R=17$ nm - силно ферромагнитно. $p=0,05$, $R=\frac{3.4nm}{0.05}=68$ nm is OK (Fe).

$h=0,19$, $H_z=4$ kOe $=400$ mT силно високо. У окол. $H_z\le 100$ mT.

Сравнен Fig.2 Arias-Mills ($m=1$) и Fig.4B ($m=3$), нов

$p=0.05$, $R=68$ nm, $h=0.19$ с наш $\omega(k)$ - Herring-Kittel с $x_n=dn/R$.

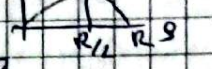
У Arias-Mills $\min\omega(k)/\omega_M\approx 0.33$, $\log(kR)\approx 0.2$. У наш $\min\omega(k)/\omega_M(k_z)$ при $\log(kR)\approx 1.35$ и $\omega(k_m)/\omega_M\approx 0.23$. Т.е. за Arias-Mills $\omega(k)$ приблизително в 1.5 пъти и завишено k_m приблизително в $10^{1.35}/10^{0.2}=4.5$ пъти

Ако $\min v_{ph}(k)$ оценяваме по $\frac{\omega(k_m)}{k_m}$, то за Arias-Mills v_{ph} приблизително в $1.5\div 4.5\approx 6$ пъти, в нашето най-typical 50 m/s далеко биха 300 m/s.

Но у Arias-Mills $m=1$, у нас $m=0$. $\sin^2\theta_k=\frac{k_0^2+k_e^2}{k^2}$, при $m=0$ $\sin^2\theta_k=\underset{(m=0)}{k_0^2/k^2}=\frac{x_n^2}{x_n^2+k^2}$ нещо по-малко от $\sin^2\theta_k(m=1)$, където $k_{ep}\neq 0$.

Мед $\omega_{m=0,n}(k)$ с ъгъли $k_e\neq 0$, $m=1$ увеличаване в област (k<kR) $\omega_{m=1,n}(k)$ Arias-Mills. $k_e\approx m/R$. Дори $m=1$ $\sin^2\theta_k=\frac{k_0^2+k_e^2}{k^2}=$

$=\frac{x_n^2+1/R^2}{x_n^2+\frac{1}{R^2}+k^2}=\frac{dn^2+1}{dn^2+1+k^2}$, $(k\to kR)$, $k_e=1/R$ ъгъл слабо повлиява $\omega(k)$.

$k_e=m/R$ нещо. $\lambda_e=\frac{2\pi\bar{R}}{m}$, $\bar{R}\approx\frac{R}{2}$ grnd $J_0(x_n s)$ 

$k_e=\frac{2\pi}{\lambda}=\frac{2\pi nm}{2\pi\bar{R}}=m/\bar{R}\approx 2m/R$. $k_e(m=1)=\frac{2}{R}$.

Дори ъгъл $k_e(m=1)=2/R$ повлиява нещо $\omega(k)$ всего на 2%. $\underset{m=1}{\omega(k)}$

// Отнесли на пункт сбора комплекти Украйна 10 дамски/мъжки с нашийник, подгъващи одеяла (1 м.+10), обувки, 1 сушилня, компресия, раси, сонички.

E-mails to Оксана о v_{ph} (Fe нанонити) и v_{gr} (DW) ≈ 1.8 km/s.

Въпроси inv. Speakers to send titles/abstracts SolSkyMag'22.

Krzysztof (Poznan) изпрати slides с simulations $\omega(k)$ Fe нанонити.

E-mail to him. Отбор Фарадей по x_2, x_3 stiffness beg. BP-DW nanowires.

(11/III) Skype с Krzysztof о dyn simulations $\omega(k)$ Fe-nanowires.

Той изпрати $\omega(k)$ grnd $R=30$ nm, $H_z=20$ mT. grnd всич SW modes.

(m=0) SW $\omega(k)$ имат min при $k_m\approx 0.9\,(10^8)$ /m и $\frac{\omega(k_m)}{2\pi}\approx 15$ GHz.

Оценка $v_{ph}(k_m)\approx\frac{15\,GHz}{2\pi\,10\,(10^8/m)}=15\,\frac{10^9}{10^8}\,\frac{m}{s}\approx 150$ m/s. Бързо

точен расчет $v_{ph}(k)=\omega(k)/k$ дава $\min v_{ph}=220$ m/s, $R=30$ nm $H=20$ mT. Изпратих изчислена таблица $\omega(k_i)/k_i$ в Origin.

Проверка множене 2π и $\omega(k)$ and k. (!)

(12/III) С А. в Carrefour Diversum. Бълг. програма SolSkyMag'2022.

E-mail Оксана. Тя взе $\min v_{ph}(k)=220$ m/s, $R=30$ nm, $H=20$ mT. Към $\omega/2\pi(k_m)\approx 15$ GHz, $k=1.4\cdot 10^8$ /m, $\lambda\approx 45$ nm. Проблемът.

(13/III) С А. и Ю. в Galeria, разговаряме. Бълг.

(14/III) Reminders to inv. speakers to send titles/abstracts. Програмата.

(15/III) SolSkyMag'22 program: 1 plenary, 21 inv., 54 orals, 12 posters. - Всего (88) talks. 4 talks with Russian affil. - Nikitov - rejected.

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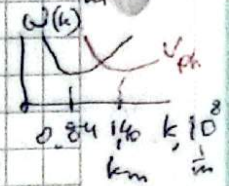
(16/III) Update Conf. website. Домашние о телефонных talks с Russian or Belorussian affiliations of the authors. SW-nanowire

Согласно Krzysztof Origin file "Fe_rod - 30nm - 20mT.opj" min $v_{ph}(k)$ для Fe nanowires есть 910,4 m/s и достигается при $k_m = 1,40 \cdot 10^8 \frac{1}{m}$ (хотя он и имеет $108 \frac{rad}{m}$). $\frac{\omega}{2\pi}(k_m) = 20,3$ GHz (!).

$v_{ph} = \frac{\omega}{k} = 2\pi \frac{f}{k}$, где frequency f is in GHz.

$\omega(k)$ имеет min при $k_m \approx 0,84 \cdot 10^8 \frac{1}{m}$ $\omega_{min} = 15,96$ GHz.



$\lambda_m = \frac{2\pi}{k_m} = \frac{2\pi}{1,40 \cdot 10^8 \frac{1}{m}} = \frac{2\pi}{1,40} 10^{-8}$ m $= 4,488 \cdot 10$ nm ≈ 45 nm.

$\frac{\omega(k_m)}{\omega_M} = \frac{15,96 \text{ GHz}}{61,94 \text{ GHz}} = 0,258$. Если считать, что $\omega_{min} = 15,96 \text{ rad/s} \cdot 10^9$

$\Rightarrow f_{min} = 15,96/2\pi$ GHz $= 2,54$ GHz.

В [Rychly, PhD 2018] $k \in [0, 100 \cdot 10^6 \frac{1}{m}] = [0, 10^8 \frac{1}{m}]$, что подтверждает мое предположение, что $k = 0,84$ и $1,40 \cdot 10^8$ есть в $(\frac{1}{m})$, а не в $(rad/m \cdot 10^8)$ units.

$k_m R = 0,84 \cdot 10^8 \frac{1}{m} \cdot 30$ nm $= 0,84 \cdot 10^8 \frac{1}{m} 30 \cdot 10^{-9}$ m $\approx 2,52$ всего.

$\log(k_m R) = \log(2,52) = 0,40$. У Arias-Mills min $\omega(k)$ при $\log(kR) \approx 0,7$ $k_m R = 2,52$ и они дают, деление k на 2π (если допустить k в rad/m). Вывод: k в γ_m, $k_{max} \approx 10^8 \frac{1}{m} = 10^6 cm^{-1}$, хотя и написано $[k] = rad/m$.

// разность фаз $\Delta\varphi = k\Delta z = k \cdot (20\pi \text{ nm}) = \pi$, $k = 10$ nm $= 1$, $k = 10^8 \gamma_m$ подтверждает это.

Вопрос: frequency in Krzysztof simulations in GHz или rad/s (надо для перевода в GHz делить на 2π)?

$\min v_{ph}(k) = (2\pi) \cdot \frac{\omega/2\pi\,(k_m)}{k_m} = 2\pi \cdot \frac{20,3 \text{ GHz}}{1,40 \frac{1}{m} \cdot 10^8} = 2\pi \frac{20,3}{1,40} \cdot 10 \frac{m}{s} = 911 \frac{m}{s}$

Без 2π (в ω) $\min v_{ph}(k) = 911/2\pi = 145$ m/s. (если $\omega(k_m) = 20,3 \cdot 10^9 \frac{rad}{s}$).

$\omega_H/\omega_M = 0,0094$ (для $H = 20$ mT, $\omega_{ex}/\omega_M = (l_e k)^2 = \begin{cases} 0,082 & (k = 0,84 \cdot 10^8 \gamma_m) \\ 0,227 & (k = 1,40 \cdot 10^8 \gamma_m) \end{cases}$

т.е. $H = 20$ mT влияние на спектр $\omega(k)$ очень мало ($R = 30$ nm Fe nanowire).

Величина H_{ex} at d_1 ($x = d_1/R$). $x_1 = \frac{3,83}{R = 30 \text{ nm}} = 128 \cdot 10^{-3} \cdot 10^9 \gamma_m$

$x_1 = 1,28 \cdot 10^8 \frac{1}{m}$. $H_{ex}/(4\pi M_s) = (l_e/R)^2 d_1^2 = l_e^2 x_1^2 = (3,4 \text{ nm})^2 \cdot (0,128 \frac{1}{nm})^2 = (3,4 \cdot 0,128)^2 = 0,188$, $H_{ex}(x_1) = 0,188 \cdot 21,36 \cdot 10^3 = 4,025$ kOe

$\omega_{ex}/2\pi\,(x_1) \approx 3 \frac{\text{GHz}}{\text{kOe}} \cdot 4,03$ kOe ≈ 12 GHz. Поэтому $\omega(0, 0.01, 30 \text{ nm}, d_1)/2\pi \approx 6$ GHz $< \omega_{ex}/2\pi$?

Запомнить Sol-Stud Mag 2022 program и начать ее уже на дорабатку (applications at glance). Это уже в Киеве.

(17/III) Очень SW-cylinder simulations by Krzysztof, хорошие comments to Maciej and Krzysztof. E-mail to A. Fert – title stated.

(18/III) С П. в Ikea Wroski и Urbil, Блин, один индеец в Champion league.

(19/III) Опять С П. в "Lidl", "Carrfour" Alakervia. SW-wires calcs.

Начал считать в file MathCad "SW_cylinder_nanowires.mcd" Было $l_{ex} = 0,84$ nm (забыл $\times 10$ для перевода в nm). Надо $l_e = 3,40$ nm (Fe) → Сильно повлияло на $\omega(k)$, увеличив ω и сместив min $\omega(k)$ к меньшим k.

γ_m $H = 20$ mT ($h = 0,01$), $R = 30$ nm $\omega(k)$ растет с $k\uparrow$, min $\omega(k)$ нет (!). $h = 0,01$ соответ. $H_z = 21,4$ mT. Min $\omega(k)$ появляется с ростом H_z (!)

γ_m $R = 30$ nm min $\omega(k)$ нет. при $h = 2,0$ ($H_z = 400$ mT) – giant field. Min $\omega(k)$ $R = 30$ nm нет в разумных H_z а min $v_{ph}(k)$ есть (!!) при $h = 0,05$ (≈ 100 mT).

γ_m $R = 50$ nm есть и min $\omega(k)$ и min $v_{ph}(k)$, $h = 0,05$.

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k/m k=0,01 min $v_{ph}(k)$ есть для всех $R = 30, 50$ км of interest.



Для $\omega(k)$ any есть $\min v_{ph}(k) = \frac{\omega(k)}{k}$

Начиная Октябрь с corrected v_{ph}-min $\approx 1{,}6$ km/s для $R = 30$ km. И 20 min analytical Herring-Kittel eq. с corrected simulated v_{ph}-min $= 910 \frac{m}{s}$.

→ Сделано: •1) Contract Ricardo; •2) travel report Gleb; •3) Caixa bank mobile •4) Excel-travel report; •5) Vortex GM thick bot MS; •6) lecture Poznań •7) Session chairmens; •8) SW nanowire; •9) Расчет с Марией об wire BP-DW gyro-dynamics •10) Booking Porto, Sevilla, Fert, Warsaw; •11) transfer prize 200 ₺ chess.com; •12) Spanish y Polish income tax.

(21-22/III) Переписка SolSkyMag 2022. переписка с Океаном по SW nanowires.

(23/III) Skype/MS Teams discussion SW nanowires с Maciej and Krzysztof. их simulation broadcasting, что Rot mode (starting с ≈ 30 GHz) и $\omega(k)$ ↓, но конечно получаем $\min v_{ph}(k) = 910$ m/s) есть azimuthally symmetric ($m=0$) только для больших k_z. Для k_z это uniform FMR mode (?!) (m_x). Blue mode (starting с ≈ 14 GHz, the lowest mode for small k_z) есть SW mode с rotating helicity (m_x) (m_z) — vortex → radial vortex (?!). Это напоминает с horiz. $\varphi=0$ m_x $\varphi=\pi/2$ Здесь Herring-Kittel, где the lowest SW mode должна быть $m=0$ mode с inhom. profile $m_x(s) \sim J_0(\alpha_1 s)$, $\alpha_1 \approx 3{,}83/R$. Блин.

(24/III) SolSkyMag 22 переписка. ? Океану о SW during BP-DW motion.

(25/III) Встреча в Доности с Марией и Julian (Tour. bureau). С 10. В банк "Caixa" to activate my cell phone. Марта обещала прин. по почте. Julian пригласил Cheese party в ресторане near S. Maria Iglesia и Велосипед Antonio Hernando в SolSkyMag program.

(26-27/III) Работа в саду. Блин. Подготовка Poznań lecture #6 "VC reversal".

(28/III) Booking hotel in Porto на 15-21 Apr. (скорее до 11 April). Лекция #6 UAM, Poznań.

(23-30/III) переписка SolSkyMag, update conf. program. Meeting с Julian. Подписан рецензия о contract Ricardo.

(31/III) подготовил и послал Cristina, UPV/EHU, travels June 2020, - Nov. 2021.

(1/IV) updated conf. program. Блин. переписка SolSkyMag.

(2/IV) Отдых дома, блин, камин.

(3/IV) Работа в саду. Блин. SolSkyMag переписка.

(4/IV) подпис. решение Select. Committee по Ricardo.

(5/IV) Отправил docs (Ricardo) в Lucia, UPV/EHU. С 17. в "Carrefour Market". Блин. Abs. Nano 2022, Sevilla.

(6/IV) Skype обед мед. Ricardo's thesis с Ruben. Slides для lecture #7. Отменил Hotel Porto, Booked Hotel in Madrid/Alcobendas. E-mail to Cristina (MICINN reports).

(8/IV) Info income tax - Poland (сдать до Apr. 30), offer Veronica о JMMM 11 paper. Proof PRA-Applied.

(10/IV-14/IV) Trip to Alcobendas с Марией. BB-Hotel.

(14/IV) Proof IEEE Magn. и "Magnonics" to Chumak.

(15/IV) E-mails to A. Fert, Hoffmann, T. Ono, Ainhoa (Ikerbasque).

(16-18/IV) Отдых дома, работа в саду. Блин. Talk to the NAP 22, Cracow.

(19/IV) Министерия FIS 2016 отчет - audit. E-mails to Maciej, Ruben (Rook)

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Nolmi (Barcelona), Miguel, Eli.

(20/IV) С.А. русский магазин - Amara, scanned 2 facturas 2017 в Yandex. E-mails to Eli и Morshad.

(21/IV) Lecture #7 Poznan. Vortex/Skyrmion STO. Теорема в дом. А. послала в Киев 300 € (200 € дом, дружу и 100 € дорге D.).

(22/IV) С.А. в русс. магазин „Amara"). Scanned 2 ордера Travel за 2017 по FIS2016 project. Теорема в дом. Отдел IVA „Constanto".

(23/IV) E-mails to Tserkovnyak, Aliev, Ricardo, IkerBasque (meeting June 1). Update Conf. program.

Сроки: 1) Travel reports Madrid и Gleb; 2) FIS2016 audit
3) Income tax Tolosa 29/IV - docs; 4) ~~200$~~ Chess.com; 5) Polish tax go 30/IV
5) Booking Sevilla (to cancel), Fert, Warsaw; 6) Lecture #8 Poznan.

(24/IV) Maixa. Отдых дома. Бины. E-mails to Ricardo, Yaroslav.

(25/IV) Facturas sent to Miguel (audit) FIS2016, отказ от Sevilla conf. NANO 2022, e-mails to Ruben и Yanko (permission to use Fig. 3, PRB 2017), IVA „Constanto". С А. и Ю. ужин в restaurant „Aranzazu".

(26/IV) Skype с Farkhad y Diego: 1) они simulated α_S stiffness для BP-DW in nanowire, надо еще α_z для $\omega_G \sim \sqrt{\alpha_S \alpha_z}$; 2) в-FMR measurements для nanowires Fe/Co с L=350 nm 2R=115 nm (BA-DW) и 2R=200 nm (Vortex), частоты сильно зависят в интервале 5-15 GHz, static M(H) с Hsat (Vortex), Hsat (BP) ≃ 6-7 kOe. Nanowires by C.Bran.

(27/IV) Дорог. Lecture #8 Poznan, High-order vortex GM. T=10 для Пороч.

(28/IV) Online Lecture #8. Poznan. Подпис. docs для Tolosa - Tax office. На сайт Polish tax office деньги не дошли, пришлось уплатить 850 PLN как tax на 10 kPLN paid in Nov. 2021.

(29/IV) Tax office - Tolosa, А. T10, Ncrera Caixa Bank. Не приняли А. T10, надо доходы минус Soc. Sec.

(30/IV) Tax office Tolosa, return 3.000 € за 2021. СА „Lidl", Mercadona.

(2/V) Meeting с Julian (зам N2 in Library), SolSkyMag. 2022. Meeting с Ricardo (Ch. 2 thesis) и Ruben. Ricardo с Oct. 1, 2022 переходит на U.C. postdoc в DIPC.

(3/V) Travel reports to Madrid (Apr 2022) и Kokarei (Feb 2022).

(4/V) SolSkyMag переписка + Program update. Бины.

(5/V) Meeting в Carlos Santa Maria library с admin. (Julian, А., Ю.): бюджет зам #2 для coffee breaks, registration и posters и ген. комнаты для ужина. Отдел Eli travel reports Gleb и Madrid.

(6/V) Conf. program. подготовка лекции #9 „Coupled vortex dyn."

(7/V) подготовка лекции #9. Paper с Farkhad „BP-DW dynamics in short NW".

(8/V) расчет ω_G BP in nanowire для paper with Farkhad. Сдано 15/II → G граница

$G_S = 4\pi R\left(\frac{M_S}{\gamma}\right)\arctg\left(\frac{1}{t}\right)$, $t = b/R$, $G_S = G_S(R, b)$, b - BP radius.

Equilibrium „b" посчитано 16/II, $b = b(R, L, A, M_S)$. $L_e = \sqrt{\frac{A}{2\pi M_S^2}}$ BP nanowire

у Фархада $A = 25$ pJ/m, $M_S = 0$ T, $4\pi M_S = 20$ kG, $M_S = 1592$ G,

$R = 60$ nm, $L = 250$ nm для stiffness α_1, α_2. $L_e = \sqrt{\frac{2.5\cdot 10^6 \text{ erg/cm}}{9\pi\,(1.592)^2 10^6 \frac{\text{erg}}{\text{cm}^3}}} = \sqrt{\frac{2.5}{9\pi\,(1.592)^2}}\cdot 10_{\text{nm}} = 3.96$ nm

$P = L/R = 250/60 = 4.167$.

Расчет ω_G показывает для $\beta = 4.167$, $R = 60$ nm, $L_e = 3.96$ nm. „Bloch_point_energy_wire.mcd"

$\omega_G = \sqrt{\alpha_S \alpha_z}\frac{1}{G_e}$, $G_S = 4\pi R\frac{M_S}{\gamma} f(t)$, $f(t) = \arctg(V_t)$, $t = b/R$.

$\omega_G = \gamma/(4\pi R M_S f)\sqrt{\alpha_S \alpha_z}$. $\frac{\omega_G}{\omega_M} = \frac{1}{f 4\pi M_S}\frac{\gamma}{4\pi M_S R f}\sqrt{\alpha_S \alpha_z} = \frac{1}{(4\pi)^2 f M_S^2 R}\sqrt{\alpha_S \alpha_z}$

Eq. $b/R = 0{,}12$ посчитано „Bloch_point_energy_wire.mcd", $R = 60$ nm.

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(9/V) Лекция Poznan #9, Coupled vortex dynamics.

WG-gate BP in nanowire $R = 60$ nm, $\beta = L/R = 4,167$, $l_0 = 3,96$ nm

$$\frac{\omega_G}{\omega_M} = \frac{1}{(4\pi)^2} \frac{1}{g(\beta) M_s^2 R} \sqrt{\varkappa \varkappa_2}, \quad g(\beta = 4,17) = 0,174, \quad \text{BP GW}$$

Forkhead data: 1) $\varkappa = 1,234 \cdot 10^{-22}$ J/nm, $\varkappa_2 = 8,085 \cdot 10^{-20}$ J/nm² without pinning

2) $\varkappa = 2,672 \cdot 10^{-22}$ J/nm² и $\varkappa_2 = 8,765 \cdot 10^{-19}$ J/nm².

1 J/nm² $= 1 \cdot 10^7$ erg $/ (10^{-7}$ cm$)^2 = 10^7 \cdot 10^{14}$ erg/cm² $= 10^{21}$ erg/cm²

Without pinning: $\frac{\omega_G}{\omega_M} = \frac{1}{(4\pi)^2} \frac{1}{0,174} \frac{(1,592)^2}{(6 \cdot 10 \cdot 10^{-7}\text{ cm})^2 10^6 \frac{\text{erg}}{\text{cm}^2}} \sqrt{1,234 \cdot 10^{-22} \cdot 10^{21} \frac{\text{erg}}{\text{cm}^2} \cdot 8,085 \cdot 10^{-20} \cdot 10^{21} \frac{\text{erg}}{\text{cm}^2}}$

$= \frac{1}{(4\pi)^2} \frac{1}{0,174 \cdot 6 \cdot (1,592)^2 \left(\frac{\text{erg}}{\text{cm}^2}\right)} \sqrt{1,234 \cdot 8,085 \cdot 10^{+0} \left(\frac{\text{erg}}{\text{cm}^2}\right)} = \frac{1}{(4\pi)^2} \frac{1}{0,174 \cdot 6 \cdot (1,592)^2} \sqrt{1,234 \cdot 8,085}$

ω_G/ω_M (no pinning) $= 7,56 \cdot 10^{-3}$, $\frac{\omega_M}{2\pi} = \frac{\gamma}{2\pi} 4\pi M_s = 3,0 \frac{\text{GHz}}{\text{kOe}} \cdot 20 \text{ kOe} = 60 \text{ GHz}$

→ $\frac{\omega_G}{2\pi}$ (no pin.) $= 7,56 \cdot 10^{-3} \cdot 60$ GHz $= 0,454$ GHz (!).

With pinning: $\frac{\omega_G}{\omega_M} = \frac{1}{(4\pi)^2} \frac{1}{0,174 \cdot 6 \cdot (1,592)^2} \sqrt{2,672 \cdot 10^{-22} \cdot 10^{21} \cdot 8,765 \cdot 10^{-19} \cdot 10^{21}} =$

$= \frac{1}{(4\pi)^2} \frac{1}{0,174 \cdot 6 \cdot (1,592)^2} \sqrt{2,672 \cdot 8,765 \cdot 10} = 0,0366$, $\left(\frac{\omega_M}{2\pi}(\text{pin.}) = 0,0366 \cdot 60 \text{ GHz} = 2,20 \text{ GHz}\right.$

→ $\frac{\omega_G}{2\pi}$ (no pin.) $= 0,45$ GHz, $\frac{\omega_G}{2\pi}$ (pin.) $= 2,20$ GHz.

Запис ωG BP in nanowire in Overleaf file и нощен Papers-word file с equations.

(10-14/V) SolSkyMag'22 конференция. Update програми. C ↑. Б. „Comsol" Olaberria „Comsol и Quantum", „AlCompo" – нощувки, хотел.

(15/V) нощен. lecture #10 Poznan. Бълг.

(16/V) Lecture #10 „magn. topological charges". Update Overleaf file Pap.

(17/V) Editing MS с Papers, „Spin dynamics short nanowires".

(18/V) Оставам 25% deposit сума rest. „Londres", Ощ. Conf. fee JEMS 2022 – 460€ сбоен credit card и нощен Запис Регистрация Eli.

(19-20/V) SolSkyMag – 2022 програма и конференция. Запис. и изпращ. Maciej docs на Overleaf у нас ПЛН за нощен. Ремонт А. сградата.

(21/V) нощен билети Warsaw/Poznan, Куриер „Lufthansa" airtickets за July 13-30 и booked Hotel in Bilbao за „30/VI – сбоен и Норе.

(22/V) Болен. Възстан. Free music в Lugo и „legend" – бълг.

(23/V) Lecture Ch.3 – thesis Ricardo. Ремонт А. сградата.

(24/V) Откъсване от meeting в NAP/ELI изпр. 30 години. Ремонт А. сградата.

(25-26/V) SolSkyMag работа. конференция с Maciej о GW scattering на ампулите (4x AIP-Mat manuscript). Submit abstract to NAP-2022, Krakow.

(27/V) Positive COVID-test in Villabona. конференция с Carla Bonilla и T. Ono.

(28/V) конференция SolSkyMag – invited speakers и с Ricardo/Ruben.

(30/V) Online meeting с Ricardo Ruben – обучение. chapt. 3, Ricardo's thesis конференция с Maciej и NAP-2022 Krakow Conference.

(31/V) Забиномиране сина „CONSTANTO" и personal in Caixabank. Откъсване от Ikerbasque workshop Bilbao чрез COVID.

(1-5/VI) Болен. Работа над SolSkyMag program, Vortex GM-paper. Бълг.

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6/VI) С Л - в Carrefour" Olaberria (check 20+€). Vortex GM - Figs.

7/VI) С-10. в „CaixaBank" (разпоредят сряда до 14/VI). С Л. в Mercadona, "Alfaain.

8/VI) Подготовка Figs, написване Vortex GM paper (Sent to Magnetism?)

9/VI) Расчет eigenfunction high-f GM modes и сравн. с $\cos(n\pi z)$, $\sin(n\pi z)$ Ding '2014.

Comm. 7,13/X сер $S_n(z) = e^{i\Phi_0} 2\cos\tilde{\Phi}(z)$ - even
и $S_n(z) = e^{i\Phi_0} \cdot 2i \sin\tilde{\Phi}(z)$ - odd, $\tilde{\Phi}(z) = \Phi(z) - \Phi_0 = \int_0^z dz \sqrt{f(z)}$, $f(z) = \frac{(1-\lambda)(z)}{y}$
$\lambda = \omega/\omega_M$. При $\beta \to 0$, $\psi = \Phi(\frac{1}{2}) - \Phi(-\frac{1}{2}) = n\pi$ и $\Phi(z) = \frac{n\pi}{2}$
Съвпадение $\cos(n\pi z)$, n - even и $\sin(n\pi z)$, n - odd as in Ding '2014.
При $\beta = 0.5 - 2$ осцилира сер $S_n(z)$ от $\cos(n\pi z)$, $\sin(n\pi z)$ Значимо ~ 10 - 30% и расчет с увеличаване β.

Old high-f modes (Ding '2014): $\frac{\omega_n(\beta)}{\omega_M} = \frac{\pi}{8} \frac{1}{\pi^2} \frac{1}{\beta^2} \left[\ln\frac{8}{R_0} + \frac{5}{4} \right] + 2\beta \int_0^\infty dk\, f_n(\beta k) I^2(k)$

$I(k) = \int_0^1 dz\, z J_1(kz)$, $f_n(k) = \frac{k}{k^2+q_n^2} \left[1 + 2 \frac{1}{1+\delta_{n0}} \frac{k}{k^2+q_n^2} \left\{ (-1)^n e^{-k} - 1 \right\} \right]$

$q_n = n\pi$, $n = 0, 1, 2, \ldots$
$n=0$ comb. the lowest GM и сходимост к $\omega_G(\beta)$ JAP '2002.
Сега $n=0$ mode станал еквивалентно. $n = 1, 2, \ldots$ и $\delta_{n0} = 0$.
$f_n(k) = \frac{k}{k^2+q_n^2} \left[1 + \frac{2k}{k^2+q_n^2} \left\{ (-1)^n e^{-k} - 1 \right\} \right]$. Изобщо подобно
old-$\omega_n(\beta)$ SciRep 2014 близки к $\omega_n(\beta)$-new из WKB method.
$\omega_n(\beta)$-WKB всяка $\forall \beta \in [0.2, 2]$ точно, чел $\omega_n(\beta)$ old - Sci.Rep. 2014

10/VI) Построение Fig. $S_n(z)$ eigenfunctions. $\omega_n(\beta)$ Math Cad good ums при $\beta \le 1.0$. Разсъждения нужни? Резюме нагоре €/$ - 1.07 до 1,05.

11/VII) Изчисляване Thiele eq. of motion given $\vec{S}(z,t)$ in thick dot и приложимост WKB - approximation.

$\vec{g} \times \dot{\vec{S}} = -\frac{\delta W}{\delta \vec{S}}$. Умножение $\frac{\delta W}{\delta \vec{S}} = \frac{\partial w}{\partial \vec{S}} - \frac{\partial}{\partial z} \frac{\partial w}{\partial(\partial \vec{S}/\partial z)}$. $\vec{S} = \vec{S}(z,t)$ is real function here.

$\vec{g} \times \vec{S} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & g_z \\ \dot{S}_x & \dot{S}_y & 0 \end{vmatrix} = \hat{y} g_z \dot{S}_x - \hat{x} g_z \dot{S}_y$. $\vec{S} = (S_x, S_y)$.

$\dot{\vec{S}} = \vec{\omega} \times \vec{S}$ и Fourier transform $\vec{S}(z,t) = \vec{S}(z,\omega) e^{i\omega t}$? Съвместимо ли?

$\vec{\omega} \times \vec{S} = |\vec{\omega} = \omega_z \hat{z}| = \omega_z \left[\hat{y} S_x - \hat{x} S_y \right]$. $\dot{\vec{S}} = \frac{d\vec{S}}{dt} = \frac{d}{dt} \vec{S}(z,t) = \frac{d}{dt} \vec{S}(z\omega) e^{i\omega t} =$
$= i\omega \vec{S}(z,\omega)$. Нямат $S_x(z,t) = S_x(z) \cos(\omega t + \varphi)$, $S_y(z,t) = S_y(z) \sin(\omega t + \varphi)$?
$S(z,t) = S_x(z,t) + i S_y(z,t) = |S_x(z) = S_y(z) = S(z)| = S(z) e^{i(\omega t + \varphi)}$.
$\vec{g} \times \dot{\vec{S}} = \hat{y} g_z S_x(z) (-1) \omega \sin(\omega t + \varphi) - \hat{x} g_z S_y(z) \omega \cos(\omega t + \varphi)$.

$\frac{1}{w}$ $\frac{1}{V} W = \frac{1}{\pi} \int dz\, w$. z in units of dot thickness L, $[w] = erg/cm^3$.

$\frac{1}{V} W = \frac{1}{\pi} \int dz\, w = \frac{1}{2} \int dz \left[\eta \left(\frac{\partial \vec{S}}{\partial z} \right)^2 - \mu \vec{S}^2(z) + \int dz' \varkappa(zz') \vec{S}(z) \cdot \vec{S}(z') \right]$

$\frac{1}{\pi} W = \frac{1}{2} \eta \left(\frac{\partial \vec{S}}{\partial z} \right)^2 - \mu \vec{S}^2(z) + \vec{S}(z) \cdot \frac{1}{2} \int dz' \varkappa(zz') \vec{S}(z')$

$\frac{1}{\pi} \frac{\delta W}{\delta \vec{S}} = \frac{1}{\pi} \left[\frac{\partial w}{\partial \vec{S}} - \frac{\partial}{\partial z} \frac{\partial w}{\partial(\partial \vec{S}/\partial z)} \right] = -\mu \vec{S}(z) + \int dz' \varkappa(zz') \vec{S}(z') - \frac{\partial}{\partial z} \eta \frac{\partial \vec{S}}{\partial z} =$
$= -\mu \vec{S}(z) - \eta \frac{\partial^2 \vec{S}}{\partial z^2} + \int dz' \varkappa(zz') \vec{S}(z') = \hat{\Gamma} \vec{S}$. $\hat{\Gamma}$ - linear operator.

$\frac{1}{\pi} \frac{\delta W}{\delta \vec{S}} = \hat{\Gamma} \vec{S}$. $\vec{g} \times \dot{\vec{S}} = -\frac{\delta W}{\delta \vec{S}} = -\pi \hat{\Gamma} \vec{S}$. $(\vec{g} \times \dot{\vec{S}})_x = -(\hat{\Gamma} \vec{S})_x$, $(\vec{g} \times \dot{\vec{S}})_y = -(\hat{\Gamma} \vec{S})_y$

Всъщност $\vec{g} \times \dot{\vec{S}} = -\omega g_z \left[\hat{y} S_x(z) \sin(\omega t + \varphi) + \hat{x} S_y(z) \cos(\omega t + \varphi) \right] = |S_x(z) = S_y(z)| =$
$= -\omega g_z \left[S(z) \cos(\omega t + \varphi) \hat{x} + S(z) \sin(\omega t + \varphi) \hat{y} \right] = -\omega g_z \left[S_x(z,t) \hat{x} + S_y(z,t) \hat{y} \right] = -\omega g_z \vec{S}(z,t)$
$\vec{g} \times \dot{\vec{S}}(z,t) = -\omega g_z \vec{S}(z,t)$ при $S_x(z,t) = u(z) \cos(\omega t + \Phi(z))$, $S_y(z,t) = u(z) \sin(\omega t + \Phi(z))$

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$S(zt) = iS_y(zt) + S_x(zt) = u(z)\, e^{i[\omega t + \Phi(z)]}$ как в MS text чем отличается фаза $\Phi(z)$. $S(zt) = S(z\omega)\, e^{i\omega t}$, где $S(z\omega) = u(z) e^{i\Phi(z)} = u(z\omega) e^{i\Phi(z\omega)}$

$|\vec{g}| = g = 2\pi M_s/\gamma$, $g_z = (\pm)g$. $\vec{g} \times \dot{\vec{S}} = -\omega g_z \vec{S}(zt) = -\hat{\Gamma}\vec{S}(zt)/\pi$.

$\omega g_z \vec{S}(zt) = \hat{\Gamma}\vec{S}(zt)$, $\omega g_z = \omega(\pm)\frac{2\pi M_s}{\gamma}$. $\lambda \vec{S}(zt) = \hat{\Gamma}\vec{S}(zt)$,

$\lambda = \frac{1}{\pi}\omega g_z = (\pm)\frac{1}{\pi}\omega\frac{2\pi M_s}{\gamma}$. Берем (+). $\lambda = 2\frac{\omega}{\gamma}M_s$ как в Ding '2014. Используем нормировку на $8\pi M_s^2$. $\lambda = 2\frac{\omega}{\gamma}M_s \to \lambda = \frac{1}{8\pi M_s^2} 2\frac{\omega}{\gamma}M_s = \frac{\omega}{4\pi^2\gamma M_s} = \frac{\omega}{\omega_M}$

Alternative: $\vec{g} \times \dot{\vec{S}} = -\frac{\delta W}{\delta \vec{S}}$. Здесь $S(zt) = S_x(zt) + iS_y(zt)$ через $S = S_x + iS_y$.

$\vec{g} \times \dot{\vec{S}} = \dot{y} g_z \dot{S}_x - \dot{x} g_z \dot{S}_y = -\frac{\delta W}{\delta \vec{S}}$ $\left\{ \begin{array}{l} -g_z \dot{S}_y = -\left(\frac{\delta W}{\delta \vec{S}}\right)_x = -\frac{\delta W}{\delta S_x} \\ g_z \dot{S}_x = -\left(\frac{\delta W}{\delta \vec{S}}\right)_y = -\frac{\delta W}{\delta S_y} \end{array} \right\}$

$g_z(\dot{S}_x + i\dot{S}_y) = -\frac{\delta W}{\delta S_y} + i\frac{\delta W}{\delta S_x}$

$g_z \dot{S} = -\frac{\delta W}{\delta S_y} + i\frac{\delta W}{\delta S_x} = 2\frac{\partial W}{\partial \bar{S}}(+i)$ — проверить. $\frac{\partial}{\partial \bar{S}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)$

$g_z \dot{S} = i\left(\frac{\delta W}{\delta S_x} + i\frac{\delta W}{\delta S_y}\right) = 2i\frac{\partial W}{\partial \bar{S}} = 2i\frac{\delta W}{\delta \bar{S}}$. $S(zt) = S(z\omega)e^{i\omega t}$, $\dot{S} = i\omega S$

$g_z i\omega S = 2i\frac{\delta W}{\delta \bar{S}}$, $g_z \omega S(zt) = 2\frac{\delta W}{\delta \bar{S}} = 2\pi\hat{\Gamma}S(zt)$, где

$\hat{\Gamma}S = -\mu S - \gamma\frac{\partial^2 S}{\partial z^2} + \int dz' \chi(zz') S(z')$, $S = S(zt) = S_x + iS_y$.

Т.е. если ненамагничено ур-е $g_z \omega S = \frac{2\delta W}{\delta \bar{S}}$, где $S = S_x + iS_y$, то надо решать $S(zt) = S(z\omega)e^{i\omega t}$. Если ур-е для S_x, S_y, то надо $S_x(zt) = u(z)\cos[\omega t + \Phi(z)]$, $S_y(zt) = u(z)\sin[\omega t + \Phi(z)]$.

Аналог sign $f(z,\beta) = \frac{\omega_n(\beta)}{\omega_M} - J(z,\beta)$, если $\sqrt{f(z,\beta)}$ в MS-B approach, $f(z\beta) \to \min$, $J(z,\beta) \to \max$ at $z=0$. $\mathrm{Sign}\left[\frac{\omega_n(\beta)}{\omega_M} - J(0,\beta)\right]$ - analog calculated $\omega_n(\beta)$ поведен, где sign $[\dots] > 0$ при $\beta \le 1.0$ для $n=1$ mode $\omega_1(\beta)$ и $\beta \le 1.5$ для $\omega_2(\beta)$ - mode пересечения ветвей $\omega_n(\beta)$, $n = 1, 2, 3$ с уровнем Sign $f(0,\beta)$.

(12/VI) Построение $\omega_n(\beta)$ и $S_{xy}(z)$ - ФФ в Origin.

(13/VI) Написание "Vortex GM paper". С "п. Alcampo" "Carrefour" "Diarteum".

(14/VI) Визит в "Caixa Bank" относительно счетов. Конвертирован все USD (21.550 €) в €. Bloomberg 1.043 $/€, "Caixa Bank" used 1.068 ratio и 40 € service fee. Новая Vortex GM-MS to Nenashev.

(15/VI) Написание Vortex GM paper. Emm.

(16/VI) С п. в "Alcampo" Diarteum. Переписка с Cristina UPV/EHU о Contract Ricardo (она предлагает new "art 23 bis" Contract, старые contracts типа PIC будут тоже отменены, чем не удобно) template.

(17/VI) Cristina прислала docs на new "art 23 bis" Contract, Julian gonjatur Заключил "Vortex GM" MS в Magnetism-template и Submitted the MS to "Magnetism"/MDPI, spec. issue "Topological spin textures". 440 €.

(18/VI - 19/VI) Запрос Ane Martinez, Leioa, о статусе PIC Contract Ricardo. Новая Oksana Work fede с Calculations SW in cylindrical harmonics, Herring-Kittel eq., $\min \mathrm{Sph}(k)$.

(20/VI) Не смог зайти в "Santander Polska" по user name/password, т.к. послали SMS code, который не пришел на мой cell phone.

(21/VI) Написал Lubos Dominiak (Santander, Poznan), у которого откр. счет. Разблокировали счет CONSTANTY в "CaixaBank", еще 2 conf (see transfers).

(22/VI) С п. и ю. в "Alcampo" Emm и "Eroski" + Urbil (Boga, >600 бут.). Добавлено по new "art 23 bis" Contract Ricardo.

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(23-28/VI) Пренощува SolSkyMag-2022. Artem - GM domes draft.

(25/VI) Заверен на пристовището и пощенската банка /преводът дошъл/ conf.

(27/VI - 1/VII) Пребиваване SolSkyMag-2022. Interview с P Fischer Ricardo. Diario Vasco" published 29/VI. Docs на new Contract "vortex GM - "magnetism" - major revision. Бележа В. Крабчук

(2/VII) Заработена personal account в "Банка": пренесен л. Sol Sad. за May, June 2022. Hogener cash Gorboga conf. : 8145 €

(3-12/VIII) Hogener IVA, Constanbo" ltd 2022 и "profit - 2021. пренощува с Jiadong Zang и Terno Ono. Пренасяне „GM black-dots" paper gmo magnetism /MDPI. Arkady: group project 2022-25 приемен, ~80 k€. Пробен: new paper on dome-shape dots GM. пренощува с Maciej (lecture honorarium и 3.9 years на ROLS project), Julian, Ruben.

(13/VII - 1/VIII) Trip to Poland Warsaw 13/VII Poznan 14-24/VII Warsaw 24-31/VII. JEMS Warsaw July 25-29 2022. Принемам и опубликувам GM paper в Magnetism/MDPI. Maciej бележи ~ 2,000 PLN per diem. Курсът в Poland (400+420+420+400) = 1640 € по сред. курсът 4.80 PLN/Euro. Intern. Poznan G. Cantala пътувам в Donostia в Jan. - June 2023. Project NW online meeting 20/VII. Faikhod 21/VII

Обсъждане Nanowire project papers с Оксана:

1) Оксана до end August доделяне и публикуване на MS BP-DW dynamics nanowire (kinetic field, SW, $\vec{G}$ - нов вектор);
2) нов numerical vortex GM-in short nanowires и сравнение с simulations Felipe;
3) нов numerical BP trajectories in nanowires by eq. Thiele и $\vec{X}(0) = 0$ (center) и $\vec{X}(t=0) = R$ give various значения α_2, α_3 - Stiffness Coefficients in $W(\vec{X}) = \frac{1}{2}\alpha_2 Z^2 + \frac{1}{2}\alpha_3(X^2+Y^2)$ - BP energy.

(1/VIII) Skype с Ruben (Ulrich reports Madrid, Donostia) и Gleb (comments to MS draft - Artem-Adelaide GM dome shape dots". Liubov Ivanko (Poznan) - new "NATO/Ukraine project (Sept. Oct. -?)

(2/VIII) Вчера. Пълнен. Отдих дома. Час „Lufthansa" за hotel, meals July 31, 2022.

(3/VIII) С А. и W. в Oslo с обедом. Skype - Gleb - Artem MS GM dome shaped dots", фирмени files.

(4/VIII) Работа над Artem GM dome-shape dots" нощен PDF file revised to Artem и Gleb. л. пренасяне Lume, Kneb - 200 €.

(5/VIII) Купуване airtickets Krakow „Lufthansa" и Booked Hotel Bilbao, отмяна до 23:50 Sept. 1. Запис MSCINN message Оксана.

(6/VIII) BP (soliton) trajectory in cylindrical nanowire (Сън 5/VIII). BP dyn. nanowire

$\vec{G} = (G_s, 0, 0)$ global gyrovector. $G_s = 4\pi R\left(\frac{M_s}{\gamma}\right) t \arctg\left(\frac{1}{t}\right)$, $t = b/R$

b - BP size in Thiaville ansatz нахождане чрез $\min W(b)$. $\min W(B)$

$\vec{G} \times \vec{\dot{X}} = \frac{\partial W}{\partial \vec{X}}$, $\vec{X}$ (BP shift from nanowire center, $x=0, y=0, z=0$)

$\vec{X} = (X_s, X_\varphi, X_z)$, $X_z = Z$. $W(\vec{X}) = \frac{1}{2}\alpha_3(X_s^2 + X_\varphi^2) + \frac{1}{2}\alpha_2 Z^2$, $\alpha_3 \to \alpha$.

$X^2 + Y^2 = X_s^2 + X_\varphi^2$. $W(\vec{X}) = W(X_s, X_\varphi, X_z)$. Верно ли $\frac{\partial W}{\partial \vec{X}} = \frac{\partial W}{\partial X_s}\hat{s} + \frac{\partial W}{\partial X_\varphi}\hat{\varphi} + \frac{\partial W}{\partial Z}\hat{z}$?

$W(\vec{r}) = W(s, \varphi, z)$, $\nabla_r W = \frac{\partial W}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial W}{\partial \varphi}\hat{\varphi} + \frac{\partial W}{\partial z}\hat{z}$ cyl. coordinates

Нулев $Z=0$. $W = \frac{1}{2}\alpha(X^2+Y^2) = \frac{1}{2}\alpha \vec{X}^2$, $\frac{\partial W}{\partial \vec{X}} = \alpha \vec{X} = \alpha\left[X_s\hat{s} + X_\varphi\hat{\varphi}\right]$

$\left(\frac{\partial W}{\partial \vec{X}}\right)_s = \alpha X_s = \frac{\partial W}{\partial X_s}$, $\left(\frac{\partial W}{\partial \vec{X}}\right)_\varphi = \alpha X_\varphi = \frac{\partial W}{\partial X_\varphi}$, $\left(\frac{\partial W}{\partial \vec{X}}\right)_z = 0$.

Сън $Z \neq 0$. $W(\vec{X}) = \frac{1}{2}\alpha(X^2+Y^2+Z^2) + \frac{1}{2}(\alpha_2 - \alpha)Z^2$.

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$W(\vec{X}) = \frac{1}{2}\varkappa \vec{X}^2 + \frac{1}{2}(\varkappa_2 - \varkappa) Z^2 = \frac{1}{2}\varkappa \vec{X}^2 + \frac{1}{2}\Delta\varkappa (\vec{X}\cdot\hat{z})^2$

$\frac{\partial W}{\partial \vec{X}} = \frac{1}{2}\varkappa \vec{X} + \frac{1}{2}\Delta\varkappa \cdot 2(\vec{X}\cdot\hat{z})\cdot\hat{z}$, $\left(\frac{\partial W}{\partial X}\right)_\varphi = \frac{1}{2}\varkappa X_\varphi$, $\left(\frac{\partial W}{\partial X_\varphi}\right)_z = \frac{1}{2}\varkappa X_\varphi = \varkappa X_\varphi$

$\left(\frac{\partial W}{\partial X}\right)_z = \frac{1}{2}\varkappa z + (\varkappa_2 - \varkappa) z = \varkappa_2 z$. $\frac{\partial W}{\partial \vec{X}} = \varkappa X_\varphi \hat{\varphi} + \varkappa X_\varphi \hat{\varphi} + \varkappa_2 Z\hat{z}$

$\partial W/\partial \vec{X}$ (ур-я III) формы.

Тяжёл. ед. с G_φ то точно даст $\frac{\partial W}{\partial X_\varphi} = \varkappa X_\varphi = 0$, $X_\varphi = 0 \Rightarrow X_\varphi(t) = 0$.

$\vec{X}(t) = X_\varphi(t)\hat{\varphi} + X_z(t)\hat{z} = X_\varphi(t)\hat{\varphi} + Z(t)\hat{z}$. тяжёл. ед.

$-G_\varphi \dot{z} = \frac{\partial W}{\partial X_\varphi} = \varkappa X_\varphi$, $G_\varphi \dot{X}_\varphi = \frac{\partial W}{\partial z} = \varkappa_2 z$. $\left\{\begin{array}{l} -G_\varphi \dot{z} = \varkappa X_\varphi \\ G_\varphi \dot{X}_\varphi = \varkappa_2 Z \end{array}\right\}$

$-G_\varphi \ddot{z} = \varkappa \dot{X}_\varphi = \varkappa \frac{\varkappa_2}{G_\varphi} z$, $\ddot{z} = -\frac{\varkappa\varkappa_2}{G_\varphi^2} z = -\omega_G^2 z$ – осциллятор,

$\omega_G^2 = \varkappa\varkappa_2 / G_\varphi^2$.

а) $\varkappa > 0$, $\varkappa_2 > 0$ oscillatory elliptic motion in φ-z plane с $\omega_G = \frac{1}{G_\varphi}\sqrt{\varkappa\varkappa_2}$.

$X_\varphi(t) = X_\varphi^0 \sin\omega_G t$, $Z(t) = Z_0 \cos\omega_G t$, $X_\varphi^0 = \sqrt{\frac{\varkappa_2}{\varkappa}} Z_0$ – elliptic trajectory.

б) $\varkappa < 0$, $\varkappa_2 < 0$ формально сохраняется elliptic trajectory oscillatory motion с $\omega = \omega_G$ Обмен $\max W(\vec{X})$.

в) $\varkappa > 0$, $\varkappa_2 < 0$, $\varkappa_2 = -|\varkappa_2|$, $\ddot{z} = \frac{\varkappa|\varkappa_2|}{G_\varphi^2} z$, $\Gamma^2 = \frac{\varkappa|\varkappa_2|}{G_\varphi^2} > 0$.

$\ddot{z} = \Gamma^2 z$, $z(t) = z_0 e^{\Gamma t}$, тогда $z(t) = z_+ e^{\Gamma t} + z_- e^{-\Gamma t}$ (ch t, sh t).

$G_\varphi \ddot{X}_\varphi = \varkappa_2 \dot{z} = \varkappa_2 \left(\frac{-1}{G_\varphi}\right) \varkappa X_\varphi$, $\ddot{X}_\varphi(t) = -\frac{\varkappa\varkappa_2}{G_\varphi^2} X_\varphi = \Gamma^2 X_\varphi(t)$.

$X_\varphi(t) = X_+ e^{\Gamma t} + X_- e^{-\Gamma t}$

Начальное условие $\vec{X}(0) = 0$, $X_\varphi(0) = 0$, $z(0) = 0$. $Z(0) = z_+ + z_- = 0$

$z(t) = z_+(e^{\Gamma t} - e^{-\Gamma t}) = 2z_+ \operatorname{sh}(\Gamma t)$. Exp. рост $z(t)$ с $t\Gamma$. $z_- = -z_+$.

– ВР убегает от точки center position $z(0) = 0$ на $\pm\infty$ будем осц O_z t.

$X_\varphi(t) = X_+(e^{\Gamma t} - e^{-\Gamma t}) = 2X_+ \operatorname{sh}(\Gamma t)$.

$X_\varphi(t)$ также exp. растёт, но $X_\varphi(t) \le R$ (наименьшее радиус).

г) $\varkappa < 0$, $\varkappa_2 > 0$. $\varkappa = -|\varkappa|$, $\ddot{z} = -\frac{\varkappa\varkappa_2}{G_\varphi^2} z = \frac{|\varkappa|\varkappa_2}{G_\varphi^2} z(t) = \Gamma^2 z(t)$.

Сводится к в). Если $X_\varphi(0) = 0$, $z(0) = 0$, то $z(t) = 2z_+ \operatorname{sh}(\Gamma t)$ и $X_\varphi(t) = 2X_+ \operatorname{sh}(\Gamma t)$.

→ Вывод: ВР oscillatory motion (ω_G) с elliptic trajectory при $\varkappa\varkappa_2 > 0$

ВР убегает к $z = \pm\infty$ и $|X_\varphi| = R$ при $t\Gamma$ при $\varkappa\varkappa_2 < 0$.

Послан e-mail о ВР trajectories in nanowire to Oksana.

(7–16/VIII) Сборы домой. Поездка на машине Орёл – Гелария. Бишк.

Submitted appl. на compensation 627 PLN to "Lufthansa", переписка. Запрос 500 PLN deposit back to Exclusive Apartahotel, Warsaw. Бишк.

travel reports Poznan – Warsaw – Kostya y Larysa. Посылка Lund A.D. 12 kg – 29€.

Самол. Гелария (14/VIII)

(17–18/VIII) Сборы домой. Переписка с Exclusive hotel, Warsaw о Airbnb. MINECO.

(19/VIII) Интервью "Diario Vasco", DIPC (A. Nikitin + guiristas, Ukraine).

(20/VIII) Вылет Орёл с 7.10. Перевод 300 € Lund, WI, Пете и Эдуарду.

(22/VIII) Фотосессия DIPC, "Diario Vasco". Lisl с A.u.n. Переписка с Javier (Dia 20/VI)

(23/VIII) E-mails to V. Kravchuk, Lufthansa, Julian, Airbnb.

(24/VIII) Запрос адреса Arantxa Inmobiliaria (reb. guest AD student, Poznan). Travel report Karaiei, San Sebastian.

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(25/VIII) Analyz simulations by Oleana / Felipe (file Resultados Oxe.docx 11/VII 2020 г.).
Vortex gyromodes in short nanowires. GM thick dots/nanowires

Nanowire (dot) length L = 150 nm ÷ 40 nm. Nanowire radius R = 50 nm.
M_s = 1591,5 G, A = 25 pJ/m ÷ 10 pJ/m. No anisotropy. H_z = 0 – 475 mT.


→ z – dynamic magnetostatic coupling between segments in multisegmented nanowire (face-to-face distance d = 50 nm среднее).
В simulations видно 6-7 eigenmodes within f-range 0 – 20 GHz. Все modes локализованы вблизи Oz-axis вблизи static vortex core.
Видно, все Spin-modes – gyrotropic oscillations of the vortex core string. Т.к. R = 50 nm мал V. core radius сравним с R, надо учитывать vortex core. Оценка $c = R_c/R = 0.15 \div 0.20$.
Все simulated modes имеют max intensity at dot faces $z = \pm L/2$. У меня только (n=0) mode is localized near $z = \pm L/2$, остальные modes – plane waves ⊥ Oz.

(6/VIII) // Расчет Vortex GM wires/dots начат 15/VII 2020 г. (see Oct. 2014. Volume magnetostatic energy within TVA (20/25/VI 2020):
GM wires

$$\frac{1}{V}W_m^{vol} = \frac{1}{2}\int dz dz' \varkappa(zz')\vec{S}(z)\cdot\vec{S}(z') + \frac{1}{2}\int dz dz' \beta(zz')\left(\frac{\partial\vec{S}}{\partial z}\cdot\frac{\partial\vec{S}}{\partial z'}\right) - \frac{1}{2}\int dz dz' P(zz')\left[\frac{\partial\vec{S}}{\partial z}\times\vec{S}(z)\right]_z$$

– содержит new β- и P-terms due to V. core contribution, $\varkappa$ is renormalized.

$$\lambda\vec{S} = \frac{1}{2}\int dz' \frac{\partial P}{\partial z}\vec{S}(z')\times\hat{z} - \mu\vec{S} - \eta\frac{\partial^2\vec{S}}{\partial z^2} + \int dz' \varkappa(zz')\vec{S}(z') - \int dz' \frac{\partial\beta}{\partial z}\frac{\partial\vec{S}(z')}{\partial z'}$$ – new eq. Thiele.

See word file „GM_thick_vortex_dot_wire.doc" (short version „GM_vortex_wire_Farkhad.doc" носила Фархад 6 2020) // P – magnetostatic term VI 2020 г. может predict pC. Осталось неясным if change $p\vec{C} \to -pC$ изменит ω_n или нет?
See speculations о P-term for GM wires/thick dots in July 5-12, 2020, more zero и переключение in Aug. 2020 на BP-Hopf index и AF-dynamics.
MathCad files: „Vortex_TM_long_wires.mcd" и „Vortex_TM_long_wires_1.mcd".

(Felipe Simul.) $L_e = \sqrt{\frac{2A}{M_s^2}} = \sqrt{\frac{2\cdot 2.5\cdot 10^{-6}\,erg/cm}{10^6 (1.592)^2\,erg/cm^3}} = \frac{\sqrt{5}}{1.592}\,10^{-6}\,cm = 14.05\,nm$

$\omega_M/2\pi = \left(\frac{\gamma}{2\pi}\right)4\pi M_s = 3.0\cdot 4\pi\cdot 1.592\,\frac{GHz}{kOe}\,kOe = 60.02\,GHz$, $B_{res}/2\pi = 30\,\frac{GHz}{kOe}$, для FeCo.
оставляем

7/VIII Все вычисления MathCad расчета [Vortex_TM_long_wires_1.mcd] повторяем, следуя расчету 9-12/VII 2020 г., где $\varphi_n(z) = \frac{1}{\sqrt{N_n}}\cos(q_n z)$

$q_n = n\pi$, $z \in [0,1]$, $S(z) = \sum C_n \varphi_n(z)$.

$\frac{\partial\varphi_n}{\partial z} = \frac{1}{\sqrt{N_n}} q_n \sin(\pi n z) = 0$ при $z = 0$ и $z = 1$. $\varphi_n(z)$ удовлетворяет $\left.\frac{\partial S(z)}{\partial z}\right|_{z=0,1} = 0$

В Fmr MathCad file была ошибка. Надо сопрягать π^2 ко второму члену в [...]-скобке. – Не, все там было верно.

Vaz's equation дает equilibrium $c = 0.685\,\frac{L_e}{R}\left(L/L_e\right)^{1/3}$ дает $c_0 = 0.424$, что немного не сравн. с simulations Felipe. Simulated $c = 0.1 - 0.2$.
Возьмем с ручкой, $C = 0.1 \to 0.2$.
N = 50 – dyn. matrix size, „Vortex_TM_long_wires_1.mcd". Получаем eigenfrequencies f_n при n = 42, 41, 40, 43, 44 и получаем eigenfunctions $S(z)$:

$f_{42} = 5.63$ (edge localized), $f_{41} = 5.94$ (two zeros), $f_{40} = 8.11$ (center localized), $f_{43} = 9.39$ (3 zeros), $f_{44} = 13.91$ GHz (4 zeros). $c = 0.1$
ане 2020

$\varphi_n(z) = \frac{1}{\sqrt{N_n}} \cos(\pi n z)$ – orthogonal complete set for $S(z)$ decomposition $n = 0, 1, 2 \ldots$

$N_n = \frac{1}{2}(1+\delta_{n0})$, $z \in [0,1]$ interval. $S(z) = \sum_n C_n \varphi_n(z)$. $\frac{\partial \varphi_n}{\partial z}\Big|_{0,1} = 0$

① Можно ли $n = 0, 1, \ldots$? Можно ли использовать $\cos(\pi n z)$ на $[0,1]$ интервал вместо $\cos(\pi n z)$ $T = 2$.

② Неудовлетворительные speculations 5/VII 2020. Собственные моды $\omega_n(\gamma_n)$ и собственные $\varphi_n(z)$ от Эрмитова $\hat{P}$ term. $\text{sign}(\hat{P}) = P\hat{C}$.

① Арфкен, 14.2.1: $\int_0^L \varphi_m(x)\varphi_n(x)\,dx = \frac{1}{2}L\,\delta_{mn}$, $\varphi_n(x) = \sin\left(n\pi\frac{x}{L}\right)$, $n = 1, 2, \ldots$

$$\int_0^L \cos\left(\pi m \tfrac{x}{L}\right)\cos\left(\pi n \tfrac{x}{L}\right)dx = \left|\tfrac{x}{L} = z,\ dx = L\,dz\right| = L\int_0^1 dz\,\cos(\pi m z)\cos(\pi n z) =$$

$$= \frac{1}{2}L\int_0^1 dz\left[\cos(\pi z(m+n)) + \cos(\pi z(m-n))\right] = \frac{1}{2}L\left[\frac{1}{\pi(m+n)}\sin(\pi z(m+n)) + \right.$$

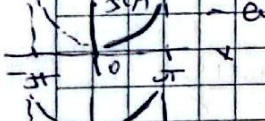
$$\left. + \frac{1}{\pi(m-n)}\sin(\pi z(m-n))\right]_0^1 = \frac{L}{2\pi}\left[\frac{\sin(\pi(m+n))}{m+n} + \frac{\sin(\pi(m-n))}{(m-n)}\right]$$

$[\ldots] = 0$ при $m \neq n$. $[\ldots] = \frac{\sin(2\pi n)}{2n} + \pi = \pi$ при $m \neq n \neq 0$, 2π, $m = n = 0$

$$\int_0^L dx\cos\left(\pi m\tfrac{x}{L}\right)\cos\left(\pi n\tfrac{x}{L}\right) = \frac{L}{2\pi}\left\{\begin{array}{l} 0, \ m \neq n \\ \pi, \ m = n \\ 2\pi, \ m = n = 0 \end{array}\right\}, \quad \int_0^1 dz\cos(\pi m z)\cos(\pi n z) = \left\{\begin{array}{l} 0, \ m \neq n \\ 1/2, \ m = n \neq 0 \\ 1, \ m = n = 0 \end{array}\right\}$$

$$\int_0^1 dz\cos(\pi m z)\cos(\pi n z) = \frac{1}{2}(1+\delta_{n0})\delta_{mn} = N_n\delta_{mn}.$$

→ Арфкен, 14.2: The set of functions $\cos(nx)$, $n = 0, 1, 2 \ldots$ forms a complete orthogonal set over $[0, \pi]$ interval. $\cos(\pi n z)$ — " — over $[0,1]$ interval.



$S(x)$ – even extension of $f(x)$ given at $[0, \pi]$ to $[-\pi, 0]$. Then use $\cos(nx)$ decomposition on $[-\pi, \pi]$ interval.

If $f(x)$ is even with period $2L$ (as a result of even extension from $[0, L]$ to $[-L, 0]$), then half-range Cosine series is

$$f(x) = \frac{1}{2}C_0 + \sum_{n=1}^{\infty} C_n \cos\left(n\pi\frac{x}{L}\right), \quad C_n = \frac{2}{L}\int_{[0,L]}^{L} f(x)\cos\left(n\pi\frac{x}{L}\right)dx, \quad n = 1, 2, 3, \ldots$$

$\frac{x}{L} \to z$ при $L = 1$, $f(z) = \frac{1}{2}C_0 + \sum_{n=1}^{\infty} C_n\cos(\pi n z) \neq \sum_{n=0}^{\infty} 2C_n\cos(\pi n z)\frac{1}{2}[1+\delta_{n0}] =$

$= \sum_{n=0} C_n[\ldots]$ причем $C_n = \frac{2}{L}\int_0^L dx\, f(x)\cos\left(\pi n\frac{x}{L}\right)$, $C_0 = \frac{1}{L}\int_0^L dx\, f(x)$, $n = 0$ (подбирается)

$$f(z) = C_0 + \sum_{n=1} C_n\cos(\pi n z) = \sum_{n=0} C_n\cos(\pi n z).$$

* ① Вообще: $n = 0, 1, 2, \ldots$ и $\varphi_n(z) = \frac{1}{\sqrt{N_n}}\cos(\pi n z)$ можно использовать на $z \in [0,1]$ interval $\int \varphi_n(z)\varphi_m(z) = \delta_{nm}$, $N_n = \frac{1}{2}(1+\delta_{n0})$. $\frac{\partial \varphi_n}{\partial z}\Big|_{z=0,1} = 0$ – Bound. conditions на $\varphi_n(z)$. $\cos(\pi n z)$ – хорошо подходит из $\frac{\partial \varphi_n}{\partial z}\Big|_{0,1} = 0$ b.c.

// $\cos(\pi n z)$ Calculation method suggested in July 2022 is correct.

Numerical c. use $c = 0.2$. Keep $N = 50$.

$f_{38} = 5.42$ (edge localized one zero at $z = 0$), $f_{37} = 5.69$ (two zeros), $f_{39} = 7.91$ (center localized), $f_{40} = 8.31$ (3 zeros), $f_{41} = 12.98$ GHz (4 zeros)

(Numerical c с $c = 0.1$ до $c = 0.2$ привело к небольшому нарушению всех fn. характер eigenfunctions $S_0(z)$ не изменился.

(28/VIII) Duncan Wood like Vortex string gyrotropic mode и нелин. theory. Skype с Oxf. Tuesday, 6/IX, 11 AM.

(29 – 30/VIII) Отдых дома. Бинц.

(31/VIII) СА и М. поехали в рес. San Antonio Getaria Zukhen; Skyrmion modes.

(1/IX) Подготовка/print travel reports Glob-Donostia и Kos-Poland в MPI/EIH.

(2/IX) Organ the travel reports to Eli. Подготовка Ruben's travel reports

Escaneado con CamScanner

(3/IX) Написание MS "BP-DW nanowires" с Фархад. Gyrovector of BP in cylindrical nanowire for Thiaville BP $\vec{m}$-ansatz, см. 15/II.

$$G_s = 4\pi \frac{M_s}{\gamma} I(L/2, b), \quad I(L/2, b) = \int_0^R d\rho\, m_z(L/2, \rho), \quad m_z(z,\rho) = \frac{z}{\sqrt{z^2+\rho^2}} \frac{b^2}{(b^2+\rho^2)}$$

$$I(L/2, b) = \frac{b}{2} t^2 \int_0^1 dz \frac{1}{\sqrt{\beta^2/4 + z^2}} \frac{1}{(z^2+t^2)}, \quad \beta = L/R, \quad t = b/R.$$

Если $\beta \gg 1$, то сводится к решению BP - $\vec{G}$ Vector potential 15/II. $(G_s = 2\pi L \left(\frac{M_s}{\gamma}\right) I_s(\beta, t))$.

$$I_s(\beta/2, t) = t^2 \int_0^1 dz \frac{1}{\sqrt{\beta^2/4 + z^2}} \frac{1}{(z^2+t^2)}. \quad \text{При } \frac{\beta}{2} \gg 1, \; I_s(\beta, t) = t^2 \frac{2}{\beta} \int_0^1 dz \frac{1}{t^2+z^2} =$$

$$= \frac{2}{\beta} t^2 \cdot t \int d(z/t) \frac{1}{t^2} \frac{1}{1+z^2/t^2} = |z = z/t| = \frac{2}{\beta} t^3 \frac{1}{t^2} \int_0^{1/t} dz' \frac{1}{1+z'^2} = \frac{2}{\beta} t \arctg z' \Big|_0^{1/t} =$$

$$= \frac{2}{\beta} t \arctg(1/t), \quad G_s(\beta \gg 1) = 2\pi \left(\frac{M_s}{\gamma}\right) L \cdot \frac{2}{\beta} b \arctg\left(\frac{1}{t}\right) = 4\pi \left(\frac{M_s}{\gamma}\right) b \arctg(1/t), \; t = \frac{b}{R}.$$

Удобнее $G_s = 2\pi L \left(\frac{M_s}{\gamma}\right) I_s(\beta, b)$, $I_s(\beta, t) = t^2 \int_0^1 dz \frac{1}{\sqrt{\beta^2/4 + z^2}} \frac{1}{(t^2+z^2)}$

to compare with $G_z(\text{vortex}) = 2\pi L (M_s/\gamma)$

Также I_s использ. в Ip - решение не нашел.

Знак $\vec{G}$. 15/II $\vec{G} = -\nabla m_z \times \nabla\Phi = \sin\theta \nabla\theta \times \nabla\Phi$

$$G_z = J \int dz \int d\vec{s}\, g_z + J \int d\vec{s} \int dz \left(-\frac{1}{s}\right) \frac{\partial m_z}{\partial s} = |\text{vortex (2D)}| = -J \cdot 2\pi L \int_0^R ds \frac{\partial m_z}{\partial s} =$$

$$= -J 2\pi L \left[m_z(\rho = R) - m_z(0) \right] = 2\pi J L\, m_z(0) = 2\pi J L p, \quad J = M_s/\gamma.$$

Gus' JMM 2008: $\vec{G} = JL \int d^2\vec{s}\, \sin\theta (\nabla\Phi \times \nabla\theta) = J \int dV \vec{g}$, $\vec{g} = \sin\theta \nabla\Phi \times \nabla\theta$

15/II использован $\vec{g} = \sin\theta \nabla\theta \times \nabla\Phi$ with opposite sign. ww

Gus' JAP 2002: $G_z = -2\pi p L (M_s/\gamma)$.

Если BP $m_z = \frac{z}{\sqrt{z^2+s^2}} \frac{b^2}{(b^2+s^2)}$, то надо сменить знак G_s, $G_s \to -G_s$.

Итого $G_s = -2\pi L \left(\frac{M_s}{\gamma}\right) I_s(\beta, t)$ для Thiaville ansatz of BP. $G_s = 4\pi \frac{M_s}{\gamma} I(L/2, b)$

$m_z > 0$ at $z > 0$, $m_z < 0$, at $z < 0$ ↑↓ — BP-DW.

sign $(m_z) = \text{Sign}(z)$. Чтобы $p = \text{sign}(m_z)$ at $z > 0$, то $(G_s = -2\pi p L \left(\frac{M_s}{\gamma}\right) I_s(\beta, t))$

$G_s = -4\pi J \int_0^R ds\, m_z(L/2, s)$. $\text{Sign}(G_s) \sim \text{Sign}(m_z)$ at $z = L/2$, top nanowire face.

(3-4/IX) Написание BP-DW paper с Фархадом (APL Mat.)

(5/IX) Skype - обсуждение BP-paper с Farzkhad и Diego - им нравится ф-ия D. 0 vortex reversal modes, мне - предлагают movies (*.avi).

(6/IX) Отдача с п.ю.о., обед в Nam. Skype с Dec. → посылаю $W(\vec{X})$, π_z (*.avi).

(7/IX) Online обсуждение mutation dynamics possible BLS-measur. с Cambridge (Paris) и Zukkra. Необходимо наблюдать mutation не для metastable skyrmions, а для stable m. vortices. Request access to GAUR.

(8/IX) Оформление (Roza Zagra) доступа в GAUR. Открыли — new password "Kostya 26" (GAUR и email), GAUR passw. - NIE number.

(9/IX) Подготовка к поездке в Krakow, NAP-2022. Завтра - 180/130.

(10/IX) Отказался от поездки в Краков, т.к. утром завтра было ≈ 130/140 с.п.ю. на ниже в Дева, обед в Getaria. Skype с Гилбом.

(11/IX) Сборы дома. Билеты. Гилб отправил NAP-certificate, receipts и online-talk 12/IX.

(12/IX) E-mails to hotel - Airport Bilbao и Аренда, Inmobiliaria.

BP BP dynamics cylindrical nanowire $W(\vec{X}) = W(0) + \frac{1}{2}\varkappa(X^2+Y^2) + \frac{1}{2}\varkappa_z Z^2$

$\vec{X}(X,Y,Z)$ BP center displacement fom nanowire center $\vec{r}(0,0,0)$.

Найти stiffness coficients $\varkappa, \varkappa_z$ в рамках "rigid BP model".

$\vec{m}(\vec{r}, \vec{X}) = \vec{m}(\vec{r} - \vec{X})$, $\vec{X} = \vec{X}(t)$, $W = W_{ex} + W_m$. Найдем $\varkappa_z = \frac{1}{2}\frac{\partial^2 W}{\partial Z^2}\Big|_{\vec{X}=0}$.

$W = W[\vec{m}(\vec{r} - \vec{X})]$. $W = \int dz d^2\vec{s}\, w[\vec{m}(x,y,z-Z)] = \int_{-L/2}^{L/2} dz\, w[\vec{m}(z-Z)]$.

$z - Z = z'$, $W(Z) = \int_{-L/2-Z}^{L/2-Z} dz'\, w[\vec{m}(z')]$, $\frac{\partial}{\partial a}\int_{\psi(a)}^{\varphi(a)} dx f(x,a) = \varphi' f[\varphi(a), a] - f[\psi(a), a]\psi'(a) + \int_\psi^\varphi dx \frac{\partial f}{\partial a}$

$$\frac{\partial W}{\partial z} = \frac{\partial}{\partial z}\left(\frac{L}{2}-z\right)\cdot\overline{W}\left[\vec{m}(z')\right]_{z'=\frac{L}{2}-z} - \frac{\partial}{\partial z}\left(-\frac{L}{2}-z\right)\cdot\overline{W}\left[\vec{m}(z')\right]_{z'=-\frac{L}{2}-z} =$$

$$= -\overline{W}\left[\vec{m}\left(\tfrac{L}{2}-z\right)\right] + \overline{W}\left[\vec{m}\left(-\tfrac{L}{2}-z\right)\right] \quad \vec{m}\left(\tfrac{L}{2}-z\right) = \vec{m}\left(\tfrac{L}{2}\right) - z\vec{m}'\left(\tfrac{L}{2}\right) + \ldots \quad \vec{m}\left(-\tfrac{L}{2}-z\right) = \vec{m}\left(-\tfrac{L}{2}\right) - z\vec{m}'\left(-\tfrac{L}{2}\right)$$

$$\overline{W}\left[\vec{m}\left(\tfrac{L}{2}-z\right)\right] = \overline{W}\left[\vec{m}\left(\tfrac{L}{2}\right) - z\vec{m}'\left(\tfrac{L}{2}\right)\right] = W\left[\vec{m}\left(\tfrac{L}{2}\right) - z\vec{m}'\left(\tfrac{L}{2}\right)\right] \quad \vec{m}' = \frac{\partial}{\partial z}\vec{m}(z)$$

$$W\left[\vec{m}(z')\right] = \int d^2\rho\, e\left[\vec{m}(x,y,z')\right], \quad z' = \pm\frac{L}{2} - z \quad W(z) = \int d^2\rho \int_{-\frac{L}{2}-z}^{\frac{L}{2}-z} e\left[\vec{m}(x,y,z')\right]$$

$$W\left[\vec{m}\left(\tfrac{L}{2}\right) - z\frac{\partial\vec{m}}{\partial z}\left(\tfrac{L}{2}\right)\right] = W\left[\vec{m}_0 + \int\vec{\mu}\right] = W(\vec{m}_0) + \int\vec{\mu}\cdot\frac{\partial W}{\partial\vec{m}}(\vec{m}_0) =$$

$$= W\left[\vec{m}\left(\tfrac{L}{2}\right)\right] - z\frac{\partial\vec{m}}{\partial z}\cdot\frac{\partial W}{\partial\vec{m}}\left(\tfrac{L}{2}\right), \quad W\left[\vec{m}\left(-\tfrac{L}{2}\right) - z\frac{\partial\vec{m}}{\partial z}\left(-\tfrac{L}{2}\right)\right] = W\left[\vec{m}\left(-\tfrac{L}{2}\right)\right] - z\frac{\partial\vec{m}}{\partial z}\cdot\frac{\partial W}{\partial\vec{m}}\left(-\tfrac{L}{2}\right)$$

$$\frac{\partial W}{\partial z} = -W\left[\vec{m}\left(\tfrac{L}{2}-z\right)\right] + W\left[\vec{m}\left(-\tfrac{L}{2}-z\right)\right] = -\left\{W\left[\vec{m}\left(\tfrac{L}{2}\right)\right] - z\frac{\partial\vec{m}}{\partial z}\cdot\frac{\partial W}{\partial\vec{m}}\left(\tfrac{L}{2}\right)\right\} +$$

$$+ W\left[\vec{m}\left(-\tfrac{L}{2}\right)\right] - z\frac{\partial\vec{m}}{\partial z}\cdot\frac{\partial W}{\partial\vec{m}}\left(-\tfrac{L}{2}\right) = \text{const}(z) + z\left\{\frac{\partial\vec{m}}{\partial z}\cdot\frac{\partial W}{\partial\vec{m}}\left(\tfrac{L}{2}\right) - \frac{\partial\vec{m}}{\partial z}\cdot\frac{\partial W}{\partial\vec{m}}\left(-\tfrac{L}{2}\right)\right\}$$

$$\frac{\partial^2 W}{\partial z^2} = \frac{\partial\vec{m}}{\partial z}\cdot\frac{\partial W}{\partial\vec{m}}\Big|_{z=\frac{L}{2}} - \frac{\partial\vec{m}}{\partial z}\cdot\frac{\partial W}{\partial\vec{m}}\Big|_{z=-\frac{L}{2}} \text{ – проверить.}$$

$m_z(z,\rho) = \frac{z}{\sqrt{z^2+\rho^2}}\frac{b^2}{(b^2+\rho^2)}$, $m_\varphi = 0$, $m_\rho = \sqrt{1-m_z^2}$ Задает $\vec{m}(\vec{\rho},z)$ для BP.

$W = \int dz \int d^2\rho\, e\left[\vec{m}(\vec{n})\right] = \int dz \int d^2\rho\, A(\nabla\vec{m})^2$ для exchange energy.

$w = \int d^2\rho\, A(\nabla\vec{m})^2 = w(\vec{n})$. – тогда $W(z) = A\int d^2\rho\,(\nabla\vec{m})^2$ – functional $\vec{m}(\vec{\rho},z)$ at given z

Вид $W_{ex}[\vec{m}(\rho,z)]$ – см. 18/II для centered BP in nanowire.

(13/IX) переписка с Valentina и Ruben о вкл. Valentina в Ricardo's Thesis tribunal (отказала). Доклад MAP-22, Krakow, 3D charges – online.

(14/IX) Предложил Л. Тартаковской to be jury member (не нужен – отказ).

(15/IX) Ruben выложил Travel reports Ruben поездок в Madrid и Bologna, послал emy files. Обед в Zaranz с Л. и Ю.

(16/IX) О-вых дома. Работа в саду. Бланш. Вопросы Alex, Felipe о GM nanowire sim.

(17/IX) Вылетел СП. в Hotel „New Bilbao airport", ужин в ресторанчике.

(18/IX) Вернулись из Bilbao. О-вых. Бланш.

(19/IX) Написал Ruben'y про Celia Sousa (Porto) согласилась быть jury member. Работа над Artem „Vortex GM dome-shape dots" manuscript.

(20/IX) Послал revised (pdf) manuscript Ruben'y и Артему. Разбор ответа Felipe по GM simulations in short (150 nm) $R = 50$ nm nanowire.

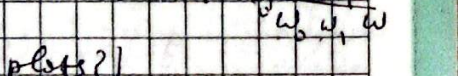
Ответ Felipe: ① mode intensity FFT(m_y)

$H_y(t) = H_0 \sin(\omega_0 t)$ – excitation mw field, $\text{sinc}(x) = \frac{\sin(x)}{x}$

$\chi(\omega) = \chi' + i\chi''(\omega)$ is calculated, namely $\chi_{yy}(\omega) = \frac{\langle M_y(\omega)\rangle}{H_y(\omega)}$,

$\langle\ldots\rangle = \int dV(\ldots)\frac{1}{V}$ – volume average. Intensity $(\omega) = |\chi''_{yy}(\omega)|$

$M_y = M_y(\vec{r},\omega)$, $\overline{M_y}(\omega) = \langle M_y(\vec{r},\omega)\rangle$



② mode spatial distributions (how is it defined in plots?)

FFT $m_y(\vec{r},\omega_n)$ is simulated at $\omega = \omega_n$. $|m_y(\vec{r},\omega_n)|$ is plotted as mode-n distribution

③ Static vortex core radius? R_c (faces, $z = \pm L/2$) ≈ 29 nm, $R_c(z=0) = 32$ nm at $R = 50$ nm.

intensity definition by Artem $I(\omega) \sim \int dV |m_i(\vec{r},\omega)|^2$, $i = x,y$.

④ Зависимости ω_n и $S_n(z,t)$ от pC (P-term). Для $n=0$ GM, $f_n = 0.537$ GHz $C = \pm 1$ chorality (P is fixed) дают одинаковую частоту ω_0 и одинаковые VC trajectories $\vec{X}(t)$, так зависимости фаз $\Phi(z)$ разных $g_n(z) = u_n(z)\,e^{i\Phi(z)} = S_n(z,\omega_n)$

При $z=0$, middle plane V. core колебывается $\parallel \vec{X}$ и создается общая общая с $m_z = -1$ edge

$n=0$ mode localization at faces $z = \pm L/2$ замечено.

Escaneado con CamScanner

Equilibrium a (edge, middle plane) = 32 nm / 500 nm = 0,64 $c = R_0/R$. $L = 150$ nm $R = 50$ nm $M_s = 1590$ G

MathCad „Vortex-TM-long-wires-1.mcd" ho opabu. c parametrom 87/nm (zavisnost od magnetizacije), $N = 50$:

$N=50$, $f_{20} = 3,70$ GHz (edge localized) $f_{30} = 4,37$ (edge localized) $f_{31} = 8,38$ (two zeros) $f_{33} = 14,31$ (3 zeros) $f_{34} = 20,33$ GHz (4 zeros)

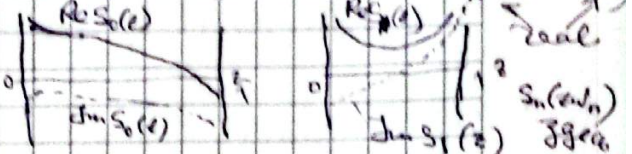
$N=100$, $f_{73} = 3,64$ GHz (edge localized) $f_{74} = 4,35$ (edge localized) $f_{75} = 8,96$ (2 zeros) $f_{76} = 14,20$ (3 zeros) $f_{78} = 20,32$ GHz (4 zeros)

0 $S_n(z)$ cygan us $|S_n(z)|$ - vonopuna MathCad cthoma. $S_n(z)$ are complex

$S_n(z) = \sum_n C_{nm} \cos(\pi m z)$, C_{nm} are complex. $S_n(z) = S_n(z,\omega) = S_n^x(z,\omega) + i S_n^y(z,\omega)$

$S_n(z,\omega) = u_n(z) e^{i\varphi(z)}$, $S_n(\omega t) = S_n(z,\omega) e^{i\omega t} e^{i\varphi(z)} = u_n(z) e^{i(\omega t + \varphi(z))} = S_n^x(t) + i S_n^y(t)$

$S_n^x(t,\omega)$ u $S_n^y(t,\omega)$ are complex in general.



Simulated $f_0 = 0,57$ GHz, calculated $f_0 = 3,04$ GHz.

Nuxo. Xoru mode profiles $|S_n(z,\omega_n)|$ nohoxu na simulated $|m_y(\vec{r},\omega_n)|$ profiles cy Felipe.

Uzaver: razlozbene po pazhoznice no $\cos(n\pi z)$, $z \in [0,1]$ yzgoguo, b $\partial M_y/\partial z|_{z=0,1} = 0$ Bound. conditions? Ltanoh zybara razlozbene no $e^{i\varphi_n z}$, φ_n resen c uepuogom $T = 1$ no z.

Gus' APL 2006: $\langle \vec{m}(\vec{r},t) \rangle_V = -\frac{1}{2} C \hat{z} \times \vec{S}(t)$, $\chi = \frac{2}{3}$ for TVA. $\chi''_{xx}(\omega)$ u/um $H_x(t)$

- linearly polarized mw field was calculated for thin dot geometry V. gyromode $\chi''(\omega) \neq 0$ us-za damping duc $\neq 0$.

Ecm no z ne yopegnuca, to $\langle \vec{m}(\vec{r},t) \rangle_S = -\frac{1}{2} C \hat{z} \times \vec{S}(z,t) = \vec{M}(z,t)$

Zeeman energy $W_H = \frac{1}{2}(C M_s/R) V [\hat{z} \times \vec{H}] \cdot \vec{X}(z,t)$, V - dot volume. Formee W_H:

$W_H = -\int dV \vec{M} \cdot \vec{H} = -\int dz \iint dS \vec{M}(z,t) \cdot \vec{H}(t) = -\int dz M_s \iint dS \vec{m}(\vec{r},t) \cdot \vec{H}(t) S =$

$= -S \int dz \langle \vec{m}(\vec{r},t) \rangle_S \cdot \vec{H} M_s = -S M_s \vec{H} \cdot \int dz (-\frac{1}{2}) C \hat{z} \times \vec{S}(z,t) = \frac{S}{R} M_s \frac{1}{2} C \int dz \vec{H} \cdot [\hat{z} \times \vec{X}(z,t)]$

$\vec{H} \cdot [\hat{z} \times \vec{X}] = \vec{c} \cdot (\vec{b} \times \vec{a}) = \vec{c} \cdot [\vec{a} \times \vec{b}] = \vec{X} \cdot [\vec{H} \times \hat{z}]$, $W_H = \frac{S}{R} M_s \frac{1}{2} C \int dz [\vec{H} \times \hat{z}] \cdot \vec{X}(z,t)$

$\frac{\delta W_H}{\delta \vec{X}} = \frac{\partial W_H}{\partial \vec{X}} = \frac{\partial}{\partial \vec{X}} \frac{S}{R} M_s \frac{1}{2} C [\vec{H} \times \hat{z}] \cdot \vec{X} = \frac{S}{R} M_s \frac{1}{2} C [\vec{H} \times \hat{z}]$, $W_H = \int dz w_H \cdot \frac{S}{R}$, $S = \pi R^2$.

Thiele eq. for given z: $\vec{G} \times \dot{\vec{s}} = -\frac{\delta W}{\delta \vec{s}}$, $W = \frac{V}{\pi} \int dz w$, z in units of L here.

Definition of W_H: $W_H = \frac{1}{2} L \int dz \frac{1}{\pi} w_H = \frac{V}{\pi} \int dz w_H$. $W_H = \frac{1}{2} M_s \frac{1}{2} C \int_0^L dz [\vec{H} \times \hat{z}] \cdot \vec{S} L$

$w_H = \frac{\pi}{V} \cdot \frac{1}{2} M_s \frac{1}{2} C L [\vec{H} \times \hat{z}] \cdot \vec{S}$ - Gus' Magnetism, 2022 Gus' APL 2022

$W_H = \pi M_s \frac{1}{2} C [\vec{H} \times \hat{z}] \cdot \vec{S}(z,t)$. $\frac{\delta W_H}{\delta \vec{S}} = \frac{\partial W_H}{\partial \vec{S}} = \pi M_s \frac{1}{2} C [\vec{H} \times \hat{z}]$

B YD Thiele Eq. (8) GM_vortex_wire_theory.doc" $\lambda \dot{\vec{s}} = \Gamma \vec{s}$ dobaBun B u/ebuyuo yacm term Zeeman $\pi M_s \frac{1}{2} C [\vec{H} \times \hat{z}]$ no zabucumocu or $\vec{S}(z,t)$. W_H gender YD gna $\vec{s}(z)$ desgno/ogeneu. $\vec{A} = [\vec{H} \times \hat{z}] = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ H_x & H_y & 0 \\ 0 & 0 & 1 \end{vmatrix} = \hat{x} H_y - H_x \hat{y}$, $\frac{\delta W_H}{\delta \vec{S}} = \pi M_s \frac{1}{2} C \vec{A}$

Com. 11/11: $g_z(\dot{s}_x + i\dot{s}_y) = (\text{terms without } \vec{H}) = -\frac{\delta W_H}{\delta s_y} + i \frac{\delta W_H}{\delta s_x} = \pi M_s \frac{1}{2} C [-A_y + iA_x]$

$g_z \dot{\vec{s}} = (\text{terms at } \vec{H}=0) + i \pi M_s \frac{1}{2} C A$, $A = A_x + iA_y = H_y + i(-H_x) = -i(H_x + iH_y) = -iH$

$g_z \dot{\vec{s}} = (\text{terms at } \vec{H}=0) + \pi M_s \frac{1}{2} C H(t)$. Fourier transform $S(t) = S(\omega) e^{i\omega t}$, $H(t) = H_0 e^{i\omega t}$

$\dot{S} = i\omega S(\omega)$. Igoiuo gna circular field $H(t) = H_0 e^{i\omega t}$

$g_z \omega S(\omega t) = 2\frac{\delta W}{\delta \vec{S}} = (H=0 \text{ term}) + 2\frac{\delta W_H}{\delta \vec{S}}$. $W_H = \pi M_s \frac{1}{2} C \vec{A} \cdot \vec{S} = \frac{1}{2} \pi M_s \frac{1}{2} C [\vec{A} S + A \vec{S}]$

$\frac{\delta W_H}{\delta \vec{S}} = \frac{\delta W_H}{\partial \vec{S}} = \frac{1}{2} \pi M_s \frac{1}{2} C A$, $g_z \omega S(\omega t) = (H=0) + \pi M_s \frac{1}{2} C A(t)$. $\Rightarrow A(t) = A_0 e^{i\omega t}$.

$g_z \omega S(\omega t) = (H=0 \text{ term}) + \pi M_s \frac{1}{2} C A_0$, $(H=0 \text{ term}) = 2\frac{\delta W}{\delta \vec{S}} + \pi \Gamma S(\omega)$. $g_z \frac{\omega}{\pi} S = \Gamma S + \frac{\pi}{\pi} M_s \frac{1}{2} C A_0$

Escaneado con CamScanner

$g_2\omega/\pi = \frac{2\pi M_s}{\gamma}\frac{\omega}{\pi} = \frac{2M_s\omega}{\gamma}$. Делим УД на $8\pi M_s^2$, $\frac{2M_s\omega}{\gamma\cdot 8\pi M_s^2} = \frac{\omega}{\omega_M} = \lambda$

$\frac{M_s}{8\pi M_s^2} = \frac{1}{8\pi M_s}$. $\lambda S = \hat{\Gamma} S + \frac{3C}{8\pi M_s} A_0$, $A(t) = -iH(t) = A_0 e^{i\omega t} = -iH_0 e^{i\omega t}$

УД с круговой circular $\vec{H}(t)$, $H = H_0 e^{i\omega t}$: $(\lambda S(z,\omega) = \hat{\Gamma} S - i\frac{3C}{8\pi M_s} H_0)$

Можно $\vec{H} = (0, H_y)$ linear polarization field.

Но $H_y(t) = H_0 \sin(\omega_c t)$ синусная симметрично, даже $\chi(\omega)$ надо искать response $\vec{m}(\omega)$ на oscillating field $\vec{H}(t)$, $\vec{H} = (0, H_0 \sin\omega t)$ линейно поляриз.

$H_y(t) = H_0 \sin c(\omega_c t)$ не есть periodic // $\vec{H} = (H_x, H_y)$, $H_x + iH_y = H_0 e^{i\omega t}$ circular periodic field.

// Quasi - $\chi(\omega) = \frac{\langle M(r,\omega)\rangle}{H(\omega)}$ даёт пики at $\omega = \omega_n$ из-за пиков $\langle M(r,\omega)\rangle$ at eigenfrequencies ω_n.

Это пики power spectral density $PSD(\omega) = |\langle M(r,\omega)\rangle|^2$.

(20/IX) Вапнярчук в Лозанне защищал habilitation, т.е. не PhD.

(21/IX) – (22/IX) E-mail Зухре о mutation term, предложила вместо Skyrmion ввести mutation in vortex GM. Работа над MS с фразой metastable BP" - послал ему almost final version.

Переписка с Оксаной и Felipe о dyn. susceptibility $\chi(\omega)$, $PSD(\omega)$ и пиками при eigenfrequencies ω_n – С п.и.ю. в „La Perla".

(23/IX) Работа в саду. Бинт.

(24/IX) Dynamical Susceptibility.

① Гур.-Мелков, гл. 1., Complex $\hat{\vec{m}}(t) = \vec{m} \cdot e^{i\omega t}$, $\hat{\vec{h}}(t) = \vec{h} e^{i\omega t}$.

$\vec{m}_x(t) = Re(\hat{\vec{m}})$, $\vec{h}_x(t) = Re(\hat{\vec{h}})$ $\vec{m}, \vec{h}$ – complex amplitudes.

$\vec{m} = \hat{\chi}(\omega)\vec{h}$ – definition of susceptibility. $\vec{h}$ is not Fourier transform of $\vec{h}(t)$ т.е. $\chi(\omega) \sim \vec{m}(\omega)$.

② R.M. White Quantum th. of magnetism, 2007. $\chi(\omega)$

Без $\vec{k}$ - wave vector and without x,y,z indexies.

$\vec{M}(t) = \vec{M}(\vec{r},t) = \frac{1}{2\pi}\int d\Omega\, \vec{M}(\Omega) e^{-i\Omega t}$, $\vec{H}(t) = \frac{1}{2\pi}\int d\omega\, \vec{H}(\omega) e^{-i\omega t}$

$M(\Omega) = \int dt\, M(t) e^{i\Omega t}$, $H(\omega) = \int dt\, H(t) e^{i\omega t}$.

$M(\Omega) = \int d\omega\, \chi(\Omega,\omega) H(\omega)$. Если $H(t) = H_0 e^{-i\omega' t}$ monochromatic (ω'),

то $H(\omega) = \int_{-\infty}^{\infty} dt\, H(t) e^{i\omega t} = H_0 \int_{-\infty}^{\infty} dt\, e^{i(\omega-\omega')t} = H_0 2\pi\delta(\omega-\omega')$

$M(\Omega) = \int d\omega\, \chi(\Omega,\omega)\delta(\omega-\omega') 2\pi H_0 = 2\pi H_0 \chi(\Omega,\omega')$

Нужно не так. Если среда is stationary, $\chi(t) = \chi(t-t')$, то $\chi(\Omega,\omega) = \chi(\omega) \times \delta(\Omega-\omega)$

– monochromatic M-response to monochromatic $H(t)$.

$\chi_{\mu\nu}(\omega) = \frac{i}{\hbar}\int_0^{\infty} dt\, e^{i\omega t}\langle[\hat{M}_\mu(t), \hat{M}_\nu(0)]\rangle$ – is Fourier transform of response function

Можно $\int_{-\infty}^{\infty} dt(\ldots)$ если сделать double-time retarded Green function

$G_{\mu\nu}(t) = \langle\langle \hat{M}_\mu(t) M_\nu(0)\rangle\rangle = i\Theta(t)\langle[\hat{M}_\mu(t), \hat{M}_\nu(0)]\rangle$ откуда FDT $\chi(\omega)$

$\langle\ldots\rangle = Tr(e^{-\beta\hat{H}}\ldots)\frac{1}{Z}$ – thermodyn. average in equilibrium.

If $f(t) = \sin c(at)$, $\hat{f}(\omega) = \int dt f(t) e^{-i\omega t} = \frac{1}{|a|} rect\left(\frac{\omega}{2\pi a}\right)$, $\sin c(x) = \sin(\pi x)/\pi x$.



Escaneado con CamScanner

$f(t) = \frac{1}{\pi}\frac{\sin(\pi a t)}{(at)}$, $\omega_m = \pi a$, $f_m = \frac{\omega_m}{2\pi} = \frac{a}{2}$. $\mathcal{F}(at) = \frac{\sin(\omega_m t)}{\omega_m t}$

Функция $F(t) = \frac{\sin(\omega_m t)}{t}$, $\frac{\omega_m}{2\pi} = 40$ GHz, t в ns, ω_m in GHz·2π

$\omega_m = 40$ GHz·2π $\approx 240 \frac{\text{rad}}{\text{ns}}$. При больших ω_m, $F(t, \omega_m)$ близка к $\delta(t)$-delta function.

(25/IV) Проверим P и Z-terms в YD для vortex string $S(z,\omega)$.

„GM_vortex_wire_Oksana.doc". Система задана с $q_n(z) = \cos(\pi n z)$ $z \in [0,1]$

$\partial q_n/\partial z|_{z=0,1} = 0$ для e^{iqnz}, $q_n = 2\pi n$, нормир по $z = 1$.

Сом. 25/IV 2020 $W_m^{vol} = -\frac{1}{2}V\int dz dz' P(zz')\left[\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z')\right]_z$, $P(zz') = P(z'z)$

$W = \frac{V}{\pi}\int dz w$, $\frac{1}{\pi}W_m^{vol} = -\frac{1}{2}\int dz' P(zz')\left[\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z')\right]_z$ - en. density без симметризации

$\left[\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z')\right]\cdot\hat{z} = \frac{\partial \vec{S}}{\partial z}\cdot\left[\vec{S}(z') \times \hat{z}\right]$, $\frac{1}{\pi}W_m^{vol} = -\frac{1}{2}\int dz' P(zz')\left[\vec{S}(z') \times \hat{z}\right]\cdot\frac{\partial \vec{S}}{\partial z}$

$\frac{1}{\pi}\frac{\delta W_m}{\delta \vec{S}(z)} = \frac{1}{\pi}\frac{\partial W_m}{\partial \vec{S}} - \frac{1}{\pi}\frac{\partial}{\partial z}\frac{\partial W_m}{\partial(\partial \vec{S}/\partial z)} = \left|\frac{\partial W_m}{\partial \vec{S}(z)} = 0\right| = -\left(+\frac{1}{2}\right)\frac{\partial}{\partial z}\frac{\partial}{\partial(\partial \vec{S}/\partial z)}$

$\int dz' P(zz')\left[\vec{S}(z') \times z\right]\cdot\frac{\partial \vec{S}}{\partial z} = \frac{1}{2}\frac{\partial}{\partial z}\int dz' P(zz')\left[\vec{S}(z') \times \hat{z}\right] = \frac{1}{2}\int dz' \frac{\partial P}{\partial z}\left[\vec{S}(z') \times \hat{z}\right]$

7/IX 2014 W_m^{vol} получена в сим. форме $\left[\frac{\partial \vec{S}}{\partial z} \times S(z')\right]\cdot\hat{z} + \left[\frac{\partial \vec{S}}{\partial z'} \times \vec{S}(z)\right]\cdot\hat{z}$

Если $\int dz dz'(\ldots)$ то нет разницы между сим. и не сим. формами.

Но при определении en. density w, $W = \frac{V}{\pi}\int dz w$, $\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z')$ и $\frac{\partial \vec{S}}{\partial z'} \times \vec{S}(z)$ не эквивалентны, нельзя $(1)+(2) = 2\cdot(1)$ как в W (1)

Надо использовать сим. форму W равно как 25/IV 2020.

$W_m = -\frac{1}{4}V\int dz dz' P(zz')\left[\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z')\right]_z - \frac{1}{4}\int dz dz' P(zz')\left[\frac{\partial \vec{S}}{\partial z'} \times \vec{S}(z')\right]_z$

$\frac{1}{\pi}W_m = -\frac{1}{4}\int dz' P(zz')\left[\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z')\right]_z + \frac{1}{4}\int dz' P(zz')\left[\frac{\partial \vec{S}}{\partial z'} \times \vec{S}(z')\right]_z$

$\hat{z}\cdot\left[\vec{S}'(z) \times \vec{S}(z')\right] = \vec{S}'(z)\cdot\left[\vec{S}(z') \times \hat{z}\right]$, $\hat{z}\cdot\underbrace{\left[\vec{S}'(z') \times \vec{S}(z)\right]}_{c} = \vec{S}'(z')\cdot\left[\vec{S}(z) \times \hat{z}\right] =$

$= c\cdot[a \times b] = \vec{S}(z)\cdot\left[\hat{z} \times \vec{S}'(z')\right]$

$\frac{1}{\pi}W_m = -\frac{1}{4}\int dz' P(zz')\left[\vec{S}(z') \times \hat{z}\right]\cdot\frac{\partial \vec{S}}{\partial z} - \frac{1}{4}\int dz' P(zz')\left[\hat{z} \times \frac{\partial \vec{S}}{\partial z'}\right]\cdot\vec{S}(z)$

$\frac{1}{\pi}\frac{\delta W_m}{\delta \vec{S}(z)} = \frac{1}{\pi}\frac{\partial W_m}{\partial \vec{S}} - \frac{1}{\pi}\frac{\partial}{\partial z}\frac{\partial W_m}{\partial(\partial \vec{S}/\partial z)} = -\frac{1}{4}\int dz' P(zz')\left[\hat{z} \times \frac{\partial}{\partial z'}\right] -$

$- \frac{\partial}{\partial z}\left[-\frac{1}{4}\int dz' P(zz')\left[\vec{S}(z') \times \hat{z}\right]\right] = -\frac{1}{4}\int dz' P\left[\hat{z} \times \frac{\partial \vec{S}}{\partial z'}\right] + \frac{1}{4}\frac{\partial}{\partial z}\int dz' P(zz')\left[\vec{S}(z') \times \hat{z}\right] =$

$= \frac{1}{4}\left[\int dz' P(zz')\frac{\partial \vec{S}}{\partial z'} + \frac{\partial}{\partial z}\int dz' P(zz')\vec{S}(z')\right] \times \hat{z}$ - далее можно дифференцировать только предполагая, как 25/IV 2020, $\vec{S}(z)|_{z=\pm 1/2} = 0$, pinning boun. conditions, несовместимо с $\partial q_n/\partial z|_{z=0,1} = 0$, $\cos(\pi n z)$ методом.

$\frac{1}{\pi}\frac{\delta W_m}{\delta \vec{S}} = \frac{1}{4}\int dz'\left[P(zz')\frac{\partial \vec{S}}{\partial z'} + \frac{\partial P}{\partial z}\vec{S}(z')\right] \times \hat{z}$. было 25/IV 2020 $\frac{1}{\pi}\frac{\delta W_m}{\delta \vec{S}} = \frac{1}{2}\int dz' \frac{\partial P}{\partial z}\vec{S}(z') \times \hat{z}$ это неверно

→ т.о. Eq. (8) Word file GM_vortex_wire_Oksana неверно. в P-члене.

Vortex string dynamics с уклоном V. core Записана in April - July 2020, вернулся к ней Aug. 25 / 2022 для paper с Oksana - Felipe.

B-term. $W_m = \frac{1}{2}V\int dz dz' \beta(zz')\left(\frac{\partial \vec{S}}{\partial z}\cdot\frac{\partial \vec{S}}{\partial z'}\right)$, $\frac{1}{\pi}W_m = \int dz' \beta(zz')\left(\frac{\partial \vec{S}}{\partial z}\cdot\frac{\partial \vec{S}}{\partial z'}\right)$

$\frac{1}{\pi}\frac{\delta W_m}{\delta \vec{S}} = \frac{1}{\pi}\frac{\partial W_m}{\partial \vec{S}} - \frac{1}{\pi}\frac{\partial}{\partial z}\frac{\partial W_m}{\partial(\partial \vec{S}/\partial z)} = \left[\frac{\partial}{\partial \vec{S}} - \frac{\partial}{\partial z}\frac{\partial}{\partial(\partial \vec{S}/\partial z)}\right]\int dz' \beta(zz')\left(\frac{\partial \vec{S}}{\partial z}\cdot\frac{\partial \vec{S}}{\partial z'}\right) =$

$= +\frac{\partial}{\partial z}\int dz' \beta(zz')\frac{\partial \vec{S}}{\partial z'} = -\int dz' \frac{\partial \beta(zz')}{\partial z}\frac{\partial \vec{S}(z')}{\partial z'}$, это совпадает с Eq. (8) word file.

Escaneado con CamScanner

Переход от $\vec{S}$ к $S = S_x + iS_y$ (представление), см. 2/V 2020.

$$\frac{1}{\pi} W_m = -\frac{1}{4}\int dz' P\left[\frac{\partial \vec{S}}{\partial z'} + \vec{S}(z')\right]_z - \frac{1}{4}\int dz' P\left[\frac{\partial \vec{S}}{\partial z'} \times \vec{S}(z')\right]_z = -\frac{1}{4}\int dz' P\left\{\left[\frac{\partial \vec{S}}{\partial z'} + \vec{S}(z')\right]_+ + \left[\frac{\partial \vec{S}}{\partial z'} \times \vec{S}(z')\right]_\times\right\} =$$

$$= -\frac{1}{4}\int dz' P\left\{\vec{S}'(z)S(z') - S'(z)\vec{S}(z') + \vec{S}'(z')S(z) - S'(z')\vec{S}(z)\right\}$$

Для чтения $\vec{S}'(z) \times \hat{z} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ S_x & S_y & 0 \\ 0 & 0 & 1 \end{vmatrix} = \hat{x}S_y - \hat{y}S_x$, $\vec{S} = \vec{S}(z,w)$

$$\lambda \vec{S} = \frac{1}{\pi}\frac{\delta W}{\delta \vec{S}} = (\delta e_3 P) + \frac{1}{\pi}\frac{\delta W_m}{\delta \vec{S}}(P) \quad \Big| \quad \frac{1}{\pi}\frac{\delta W_m(P)}{\delta \vec{S}'} = \frac{1}{4}\int dz' \left[P\frac{\partial \vec{S}}{\partial z'} + \frac{\partial P}{\partial z}\vec{S}(z')\right] \times \hat{z} =$$

$$= \frac{1}{4}\int dz' \left[P\frac{\partial}{\partial z'} + \frac{\partial P}{\partial z}\right]\vec{S}'(z') \times \hat{z} = \frac{1}{4}\int dz' \left[P\frac{\partial}{\partial z'} + \frac{\partial P}{\partial z}\right]\left(\hat{x}S_y(z') - \hat{y}S_x(z')\right)$$

$$\lambda S_x = \frac{1}{4}\int dz' \left[P\frac{\partial}{\partial z'} + \frac{\partial P}{\partial z}\right] S_y(z'), \quad \lambda S_y = -\frac{1}{4}\int dz' \left[P\frac{\partial}{\partial z'} + \frac{\partial P}{\partial z}\right] S_x(z')$$

$$\lambda(S_x + iS_y) = \lambda S(z,w) = \frac{1}{4}\int dz' \left[P\frac{\partial}{\partial z'} + \frac{\partial P}{\partial z}\right]\left[S_y(z') - iS_x(z')\right] =$$

$$= -\frac{i}{4}\int dz' \left[P\frac{\partial}{\partial z'} + \frac{\partial P}{\partial z}\right] S(z'), \quad \left(\lambda S = (\delta e_3 P) - \frac{i}{4}\int dz' \left[P\frac{\partial}{\partial z'} + \frac{\partial P}{\partial z}\right] S(z')\right)$$

Далее было в word file $\lambda S = (\delta e_3 P) - \frac{i}{2}\int dz' \frac{\partial P}{\partial z} S(z')$

Смысл и сф перенесена в new P-term.

Роль pC-sign в P-term, см. 5/VII 2020, ω_n-те же, сф меняются.

$$\frac{1}{\pi}\frac{\delta W_m^\partial}{\delta \vec{S}} = -\int dz' \frac{\partial \beta(zz')}{\partial z}\frac{\partial \vec{S}(z')}{\partial z'}, \quad \int dz' \frac{\partial \beta}{\partial z'}\vec{S}'(z') = -\int dz' \frac{\partial \beta}{\partial z'}\vec{S}'(z') =$$

$$= \left| \int dz' \frac{\partial \beta}{\partial z'} = d\nu,\ \nu = \beta,\ \frac{\partial \vec{S}}{\partial z'} = u,\ u' = \vec{S}''(dz') \right| = -\left[\frac{\partial \vec{S}(z')}{\partial z'}\beta(zz')\Big|_{z'=0}^{1} - \int dz' \beta(zz')\vec{S}''(z')\right] = \left| \text{если } \frac{\partial \vec{S}}{\partial z'} = 0 \text{ at } z' = 0,1 \right| = +\int dz' \beta(zz')\frac{\partial^2 \vec{S}}{\partial z'^2}$$

$$\frac{1}{\pi}\frac{\delta W_m^\partial}{\delta \vec{S}} = -\int dz' \beta(zz')\frac{\partial^2 \vec{S}}{\partial z'^2}(z'), \text{ см. 26/IV, 9/VII 2020, если } \frac{\partial \beta(z)}{\partial z}\Big|_{z=0,1} = 0 \text{ граничное.}$$

(*) $$\lambda S = \left[-v\frac{\partial}{\partial z} - \mu\right] S(z) + \int dz' \varkappa(zz') S(z') - \frac{i}{4}\int dz' \left[P\frac{\partial}{\partial z'} + \frac{\partial P}{\partial z}\right] S(z') - \int dz' \beta(zz')\frac{\partial^2 S}{\partial z'^2}$$

Спектр eq. с corrected P-term и с $\frac{\partial S}{\partial z}\Big|_{z=0,1} = 0$ для β-term, подходящие для нахождения расчета с $\omega_n(z) = \frac{1}{\sqrt{N}}\cos(\pi n z)$.

? Роль W_m^{face} в V. core dynamics нельзя — см. 25/IV 2020. Vortex GM wires

(26/IX) Вчера Eli signed travel reports of Ruben (Madrid, Donostia). Вчера также meeting был с Julian, рассказал Gerard 29/IX. Состав комиссии (jury) Ricardo defence: Hillebrandt, Chantrell, Augustina, Luis Lopez C. Sousa. Julian внесена будет не как/если внешний 2-й foreigners (Hillebrands, Chantrell) и нужно это donation был + предложить как/если в jury (заменить C. Sousa на Marcia Blanco?). Сл. и Correction: Diagram.

(27/IX) Начало современного расчета на $\varphi_n(z) = \frac{1}{\sqrt{N}}\cos(\pi n z)$ — 9/VII 2020.

В Shiela eq. будет $\int dz' P(zz')\frac{\partial S(z')}{\partial z'} = \left| d\nu = dz' P,\ \nu = \int dz' P(zz'),\ \frac{\partial S}{\partial z'} = u \right| =$

$$= \frac{\partial S(z')}{\partial z'}\int dz' P(zz')\Big|_{z'=0,1} - \int dz' \left[\int dz' P(zz')\right]\frac{\partial^2 S}{\partial z'^2} = \left| \frac{\partial S(z')}{\partial z'}\Big|_{z'=0,1} = 0, \text{ free b.c.} \right| =$$

$$= -\int dz' \left[\int_0^{z'} dz\, P(z,z)\right]\frac{\partial^2 S(z')}{\partial z'^2} = -\int dz' \frac{\partial^2 S(z')}{\partial z'^2}\int_0^{z'} dz\, P(z,z), \quad P(zz') = 8\pi m_s^2 c\rho C \int dk e^{ik|z-z'|}\Lambda(k)\tilde{N}(k)$$

$$\nu(zz') = \int_0^{z'} dz\, P(zz) = \int_{z_0}^{z'} dz\, P(z,z),\ z_0 = \text{const, undefined.}\ d\nu = P(zz')dz'.$$

$\int dz'(\dots) = \int_0^1 dz'(\dots)$, т.к. $z, z' \in [0,1]$ и $\cos(\pi n x)$ четная.

При дискретизации dyn. matrix $\Lambda_{nn'}$ берутся $(\dots)_{nn'}$ matrix elements от конкретного члена в $\nu D(z)$. Далее брал $-\frac{i}{2}\left(\frac{\partial P}{\partial z}\right)_{nn'}$ на $\varphi_n = \frac{1}{\sqrt{N}}\cos(\pi n z)$

$-\frac{i}{2}\left(\frac{\partial P}{\partial z}\right)_{nn'} = -\frac{i}{2}\int dz \int dz' \frac{\partial P}{\partial z}\varphi_n(z)\varphi_{n'}(z')$, $K_{nn'} = \int dz dz' \varkappa(zz')\varphi_n(z)\varphi_{n'}(z')$ etc.

$z = [0,1]$. Все члены в νD содержали $S(z)$ или $\partial^2 S/\partial z^2$, обе $\sim \cos(\pi n z)$.

Escaneado con CamScanner

$f(t) = \frac{1}{\pi}$ gives $f(\omega) = \frac{\sin(\omega_m t)}{\omega_m t}$, $\omega_m = \pi a$, $f_m = \frac{\omega_m}{2\pi} = \frac{a}{2}$, $f(a) = \frac{\sin(\omega_m t)}{\omega_m t}$

Gibbs: $f(t) = \frac{\sin(\omega_m t)}{\omega_m t}$, $\frac{\omega_m}{2\pi}$ = 40 GHz, A & ω_m, ω_m in GHz·2π

ω_m = 40 GHz·2π = 240 rad/ns. W_m Boltzmann ω_m, $f(A, \omega_m)$ Gibbs ω_m & $f(A)$.

Delta function.

(25/IV) Найдем P – 2 terms в YD для vortex string $S(z,\omega)$.

GM vortex wire Physical Review, Riemann Surface σ $\sigma_n(z) = \cos(\pi n z)$ $z \in [0,1]$

Definite $\sigma_{z=0,1} = 0$ на сфере, σ_n – basis, нормировка $\sigma_{n=1} = 1$.

Сам 25/IV 2020 $W_m^{vol} = \frac{1}{2} V \int dz dz' P(zz') \left[\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z')\right]_z$, $P(zz') = P(z/z)$

$W = \frac{V}{\pi} \int dz w$, $\frac{1}{\pi} W_m^{vol} = \frac{1}{2} \int dz P(zz') \left[\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z')\right]_z$ – on density for a magnetic Riemann

$\left[\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z')\right] \cdot \hat{z} = \frac{\partial \vec{S}}{\partial z} \cdot \left[\vec{S}(z') \times \hat{z}\right]$, $\frac{1}{\pi} W_m^{vol} = \frac{1}{2} \int dz P(zz') \left[\vec{S}(z') \times \hat{z}\right] \cdot \frac{\partial \vec{S}}{\partial z}$

$\frac{1}{\pi} \frac{\delta W_m}{\delta \vec{S}(z)} = \frac{1}{\pi} \frac{\partial W_m}{\partial \vec{S}} = \frac{1}{\pi} \frac{\partial W_m}{\partial z} \frac{\partial z}{\partial (\partial \vec{S}/\partial z)} = \left|\frac{\partial W_m}{\partial z} = -\left(\frac{1}{2}\right) \frac{\partial z}{\partial (\partial \vec{S}/\partial z)}\right.$

$\int dz' P(zz') \left[\vec{S}(z') \times \hat{z}\right] = \frac{1}{2} \frac{\partial}{\partial z} \int dz' P(zz') \left[\vec{S}(z') \times \hat{z}\right] = \frac{1}{2} \int dz' \frac{\partial P}{\partial z} \left[\vec{S}(z') \times \hat{z}\right]$

$\left[\hat{z} \times \vec{S}(z)\right] \cdot \hat{z}$

25/IV 2020 W_m^{vol} можно взять в сам оператор $\left[\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z')\right] \cdot \hat{z} + \left[\frac{\partial \vec{S}}{\partial z'} \times \vec{S}(z)\right] \cdot \hat{z}$

Если $\int dz dz' (\ldots)$ то нет разницы можно сами и на сам оператор.

На пути вычисления on density w, $W = \frac{V}{\pi} \int dz w$, $\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z)$ и $\frac{\partial \vec{S}}{\partial z'} \times \vec{S}(z)$ не зависят от z и z' соответственно, поэтому $(1) \pm (2) = 2 \cdot (1)$ как в W. (1) (2)

Дальше использовать сам оператор W обычно как 25/IV 2020.

$W_m = -\frac{1}{4} V \int dz dz' P(zz') \left[\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z')\right]_z - \frac{1}{4} \int dz dz' P(zz') \left[\frac{\partial \vec{S}}{\partial z'} \times \vec{S}(z)\right]_z$

$\frac{1}{\pi} W_m = -\frac{1}{4} \int dz' P(zz') \left[\frac{\partial \vec{S}}{\partial z} \times \vec{S}(z')\right]_z - \frac{1}{4} \int dz P(zz') \left[\frac{\partial \vec{S}}{\partial z'} \times \vec{S}(z)\right]_z$

$\hat{z} \cdot \left[\vec{S}(z) \times \vec{S}(z')\right] = \vec{S}'(z) \cdot \left[\vec{S}(z') \times \hat{z}\right]$, $\hat{z} \cdot \left[\vec{S}'(z') \times \vec{S}(z)\right] = \vec{S}'(z') \cdot \left[\vec{S}(z) \times \hat{z}\right] =$

$= c \cdot [a \times b] = \vec{S}(z) \cdot \left[\vec{S}'(z') \times \hat{z}\right]$

$\frac{1}{\pi} W_m = -\frac{1}{4} \int dz' P(zz') \left[\vec{S}(z) \times \hat{z}\right] \cdot \frac{\partial \vec{S}}{\partial z'} - \frac{1}{4} \int dz' P(zz') \left[\hat{z} \times \frac{\partial \vec{S}}{\partial z'}\right] \cdot \vec{S}(z)$

$\frac{1}{\pi} \frac{\delta W_m}{\delta \vec{S}(z)} = \frac{1}{\pi} \frac{\partial W_m}{\partial \vec{S}} - \frac{1}{\pi} \frac{\partial z'}{\partial (\partial \vec{S}/\partial z')} \frac{\partial W_m}{\partial z'} = -\frac{1}{4} \int dz' P(zz') \left[\hat{z} \times \frac{\partial \vec{S}}{\partial z'}\right] -$

$- \frac{\partial}{\partial z} \left[-\frac{1}{4} \int dz' P(zz') \left[\vec{S}(z') \times \hat{z}\right]\right] = -\frac{1}{4} \int dz' P \left[\hat{z} \times \frac{\partial \vec{S}}{\partial z'}\right] + \frac{1}{4} \frac{\partial}{\partial z} \int dz' P(zz') \left[\vec{S}(z') \times \hat{z}\right] =$

$= \frac{1}{4} \left[\int dz' P(zz') \frac{\partial \vec{S}}{\partial z'} + \frac{\partial}{\partial z} \int dz' P(zz') \vec{S}(z')\right] \times \hat{z}$ – далее можно упростить

только предполагая, как 25/IV 2020, $\vec{S}(z)\big|_{z=\pm 1/2} = 0$, pinning boundary conditions,

необходимые с $\partial P/\partial z\big|_{z=0,1} = 0$, $\cos(\pi n z)$ например.

$\frac{1}{\pi} \frac{\delta W_m}{\delta \vec{S}} = \frac{1}{4} \int dz' \left[P(zz') \frac{\partial \vec{S}}{\partial z'} + \frac{\partial P}{\partial z} \vec{S}(z')\right] \times \hat{z}$. Было 25/IV 2020 $\frac{1}{\pi} \frac{\delta W_m}{\delta \vec{S}} = \frac{1}{2} \int dz' \frac{\partial P}{\partial z} \vec{S}(z') \times \hat{z}$, это неверно

→ т.о. Eq. (8) Word file GM_vortex_wire. Оксана неверно в P-члене.

Vortex string dynamics с уч. V. core замечания in April – July 2020, вернулся к ним Aug. 25 / 2020 для paper с Оксана – Felipe.

2-term. $W_m = \frac{1}{2} V \int dz dz' \vartheta(zz') \left(\frac{\partial \vec{S}}{\partial z} \cdot \frac{\partial \vec{S}}{\partial z'}\right)$, $\frac{1}{\pi} W_m = \int dz' \vartheta(zz') \left(\frac{\partial \vec{S}}{\partial z} \cdot \frac{\partial \vec{S}}{\partial z'}\right)$

$\frac{1}{\pi} \frac{\delta W_m}{\delta \vec{S}} = \frac{1}{\pi} \frac{\partial W_m}{\partial \vec{S}} - \frac{1}{\pi} \frac{\partial}{\partial z} \frac{\partial W_m}{\partial (\partial \vec{S}/\partial z)} = \left[\frac{\partial}{\partial z'} - \frac{\partial}{\partial z} \frac{\partial z}{\partial (\partial \vec{S}/\partial z)}\right] \int dz' \vartheta(zz') \left(\frac{\partial \vec{S}}{\partial z} \cdot \frac{\partial \vec{S}}{\partial z'}\right) =$

$= -\frac{\partial}{\partial z} \left[\int dz' \vartheta(zz') \frac{\partial \vec{S}}{\partial z'}\right] = -\int dz' \frac{\partial \vartheta(zz')}{\partial z} \frac{\partial \vec{S}(z')}{\partial z'}$, это совпадает с Eq. (8) word file.

Escaneado con CamScanner

Escaneado con CamScanner

Тогда $-\frac{i}{4}\int dz' P(zz') \frac{\partial S(z')}{\partial z'}$, $\frac{\partial S}{\partial z'}$ меняет φ_n на $\frac{\partial \varphi_n}{\partial z} = \frac{-1}{\sqrt{\nu_n}} \pi n \sin(\pi n z)$

не забываем $\varphi_n(z) = \frac{1}{\sqrt{\nu_n}} \cos(\pi n z)$.

Минск 9/VII 2020: $S(z) = \sum_n C_n \varphi_n(z)$ подставляем сюда $\hat{\Lambda}\vec{C} = \lambda \vec{C}$, $\lambda = \omega/\omega_M$. $\lambda S(z) = (\text{other}) - \frac{i}{4}\int dz' P(zz') \frac{\partial S(z')}{\partial z'}$

$\lambda \sum_n C_n \varphi_n(z) = -\frac{i}{4}\int dz' P \frac{\partial}{\partial z'} \sum_n C_n \varphi_n(z')$, умножим на $\varphi_{n'}(z)$ и $\int dz(\dots)$.

$\lambda \sum_n C_n \int dz\, \varphi_n(z)\varphi_{n'}(z) = -\frac{i}{4}\sum_n C_n \int dz \int dz' P \frac{\partial \varphi_n(z')}{\partial z'} \varphi_{n'}(z)$, $\int_0^1 dz\, \varphi_n \varphi_{n'} = \delta_{nn'}$

$\lambda C_n = (\text{other}) - \frac{i}{4}\sum_n C_n \int dz dz' P(zz') \frac{\partial \varphi_n(z')}{\partial z'} \varphi_{n'}(z) = -\frac{i}{4}\sum_n C_n P_{nn'}$

$P_{nn'} = \int dz \int dz' P(zz') \frac{\partial \varphi_n(z')}{\partial z'} \varphi_{n'}(z)$, $\Lambda_{nn'} = (\text{without}) + (P)$. $n \rightleftarrows n'$.

$\lambda C_n = (\text{other}) - \frac{i}{4}\sum_{n'} C_{n'} \int dz dz' P\, \varphi_n(z) \frac{\partial \varphi_{n'}(z')}{\partial z'} = (\text{other}) - \frac{i}{4}\sum_{n'} C_{n'} P_{nn'}$

$P_{nn'} = \int dz \int dz' P(zz') \varphi_n(z) \frac{\partial \varphi_{n'}(z')}{\partial z'}$, $\Lambda_{nn'} = (\rho_3 P) - \frac{i}{4} P_{nn'} - \frac{i}{4}\left(\frac{\partial P}{\partial z}\right)_{nn'}$

In units $\omega_M s^2$ (divide by $\omega_M s^2$): $P = c\rho\mathbb{C}\int dk\, e^{-\beta k|z-z'|}\Lambda(k)\Lambda'(k)$.

$P_{nn'} = \int dz dz' \frac{1}{\sqrt{\nu_n \nu_{n'}}} c\rho\mathbb{C}\int dk e^{-\beta k|z-z'|}\Lambda(k)\Lambda'(k)\cos(\pi n z)(-1)^{n'}\sin(\pi n' z')$

$x = \beta k$

$P_{nn'} = -\frac{\pi n' \cdot c\rho\mathbb{C}}{\sqrt{\nu_n \nu_{n'}}} \int_0^1 dz dz' \int dk e^{-\beta k|z-z'|}\Lambda(k)\Lambda'(k)\cos(\pi n z)\sin(\pi n' z')$, $\nu_n = \frac{1}{2}(1+\delta_{n0})$

$g_{nn'}(x) = \int_0^1 dz\, dz'\, e^{-x|z-z'|}\cos(\pi n z)\sin(\pi n' z') = \int_0^1 dz \cos(\pi n z)\int_0^1 dz' e^{-x|z-z'|}\sin(\pi n' z') =$

$= \int_0^1 dz \cos(\pi n z)\left[\int_0^z dz' e^{-x(z-z')}\sin(\pi n' z') + \int_z^1 dz' e^{-x(z'-z)}\sin(\pi n' z')\right] =$

$= \int_0^1 dz \cos(\pi n z)\left[e^{xz}\int_0^z dz' e^{xz'}\sin(\pi n' z') + e^{xz}\int_z^1 dz' e^{-xz'}\sin(\pi n' z')\right] = \left|\begin{array}{l}\pi n' z' = t \\ \pi n' dz' = dt\end{array}\right|$

$[\dots] = e^{-xz}\int_0^{\pi n' z} \frac{dt}{\pi n'} e^{(x/\pi n')t}\sin(t) + e^{xz}\int_{\pi n' z}^{\pi n'} \frac{dt}{\pi n'} e^{-xt/\pi n'}\sin(t) =$

$= e^{-xz}\frac{1}{\pi n'}\frac{e^{xt/\pi n'}}{\left(\frac{x}{\pi n'}\right)^2+1}\left\{\frac{x}{\pi n'}\sin t - \cos t\right\}_0^{\pi n' z} + e^{xz}\frac{e^{-xt/\pi n'}}{\left(\frac{x}{\pi n'}\right)^2+1}\left\{-\frac{x}{\pi n'}\sin t - \cos t\right\}_{\pi n' z}^{\pi n'} =$

$= \frac{1}{\pi n'}e^{-xz}\frac{1}{(\cdot)^2+1}\left[e^{xz}\left\{\frac{x}{\pi n'}\sin(\pi n' z) - \cos(\pi n' z)\right\} - \{-1\}\right] +$

$+ \frac{1}{\pi n'}e^{xz}\frac{1}{(\cdot)^2+1}\left[e^{-x}\{0-(-1)^{n'}\} - e^{-xz}\left\{-\frac{x}{\pi n'}\sin(\pi n' z) - \cos(\pi n' z)\right\}\right] =$

$= \frac{1}{\pi n'}\frac{1}{(\cdot)^2+1}\left[\frac{x}{\pi n'}\sin(\pi n z) - \cos(\pi n' z) + e^{-xz} - (-1)^{n'}e^{-x}e^{xz} + \frac{x}{\pi n'}\sin(\pi n' z) + \cos(\pi n' z)\right] =$

$= \frac{1}{\pi n'}\frac{1}{(x/\pi n')^2+1}\left[\frac{2x}{\pi n'}\sin(\pi n' z) + e^{-xz} - (-1)^{n'}e^{-x}e^{xz}\right]$, $n' \neq 0$, иначе $g_{nn'} = 0$.

$g_{nn'}(x) = \int_0^1 dz \cos(\pi n z)[\dots] = \int_0^1 dz \cos(\pi n z)\frac{1}{\pi n'}\frac{1}{(x/\pi n')^2+1}\left[\frac{2x}{\pi n'}\sin(\pi n' z) + e^{-xz} - (-1)^{n'}e^{-x}e^{xz}\right]$

$\int_0^1 dz \cos(\pi n z)\sin(\pi n' z) = \frac{1}{2}\int_0^1 dz\left[\sin(\pi(n+n')z) + \sin(\pi(n'-n)z)\right] = \frac{1}{2}\frac{i(-1)}{\pi(n+n')}\cos\pi(n+n')z\Big|_0^1 +$

$+ \frac{1}{2}\frac{1\cdot(-1)}{\pi(n'-n)}\cos\pi(n'-n)z\Big|_0^1 = \frac{1}{2}\frac{1\cdot(-1)}{\pi(n+n')}\left[(-1)^{n+n'}-1\right] + \frac{1}{2}\frac{1\cdot(-1)}{\pi(n'-n)}\left[(-1)^{n'-n}-1\right] =$

$= \left[(-1)^{n+n'}-1\right]\frac{1(-1)}{2\pi}\left[\frac{1}{(n+n')} + \frac{1}{(n'-n)}\right] = \frac{\left[(-1)^{n+n'}-1\right]}{2\pi}\frac{2n'(-1)}{(n'^2-n^2)}$, $= 0$ при $n = n'$

$g_{nn'}(x)$ содержит ускорение сингулярности - много для Math Cad.

подставим исходное P в Math Cad, без аналитики $\int dz dz'$.

File "Vortex-TM-long-wires-1.mcd". Math Cad $\int dz \int dz' \int dk(\dots)$ считает очень долго.

$J_1 = \int_0^1 dz' e^{-x|z-z'|}\sin(\pi n' z') = \frac{1}{\pi n'}\frac{1}{(x/\pi n')^2+1}\left[2x\frac{\sin(\pi n' z)}{\pi n'} + e^{-xz} - (-1)^{n'}e^{-x}e^{xz}\right]$

$J_1(x)$ берем. $J_2 = \int_0^1 dz \cos(\pi n z) J_1(z) = g_{nn'}(x)$

Escaneado con CamScanner

$$g_{nn'}(x) = \frac{\pi n'}{x^2+(\pi n')^2}\int_0^1 dz\cos(\pi n z)\left[\frac{2x}{\pi n'}\sin(\pi n' z) + e^{-xz} - (-1)^{n'}e^{-x}e^{xz}\right]$$

$$\int_0^1 dz\cos(\pi n z)\sin(\pi n' z) = \frac{1}{\pi}\left[(-1)^{n+n'}-1\right]\frac{n'}{n'^2-n^2},\ n\neq n';\quad 0 \text{ при } n=n'$$

$$\int_0^1 dz\cos(\pi n z)e^{-xz} = \frac{e^{-xz}}{(\pi n)^2+1}\left[\pi n\cos(\pi n z)+\sin(\pi n z)\right]\Big|_0^1 =$$

$$\int_0^1 dz\cos(\pi n z)e^{-xz} = \left|\pi n z = t\right| = \int_0^{\pi n}\frac{dt}{\pi n}\cos(t)e^{-(x/\pi n)t} = \frac{1}{\pi n}\frac{e^{-xt/\pi n}}{(x/\pi n)^2+1}\times$$

$$\times\left[-\frac{x}{\pi n}\cos t+\sin t\right]\Big|_0^{\pi n} = \frac{1}{\pi n}e^0\frac{1}{(1/\pi n)^2+1}\left[e^{-x}\left(-\frac{x}{\pi n}(-1)^n+0\right)-\left(-\frac{x}{\pi n}\right)\right] =$$

$$= \frac{\pi n}{x^2+(\pi n)^2}\frac{x}{\pi n}\left(1-(-1)^n e^{-x}\right),\quad \int_0^1 dz\cos(\pi n z)e^{xz} = \frac{\pi n}{x^2+(\pi n)^2}\frac{(-x)}{\pi n}\left[1-(-1)^n e^{x}\right]$$

$$g_{nn'}(x) = \frac{\pi n'}{x^2+(\pi n')^2}\left[\frac{2x}{\pi n'}\frac{1}{\pi}\left\{(-1)^{n+n'}-1\right\}\frac{n'}{(n'^2-n^2)} + \frac{\pi n}{x^2+(\pi n)^2}\frac{x}{\pi n}\left\{1-(-1)^n e^{-x}\right\} -\right.$$

$$-(-1)^{n'}e^{-x}\cdot\frac{(-x)}{x^2+(\pi n')^2}\left\{1-(-1)^n e^{x}\right\}\Big] = \frac{\pi n'}{x^2+(\pi n')^2}\left[(\ldots)+\frac{x}{x^2+(\pi n)^2}\left\{1-(-1)^n e^{-x}\right\}+\right.$$

$$+\frac{x}{x^2+(\pi n)^2}(-1)^n e^{-x}\left\{1-(-1)^n e^{x}\right\}\Big] = \frac{\pi n'}{x^2+(\pi n')^2}\left[(\ldots)+\frac{x}{x^2+(\pi n)^2}\left\{1-(-1)^n e^{-x}+\right.\right.$$

$$\left.\left.+(-1)^n e^{-x}-1\right\}\right] = \frac{\pi n'}{x^2+(\pi n')^2}\frac{2x}{\pi^2 n'}\left\{(-1)^{n+n'}-1\right\} \rightsquigarrow \frac{1}{\pi}\frac{2x}{x^2+(\pi n')^2}\left\{(-1)^{n+n'}-1\right\}$$ НЕТ

(28/IV) $$g_{nn'}(x) = \frac{2}{\pi}\frac{x}{(x)^2+(\pi n')^2}\left\{(-1)^{n+n'}-1\right\}$$ — наконец-то. MathCad не подтвердил.

Проведем вычисления в $g_{nn'}(x)$:

$$g_{nn'}(x) = \frac{\pi n'}{x^2+(\pi n')^2}\frac{2x}{\pi n'}\frac{1}{\pi}\left\{(-1)^{n+n'}-1\right\}\frac{n'}{(n'^2-n^2)} = \frac{2}{\pi}\frac{x}{x^2+(\pi n')^2}\left\{(-1)^{n+n'}-1\right\}\frac{n'}{(n'^2-n^2)}$$

Еще одно не так:

$$\int_0^1 dz\cos(\pi n z)\left[e^{-xz}-(-1)^{n'}e^{-x}e^{xz}\right] = \left|n\pi z = t\right| = \frac{1}{n\pi}\int_0^{n\pi}dt\,e^{-xt/n\pi}\cos t -$$

$$-\frac{1}{n\pi}\int_0^{n\pi}dt\,e^{xt/n\pi}\cos t\cdot(-1)^{n'}e^{-x} = \frac{1}{n\pi}\left[-\frac{x}{n\pi}\cos t+\sin t\right]_0^{n\pi}\frac{e^{-xt/n\pi}}{(x/n\pi)^2+1} -$$

$$-\frac{1}{n\pi}(-1)^{n'}e^{-x}\frac{e^{xt/n\pi}}{(x/n\pi)^2+1}\left[\frac{x}{n\pi}\cos t+\sin t\right]_0^{n\pi} = \frac{1}{n\pi}\frac{1}{(x/n\pi)^2+1}\left[e^{-x}\frac{(-x)}{n\pi}(-1)^n -\right.$$

$$\left.-\frac{(-x)}{n\pi}-(-1)^{n'}e^{-x}\left\{e^{x}\frac{x}{n\pi}(-1)^n-\frac{x}{n\pi}\right\}\right] = \frac{x}{(n\pi)^2}\frac{1}{(x/n\pi)^2+1}\left[-e^{-x}(-1)^n+1-1+(-1)^{n'}e^{-x}\right] = 0$$ (?)

$$g_{nn'}(x) = \frac{\pi n'}{x^2+(\pi n')^2}\frac{2x}{\pi n'}\int_0^1 dz\cos(\pi n z)\sin(\pi n' z) = \frac{2x}{x^2+(\pi n')^2}\frac{1}{\pi}\frac{n'}{(n'^2-n^2)}\left[1-(-1)^{n+n'}\right]$$ (2a)

MathCad $g_{nn'}(x)$ и аналитич. $g_{nn'}(x)$ разошлись. Вычислено [sinc_function.mcd].

$$\int_0^1 dz\cos(\pi n z)e^{-xz} = \left|z = 1-z'\right| = -\int_0^1 dz'\cos\left[\pi n(1-z')\right]e^{-x(1-z')} =$$

$$= \int_0^1 dz'\cos(\pi n)\cos(\pi n z')e^{-x(1-z')} = \int_0^1 dz\cos(\pi n z)e^{xz}(-1)^n e^{-x}$$

$$g_{nn'}(x) = \int_0^1 dz\cos(\pi n z)\int_0^1 dz' e^{-x|z-z'|}\sin(\pi n' z') = \int_0^1 dz\cos(\pi n z)\frac{2x}{x^2+(\pi n')^2}\sin(\pi n' z) =$$

$$= \frac{2x}{x^2+(\pi n')^2}\int_0^1 dz\cos(\pi n z)\sin(\pi n' z).$$ Одинаковая $(-1)^n$ в $[\ldots]$, поэтому $(-1)^{n'}$.

$$\int_0^1 dz\cos(\pi n z)\left[e^{-xz}-(-1)^{n'}e^{-x}e^{xz}\right] = \frac{x}{x^2+(n\pi)^2}\left[-e^{-x}(-1)^n+1-(-1)^{n+n'}+(-1)^{n'}e^{-x}\right] =$$

$$= \frac{x}{x^2+(n\pi)^2}\left[1-(-1)^{n+n'}+e^{-x}\left\{(-1)^{n'}-(-1)^n\right\}\right] = (\ldots)\left[1-(-1)^{n+n'}+e^{-x}(-1)^{n'}\left\{1+(-1)^{n+n'}\right\}\right] =$$

$$= \frac{x}{x^2+(n\pi)^2}\left[1+e^{-x}(-1)^{n'}\right]\left[1-(-1)^{n+n'}\right]$$

$$g_{nn'}(x) = \frac{2x}{x^2+(\pi n')^2}\frac{1}{\pi}\left[1-(-1)^{n+n'}\frac{n'}{(n'^2-n^2)}+\frac{\pi n'}{x^2+(\pi n')^2}\int_0^1 dz\cos(\pi n z)\left[e^{-xz}-(-1)^{n'}e^{-x}e^{xz}\right]\right] =$$

$$= \frac{2x}{x^2+(\pi n')^2}\frac{1}{\pi}\left[1-(-1)^{n+n'}\right]\frac{n'}{(n'^2-n^2)}+\frac{\pi n'}{x^2+(n'\pi)^2}\frac{x}{x^2+(n\pi)^2}\left[1-(-1)^{n+n'}\right]\left[1+(-1)^{n'}e^{-x}\right]$$

$$\rightarrow g_{nn'}(x) = \frac{x}{x^2+(\pi n')^2}\left[1-(-1)^{n+n'}\right]\left[\frac{2}{\pi}\frac{n'}{(n'^2-n^2)}+\frac{\pi n'}{x^2+(n\pi)^2}\left\{1+(-1)^{n'}e^{-x}\right\}\right]$$ — верно.

Escaneado con CamScanner

$P_{nn'} = -\frac{\pi i c P\Gamma}{\sqrt{N_n N_{n'}}} \int_0^\infty dk\, g_{nn'}(\beta k)\, \Lambda(k)\, \Lambda'(k)$, $x = \beta k$

Чтобы $\Lambda_{nn'} = (\delta_{z_3} P) - \frac{i}{4} P_{nn'} - \frac{i}{4}\left(\frac{\partial P}{\partial z}\right)_{nn'}$ была эрмитова и

λ_n – real (ω_n-real) надо $P_{nn'} = -P_{n'n}$ – была антисимметрична.

А это не так. $h_{nn'} = n' g_{nn'}$, $h_{nn'} = \frac{1}{2}(h_{nn'} + h_{n'n}) + \frac{1}{2}(h_{nn'} - h_{n'n})$

Замечание: Берем антисим. часть $a(h_{nn'}) = \frac{1}{2}(h_{nn'} - h_{n'n})$ в $P_{nn'}$.

$a(h_{n'n}) = (n'_{n' \to n}) = \frac{1}{2}(h_{n'n} - h_{nn'}) = -a(h_{nn'})$.

$a(h_{nn'}) = \frac{1}{2}(h_{nn'} - h_{n'n}) = \frac{1}{2}(n' g_{nn'} - n g_{n'n})$.

$N=100$ $f_{75} = 3{,}53$ GHz, $f_{76} = 4{,}23$, $f_{78} = 8{,}98$, $f_{79} = 14{,}32$ $f_{80} = 20{,}35$ GHz

(edge localized) (edge loc. с min $z=0$) (2 zeros) (3 zeros) (4 zeros)

→ f_n численно те же. 250 ×0 и old YD 19/IV, локализация та же.

А.е. учет более точный P-term и artificial антисимметризация $P_{nn'}$ матрицы ничего нового не дали. посылка Л.Д. 9 kg, 25 €

Выводы: 1) сменить базис с $\cos(\pi n z)$, $z \in [0,1]$ на $\exp(i q_n z)$, $z \in [-\frac{1}{2}, \frac{1}{2}]$ т.е. сходится ф $\frac{\partial \varphi_n}{\partial z}\big|_{z=0,1} = 0$ b. conditions. При этом баг в P-term исчезнет. $q_n = 2\pi n$

2) попробовать для $n=0$ localized mode local approximation, как в Gus' Magnetism '2022.

(29-30/IV) Cover letter and resubmission Саше – metastable BP nanowires" в Ph. Rev. Appl. переписка с Зухрой о "vortex nutation frequency.

(2-13/V) поездка в Порто. Беседы с Л. Написание "GM in dome-shape nanodots" с Рубеном.

(14/V) Subsanation MAT-2013 project. переписка с Ruben, Julián, Arantza (Doctorado dept.) по составу tribunal для защиты PhD thesis Ricardo.

(15-16/V) Дворк дома. Travel reports Krakow y Porto.

(17/V) переписка с Julian, Nena T., Аркадием (Quantum Center appl.).

(18/V) Questions to Chapters 2, 3 Ricardo's thesis. Jury members, anti-plagiarism program.

(19/V) Организация jury, theme thesis, ремонт. Turnitin (anti-plagiat).

(20/V) E-mail to Celia Sousa (jury), переписка с Ruben y Ricardo. Сдан travel reports to Eli. С Л. в кабинете Германа.

(21/V) Nagorer refs, of A. Fert to Gonzalez-Gutierrez papers (всем одна citation на J. Phys. 2011) переписка с Ricardo. Theme thesis. (Ch. 3). Надо: Eq. (3.53) и σ_{max}, v_w (1) (3.40) – (3.42).

thes. Ricardo, Eq. (3.38) DW velocity in biaxial FM, $\lambda = K_x/K_z$:

$$v(h) = \frac{\gamma \Delta_0}{2} \frac{H_w h}{\sqrt{1 + \frac{\lambda}{2}(1 - \sqrt{1-h^2})}}, \quad h = \frac{H}{H_w}, \quad H_w = \frac{1}{2} \alpha H_k, \quad H_K = \frac{2|K_x|}{M_s}$$

Δ_0 – DW width at rest, $v=0$ $E_{ex} = J a_0^2 (\nabla\Theta)^2 = A(\nabla\theta)^2$, $A = J a_0^2$.

$\delta=0$, $\frac{J a_0^2}{\Delta_0} - (K_z + K_x \sin^2\Phi)\Delta_0 = 0$, $\Delta_0^2 = A/(K_z + K_x \sin^2\Phi)$, easy plane "yz"

$E_A = -K_z \cos^2\Theta + K_x \sin^2\Theta \sin^2\Phi = -K_z m_z^2 + K_x m_y^2$, $K_z, K_x > 0$.

Но должно $E_A = -K_z m_z^2 + K_x m_x^2$, $m_y = \sin\theta \cos\Phi$.

Ricardo (3.14) берет $E_A = K_z \sin^2\Theta + K_x \sin^2\Theta \sin^2\Phi$ – либо неверно, либо Φ отсчитывается от oy-axis. Скорее Φ от oy. $\Phi_{start} = 0$ и $\Delta_0^2 = A/K_z$.

Escaneado con CamScanner

(22/2) Max and Walker velocities in biaxial FM. DW velocity

$\frac{\gamma \Delta_0 H_w}{\alpha} = \frac{\gamma}{\alpha}\Delta_0 H_w = \frac{\gamma}{\alpha}\sqrt{A/K_2}\,\frac{1}{2}\alpha H_K = \frac{\gamma}{2} H_K \sqrt{A/K_2}$

Thesis Ricardo $v_R(h) = \frac{\gamma \Delta_0 H_w}{\alpha} \frac{h}{\sqrt{1+\frac{\lambda}{2}(1-\sqrt{1-h^2})}}$. При $h=1$, $H=H_w$

Walker velocity $v_R^w(h=1)$: $v_R^w(h=1) = \frac{\gamma \Delta_0 H_w}{\alpha} \frac{1}{\sqrt{1+\frac{\lambda}{2}}}$

Ricardi PRB'2000: $v_w = \frac{\lambda}{\sqrt{2(2+\lambda)}}$ (взято из Sobolev, JMMM'1995). в единицах $v_0 = \frac{2\gamma}{M_s}\sqrt{AK_2}$. $\frac{\gamma \Delta_0 H_w}{\alpha} = \frac{\gamma}{2} H_K \sqrt{\frac{A}{K_2}} = \frac{\gamma}{2}\cdot\frac{2K_x}{M_s}\frac{1}{K_2}\sqrt{AK_2} = \frac{\gamma}{M_s}\lambda\sqrt{AK_2} =$

$= \lambda \cdot \frac{v_0}{2}$. $v_R^w = \frac{1}{2}\lambda v_0 \frac{1}{\sqrt{1+\lambda/2}} = v_0 \cdot \frac{\lambda}{\sqrt{2(2+\lambda)}}$ Совпадает с Rama-Eriksa PRB 2000

Ratio $\frac{v_-}{v_w} = \frac{\sqrt{2(2+\lambda)}}{\lambda}\left[\sqrt{1+\lambda} - 1\right] \to \sqrt{2}$ при $\lambda \to \infty$. $v_-(\lambda) = v_{max}(\lambda)$

v_{max} - is maximal DW velocity at $h \in [0,1]$, max DW velocity v_- solution theory.

Sobolev JMMM'1995: $v(h) = \frac{\varepsilon h}{\sqrt{2(1+\varepsilon-\varepsilon\sqrt{1-h^2})}}$ in units $\Delta_B \omega_A$, $\varepsilon = K_x/K_z = \lambda$.

$\Delta_B \omega_A = \Delta_B \gamma H_{K2} = \frac{2\gamma K_2}{M_s}\Delta_B = \left|\Delta_B = \sqrt{AK_2}\right| = \frac{2\gamma}{M_s}K_2\sqrt{\frac{A}{K_2}} = v_0$

$v_{Sob}(h) = v_0 \frac{\lambda h}{\sqrt{2(2+\lambda-\lambda\sqrt{1-h^2})}} = \frac{1}{2}v_0\lambda\frac{h}{\sqrt{1+\frac{\lambda}{2}(1-\sqrt{1-h^2})}} = v_R(h)$, т.к. R. scale $= \frac{1}{2}\lambda v_0$.

→ Вывод: Eq. (3.38) thesis Ricardo дает $v(h)$ верное.

$v(h)$ Schryer-Walker, JAP 1974: $v(h) = \frac{\gamma H}{\alpha}\frac{1}{\sqrt{1+\lambda\cos^2\varphi_0}}$, $\sin 2\varphi_0 = \frac{H}{H_w}$

$\cos^2\varphi_0 = \frac{1}{2}(1+\cos 2\varphi_0) = \left|\cos 2\varphi_0 \le 0, \text{т.к. } \varphi_0 \text{ больше } \frac{\pi}{2} \text{ для } \lambda > 1\right| = \frac{1}{2}\left(1-\sqrt{1-\sin^2 2\varphi_0}\right) =$

$= \frac{1}{2}(1-\sqrt{1-h^2})$ и $v(h) = \frac{\gamma H}{\alpha}\frac{1\cdot\Delta_0}{\sqrt{1+\frac{\lambda}{2}(1-\sqrt{1-h^2})}}$, $\frac{\gamma H}{\alpha} = \frac{\gamma H_w}{\alpha}\left(\frac{H}{H_w}\right) = \frac{\gamma}{\alpha}\frac{1}{2}\alpha H_K h =$

$= \frac{\gamma}{2}H_K h = \frac{\gamma}{2}h\frac{2K_x}{M_s}\cdot\sqrt{\frac{A}{K_2}} = \frac{\gamma}{2}h\frac{2}{M_s}\frac{K_x}{K_z}K_z\sqrt{A/K_z} = \frac{\gamma h}{M_s}\lambda\sqrt{AK_z} = \frac{1}{2}h\lambda v_0$. (units Δ_0)

Schryer-Walker $v_{SW}(h) = \frac{1}{2}v_0\frac{h\lambda}{\sqrt{1+\frac{\lambda}{2}(1-\sqrt{1-h^2})}}$, что совпадает с Sobolev 1995 и Ricardo's Eq. (3.38).

Ricardo's (3.40) $v_{max} = \gamma\Delta_0\frac{H_K}{\lambda}(\sqrt{1+\lambda}-1) = \gamma\sqrt{\frac{A}{K_2}}\frac{K_2}{K_x}\frac{2K_x}{M_s}(\dots) = v_0(\sqrt{1+\lambda}-1)$ - верно. (3.40)

Sobolev 1995: $v_{max} = v(h_m)$, h_m - точка при которой достигается v_{max}, $h_m = h_0$,

$h_0^2 = \frac{4}{\lambda^2}\left[(2+\lambda)\sqrt{1+\lambda} - 2(1+\lambda)\right]$, при $\lambda \to 0$ $h_m(\lambda) \to$ (должно) 1.

$[\dots] = (2+\lambda)\sqrt{1+\lambda} - 2(1+\lambda) = (2+\lambda)\left(1+\frac{\lambda}{2}\right) - 2(1+\lambda) = 2+\frac{\lambda}{2}2+\lambda+\frac{\lambda^2}{2}-2-2\lambda = \frac{1}{2}\lambda^2$

$h_0^2(\lambda\to 0) = \frac{4}{\lambda^2}\cdot\frac{1}{2}\lambda^2 = 2$ (? должно $h_0(\lambda\to 0) = 1$). $h_m(\lambda)$ при $\lambda \gg 1$,

$h_0^2(\lambda \gg 1) = \frac{4}{\lambda^2}\left[\lambda\sqrt{\lambda} - \lambda\right] = \frac{4}{\lambda^2}\lambda\sqrt{\lambda} = \frac{4}{\sqrt{\lambda}} \to 0$ при $\lambda \to \infty$.

$v(h) = \frac{1}{2}v_0\lambda\delta(h)$, $\delta(h) = \frac{h}{\sqrt{1+\frac{\lambda}{2}(1-\sqrt{1-h^2})}}$, ищем h_m, $\max\delta(h)$.

$\delta'(h) = \frac{1}{\sqrt{(\dots)}} + h\frac{\partial}{\partial h}\frac{1}{\sqrt{q}} = \frac{1}{\sqrt{q}} + h\left[-\frac{1}{2}q^{-3/2}\frac{\lambda}{2}\left(-\frac{1}{2\sqrt{1-h^2}}(-2h)\right)\right] =$

$= \frac{1}{\sqrt{q}} - \frac{1}{4}\lambda h^2 q^{-3/2}\frac{1}{\sqrt{1-h^2}} = \frac{1}{\sqrt{q}}\left[1 - \frac{1}{4}\frac{\lambda}{q}\frac{h^2}{\sqrt{1-h^2}}\right] = 0$, $q = 1+\frac{\lambda}{2}(1-\sqrt{\dots})$

$1+\frac{\lambda}{2}(1-\sqrt{1-h^2}) = \frac{\lambda}{4}\frac{h^2}{\sqrt{1-h^2}}$, $1-h^2 = z$, $1+\frac{1}{2}(1-\sqrt{z}) = \frac{\lambda}{4}\frac{1+z}{\sqrt{z}}$

$\sqrt{z}+\frac{\lambda}{2}(\sqrt{z}-z) = \frac{\lambda}{4}(1-z)$, $\left(1+\frac{\lambda}{2}\right)\sqrt{z} = \frac{\lambda}{4}(1-z) + \frac{\lambda}{2}z = \frac{\lambda}{4}+\frac{1}{4}\lambda z = \frac{\lambda}{4}(1+z)$,

$\left(1+\frac{\lambda}{2}\right)^2 z = \frac{\lambda^2}{16}(1+2z+z^2)$, $z^2 + \left[2+\frac{4}{\lambda^2}(1+\frac{\lambda}{2})^2\right]z + 1 = 0$, квадратное ур-е $z = 1-h^2$.

$z \in [0,1]$

Escaneado con CamScanner

$2z_{1,2} = -2 + \frac{4}{\lambda^2}(1+\frac{\lambda}{2})^2 \pm \sqrt{\left[2 - \frac{4}{\lambda^2}(1+\lambda/2)^2\right]^2 - 4}$, $D = [\dots]^2 - 4 =$

$= 4 - \frac{16}{\lambda^2}(1+\frac{\lambda}{2})^2 + \frac{16}{\lambda^4}(1+\lambda/2)^4 - 4 = \frac{16}{\lambda^2}(1+\frac{\lambda}{2})^2\left[-1 + \lambda^{-2}(1+\frac{\lambda}{2})^2\right] =$

$= \frac{16}{\lambda^4}(1+\frac{\lambda}{2})^2\left[-\lambda^2 + (1+\frac{\lambda}{2})^2\right] = \frac{16}{\lambda^4}(1+\lambda/2)^2\left[1 + \lambda - \frac{3}{4}\lambda^2\right]$

$2z_{1,2} = -2 + \frac{4}{\lambda^2}(1+\frac{\lambda}{2})^2 \pm \sqrt{D} = -2 + \frac{4}{\lambda^2}(1+\frac{\lambda}{2})^2 \pm \frac{4}{\lambda^2}(1+\frac{\lambda}{2})\sqrt{1+\lambda-\frac{3}{4}\lambda^2}$

$\Rightarrow z_{1,2}$ невероятно, $D(\lambda) < 0$ при $\lambda > 1$. Нашли ошибку.

Надо $(1+\frac{\lambda}{2})^2 z = \frac{\lambda^2}{16}(1+z+z^2)$, $z^2 + \left[2 - \frac{16}{\lambda^2}(1+\frac{\lambda}{2})^2\right]z + 1 = 0$

$D = \left[2 - \frac{16}{\lambda^2}(1+\frac{\lambda}{2})^2\right]^2 - 4 = 4 - \frac{64}{\lambda^2}(1+\frac{\lambda}{2})^2 + \frac{16^2}{\lambda^4}(1+\lambda/2)^4 - 4 =$

$= \frac{16^2}{\lambda^4}(1+\frac{\lambda}{2})^2\left[(1+\frac{\lambda}{2})^2 - \frac{1}{4}\right]$, $2z_{1,2} = -2 + \frac{16}{\lambda^2}(1+\frac{\lambda}{2})^2 \pm \sqrt{D} =$

$= -2 + \frac{16}{\lambda^2}(1+\frac{\lambda}{2})^2 \pm \frac{16}{\lambda^2}(1+\frac{\lambda}{2})\sqrt{(1+\frac{\lambda}{2})^2 - \frac{1}{4}}$, $2z_{1,2} = -b \pm \sqrt{b^2-4}$, берем "−"

При $\lambda \to 0$ $2z_{1,2} = -2 + \frac{16}{\lambda^2} - \frac{16}{\lambda^2}\sqrt{3/4}$ – сингулярно, неверно.

При $\lambda \to \infty$ $2z_{1,2} = -2 + \frac{16}{\lambda^2}\frac{\lambda^2}{4} - \frac{16}{\lambda^2}\frac{\lambda}{2}\frac{\lambda^2}{2} = -2$, $z_2 = -1$, $h_m^2 = 1 - z_2 = 2$ –

– очень неверно.

$b = 2 - \frac{16}{\lambda^2}(1+\lambda/2)^2$, $2z_{1,2} = -b \pm \sqrt{b^2-4}$. При $\lambda \to 0$, $b \approx -\frac{16}{\lambda^2} \gg 1$

$2z_{1,2} = \frac{16}{\lambda^2} - 2 \pm \sqrt{b^2-4} = \frac{16}{\lambda^2} - |b| = \frac{16}{\lambda^2} - \frac{16}{\lambda^2} = 0$, $z_2 = 0$, $h_m = 1$ (верно).

При $\lambda \to \infty$, $b(\lambda) = 2 - \frac{16}{\lambda^2}\frac{\lambda^2}{4} = -2$, $2z_2 = 2$, $z_2 = 1$, $h_m^2 = 1 - z_2 = 0$ (верно).

Schuyler'74: $\frac{v(h)}{v_w} = \frac{1}{2}v_0\lambda f(h)\frac{1}{v_0\lambda}\sqrt{2(2+\lambda)} = \frac{1}{2}\sqrt{2(2+\lambda)}\,\frac{h}{\sqrt{1+\frac{\lambda}{2}(1-\sqrt{1-h^2})}}$

(Eq. 2)

$v(1)/v_w = 1$. $v(0)/v_w$ – нельзя. $h=0$ нужен на границе при $\lambda \to \infty$ и λ конечно.

$h_m^2 = 4\sqrt{\lambda+1}\left[2 + \lambda + \sqrt{\lambda+1}\right]$, $h_m^2(\lambda \to 0) = 4 \cdot 3 = 12$, это неверно

$h_m(\lambda \gg 1) \approx 4\sqrt{\lambda}\,\lambda = 4\lambda^{3/2}$, ~~это тоже~~ неверно. $v_w = v_0\frac{1}{\sqrt{2(2+\lambda)}}$

//MathCad показал, что все $z^2 + \left[2 - \frac{16}{\lambda^2}(1+\lambda/2)^2\right]z + 1 = 0$, $z = 1 - h_m^2$

и Eq. (6) Sobolev 1995, $h_m^2 = \frac{4}{\lambda^2}\left[(2+\varepsilon)\sqrt{1+\varepsilon} - 2(1+\varepsilon)\right]$, $\varepsilon = \lambda = K_x/K_z$

equations совпадают, дают тот же $h_m(\lambda)$ — т.е. обе верны.

«DW_velocity_FM.mcd».

→ Выводы: Eq. (3.99) Ricardo's thesis дает $h_m(\lambda)$ неверно, $h_m(\lambda)$ Sobolev'ят верно.

Eq. (3.100) тоже Ricardo – верное.

И-з верного Eq. (3.38) Ricardo's thesis дает $v(h)$ следует верное $v_w^R = v(h=1)$

$v_w^R = \frac{\gamma\Delta_0 H_w}{\alpha}\frac{1}{\sqrt{1+\lambda/2}} = v_0\frac{\lambda}{\sqrt{2(2+\lambda)}}$. Eq. (3.41) – Ricardo (неверное), и неверно

$v_w^{41} = \frac{1}{2}\gamma\Delta_0 H_K = \frac{1}{2}\gamma\Delta_0\frac{2H_w}{\alpha} = \frac{\gamma\Delta_0 H_w}{\alpha} = \frac{1}{2}\lambda v_0$ – неверное.

Надо $v_w = v_0\frac{\lambda}{\sqrt{2(2+\lambda)}}$,

(23/X) Тема Thesis Ricardo. Подготовка IVA „Конкурс" до III кв. 2022. Zoom meet. нанолизы

(24/X) Meeting с Ricardo: надо им Thesis file to check plagiat, отсюда мне Thesis text, defence date и состав jury. с Л. Б. Ерохи"

(25/X) Расчет IVA „Конкурс". Invitation G. Cantele to Donostia. Registered persons for IkerBasque workshop, Bilbao, Nov. 28, monday, 2022

(26/X) Подготовка и чтение Note и Maciej Report Task + Pole Project „BP dyn."

(27/X) Anti-plagiarism report для Ricardo's thesis.

(28/X) Зарплата 1210€ (1000 + 21% IVA) Salary Antony за Окт 2022. GM thick dot

Нужно ли учитывать отклонение от круга $g=c$ при расчете W_{ex}, W_m^{vol} of thin(ish) wires in thick dot? Ср. 7/X 2022. Не нужно.

$W_m^{vol} = \frac{1}{2}M_s^2 \int dV \int dV' \frac{d\vec{m}(r) d\vec{m}(r')}{|r-r'|}$, $d\vec{m} \sim S d$, $d\vec{m}\cdot d\vec{m}(r') \sim \frac{1}{r^2}$ и

в $\int_0^{2\pi} d\varphi g(\cdot)$ интеграл у. кого можно брать как по кругу $g=c$, $c=c(z)$.

Сегодня 18+19 IX 2020. $W_{ex}(\vec{m}) \sim s^2$, $W_{ex}\left(\frac{\partial \vec{m}}{\partial z}\right) \sim \left(\frac{\partial s}{\partial z}\right)^2$ и грат отклонения VC (radius) border от круга $g=c$ не играет в s^2-приближении.

(28/X) - (2/XI) Переписка с Оскаром, Felipe, Ruben, Jose по BP-DW dynamics $s(H)$ in nanowires. Переписка с Авс. о BP gyrotropes nanowires.

(1/XI) Встреча со Леной/Мацеем на new Polish project. Итог. docs.

(5/XI) С П. в „Lidl", „Carrefour", Olaberia. Блины.

(7/XI) Skype с Гжесем (Polish grant, заявка Ricardo, paper с Artem-GM).

(8/XI) Skype session с Леной, Гжесем, Мацеем о new NCN-Poland project. OPUS-3-й конкурс, Лена full-time PI, >1M PLN, с exp. international collaborator (Germany, Austrian?), мне и Гжесю 2.5 k PLN/month, по 6 командировок per-diem для travels to Poznan, тема – Curvilinear magnetic structures. Обзор Макаров' 2022, Adv. Materials.

(9/XI) E-mail Оскару: progress on vortex-GM wire и вопросы к BP-DW dynamics paper. Лена paper Sheka Commun. Phys. 2020 и предложение new о curvilinear magnetostatics для OPUS proposal.

(10/XI) Наше book, Curvilinear magnetism, Springer, Sp. Appl. Phys. 2022. Skype с Гжесем: exp. collaboration OPUS project (Dobrovolskiy's оказался, вернулся? Назад?).

(11/XI) Назван пригласил. invited speakers Solsky Meg' 2023.

Работа над Vortex nutation manuscript с Зуркой. Блины. Vortex nutation

Гур'янов: $\chi_\pm(\omega) = \gamma M_0/(\omega_H \mp \omega)$, $\omega_H = \gamma H_0$. только $\chi_+(\omega)$ имеет резонанс. или в circular mag. field, ccw(h_+) или cw(h_-) rotating mag. fields.

$\chi_+(\omega) \sim 1/[\omega_0 - \omega + \eta\omega^2]$ Зуркин ($\alpha=0$, damping) имеет резонансный или при $\omega_0 - \omega + \eta\omega^2 = 0$, $\eta\omega^2 - \omega + \omega_0 = 0$, два корня, res. frequencies

$\omega_{1,2} = \frac{1}{2\eta}\left[1 \pm \sqrt{1 - 4\eta\omega_0}\right]$, $\eta\omega_0 \ll 1$, $\eta = \alpha\tau$, $\alpha\omega_0\tau \ll 1$,

$\alpha \sim 0{,}01$, $\omega_0 = 2\pi \cdot 1$ GHz, $\tau \approx 10$ ps, $\alpha\omega_0\tau = 10^{-2} 2\pi \cdot 10^{9} \frac{1}{s} 10^{-12} s \cdot 10 \approx 10^{-3} \ll 1$.

τ – nutation time.

$\sqrt{1-4\eta\omega_0} = 1 - \frac{1}{2}(4\eta\omega_0) - \frac{1}{8}(4\eta\omega_0)^2 + O((\eta\omega)^3) = 1 - 2\eta\omega_0 - 2\eta^2\omega_0^2$, $\omega_0 > 0$ def.

$\omega_1 = \frac{1}{2\eta}\left[1 + \sqrt{\dots}\right] = \frac{1}{2\eta}\left[1 + (1 - 2\eta\omega_0)\right] = \frac{1}{\eta} - \omega_0 \approx \frac{1}{\eta}$, $\omega_1 > 0$

$\omega_2 = \frac{1}{2\eta}\left[1 - \sqrt{\dots}\right] = \frac{1}{2\eta}\left[1 - (1 - 2\eta\omega_0 - 2\eta^2\omega_0^2)\right] = \omega_0 + \eta\omega_0^2$, $\omega_2 > 0$

// Buettner, Nat. Phys. 2015: vortex mass $+M\ddot{\vec{X}} + \vec{G}\times\dot{\vec{X}} - \frac{\partial W}{\partial \vec{X}} = 0$.

$\vec{G} = (0, 0, G_z)$, $G_z = -2\pi\frac{M_s}{\gamma} Lp < 0 = -|G|$, $\vec{G}\times\dot{\vec{X}} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & G_z \\ \dot{X} & \dot{Y} & 0 \end{vmatrix} = \hat{y} G_z \dot{X} - \hat{x} G_z \dot{Y}$

$\begin{cases} -M\ddot{X} - G_z\dot{y} - \varkappa X = 0 \\ -M\ddot{Y} + G_z\dot{X} - \varkappa Y = 0 \end{cases}$, $Z = X + iY$, $-M\ddot{Z} + iG_z\dot{Z} - \varkappa Z = 0$.

$Z(t) = Z(\omega)e^{i\omega t}$, $-M(-\omega^2 Z) + iG_z i\omega Z - \varkappa Z = 0$, $\omega^2 M - G_z\omega - \varkappa = 0$.

Берем $p>0$, $G_z = -|G|$, $\omega^2 M + |G|\omega - \varkappa = 0$, $\omega_0 = \frac{\varkappa}{|G|} > 0$, $\frac{M}{|G|} = \mu > 0$.

$\mu\omega^2 + \omega - \omega_0 = 0$ отличается от $\eta\omega^2 - \omega + \omega_0 = 0$ на $\omega \to -\omega$ Зуркин значит

Vortex yD with mass: $\mu\omega^2 + \omega - \omega_0 = 0$, $\omega_{1,2} = \frac{1}{2\mu}\left[-1 \pm \sqrt{1 + 4\mu\omega_0}\right]$ (ω-ψ).

$\omega_1 = \frac{1}{2\mu}\left[-1 + (1 + 2\mu\omega_0)\right] \approx \omega_0$, $\omega_2 = \frac{1}{2\mu}\left[-1 - (1 + 2\mu\omega_0)\right] = -\frac{1}{\mu} < 0$. т.е.

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Two vortex with small mass: $\omega_1 \times \omega_0 > 0$, $\omega_2 = -\frac{1}{\gamma} < 0$, ω_1 – CCW, ω_2 – CW rotation

Согласно $\vec{z}(t) = e^{i\omega_{1,2} t} z_0$. Зуэра: $\vec{V} = \dot{\vec{X}} = \vec{\omega} \times \vec{X}$, $\vec{\omega} = \omega \hat{z}$.

$\vec{G} \times \vec{V} = \vec{G} \times \frac{d\vec{X}}{dt} = \vec{G} \times (\vec{\omega} \times \vec{X}) = \vec{\omega}(\vec{G} \cdot \vec{X}) - \vec{X}(\vec{G} \cdot \vec{\omega}) = -\vec{X}(\vec{G} \cdot \vec{\omega})$

$\frac{d\vec{V}}{dt} = \vec{\omega} \times \vec{X} = \vec{\omega} \times (\vec{\omega} \times \vec{X}) = \vec{\omega}(\vec{\omega} \cdot \vec{X}) - \vec{X}(\vec{\omega} \cdot \vec{\omega}) = -\omega^2 \vec{X}$.

Eq. (e) Зуэра верное: $\vec{G} \times \vec{V} - \frac{\partial W}{\partial \vec{X}} - N_0 \frac{d\vec{V}}{dt} = 0$ (без damping field ii)

$-\vec{X}(\vec{G} \cdot \vec{\omega}) - \varkappa \vec{X} - N_0(-\omega^2 \vec{X}) = 0$, $N_0 \omega^2 \vec{X} - \vec{X}(\vec{G} \cdot \vec{\omega}) - \varkappa \vec{X} = 0$

$N_0 \omega^2 - (\vec{G} \cdot \vec{\omega}) - \varkappa = 0$, $\vec{\omega} = \omega \hat{z}$. $\vec{G} \cdot \vec{\omega} = (\vec{G} \cdot \hat{z})\omega = G_z \omega = -|G|\omega$, $p > 0$

$N_0 \omega^2 + |G|\omega - \varkappa = 0$, $\frac{N_0}{|G|}\omega^2 + \omega - \omega_0 = 0$ – верное УД сила. CCW

Близкие с УД Thiele g.e vortex with mass. $\gamma_0 \omega^2 + \omega - \omega_0 = 0$ дает ($\gamma_0 = N_0/|G|$)

при $\gamma_0 \omega_0 \ll 1$ $\omega_1 \approx \omega_0$, $\omega_2 = -\frac{1}{\gamma_0}$. $\omega_1 > 0$ (CCW), $\omega_2 < 0$ (CW), т.е.

предполагаем $\omega_0 > 0$ (CCW) ($p > 0$), получим мутацию v. frequency $\omega_2 < 0$, $|\omega_2| \gg \omega_0$.

Достаточно рассм. полюса $\mathcal{J}_+(\omega)$, чтоб найти $\omega_1 > 0$, $\omega_2 < 0$, (CW). $\mathcal{J}_-(\omega)$ не нужна для расчета.

→ Выводы: мутация eq. Thiele сверяется к mass eq. Thiele $\vec{X}(t)$

мутация analog to v. mass; 2) в Eq (4) Зуэра знаки обеих первых членов не верны; 3) полюсов $\mathcal{J}_+(\omega)$ достаточно, $\omega_1 > 0$, $\omega_2 < 0$.

Новая paper draft with comments Зуэра.

Новая приглашение invited speakers to SolSkyrmag 2023.

(13-15/II) Invitation speakers SolSkyrmag'2023 и переписка с ними. Vortex

LLG с motion term $\dot{\vec{m}} = -\gamma \vec{m} \times \vec{H} + \alpha \vec{m} \times \dot{\vec{m}} + \frac{\gamma}{M_s} \vec{m} \times \ddot{\vec{m}}$ Nutation

Зуэра убедился, что у-term берет к $\vec{G} \times \vec{V} - \nabla_{\vec{X}} W + \hat{\mathcal{D}} \cdot \vec{V} - \hat{N}_0 \cdot \frac{d\vec{V}}{dt} = 0$

$(\hat{\mathcal{D}})_{ij} = \mathcal{D}\delta_{ij}$, $(\hat{N}_0)_{ij} = N_0 \delta_{ij}$, $\mathcal{D} = -\frac{dM_s L}{\gamma} \int d^2\vec{s} \left[(\nabla\Theta)^2 + \sin^2\Theta (\nabla\Phi)^2\right]$ и

$\vec{m} = \vec{m}(\Theta, \Phi)$. $N_0 = \gamma \frac{L M_s}{\gamma} \int d^2\vec{s} \left[(\nabla\Theta)^2 + \sin^2\Theta (\nabla\Phi)^2\right]$, $\vec{M}(\vec{s}, \vec{X})$

Sukh Nanolett 2014: $\mathcal{D}_{\alpha\beta} = -\alpha \frac{M_s}{\gamma} \int d^3\vec{r}\, \partial_\alpha \vec{m} \cdot \partial_\beta \vec{m}$, $\partial_\alpha = \partial/\partial X_\alpha$, $\vec{X}(t)$ – vc. position

Gus' PRL'2006: $\vec{G} = -G\hat{z} = G_{xy}\hat{z}$, $G_z = -G = -G_{xy}$, $G_{ij} = \int d^3\vec{r}\, \vec{m} \cdot [\partial_x \vec{m} \times \partial_y \vec{m}] \cdot \frac{M_s}{\gamma}$

$G = 2\pi p L M_s/\gamma > 0$ при $p > 0$, $\gamma = |\gamma|$.

$-\frac{1}{\gamma}\frac{dM_i}{dt} + \frac{\alpha}{\gamma M_s} \varepsilon_{ijk} M_j \frac{dM_k}{dt} = \varepsilon_{ijk} M_j H_k$. $g_{ij} = \frac{1}{M_s^2 \gamma} \varepsilon_{kmn} M_k \frac{\partial M_m}{\partial x_i} \frac{\partial M_n}{\partial y_j}$

$\frac{dM_i}{dt} = \frac{dM_i}{\partial X_j} \frac{\partial X_j}{\partial t}$, $f_i^g = g_{ij} \frac{dX_j}{dt} = \left(\vec{g} \times \frac{d\vec{X}}{dt}\right)_i = \varepsilon_{ijk} g_j \frac{dX_k}{dt}$, $g_i = -\frac{1}{2}\varepsilon_{ijk} g_{jk}$

$\left(\vec{g} \times \frac{d\vec{X}}{dt}\right)_i = \varepsilon_{ijk}\left(-\frac{1}{2}\right)\frac{dX_k}{dt} \varepsilon_{jlm} g_{lm}$. $\frac{dM_k}{dt}$? умножим LLG by $\vec{M} \times$ - cross product (векторно) и на $M_s^{-1}\frac{\partial \vec{M}}{\partial X_i}$ скалярно.

$\dot{\vec{M}} = -\gamma \vec{M} \times \vec{H} + \frac{\alpha}{M_s} \vec{M} \times \dot{\vec{M}} + \frac{\gamma}{M_s} \vec{M} \times \ddot{\vec{M}}$, $\vec{M} \times \dot{\vec{M}} = -\gamma \vec{M} \times \vec{M} \times \vec{H} + \frac{\alpha}{M_s} \vec{M} \times \vec{M} \times \dot{\vec{M}} +$

$+ \frac{\gamma}{M_s} \vec{M} \times \vec{M} \times \ddot{\vec{M}} = -\gamma\left[\vec{M}(\vec{M} \cdot \vec{H}) - \vec{H}(\vec{M} \cdot \vec{M})\right] + \frac{\alpha}{M_s}\left\{\vec{M}(\vec{M} \cdot \dot{\vec{M}}) - \dot{\vec{M}}(\vec{M} \cdot \vec{M})\right\} +$

$+ \frac{\gamma}{M_s}\left[\vec{M}(\vec{M} \cdot \ddot{\vec{M}}) - \ddot{\vec{M}} M_s^2\right]$ // $\frac{1}{M_s}\frac{\partial \vec{M}}{\partial X_i} \cdot (\vec{M} \times \dot{\vec{M}}) = -\gamma \frac{\partial \vec{M}}{M_s \partial X_i} \cdot \left[\vec{M}(\vec{M} \cdot \vec{H}) - \vec{H} M_s^2\right] +$

$+ \frac{\alpha}{M_s^2}\frac{\partial \vec{M}}{\partial X_i} \cdot \left[\vec{M}(\vec{M} \cdot \dot{\vec{M}}) - \dot{\vec{M}} M_s^2\right] + \frac{\gamma}{M_s}\frac{\partial \vec{M}}{\partial X_i}\left[\vec{M}(\vec{M} \cdot \ddot{\vec{M}}) - \ddot{\vec{M}} M_s^2\right]$, $\frac{\partial \vec{M}}{\partial X_i} \cdot \vec{M} = 0$, $\vec{M} \cdot \dot{\vec{M}} = 0$, $\vec{M}^2 = \text{const}$

$\frac{1}{M_s}\frac{\partial \vec{M}}{\partial X_i} \cdot (\vec{M} \times \dot{\vec{M}}) = \gamma M_s \frac{\partial \vec{M}}{\partial X_i} \cdot \vec{H} - \alpha \dot{\vec{M}} \cdot \frac{\partial \vec{M}}{\partial X_i} - \gamma \frac{\partial \vec{M}}{\partial X_i} \cdot \ddot{\vec{M}}$, $\dot{\vec{M}} = \frac{d}{dt}\vec{M}(\vec{r}, \vec{X}) = \frac{\partial \vec{M}}{\partial X_j}\frac{dX_j}{dt}$. $\vec{V} = \frac{d\vec{X}}{dt}$

$\frac{1}{M_s}\frac{\partial \vec{M}}{\partial X_i} \cdot \left[\vec{M} \times \frac{\partial \vec{M}}{\partial X_j} V_j\right] = \gamma M_s \frac{\partial \vec{M}}{\partial X_i} \cdot \vec{H} - \alpha \frac{\partial \vec{M}}{\partial X_i} \cdot \frac{\partial \vec{M}}{\partial X_j} V_j - \gamma \frac{\partial \vec{M}}{\partial X_i} \cdot \ddot{\vec{M}}$, см. 13/II

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(15/XI 2022) Нови работи (от 12/2021):

1) Vortex GM dome-shaped dots - Artem - Intro/Summary → Small
2) Nutation Vortex GM, Бухра → Nanomaterials, subm. 14/12/2022, publ. 23/I
3) BP domain wall, Фархад, subm. PRA 30/IX 22, resubm. 17/I
4) Vortex string GM, Оксана-Felipe, нерабочее об-ие
5) BP gyrotropic dynamics, cylinder, Окс., x_2, x - pds ones
6) BP-DW dynamics, нанопоста Оксана, Ssw(H) Overleaf, subm. 24/I
7) Invitations speakers SolStyMag. 2023 - з/на. Ansona
8) pap. IEEE TM, vortex M(H)
9) Ricardo's contract: appl. finish 31/12/2022 → 11/01/2023
10) New curvilinear structures proposal с Ларой, subm. OPUS 15/12/2022
11) Paper no 4-task PdLS project (hopfions)
12) trip to Madrid, change 23/XI → 30/XI, hotel deport
13) Air tickets y hotel Poznan, February 2023 - hotel os.нема до 25/I Berlin 4-15/II 2023
14) New MICINN proposal с Оксана
15) Abs. HMM, Vienna, June 5-7, 2023, до Feb. 28
16) PM' 2023, Poznan, June 26-30, 2023, abs. до 28/II 2023
17) JEMS'23, Madrid, Aug. 27 - Sept. 1, 2023, abs. до 3/III 2023
18) TMAG'23, Rome, Italia, Sept. 4-8, 2023, abs. до 1/V 23г.
19) Ikerbasque report за 2022
20) NAP-2023, Bratislava, Sept. 10-15, 2023. Abs. 1/V 2023.
21) passport Ansona
22) Distinguished Scientist UPV/EHU appl.

(16/XI) С А.Вчера за срещата нерабочее SolStyMag. 2023.
E-mail до Ricardo about progress calc. SW in cylindrical wires.
Реферат IEEE TM Strelkov et al. Vortex M(H) curves - fitting.

(17/XI) нерабочее с Арантха. Вывод eq. Thiele с mutation term:

$$\frac{\partial \vec{m}}{\partial x_i}\cdot\left[\vec{m}\times\frac{\partial \vec{m}}{\partial x_j}\right] = \vec{m}\cdot\left[\frac{\partial \vec{m}}{\partial x_i}\times\frac{\partial \vec{m}}{\partial x_j}\right] = -\vec{m}\cdot\left[\frac{\partial \vec{m}}{\partial x_i}\times\frac{\partial \vec{m}}{\partial x_j}\right],\quad g_{ij} = \frac{1}{M_s^2\gamma}\vec{m}\cdot\left[\frac{\partial \vec{m}}{\partial x_i}\times\frac{\partial \vec{m}}{\partial x_j}\right]$$

$$\frac{1}{M_s}V_j\frac{\partial \vec{m}}{\partial x_i}\cdot\left[\vec{m}\times\frac{\partial \vec{m}}{\partial x_j}\right] = \frac{1}{M_s}V_j(-)\vec{m}\cdot\left[\frac{\partial \vec{m}}{\partial x_i}\times\frac{\partial \vec{m}}{\partial x_i}\right] = -\frac{1}{M_s}V_j\gamma M_s^2 g_{ij} = -\gamma M_s g_{ij}V_j =$$

$$= -\gamma M_s(\vec{g}\times\vec{V})_i,\quad -\gamma M_s(\vec{g}\times\vec{V})_i = \gamma M_s\frac{\partial \vec{m}}{\partial x_i}\cdot\vec{H} - \alpha\left(\frac{\partial \vec{m}}{\partial x_i}\cdot\frac{\partial \vec{m}}{\partial x_j}\right)V_j - (\eta\text{-term})$$

$\vec{H} = -\frac{\partial W}{\partial \vec{m}}$ - effective field, $\frac{\partial \vec{m}}{\partial x_i}\cdot\vec{H} = -\frac{\partial \vec{m}}{\partial x_i}\cdot\frac{\partial W}{\partial \vec{m}} = -\frac{\partial W}{\partial x_i}$

$$-(\vec{g}\times\vec{V})_i = \frac{\partial \vec{m}}{\partial x_i}\cdot\vec{H} - \frac{\alpha}{\gamma M_s}\left(\frac{\partial \vec{m}}{\partial x_i}\cdot\frac{\partial \vec{m}}{\partial x_j}\right)V_j - \frac{1}{\gamma M_s}(\eta\text{-term}).\quad d_{ij} = \left(\frac{\partial \vec{m}}{\partial x_i}\cdot\frac{\partial \vec{m}}{\partial x_j}\right)\frac{1}{M_s^2}$$

$$-(\vec{g}\times\vec{V})_i = -\frac{\partial W}{\partial x_i} - \frac{\alpha}{\gamma M_s}M_s^2 d_{ij}V_j - \frac{1}{\gamma M_s}(\eta\text{-term})$$

$$-\vec{g}\times\vec{V} = -\frac{\partial W}{\partial \vec{x}} - \frac{\alpha M_s}{\gamma}\hat{d}\cdot\vec{V} - \frac{1}{\gamma M_s}(\eta\text{-term}) - \text{eq. Thiele nocna } \int d^3\vec{r}(\ldots).$$

$$-\vec{G}\times\vec{V} = -\frac{\partial W}{\partial \vec{X}} + \hat{D}\cdot\vec{V} - \frac{1}{\gamma M_s}\int d^3\vec{r}\,(\eta\text{-term}),\quad \hat{D} = -\int\frac{dM_s}{\gamma}d\vec{a}\vec{r}$$

$$(\hat{D})_{ij} = -\frac{\alpha M_s}{\gamma}\int d^3\vec{r}\,d_{ij} = -\frac{\alpha M_s}{\gamma}\int d^3\vec{r}\left(\frac{\partial \vec{m}}{\partial x_i}\cdot\frac{\partial \vec{m}}{\partial x_j}\right),\quad \dot{\vec{m}} = \frac{\partial \vec{m}}{\partial x_j}\frac{dx_j}{dt}$$

$$(\eta\text{-term}) = \eta\frac{\partial \vec{m}}{\partial x_i}\cdot\ddot{\vec{m}} = \eta\frac{\partial \vec{m}}{\partial x_i}\cdot\frac{d}{dt}\dot{\vec{m}}(\vec{r},\vec{X}) = \eta\frac{\partial \vec{m}}{\partial x_i}\cdot\frac{d}{dt}\left[\frac{\partial \vec{m}}{\partial x_j}\frac{dx_j}{dt}\right] = \eta\frac{\partial \vec{m}}{\partial x_i}\cdot$$

$$\cdot\left[\frac{d}{dt}\left(\frac{\partial \vec{m}}{\partial x_i}\right)V_j + \frac{\partial \vec{m}}{\partial x_j}\frac{d^2x_j}{dt^2}\right] = \eta\left(\frac{\partial \vec{m}}{\partial x_i}\cdot\frac{\partial \vec{m}}{\partial x_j}\right)\frac{dV_j}{dt} + \eta\frac{\partial \vec{m}}{\partial x_i}\cdot\frac{d}{dt}\left(\frac{\partial \vec{m}}{\partial x_j}\right)\cdot V_j$$

$$\frac{\partial \vec{m}}{\partial x_i}\cdot\frac{d}{dt}\left(\frac{\partial \vec{m}}{\partial x_j}\right) = \frac{\partial \vec{m}}{\partial x_i}\cdot\frac{\partial}{\partial x_j}\left(\frac{d\vec{m}}{dt}\right) = \frac{\partial \vec{m}}{\partial x_i}\cdot\frac{\partial}{\partial x_j}\left(\frac{\partial \vec{m}}{\partial x_k}\frac{dx_k}{dt}\right) = \frac{\partial \vec{m}}{\partial x_i}\cdot\frac{\partial^2 \vec{m}}{\partial x_j\partial x_k}V_k$$

$$(\eta\text{-term}) = \eta\left(\frac{\partial \vec{m}}{\partial x_i}\cdot\frac{\partial \vec{m}}{\partial x_j}\right)\frac{dV_j}{dt} + \eta V_j\left(\frac{\partial \vec{m}}{\partial x_i}\cdot\frac{\partial^2 \vec{m}}{\partial x_j\partial x_k}\right)V_k$$

$$\vec{G}\times\vec{V} = \frac{\partial W}{\partial \vec{X}} - \hat{D}\cdot\vec{V} + \frac{\eta}{\gamma M_s}\int d^3\vec{r}\left[\left(\frac{\partial \vec{m}}{\partial x_i}\cdot\frac{\partial \vec{m}}{\partial x_j}\right)\frac{dV_j}{dt} + \left(\frac{\partial \vec{m}}{\partial x_i}\cdot\frac{\partial^2 \vec{m}}{\partial x_j\partial x_k}\right)V_jV_k\right]$$

$$\vec{G}\times\vec{V} = \frac{\partial W}{\partial \vec{X}} - \hat{D}\cdot\vec{V} + N\frac{d\vec{V}}{dt} + (\eta/\gamma M_s)\int d^3\vec{r}\left(\frac{\partial \vec{m}}{\partial x_i}\cdot\partial^2\vec{m}/\partial x_j\partial x_k\right)V_jV_k,\ \text{до нерабочее}$$

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Ньютон: члены $\sim V_i V_k = \frac{dX_i}{dt}\frac{dX_k}{dt}$ как члены $\sim X^2$ в linear Thiele equation.

$\left(\vec{G}\times\vec{V} = \frac{\partial W}{\partial \vec{X}} - \hat{D}\vec{V} + \hat{N}\frac{d\vec{V}}{dt}\right)$, $(\hat{N})_{ij} = \frac{\gamma M_s}{\gamma} L \int d^2\vec{s}\left(\frac{\partial \vec{m}}{\partial X_i}\cdot\frac{\partial \vec{m}}{\partial X_j}\right)$

$\hat{D}$ и $\hat{N}$ are diagonal in linear approx. on $\vec{X}$, $D_{xx} = D_{yy} = D$, $N_{xx} = N_{yy} = N_0$,

$D_{ij} = 0$, $N_{ij} = 0$ при $i \neq j$.

N_0, D зависят от модели $\vec{M}(\vec{s}, \vec{X})$ of shifted vortex: rigid, TVA

$D_{ij} = +\alpha\frac{M_s L}{\gamma}\int d^2\vec{s}\left(\frac{\partial \vec{m}}{\partial X_i}\cdot\frac{\partial \vec{m}}{\partial X_j}\right)$, B rigid v. model $\vec{m}(\vec{s}-\vec{X})$ $\frac{\partial \vec{m}}{\partial X_i}\frac{\partial \vec{m}}{\partial X_j} = \frac{\partial \vec{m}}{\partial x_i}\frac{\partial \vec{m}}{\partial x_j}$

$D_{xx} + D_{yy} = -\frac{\alpha M_s}{\gamma} L \int d^2\vec{s}\left[\left(\frac{\partial \vec{m}}{\partial x}\right)^2 + \left(\frac{\partial \vec{m}}{\partial y}\right)^2\right] = -\frac{2M_s}{\gamma} L \int d^2\vec{s}\,(\nabla\vec{m})^2 =$

$= -\frac{\alpha M_s L^2}{\gamma}\cdot E_{ex}/A$ $E_{ex} = AL\int d^2\vec{s}\,(\nabla\vec{m})^2 = AL\int d^2\vec{s}\left[(\nabla\Theta)^2 + \sin^2\Theta(\nabla\Phi)^2\right]$

Guslienko, Nano RL '2014, $d = -\frac{D}{|G|} = d_G\frac{1}{8}\left[5 + 4\ln(R/R_c)\right]$

$D = -|G|d = -\left(2\pi M_s\frac{L}{\gamma}\right)\cdot\frac{d_G}{8}\left[5 + 4\ln(R/R_c)\right]$ within TVA ansatz

Guslienko, APL, Book: $D = -\left(2\pi M_s\frac{L}{\gamma}\right)d_G\left[1 + \frac{1}{2}\ln(R/R_c)\right]$ within rigid vortex ansatz

→ D (TVA) немного меньше D (rigid vortex).

Компьютерное моделирование Vortex mutation paper с Зуевой.

(2/24) E-mails to Ricardo (2W nanowires) и Зуевой (Figs).

Vortex Zeeman energy (in-plane field) $W_H = -\langle\vec{M}(\vec{r},\vec{X})\rangle_V \vec{H} V$, $V = \pi R^2 L$.

$\langle\vec{M}(\vec{r},\vec{X})\rangle_V = M_s\langle\vec{m}(\vec{r},\vec{s})\rangle_V(\vec{H}) = M_s\cdot(-1)\frac{1}{2}C(\hat{z}\times\vec{s}) = -\frac{M_s}{R}\frac{1}{2}C(\hat{z}\times\vec{X})$

$W_H(\vec{X}) = \frac{M_s}{R}\frac{1}{2}C(\hat{z}\times\vec{X})\cdot\vec{H}V = \frac{M_s}{R}\frac{1}{2}CV\vec{H}\cdot(\hat{z}\times\vec{X}) = \mu\vec{X}\cdot(\vec{H}\times\hat{z})$

$\mu = \frac{1}{R}\frac{1}{2}CM_s V$. В Gus' APL $W_H = \mu\vec{X}\cdot(\hat{z}\times\vec{H})$ и $\mu = \frac{1}{R}\frac{1}{2}CM_s$, т.е. в APL 2006 я использовал wrong sign W_H (это неважно из-за $C = \pm 1$) и не учитывал μ на dot volume V (in W_H). Верно $\mu = \frac{1}{R}\frac{1}{2}CM_s V$.

$\mu = \frac{1}{2}CM_s\frac{1}{R}\pi R^2 L = \frac{1}{2}CM_s\pi RL$. $\vec{F} = -\frac{\partial}{\partial\vec{X}}\left[W + W_H\right] = -\frac{\partial W}{\partial\vec{X}} - \frac{\partial}{\partial\vec{X}}\left[\mu\vec{X}\cdot(\vec{H}\times\hat{z})\right] =$

$= -\frac{\partial W}{\partial\vec{X}} - \mu(\vec{H}\times\hat{z})$, $\vec{F} = -\nabla_{\vec{X}}E - \mu(\vec{H}\times\hat{z})$, $\vec{F}_H = -\frac{\partial W_H}{\partial\vec{X}} = -\mu(\vec{H}\times\hat{z}) =$

$\langle\vec{m}(\vec{r},\vec{X})\rangle_V = -\frac{M_s}{R}C(\hat{z}\times\vec{X}) = -\frac{1}{V}\mu(\hat{z}\times\vec{X})$. $\hat{z}\times\vec{X} = \hat{y}X - \hat{x}Y$.

УД для W_H: $-N\omega^2\vec{X} - G\omega\vec{X} + \kappa\vec{X} - D(\vec{\omega}\times\vec{X}) = 0$, $\vec{\omega} = \omega\hat{z}$.

$\left(\omega_0 - \omega - \frac{N}{G}\omega^2\right)X + d\omega Y = 0$, $+d\omega X + Y\left(\omega_0 - \omega - \frac{N}{G}\omega^2\right) = 0$

$Z = X + iY$, $\left(\omega_0 - \omega - y\omega^2\right)Z + id\omega Z = 0$, $y = N/G$, $d = -D/G > 0$.

$\left(\omega_0 - \omega - y\omega^2 + id\omega\right)Z = 0$, $y\omega^2 + (1 - id)\omega - \omega_0 = 0$ – res. frequency

$\nu = (1 - id)\omega$, $|\vec{v}|^2 = (1 + d^2)\omega^2$, $\omega = \frac{\nu}{(1-id)} = \nu\frac{(1+id)}{(1+d^2)}$, $d \approx 3d_G = 0{,}03 \ll 1$

Ищем решение сер. ур. $\omega = \omega' + i\omega''$, $\omega'' \ll \omega'$.

$y(\omega'^2 + 2i\omega'\omega'') + (1 - id)(\omega' + i\omega'') - \omega_0 = 0$, $(1-id)(\omega' + i\omega'') = \omega' - id\omega' + i\omega' + d\omega''$

$y(\omega'^2 + 2i\omega'\omega'') + (\omega' - id\omega' + i\omega'') - \omega_0 = 0$.

Re: $y\omega'^2 + \omega' - \omega_0 = 0$, Im: $y\,2\omega'\omega'' - d\omega' + \omega'' = 0$

Ур-е для ω' совпадает с ур. для ω: $y\omega^2 + \omega - \omega_0 = 0$ см. 11/21.

$(2y\omega' + 1)\omega'' = d\omega'$, $\omega'' = \frac{d\omega'}{(1 + 2y\omega')}$. $\omega_{res} = \omega'\left[1 + \frac{id}{(1+2y\omega')}\right]$

Vortex, GM: $\omega' \approx \omega_0$, $\omega_{res} = \omega_0\left[1 + \frac{id}{(1+2y\omega_0)}\right] \approx \omega_0(1 + id)$, т.к. $y\omega_0 \ll 1$.

Escaneado con CamScanner

Нутация GM: $\omega' = -\frac{1}{\gamma_G} e^{i\omega_{res}t}$, $\omega_{res} = -\frac{1}{\gamma_G}\left[1 + \frac{id}{\left[1+2\gamma_G\left(-\frac{1}{\gamma_G}\right)\right]}\right] = -\frac{1}{\gamma_G}[1-id]$

Vortex GM $Z(t) = Z_0 e^{i\omega_{res}t} = Z_0 \exp[i(\omega_0 + id\omega_0)t] = Z_0 e^{i\omega_0 t} e^{-d\omega_0 t}$

Нутация GM $Z(t) = Z_0 \exp\left[i\left(-\frac{1}{\gamma_G}\right)(1-id)t\right] = Z_0 e^{-i\frac{1}{\gamma_G}t} e^{-\frac{d}{\gamma_G}t}$

$Z_n(t) = Z_0 e^{i\omega_n t} e^{-(t/\tau)t}$, осциллирует с $\omega_n = -\frac{1}{\gamma_G}$ и затухает как $e^{-\frac{d}{\gamma_G}t}$ или $e^{-d|\omega_n|t}$

$\gamma = \dot{\gamma}_{xz} = \frac{N}{G}$, $d = -\frac{D}{G}$. $D = D_{xx} = -\frac{dM_s L}{\gamma}\int d^2\vec{s}\left[\left(\frac{\partial\Theta}{\partial x}\right)^2 + \sin^2\Theta\left(\frac{\partial\Phi}{\partial x}\right)^2\right]$

$N = N_{xz} = \gamma\frac{M_s L}{\gamma}\int d^2\vec{s}\left[(\partial_x\Theta)^2 + \sin^2\Theta(\partial_x\Phi)^2\right]$.

$\frac{d}{\gamma_G} = -\frac{D}{G}\frac{G}{N} = -\frac{D}{N} = \frac{dM_s L}{\gamma}\cdot\frac{1}{\gamma\left(\frac{M_s L}{\gamma}\right)J} = \frac{d}{\gamma}$. $\gamma = d\tau$ – есть инерция, то $d/\gamma_G = \frac{1}{\tau}$.

Нутация VC motion $Z_n(t) = Z_0 \exp(i\omega_n t) e^{-t/\tau}$ и

нутация vortex mode Z затухает Z_0 время $\tau = 10\text{–}10^2$ fs $= 0{,}01\text{–}0{,}1$ ps $\quad 1-10^2$ ps

$\omega_n \approx \frac{1}{d\tau} = \frac{1}{0{,}01} \frac{1}{10\text{ fs}} = \frac{10^2}{10\cdot10^{-15}\text{ s}} = 10^{16}\frac{1}{s}$, $\nu_n = \frac{\omega_n}{2\pi} = 10^{15}\text{ Hz} = 10^3$ THz –

– это то не то, ν_n должна ~ 1 THz, есть эксперимент K. Neeraj et al. Nat. Phys. 2021 на GdFeB и FeNi, где $\omega_n/2\pi \approx 0{,}6$ THz.

Надо $\tau = 10\text{–}100$ ps (10 ps), тогда $\omega_n/2\pi \approx 1$ THz, разумное.

$\Delta\omega_n \approx \omega_n$ (exp Neeraj 2021) $\approx 0{,}6$ THz, широкий mutation resonance.

Вывод: $Z_n(t) \sim e^{-t/\tau}$, $\tau \approx 10$ ps, vortex core mutation resonance очень быстро (за ~ 10 ps) затухает. Если смещать $|\vec{X}_n| \sim \chi''(\omega_n)$, то mutation V. core motion under very small amplitude, < 1 nm.

→ Vortex mutation mass: $N\frac{d\vec{V}}{dt}$ член можно ошибать как $M_V\frac{d\vec{V}}{dt}$, M_V – mass.

$M_V = N = \gamma\frac{M_s L}{\gamma}\int d^2\vec{s}\left[(\partial_x\Theta)^2 + \sin^2\Theta(\partial_x\Phi)^2\right]$, $D(\text{VA}) = -d\left(\frac{2\pi M_s L}{\gamma}\right)\left[5/8 + \frac{1}{2}\ln(R/R_c)\right]$

$D = -\frac{dM_s L}{\gamma}\int d^2\vec{s}[\ldots]$, откуда $\int d^2\vec{s}[\ldots] = 2\pi\left[5/8 + \ln(R/R_c)\right]$

$M_V = \gamma\frac{M_s L}{\gamma}\cdot 2\pi\left[5/8 + \ln R/R_c\right]$, $\gamma = d\tau$, $\tau \sim 10$ ps.

V. Mass: $M_V = \frac{L}{\gamma^2}\gamma M_s\gamma\cdot 2\pi\left[5/8 + \ln R/R_c\right] = \frac{L}{\gamma^2}\frac{1}{2}\omega_M\gamma\left[\frac{5}{8} + \ln\frac{R}{R_c}\right] = \frac{L}{\gamma^2}\frac{1}{2}d\omega_M\tau\left[\frac{5}{8} + \ln\frac{R}{R_c}\right]$

FeNi: $d\omega_M\tau \approx 0{,}01\cdot 2\pi\cdot 30\text{ GHz}\cdot 10\text{ ps} = 10^{-2}\cdot 2\pi\cdot 3\cdot 10^{10}\frac{1}{s}\cdot 10^{-11}\text{ s} \approx 6\pi\cdot 10^{-3}$

Сочетание 19/XI 2014 г. $\frac{1}{\gamma^2} = 2{,}95\cdot 10^{-15}\frac{g}{cm} = 2{,}95\cdot 10^{-22}\frac{g}{nm}$ дает $\gamma/2\pi = 2{,}93\frac{\text{GHz}}{\text{Oe}}$

Нутационная масса $M_V = \frac{L}{\gamma^2}\frac{1}{2}\left[\frac{5}{8} + \ln\frac{R}{R_c}\right]d\omega_M\tau = \frac{L}{\gamma^2}\frac{1}{2}(0{,}63+2{,}30)\cdot 6\pi\cdot 10^{-3} =$

$= \frac{L}{\gamma^2}27{,}6\cdot 10^{-3} \approx 28\cdot 10^{-3}\frac{L}{\gamma^2}$, $L = 10$ nm, $M_V(10_{nm}) = 28\cdot 10^{-3}\cdot 10\frac{}{nm}\cdot 3{,}0\cdot 10^{-22}\frac{g}{nm} = 0{,}84\cdot 10^{-22}$ g

Mass density $M_V/L = \frac{1}{\gamma^2}\cdot 0{,}84\cdot\frac{1}{2}(0{,}63+2{,}30)\cdot 6\pi\cdot 10^{-3} = 0{,}84\cdot 10^{-23}\frac{g}{nm}$.

$M_V \sim LM_s t d$, M_V можно убеждать до сих пор для M_s, τ, d.

Gus' Sci. Rep. 2015. $M_V/L = (1\text{–}10)/\gamma^2$ при $L = 20\text{–}100$ nm. // Посылка A.D. 14 kg и помпон 10 kg – 5 B€

Нутационная mass density $M_V/L = 27{,}6\cdot 10^{-3}\frac{1}{\gamma^2} = 0{,}028\frac{}{\gamma^2} \ll M_V/L$ (dyn. vortex SW-vort. interact)

19/XI: Вбегаем $Z = X + iY$ и находим $\chi(\omega)$ из уравнений для $Z(t)$, при $\vec{H} \neq 0$.

(1) $(\omega_0 - \omega - \gamma_G\omega^2)X - d\omega Y = \frac{M}{G}H_y$, (2) $d\omega X + (\omega_0 - \omega - \gamma_G\omega^2)Y = -\frac{M}{G}H_x$, умн. (2) на i и сложим (1)

$(\omega_0 - \omega - \gamma_G\omega^2 + id\omega)Z = -\frac{M}{G}i(H_x + iH_y) = -\frac{M}{G}iH$, $H = H_x + iH_y$

$Z(\omega) = -i\frac{M}{G}H/\left[\omega_0 - \omega - \gamma_G\omega^2 + id\omega\right]$, $\langle\vec{M}\rangle = -\frac{M_s}{R}\int(\hat{z}\times\vec{V})$, $\hat{z}\times\vec{V} = \hat{y}X - \hat{x}Y$

Escaneado con CamScanner

$\langle \vec{M} \rangle_V = -\frac{M}{V}(\hat{z}+\vec{X})$, $\langle M_x \rangle = -\frac{M}{V}(-y)$, $\langle M_y \rangle = -\frac{M}{V}x$

$\langle M \rangle = \langle M_x + iM_y \rangle = -\frac{M}{V}(-y+ix) = -i\frac{M}{V}(x+iy) = -i\frac{M}{V}z$

$\langle M \rangle = -i\frac{M}{V}z = -i\frac{M}{V}\left(-\frac{M}{G}iH\right)/[\omega_0 - \omega - \gamma_G\omega^2 + i d\omega]$

$\langle M_+ \rangle_V = H\frac{M^2}{VG} H/[\omega_0 - \omega - \gamma_G\omega^2 + i d\omega]$, $H_{(t)} = H_x + iH_y = H_0 e^{i\omega t} = H(\omega)e^{i\omega t}$

$\langle M_+ \rangle \to M_+$. $M_+(t) = M_x + iM_y = M_0 e^{i\omega t} = M_+(\omega)e^{i\omega t}$

$M_+(\omega) = H\frac{M^2}{VG}\frac{H(\omega)}{[\omega_0 - \omega - \gamma_G\omega^2 + i d\omega]} = \chi(\omega)H(\omega)$, $\chi(\omega) = \frac{i\gamma M^2}{VG[\omega_0 - \omega - \gamma_G\omega^2 + i d\omega]}$

При $\omega = 0$ (static limit) $M_+(0) = H\frac{M^2}{VG}H(0)\frac{1}{\omega_0} = \chi(0)H(0)$, $\chi(0) = \pm\frac{M^2}{VG}\frac{1}{\omega_0}(-1)$, знак = ?

$\langle \vec{M} \rangle_V = -\frac{M}{V}(\hat{z}\times\vec{X})$ – вернов eq.

При $\omega \to 0$ $\vec{z}\times\vec{X} = \mu(\hat{z}\times\vec{H}) = \mu[-\hat{x}H_y + \hat{y}H_x]$, $X = \frac{M}{z}(-1)H_y$, $Y = \frac{M}{z}H_x$

Eq.(4) верное, а в Eq.(5) знак перепутан знак μ (или $\vec{H}$).

Посчитаем $Z(\omega) = i\frac{M}{G}H/[\omega_0 - \omega - \gamma_G\omega^2 + i d\omega]$ и $M_+(\omega) = \frac{M^2}{VG}H\frac{1}{[\dots]}$,

$\to \chi_{++}(\omega) = \frac{\partial M_+(\omega)}{\partial H_+(\omega)} = \frac{M_+(\omega)}{H_+(\omega)} = \frac{M^2}{VG}\frac{1}{[\omega_0 - \omega - \gamma_G\omega^2 + i d\omega]}$

$\chi_{++}(\omega=0) = \chi(0) = \frac{M^2}{VG\omega_0}$, $\frac{M^2}{VG} = \frac{1}{R^2}\xi^2 M_s^2 V^2 \frac{1}{VG} = \frac{\xi^2 M_s^2}{R^2}\frac{\pi R^2 L}{2\pi\frac{M_s}{\gamma}L} =$

$= \xi^2 M_s^2 \frac{\gamma}{2M_s} = \frac{1}{2}\gamma M_s \xi^2$, $\chi(0) = \frac{1}{2}\frac{\gamma M_s \xi^2}{\omega_0}$ – ответ с JAP'2002 и APL'2006.

$\chi(\omega) = \frac{M^2}{VG}\frac{1}{[\omega_0 - \omega - \gamma_G\omega^2 + i d\omega]}$, $\chi''(\omega) = \frac{M^2}{VG}\frac{(+d)\omega}{(\omega_0 - \omega - \gamma_G\omega^3)^2 + d^2\omega^2}$

$\chi(\omega) = \frac{M^2}{VG}\frac{(\omega_0 - \omega - \gamma_G\omega^2 - i d\omega)}{[(\omega_0 - \omega - \gamma_G\omega^2)^2 + d^2\omega^2]}$, $\chi''(\omega) = -d\frac{\omega\omega_0}{(\omega_0 - \omega - \gamma_G\omega^2)^2 + d^2\omega^2}\chi(0)$

$\frac{M^2}{VG} = \chi(0)\omega_0$, ω_0 – GM frequency, $\omega_0 = \frac{2\pi}{G}$, $G > 0$ by definition of PQ-sign. $G = -G_z$, $\vec{G} = G_z\hat{z}$.

(20/11) $L_e = \sqrt{\frac{2A}{M_s^2}} = \sqrt{\frac{2\cdot 1{,}1\cdot 10^{-6}\,\text{erg/cm}}{(0{,}8)^2\cdot 10^6\,\frac{\text{erg}}{\text{cm}^3}}} = \sqrt{\frac{2{,}2}{0{,}64}}\,10^{-6}\,\text{cm} \approx 18{,}5\,\text{nm}$, $M_s = 800$ G, $R = 100$ nm, $L = 10$ nm, Py

$R_c = 0{,}68\,L_e(L/L_e)^{1/3} = 0{,}68\cdot 18{,}5\,\text{nm}\,(10/18{,}5)^{1/3} = 0{,}68\cdot 18{,}5\,\text{nm}\cdot 0{,}816 = 10{,}3\,\text{nm}$

$\ln(R/R_c) = \ln(100\,\text{nm}/10{,}3\,\text{nm}) = 2{,}27$, $d = \alpha\left[\frac{5}{8} + \frac{1}{2}\ln(R/R_c)\right] =$

$= 0{,}01\cdot\left[\frac{5}{8} + \frac{1}{2}\cdot 2{,}27\right] = 0{,}01\cdot 1{,}76 = 0{,}018 \approx 0{,}02$. $d \approx 0{,}02$

$\omega_0 \approx \frac{20}{9}\gamma M_s\beta$, $\omega_0/2\pi = \frac{20}{9}\left(\frac{\gamma}{2\pi}\right)M_s\frac{L}{R} = \frac{20}{9}2{,}95\,\frac{\text{GHz}}{\text{kOe}}\,0{,}8\,\text{kOe}\cdot 0{,}1 = 0{,}52\,\text{GHz}$

$\chi''(\omega_0) = -\chi(0)\frac{d\omega_0^2}{\gamma_G^2\omega_0^4 + d^2\omega_0^2} = |\gamma_G\omega_0 \ll 1| = -\chi(0)\frac{1}{d}$ – GM resonance peak

$\chi''(\omega_y) \approx -\chi(0)\frac{\omega_y\omega_0 d}{0 + d^2\omega_y^2} = -\chi(0)\frac{1}{d}\frac{\omega_0}{\omega_y}$ – interlation resonance peak amplitude.

Ratio $\chi''(\omega_y)/\chi''(\omega_0) = \omega_0/\omega_y \ll 1$, т.к. $|\omega_y| = \frac{1}{\gamma_G} \gg \omega_0$.

$|\omega_y| = \frac{1}{\gamma_G} = \frac{G}{N} = 2\pi M_s\frac{L}{\gamma}\frac{1}{N}$, $N = N_{xx} = N_{yy} = \frac{1}{2}\pi c(N)$, $\pi c(N) = \frac{4\pi M_s L}{\gamma}y \times$

$\times \int d^2\vec{s}\,[(\nabla_x\theta)^2 + \sin^2\theta(\nabla_x\varphi)^2]$, $J = \int d^2\vec{s}\,[(\nabla\theta)^2, \sin\theta(\nabla\varphi)^2]$ Брюно ур

Коэффициент демпфинга $D = -(2\pi M_s\frac{L}{\gamma})\alpha[5/8 + \frac{1}{2}\ln R/R_c] = -2\frac{M_s L}{\gamma}J$

$J = 2\pi[\frac{5}{8} + \frac{1}{2}\ln R/R_c]$. Или $N/D = -\gamma/\alpha$. $N = -\frac{\gamma}{\alpha}D = (2\pi M_s\frac{L}{\gamma})\gamma[\frac{5}{8} + \ln\frac{R}{R_c}]$

$N = G\gamma[5/8 + \ln R/R_c]$, $|\omega_y| = \frac{G}{N} = \frac{1}{\gamma}\frac{1}{[5/8 + \ln R/R_c]}$, $\gamma = \alpha\tau \approx 10^{-2}\,10\,\text{ps} = 10^{-13}\,\text{s}$

Escaneado con CamScanner

Zhu FeNi, $l = 10$ nm, $R = 100$ nm $[5/8 + \ln R/R_c] \simeq 1{,}76 \simeq 1{,}8$.

$\omega_y \simeq \frac{1}{2\pi}\frac{1}{\gamma}\left[5/8 + \ln\frac{R}{R_c}\right] = \frac{1}{2\pi}\, 10^{13}\,\frac{1}{s}\,\frac{1}{1{,}8} = \frac{1}{11{,}3}\,10\ \text{THz} \approx 0{,}9\ \text{THz}$. (0,90 μm 1,76)

$D_G = \frac{N}{G} = \frac{1}{(2\pi m_s \frac{L}{\gamma})}\,(2\pi m_s \frac{L}{\gamma})\,\gamma\left[5/8 + \ln\frac{R}{R_c}\right] = \gamma\left[\frac{5}{8} + \ln\frac{R}{R_c}\right] \approx 2\gamma$,

$\gamma \approx 10^{-13}$ ps. Numerically noted $\gamma_G \times 10^{11}$ in Fig. 1 unreadable as 0 or 1, и ниже $\gamma_G = 5\cdot 10^{-11}$ или $5\cdot 10^{-12}$ s in Fig. 2. Верно $\gamma_G \simeq 2\gamma \simeq 2\cdot 10^{-13}$ s меньше.

Новая revised paper „Vortex mutation" paper Зуркле.

(21/IV) Переписка с Зуркле, Ricardo, Lena, Celia Sousa, Frais.

(22/IV) Ответ с Зуркле о Mutation paper. Figs ω_1, ω_y (γ) перегляда ниже γ_G, Fig. 3 перегляда, там ω_y дошла бъде ~ 1 THz и $I(\omega_y) < I_0(\omega_0)$ далеко, другите аппроксимации да ware used. Ние проверихме signs in Thiele equations (2), (3), (4) и добавихме Alass Dw by Doring, проверих γ-value.

$\omega_{DW}(\text{Döring}) = \frac{1}{2\pi\gamma^2\Delta_0}$, $\Delta_0 = \sqrt{A/K}$ - DW width. $\omega_{DW} = M_{DW}/G$ - surface of DW.

① по вихрен dot дне оценки $S = 2R\cdot L$ (T, R, L) 2x - cross section.

$\omega_{DW} \to M_{DW} = \frac{2RL}{2\pi\gamma^2\Delta} = \frac{1}{\pi}\frac{L}{\gamma^2}\frac{R}{\Delta}$, DW width дне vortex in cyl. дом $\Delta \sim R$, $M_v = M_{DW} \sim \frac{1}{\pi}\frac{L}{\gamma^2}$. Дне mutation mass $M_v = 0{,}03\frac{L}{\gamma^2}$ ($\tau = 10$ ps) (18/IV).

$D_G = \frac{N}{G} = |N_{xx} = N_{yy} = N| = \frac{1}{2\pi M_S L}\,\gamma\frac{M_S}{\gamma}L\int d^3\vec{r}\left[(\partial_x\theta)^2 + \sin^2\theta(\partial_x\varphi)^2\right] =$

$= \frac{\gamma}{2\pi}2\pi\left[5/8 + \frac{1}{2}\ln R/R_c\right] = \gamma\left[\frac{5}{8} + \frac{1}{2}\ln\frac{R}{R_c}\right]$. Или сравнено 18/IV $\frac{1}{\gamma_G} = \frac{1}{\tau}$

$\gamma_G = d\tau = d\left[\frac{5}{8} + \frac{1}{2}\ln\frac{R}{R_c}\right]\gamma = \gamma d\left[\frac{5}{8} + \frac{1}{2}\ln\frac{R}{R_c}\right]$, $\gamma = d\tau$.

$\gamma_G = \left[\frac{5}{8} + \frac{1}{2}\ln\frac{R}{R_c}\right]\gamma$, $[\dots] \simeq 2$, $\gamma_G \simeq 2\gamma$. $[\gamma_G] = [\gamma] = [d\tau] = [\tau] = $ ps

Values of $\gamma_G \simeq 2\tau = 2d\tau = 2\cdot 0{,}01\cdot 10\,\text{ps} \sim 2\cdot 10^{-13}$ s.

(23/IV) E-mails to Cristina UPV, Ricardo, Maciej, Arkady, Ruben, Solstyagin 2023

(24/IV) meeting с Anantha Bosne University (gas leakage) и briefly с Ricardo-SW с Л. Б. Lidl" Hernani и „Correspondence" market. V mutation sent to Зуркле.

(25/IV) Ю. день Young - Estado de cuentas Ministerio project. Предложения hopefully topic to Zukunft.

BP EM dynamics

(26/IV) Domain stiffness α_1, α_2 дне BP in nanowire width rigid model, ам. 12/IV. $W(\vec{X}) = W(0) + \frac{1}{2}\alpha(X^2+Y^2) + \frac{1}{2}\alpha_z Z^2$, $\vec{X}(X,Y,Z)$ displacement from nanowire center $\vec{r} = (x,y,z) = (0,0,0)$. $\vec{m}(\vec{r},\vec{X}) = \vec{m}(\vec{r}-\vec{X})$

$\alpha_z = \frac{1}{2}\frac{\partial^2 W}{\partial Z^2}\Big|_{\vec{X}=0}$, $W = W[\vec{m}(\vec{r}-\vec{X})] = W[\vec{m}(\vec{\rho}, z-Z)] =$

$= \int dz\, d^2\vec{\rho}\; e[\vec{m}(\vec{\rho}, z-Z)]$. Def. $W[\vec{m}(z)] = \int d^2\vec{\rho}\; e[\vec{m}(\vec{\rho},z)] = W(z)$.

$m_x = m_\rho\cos\varphi - m_\varphi\sin\varphi$, $m_y = m_\rho\cos\varphi + m_\varphi\sin\varphi$. Дне Thiaville ansatz $m_\rho = 0$, $m_x = -m_\varphi\sin\varphi$, $m_y = m_\varphi\cos\varphi$, $\Phi = \arctg\left(\frac{m_y}{m_x}\right) = \arctg\left(\frac{\cos\varphi}{-\sin\varphi}\right) =$

$= -\arctg\left(\frac{x}{y}\right)$ как 16/IV. Очевидно, че за tilted cyl. BP на Z вдясно от center $m_\rho = 0$.

Escaneado con CamScanner

$N = \mu_0 M_s^2 V$; $\frac{1}{N} W_{ex} = \left(\frac{L_0}{R}\right)^2 \int_{-L/2}^{L/2} dz \int d^2x \left[\frac{1}{(1-m_z^2)} (\nabla m_z)^2 + \frac{1}{\rho^2} (1-m_z^2) \right]$; $z = \zeta / R$

$m_z(\zeta, z) = \beta \frac{z}{\sqrt{\beta^2 z^2 + \rho^2}} \cdot \frac{L^2}{(L^2 + z^2)}$; $W(z) = \int_{-L/2-z}^{L/2-z} w[\vec{m}(z')] dz'$.

$\frac{\partial W}{\partial z} = -w\left[\vec{m}\left(\frac{L}{2} - z\right)\right] + w\left[\vec{m}\left(-\frac{L}{2} - z\right)\right]$; $\vec{m}\left(\frac{L}{2} - z\right) = \vec{m}\left(\frac{L}{2}\right) - z \frac{\partial \vec{m}}{\partial z}\Big|_{z=L/2}$; $\frac{\partial \vec{m}}{\partial z} = \vec{m}'$

$\frac{\partial W}{\partial z} = -w\left[\vec{m}\left(\frac{L}{2}\right) - z\vec{m}'\left(\frac{L}{2}\right)\right] + w\left[\vec{m}\left(-\frac{L}{2}\right) - z\vec{m}'\left(-\frac{L}{2}\right)\right]$; $\vec{m}\left(-\frac{L}{2} - z\right) = \vec{m}'\left(+\frac{L}{2}\right) - z\vec{m}''(z)\Big|_{z=-\frac{L}{2}}$

$W(z) = W[\vec{m}(\zeta, z-z)] = \int dz\, w[\vec{m}(z-z)]$, $w[\vec{m}(z)] = \int d^2\zeta\, A \left(\nabla \vec{m}(\zeta, z)\right)^2$

$w\left[\vec{m}\left(\frac{L}{2} - z\right)\right] = w\left[\vec{m}\left(\frac{L}{2}\right) - z\vec{m}'\left(\frac{L}{2}\right)\right] = A \int d\zeta^2 \left\{ \nabla \vec{m} \right\}^2_{\text{resto}}$ exchange

$(\nabla \vec{m})^2 = (\nabla \theta)^2 + \sin^2\theta (\nabla \varphi)^2 = \frac{1}{(1-m_z^2)} (\nabla m_z)^2 + \frac{1}{\zeta^2} (1-m_z^2)$

$m_z(z) \to m_z\left(\frac{L}{2} - z\right) = m_z\left(\frac{L}{2}\right) - z m_z'(L/2)$, $m_z(\zeta, z) \to m_z(\zeta, z = \text{const})$

$\nabla m_z = \frac{\partial m_z}{\partial \zeta} \hat{\zeta} + \frac{1}{\zeta} \frac{\partial m_z}{\partial \varphi} \hat{\varphi} + \frac{\partial m_z}{\partial z} \hat{z} = \frac{\partial m_z}{\partial \zeta} \hat{\zeta}$, $(\nabla m_z)^2 = \left(\frac{\partial m_z}{\partial \zeta}\right)^2$

$w\left[\vec{m}\left(\frac{L}{2} - z\right)\right] = A \int d^2\zeta\, (\nabla \vec{m})^2 \overset{=0}{=} A \int d^2\zeta \left[\frac{(\nabla m_z)^2}{(1-m_z^2)} + \frac{1}{\zeta^2}(1-m_z^2) \right]$,

$\to$ Ewga nogcarbuck $m_z = m_z\left(\frac{L}{2}\right) - z m_z'\left(\frac{L}{2}\right)$, $(\nabla m_z)^2 = \left(\frac{\partial m_z}{\partial \zeta}\right)^2 + \ldots$

ygepxanoro dexmus $\sim z$. $\nabla m_z = \frac{\partial m_z}{\partial \zeta} \hat{\zeta} = \hat{\zeta} \left[\frac{\partial m_z}{\partial \zeta} - z \frac{\partial m_z'}{\partial \zeta} \right]_{z = L/2}$.

$(\nabla m_z)^2 = \left(\frac{\partial m_z}{\partial \zeta} - z \frac{\partial m_z'}{\partial \zeta} \right)^2 = \left(\frac{\partial m_z}{\partial \zeta}\right)^2 - 2z \frac{\partial m_z}{\partial \zeta} \cdot \frac{\partial m_z'}{\partial \zeta} + O(z^2)$, $z = L/2$.

$1 - m_z^2 = 1 - \left[m_z - z m_z' \right]^2_{z=L/2} = 1 - m_z^2 + 2 m_z m_z' z$, $z = L/2$.

Alternatively: разложить $(\nabla \vec{m})^2$ по z, а потом по z^2, сделать $\int dz \int dz (\ldots)$.

$W_{ex} = A \int dz\, d\zeta^2 (\nabla \vec{m})^2 = A \int dz \int d^2\zeta (\nabla \vec{m}) = 2\pi A \int dz \int d\zeta\, \zeta \left[\frac{(\nabla m_z)^2}{(1-m_z^2)} + \frac{1}{\zeta^2}(1-m_z^2) \right]$

где $\vec{m} = \vec{m}(\zeta, z)$. $(\nabla m_z)^2 = (\partial m_z / \partial \zeta)^2 + \left(\frac{\partial m_z}{\partial z}\right)^2$, $\frac{\partial m_z}{\partial z} = \frac{b_2 R}{(b^2 + \zeta^2)^2} \frac{\Lambda}{(z^2 + \zeta^2)^{3/2}}$

$\frac{\partial m_z}{\partial \zeta} = -\frac{b_2 z \zeta}{(z^2 + \zeta^2)^{3/2}} \frac{\Lambda}{(b^2 + \zeta^2)^2} \left[b^2 - \zeta^2 - 2z^2 \right]$

$\vec{m}(z - z) = \vec{m}(z) - z\vec{m}'(z) + \frac{1}{2} z^2 \vec{m}''(z)$, $\vec{m}' = \partial \vec{m} / \partial z$, $\vec{m}'' = \partial^2 \vec{m} / \partial z^2$

$(\nabla \vec{m})^2 = (\nabla \vec{m}(z))^2 - 2z (\nabla \vec{m}) \cdot (\nabla \vec{m}') + z^2 (\nabla \vec{m}')^2 + z^2 (\nabla \vec{m}) \cdot (\nabla \vec{m}'') + O(z^3)$

Якоби. Уравн. Эйлера $(\nabla \vec{m})$ через $(m_\zeta, m_\varphi, m_z)$.

$W = A \int dV (\nabla \vec{m})^2$, $\vec{m}(\vec{r}) = \vec{m}(\zeta, z) \to \vec{m}(\zeta, z - z)$. $(\nabla \vec{m})^2 = -\vec{m} \cdot \Delta \vec{m}$

$\frac{\partial W}{\partial z} = A \int dV\, 2(\nabla \vec{m}) \cdot \frac{\partial}{\partial z} (\nabla \vec{m}) = 2A \int dV\, \nabla \vec{m} \cdot \nabla \left(\frac{\partial \vec{m}}{\partial z} \right)$. $\operatorname{div} \vec{m} = \frac{\partial m_z}{\partial z}$

Hubert-Schäfer (3.2.9) при $\vec{m}(\zeta, z)$: $(\nabla \vec{m})^2 = \left(\frac{\partial m_\zeta}{\partial \zeta}\right)^2 + \left(\frac{\partial m_z}{\partial \zeta}\right)^2 + \left(\frac{\partial m_\zeta}{\partial z}\right)^2 +$ $\leftarrow$ initial file

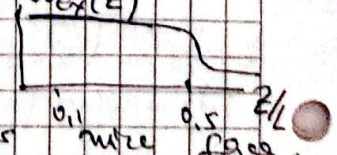
$+ \left(\frac{\partial m_z}{\partial z}\right)^2 + \frac{1}{\zeta^2} \left[m_\zeta^2 + m_\varphi^2 \zeta^2 + \left(\frac{\partial m_z}{\partial \varphi}\right)^2 \right]$, $m_\varphi = 0$ $= \left(\frac{\partial m_\zeta}{\partial \zeta}\right)^2 + \left(\frac{\partial m_z}{\partial \zeta}\right)^2 + \left(\frac{\partial m_\zeta}{\partial z}\right)^2 + \left(\frac{\partial m_z}{\partial z}\right)^2 + \frac{1}{\zeta^2} m_\varphi^2$,

$\operatorname{rot} \vec{m} = -\frac{\partial m_\varphi}{\partial z} \hat{\zeta} - \frac{\partial m_z}{\partial \zeta} \hat{\varphi} + \frac{1}{\zeta} \frac{\partial}{\partial \zeta} (\zeta m_\varphi) \hat{z}$.

$\nabla^2 \vec{m} = \Delta \vec{m} = \left(\nabla^2 m_\zeta - \frac{m_\zeta}{\zeta^2} - \frac{2}{\zeta^2} \frac{\partial m_\varphi}{\partial \varphi} \right) \hat{\zeta} + \nabla^2 m_\varphi \hat{\varphi} + \nabla^2 m_z \hat{z}$, $\vec{m} = m_\zeta \hat{\zeta} + m_z \hat{z}$

$\vec{m} \cdot \Delta \vec{m} = m_\zeta \left(\nabla^2 m_\zeta - \frac{m_\zeta}{\zeta^2} \right) + m_z \nabla^2 m_z = -(\nabla \vec{m})^2$.

// MathCad $\frac{1}{N} W_{ex}(z) = \left(\frac{L_0}{R}\right)^2 \int_{-L/2-z/L}^{1/2-z/L} dz \int d\zeta\, \zeta (\nabla \vec{m})^2$



$W_{ex}(z)$ negaet слабо с z при $-1/2 - z/L$, достигает минимума при $z = \frac{L}{2}$, т.е. достигает min при $z/L = 0.5$, т.е. при $z/L \ll 1$ $\Delta E < 0$. Но это только E_{exch} вклад. $W_m(z)$ MathCad считает очень долго (> 1 day).

23/VI Iberbasque meeting (15 years) in Gutenheim, Bilbao. Встречались. Перезвонить с Кристиной, Рикардо (contract extension) и Мацеем.

(29/XI) Вернулся Curvilinear proposal. Переписка с Cristina, Ruben.

(30/XI) – (3/XII) – trip to Madrid. Meeting с Пархаг и Diego Cao, в том числе с Пархагом.

(4/XII) – Телефонный разговор с PhD Ricardo (1h 10 min final 40 min). Many comments by me and Ruben. Degar monitor by Javier / Ruben. С п. в «Correfoc» Diada Sun.

(6–9/XII) Работа над Vortex initiation manuscript. Переписка с Бухрей и Nanomaterials. Open proposal editing give News. Travel report – Madrid. Получил ответ – Л. П. 20 kg + ручн. 12 kg, 74 €. 9/XII Отчет за Аллочку 2nd payment за credits UPV/EHU = 1,100 €.

(10/XII) Написание статьи Tejo et al. BP-DW motion in nanowires. Overleaf. Comments to co-authors.

(11/XII) Что определяет max DW velocity in antiferromagnet!

Max v_{gr} of spin waves (Ruben), min v_{phase} (Kostya)?

• Kosevich, Ph. Reports 1990: для AFM $\omega^2(k) = \omega_0^2 + c^2k^2$, где $c = \frac{2\mu_B}{\hbar} M_s \sqrt{A\alpha}$ $W(\vec{M},\vec{L}) = \frac{1}{2}A\vec{M}^2 + \frac{1}{8}\alpha\left(\frac{\partial \vec{L}^2}{\partial x_i}\right)^2 + \dots$, A и α – (A – J) uniform and non-uniform AF exchange parameters.

ω_0 – anisotropy gap. $\omega = \sqrt{\omega_0^2 + c^2k^2}$ – uniaxial AFM.

$v_{ph} = \frac{\omega}{k} = \sqrt{\frac{\omega_0^2}{k^2} + c^2}$, v_{ph} → c, min $v_{ph}(k) = c$.

$v_{gr} = \frac{\partial \omega}{\partial k} = \frac{\partial}{\partial k}\sqrt{\omega_0^2 + c^2k^2} = \frac{1}{2\sqrt{\omega_0^2 + c^2k^2}} \cdot 2c^2k = c\frac{ck}{\sqrt{\omega_0^2 + c^2k^2}}$

Max $v_{gr}(k) = c$.

→ D. в uniaxial AFM $\min v_{ph}(k) = \max v_{gr}(k) = c$, обнаружил, $c = \sqrt{A\alpha}$

• Kim, Tserkovnyak, Tchernyshyov PRB 90, 104406 (2014)

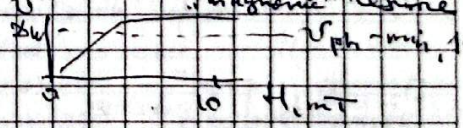
$L = \frac{1}{2}s(\dot{\vec{n}})^2 - \frac{1}{2}A\left(\frac{\partial \vec{n}}{\partial x}\right)^2$, «speed of light» – $\max v_{gr} = s = \sqrt{A/s}$

$\omega^2(k) = \omega_0^2 + s^2k^2$, the same as in Kosevich 1990.

Зачем AFM limiting velocity of DW, если работаем с FM, nanowire? Наго Walker limit speed или spin Cherenkov – limit speed (v_{ph} of SW).

• C. Andreas, A. Kakay, R. Hertel Ph. R. B 89, 134403 (2014):

BP-DW dynamics in soft cylindrical nanowire, $v_{DW}(H)$ up to 1200 m/s, limit-velocity is $\min v_{ph}(!)$ of SW.



• R. Hertel, IPM C. Mat. 28 (2016) 483002.

DW motion in nanotubes $v_{DW}(H)$, Walker regime and «magnonic regime» limiting velocity есть $\min v_{ph}(k)$ (!). Max $\omega(k)$ и $v_{ph}(k)$ similar to Fig. 6 Herrel для nanotubes. Calculated (nanowires) $v_{ph}(k)$ имеет min, а $v_{gr}(k)$ монотонно растет с k и max $v_{gr}(k)$ отсутствует (!). Если о max $v_{gr}(k)$ нет, только min $v_{ph}(k)$.

(12/XII) Editing Artem's paper. Ок in track dots. Встреча с Гнев и SMS-Artem. Отдел Madrid travel report to Ele. Оплачено e-mail, no change in Tejo MS.

(13/XII) Новые Refs. по limiting DW velocity. Recent papers Oka. / Ruben:

• Baryakhtar, Kharov, Kosev. Phys. 1979, с in AFM – min phase velocity

• Baryakhtar – Ivanov JETP Lett. 1982 Spin Cherenkov SW emission at $v = v_{ph}$.

• Baryakhtar, УФН 1985 (rus, to JETP Lett. 82), Sec. 4, DW emits SW because or moving DW если $v_{ph} < v_g$ и Hagen DW если $v_{ph} > v_g$. Emission of SW with v_{gr}(!); energy transfer. Проверим MathCad $\min v_{ph} = v_{gr}$! at $k = k_{min}$,

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т.е. special case, domyckaetcя SW emission и bnefeg u nagag
bs moving DW. $v_{ph}(k_{min}) = v_{gr}(k_{min})$, $v_{ph}(k_{min}) = \min v_{ph}(k)$.

(14/VII) Online обучающие курсы "0-DW motion in Mn_2Au antiferromagnet" с Ricardo Rubén Jaimes. Skype с Luis о GM-finish dod MS.

(15/VII) Сделал. Origin plots $\alpha(k)$, $v_{ph}(k)$, $v_{gr}(k)$, $\min v_{ph}(R,k)$ (Arrow). SW cyl. nanowire и послал их Оксане и Felipe.

(16/VII) Produced A-Item's MS "Engineering zero m. field GHz excitations in ferromagnetic nanodots".

BP 0-M dynamics

Разные stiffness coefficients χ_z, χ_z для Bloch point in cyl. nanowire, см. 26/XI. χ_z (exch) < 0 посчитано в MathCad.

χ_z (magn.) имеет 2 вклада: от volume. div $\vec{m}$ и face mag. charges.

$\mathrm{div}\vec{m} = \frac{1}{s}\frac{\partial}{\partial s}(m_s s) + \frac{1}{s}\frac{\partial m_\varphi}{\partial \varphi} + \frac{\partial m_z}{\partial z}$, BP position $\vec{X} = (X, Y, Z)$.

если $X=0, Y=0$, то BP-DW has azimuthal symmetry as in Thiaville ansatz. $m_\varphi = 0$, $\vec{m} \neq f(\varphi)$, $\frac{\partial m_\varphi}{\partial \varphi} = 0 \to \mathrm{div}\vec{m} = \frac{\partial m_z}{\partial z}$. см. 16/II.

$m_z = \frac{2}{\pi}\frac{b^2}{(b^2+s^2)}$, $\frac{\partial m_z}{\partial z} = \frac{B^2 s^2}{(b^2+s^2)}\frac{1}{(r^2+s^2)^{3/2}}$, $s^2 = x^2+y^2$, $r^2 = s^2+z^2$.

$W_m^{vol} = 2\pi M_s^2 V \beta L^4 \int_0^1 dz \int_0^1 dz' \int dk\, e^{-\beta k|z-z'|} I(k,z) I(k,z')$, $t = b/R$, $\beta = L/R$, $z \to z/L$, $z \in [0,1]$

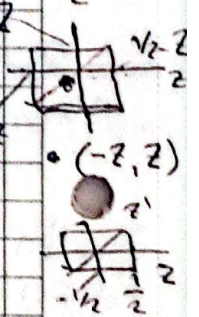
$I(k,z) = \int_0^1 dr\, r J_0(kr) \frac{b^2}{(r^2+t^2)} \frac{1}{[\beta^2 z^2 + r^2]^{3/2}}$. 26/XI "Bloch point stiffness. mod"

Считаем χ_z (exch) и χ_z (vol. m.) χ_z (vol. m.) не считает в MathCad.

В Thiaville ansatz $m_z = \frac{2}{\pi}\frac{b^2}{(b^2+s^2)}$, $z \in [-\frac{1}{2}, \frac{1}{2}]$ в eg. L, wire length.

Надо $z \in [-\frac{1}{2}, \frac{1}{2}]$ $\frac{1}{(4\pi M_s^2 V)} W_m(vol) = \frac{1}{2}\beta L^4 \int_{-1/2}^{1/2} dz \int_{-1/2}^{1/2} dz' \int dk\, e^{-\beta k|z-z'|} I(k,z) I(k,z')$

$W_m^{vol}(z) = \frac{1}{2}\beta L^4 \int_{-1/2-z}^{1/2-z} dz \int_{-1/2-z}^{1/2-z} dz' \int dk\, e^{-\beta k|z-z'|} I(k,z) I(k,z')$.



$W_m^{vol}(z) = \frac{1}{2}\beta L^4 \int dk (\dots)\, e^{-\chi|z-z'|} = e^{-\chi(z-z')}$ и $e^{-\chi(z'-z)}$, $z>z'$, $z'>z$

$W_m^{vol}(z) = \frac{1}{2}\beta L^4 \int_{-1/2}^{1/2} dz \int_{-1/2}^{1/2} dz' \int dk\, e^{-\beta k|z-z'|} I(k, z-Z) I(k, z'-Z) =$

$= (\dots) \int_{-1/2}^{1/2} dz \left[\int_{-1/2}^{z} dz' \ (z>z') + \int_{z}^{1/2} dz' \ (z'>z) \right]$

$J_z = \int_{-1/2}^{1/2} dz \int_{-1/2}^{1/2} dz' e^{-\chi|z-z'|} f(z) f(z')$, $f(z) = I(k, z-Z)$, $\chi = \beta k$

$J_z = \int_{-1/2}^{1/2} dz \left[\int_{-1/2}^{z} dz' e^{-\chi(z-z')} f(z) f(z') + \int_{z}^{1/2} dz' e^{-\chi(z'-z)} f(z) f(z') \right] =$

$= \int_{-1/2}^{1/2} dz \left[e^{-\chi z} f(z) \int_{-1/2}^{z} dz' e^{\chi z'} f(z') + e^{\chi z} f(z) \int_{z}^{1/2} dz' e^{-\chi z'} f(z') \right]$, $I(k,z) = \frac{1}{\beta^3} \int dr F(kr) \varphi(z)$

$\int_{z}^{1/2} dz' e^{-\chi z'} f(z') = \left| \xi = z' - z - 1/2 \right| = \int_{-z}^{-1/2} d\xi\, e^{-\chi(\xi + z + \frac{1}{2})} f(\xi + z + \frac{1}{2})$

$J_z = \int dz f(z) \left[e^{-\chi z} \int_{-1/2}^{z} dz' e^{\chi z'} f(z') + e^{-\chi/2} \int_{-z}^{-1/2} d\xi\, e^{-\chi\xi} f(\xi + z + \frac{1}{2}) \right]$, и 2ое?

$z = z - z'$ не годится z. $J_z = \int_{-1/2}^{1/2} dz \int_{-1/2}^{1/2} dz' e^{-\chi|z-z'|} f(z) f(z') = \left| z' = z - \xi, \ z - z' = \xi \right| =$

$= \int_{-1/2}^{1/2} dz \int_{z-1/2}^{z+1/2} d\xi\, f(z) f(z-\xi)$ /// $I(k,z) = \int_0^1 dr\, r J_0(kr) \frac{r^2}{(r^2+t^2)} \frac{1}{[\beta^2 z^2 + r^2]^{3/2}} =$

$= \int dr\, F(k,r) \frac{1}{[\beta^2 z^2 + r^2]^{3/2}} = \frac{1}{\beta^2} \int dr\, F(k,r) \frac{1}{[z^2 + r^2/\beta^2]^{3/2}}$, $\frac{1}{[(z-Z)^2 + c^2]^{3/2}}$

разложим по $z \ll 1$, $\varphi(z) = \frac{1}{[z^2+c^2]^{3/2}}$, $\frac{\partial \varphi}{\partial z} = -\frac{3}{2}[z^2+c^2]^{-5/2} 2z$,

$\varphi(z-Z) = \varphi(z) + \varphi' Z + \frac{1}{2}\varphi'' Z^2$, $W_m^{vol}(z) = \frac{1}{2}\beta t^4 \int_{-1/2}^{1/2} dz \int_{-1/2}^{1/2} dz' e^{-\beta k |z-z'|} \times$

$\nu \frac{1}{\beta^3}\int dr F(k,r)\varphi(z-Z)\frac{1}{\beta^3}\int dr' F(k,r')\varphi(z'-Z) =$

$= \frac{1}{2}\beta t^4 \frac{1}{\beta^6}\int dk \int dz \int dz' e^{-\beta k|z-z'|}\int dr F(k,r)\varphi(z-Z,r)\int dr' F(k,r')\varphi(z'-Z,r')$

$\varphi(z-Z)\varphi(z'-Z) = \left[\varphi(z) + \varphi'(z)Z + \frac{1}{2}\varphi''(z)Z^2\right]\left[\varphi(z') + \varphi'(z')Z + \frac{1}{2}\varphi''(z')Z^2\right] =$

$= \varphi(z)\varphi(z') + \varphi(z)\frac{1}{2}\varphi''(z')Z^2 + \varphi'(z)\varphi'(z')Z^2 + \frac{1}{2}\varphi''(z)\varphi(z')Z^2$, Z выносим $\sim Z$.

$W_m^{vol}(Z) = \frac{1}{2}\beta t^4 \frac{1}{\beta^6}\int dz \int dz' e^{-\beta k|z-z'|}\int dk \int dr \int dr' F(k,r)F(k,r')\varphi(z-Z,r)\varphi(z-Z,r') =$

$= \frac{1}{2}\beta t^4 \frac{1}{\beta^6}\int dz \int dz' e^{-\beta k|z-z'|}\int dk \int dr \int dr' F(k,r)F(k,r')\left[\frac{1}{2}\varphi(z,r)\varphi''(z',r') + \varphi'(z,r)\varphi'(z',r') + \right.$

$\left. + \frac{1}{2}\varphi''(z,r)\varphi(z',r')\right]Z^2$, $W_m^{vol}(Z=0)$ вот формула. $W_m^{vol}(Z) = \frac{1}{2}\varkappa_z Z^2$

$\varkappa_z = \frac{t^4}{\beta^5}\int_{-1/2}^{1/2} dz dz' \int dk e^{-\beta k|z-z'|}\int_0^\infty dr \int_0^\infty dr' F(k,r)F(k,r')\left[\frac{1}{2}\varphi(z,r)\varphi''(z',r') + \frac{1}{2}\varphi''(z,r)\varphi(z',r') + \right.$

$\left. + \varphi'(z,r)\varphi'(z',r)\right]$. $\int dk e^{-\beta k|z-z'|}F(k,r)F(k,r') = G(z,z',r,r')$

$\varkappa_z = \frac{t^4}{\beta^5}\int dz dz' \int_0^\infty dr \int_0^\infty dr' G(z,z',r,r')\left[\frac{1}{2}\varphi(z,r)\varphi''(z',r') + \frac{1}{2}\varphi''(z,r)\varphi(z',r) + \varphi'(z,r)\varphi'(z',r)\right]$

$\varphi(z,r) = \frac{1}{\left[z^2 + \frac{1}{\beta^2}r^2\right]^{3/2}}$, $F(k,r) = r J_0(kr)\frac{r^2}{(r^2+t^2)}$, $\varphi' = \frac{\partial\varphi}{\partial z}$, $\varphi'' = \frac{\partial^2\varphi}{\partial z^2}$.

Ядро $G(zz',rr')$ сим. относительно $z \rightleftarrows z'$, $r \rightleftarrows r'$ данных.

$\varkappa_z = \frac{t^4}{\beta^5}\int_{-1/2}^{1/2} dz dz' \int_0^\infty dr dr' G(zz',rr')\left[\varphi(z,r)\varphi''(z',r') + \varphi'(z,r)\varphi'(z',r)\right]$

$\varphi'(z) = -3z\left[z^2+c^2\right]^{-5/2}$, $\varphi''(z) = -3\left[z^2+c^2\right]^{-5/2} - 3z\left(-\frac{5}{2}\right)\left[z^2+c^2\right]^{-7/2}2z =$

$= -3\left[z^2+c^2\right]^{-5/2}\left\{1 - 5z^2\frac{1}{(z^2+c^2)}\right\}$, $c = r/\beta$.

Л. Ирония тех основ инфляции из-за (раз в 2 года). Они этого 15/XII заметили бозона и все уничтожили, энергия ушла в „FeuVert" - 386€.
Edited BP-DW V(4) manuscript и после Оксаны comments о kinetic (gyro-) field. Она так и не объяснила откуда цифры in simulations Ukii up to 120 T - giant value.

(17/XII) Уборка в доме. Установка ежи.

Вчера $\varkappa_z(\beta,t)$ equations в MathCad „Block point stiffness.mcd"
MCad не считает. Возможен $\int dk e^{-\beta k|z|}(...)$.
J.T. Conway. IEEE TM 37, 2973, 2001. $I_{\mu\nu}(R,r,|z|) = \int_0^\infty s^{\lambda} J_\mu(sR) J_\nu(sr) e^{-s|z|} ds$

$I_{000}(R,r,|z|) = \int_0^\infty ds e^{-s|z|} J_0(sR) J_0(sr) = \frac{1}{\pi\sqrt{rR}} K(k)$, $k^2 = \frac{4rR}{(r+R)^2+z^2}$.

$K(k) = \int_0^{\pi/2} \frac{dx}{\sqrt{1-k^2\sin^2 x}}$ - complete el. integral of the 1st kind.

$G(z,z',r,r') = \int_0^\infty dk e^{-\beta k|z-z'|}F(k,r)F(k,r') = \int_0^\infty dk e^{-\beta k|z-z'|}J_0(kr)J_0(kr')\frac{r^3 r'^3}{(r^2+t^2)(r'^2+t^2)} =$

$= \frac{r^3 r'^3}{(r^2+t^2)(r'^2+t^2)}\frac{1}{\pi\sqrt{rr'}}K(k)$, Ключ $k^2 = \frac{4rr'}{(r+r')^2 + \beta^2(z-z')^2}$

MathCad не считает $\varkappa_z(\beta,t)$ долго < зависимость будет $G(z,z',r,r')$.

(18/XII) Используем $G_0(z,z',r,r')$ при $\beta \gg 1$ (narrow integrand under sharp max at $z=z'$. $\int dz dz' G(zz')F(zz') \simeq \int dz F(zz)\int dz' G(zz') = \int dz g(z)F(zz)$,

где $g(z,rr') = \int dz' G(zz',rr')$ is reduced in-kernel.

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$F(zz') = \varphi(z,z)\,\varphi''(z',z') + \varphi'(z,z)\,\varphi'(z',z')$. / Math Cad все равно не считает (очевидно долго).

$r_z \simeq \frac{4\pi}{\beta^5} \int_{-l/2}^{l/2} dz \int_0^1 dr \int_0^1 dr'\, g(r,r,r')\, F(z,z,rr')$.

(19)/XII Перемножая с Оксаной о kinetic field (gyrofield) $\vec{H}_{kin}$. с П. в "Lida" и Mercadona: Hermann. Оксана говорила наверное $\mathcal{L}_{kin}$ in the manuscript (без гамильтониана), $\mathcal{L}_{kin} = \frac{M_s}{\gamma}(\vec{m}\times\dot{\vec{m}})\cdot\hat{n}$, чтобы рождался "zoo" в micromag. simulation она считала Лагранжиан $\mathcal{L}_{kin}$, находила из него $\vec{H}_{kin}$ и строила in Fig. 5 $|\vec{H}_{kin}|$ (!?). $\vec{H}_g$

(29)/XII $\mathcal{L}_{kin} = J\left(\frac{M_s}{\gamma}\right)\frac{\dot{\vec{n}}\cdot(\vec{m}\times\dot{\vec{m}})}{(1+\vec{m}\cdot\vec{n})}$ Вернее definition, записано w/m $\vec{m}(\theta,\varphi)$

$\mathcal{L}_{kin} = \frac{M_s}{\gamma}(1-\cos\theta)\dot{\varphi}$, $J = M_s/\gamma$. Artificial gyrofield $\vec{h}_g = \frac{1}{M_s}\frac{\partial \mathcal{L}_{kin}}{\partial \vec{m}}$

дано Gus' PRL '2008, см. 3/III. $\vec{H}_{kin} = \frac{1}{M_s}\frac{\partial \mathcal{L}_{kin}}{\partial \dot{\vec{m}}}$, $\vec{H}_{kin} = \vec{h}_g$, $J = M_s/\gamma$.

$\frac{\partial \mathcal{L}_{kin}}{\partial \dot{\vec{m}}} = J\frac{(\vec{m}\times\vec{n})}{(1+m_z)} - \vec{n}J\frac{\vec{m}\cdot(\vec{m}\times\vec{n})}{(1+m_z)^2}$, где вектор $\vec{n} = \hat{z}$ и $1 \to P$ в знаменателе. $m_z = (\vec{m}\cdot\vec{n})$

$\vec{H}_{kin} = \frac{1}{M_s}\frac{\partial \mathcal{L}_{kin}}{\partial \dot{\vec{m}}} = \frac{1}{\gamma}\frac{1}{(1+\vec{m}\cdot\vec{n})^2}\left[(\dot{\vec{m}}\times\vec{n})(1+\vec{m}\cdot\vec{n}) - \vec{n}\,\vec{m}\cdot(\dot{\vec{m}}\times\vec{n})\right]$

$\vec{H}_{kin}$ имеет longitudinal ($\parallel\vec{n}$) component и transverse ($\perp\vec{n}$) component.

$H^{\ell}_{kin} = -\frac{1}{\gamma}\frac{1}{(1+\vec{m}\cdot\vec{n})^2}\vec{m}\cdot(\dot{\vec{m}}\times\vec{n}) = (\vec{H}_{kin}\cdot\vec{n})$. $\vec{n}\cdot(\dot{\vec{m}}\times\vec{n}) \equiv 0$.

$\vec{H}_{kin}\times\vec{n} = \frac{1}{\gamma}\frac{1}{(1+\vec{m}\cdot\vec{n})^2}(1+\vec{m}\cdot\vec{n})(\dot{\vec{m}}\times\vec{n})\times\vec{n}$, $(\dot{\vec{m}}\times\vec{n})\times\vec{n} = -\vec{n}\times(\dot{\vec{m}}\times\vec{n}) = \vec{n}\times(\vec{n}\times\dot{\vec{m}}) =$

$= \vec{n}(\vec{n}\cdot\dot{\vec{m}}) - \dot{\vec{m}}(\vec{n}\cdot\vec{n}) = \vec{n}(\vec{n}\cdot\dot{\vec{m}}) - \dot{\vec{m}}$, $\vec{H}^{t}_{kin} = \vec{H}_{kin} + \vec{n}$ – transverse part of $\vec{H}_{kin}$.

$\vec{H}^{t}_{kin} = \frac{1}{\gamma}\frac{1}{(1+\vec{m}\cdot\vec{n})}\vec{n}\times(\vec{n}\times\dot{\vec{m}})$. $\vec{H}_{kin} = H^{\ell}_{kin}\vec{n} + \vec{H}^{t}_{kin} = (\vec{H}_{kin}\cdot\vec{n})\vec{n} + \vec{H}_{kin}\times\vec{n}$

$|\vec{H}_{kin}|^2 = (\vec{H}_{kin})^2 = \left[(\vec{H}_{kin}\cdot\vec{n})\vec{n} + \vec{H}_{kin}\times\vec{n}\right]^2 = (\vec{H}_{kin}\cdot\vec{n})^2 + (\vec{H}_{kin}\times\vec{n})^2 = (H^{\ell}_{kin})^2 + (H^{t}_{kin})^2$

Оксана говорила, что она simulated $|\vec{H}_{kin}|$, $\vec{H}_{kin} = \frac{1}{M_s}\frac{\partial \mathcal{L}_{kin}}{\partial \dot{\vec{m}}}$ (Gus' PRL '08)

Другая более ранняя работа, definition of kinetic (gyro-) field и дана в EPL '16:

$\vec{H}_g = \frac{1}{\gamma}(\vec{m}\times\dot{\vec{m}})$ из $\mathcal{L}_{int}$ (soliton-DW) $= M_s\vec{m}_s\cdot\vec{H}_g$, $\vec{m}_s$ – DW magnetization.

$\vec{n}$ – direction of Dirac string is arbitrary, для нанопровода можно $\vec{n}\parallel$ wire axis (Ox).

In vortex Gus PRL '08 я использовал longitudinal component: $H^{\ell}_{kin} = (\vec{H}_{kin}\cdot\vec{n})$ с $\vec{n} = \hat{z}$.

$H^{z}_{kin} = (\vec{H}_{kin}\cdot\hat{z}) = -\frac{1}{\gamma}\frac{(\dot{\vec{m}}\times\vec{m})_z}{(p+m_z)^2}$, $p = \pm 1$ – vortex polarization.

Note that $\vec{H}_g^2 = H_g^2 = \frac{1}{\gamma^2}(\vec{m}\times\dot{\vec{m}})^2 = \frac{1}{\gamma^2}\left[\vec{m}^2(\dot{\vec{m}})^2 - (\vec{m}\cdot\dot{\vec{m}})^2\right] = \frac{1}{\gamma^2}(\dot{\vec{m}})^2$, $d\vec{m}^2/dt = 0$

$\vec{H}_{kin}(wire) = \frac{1}{\gamma}[1+m_x]^{-2}\left[(\dot{\vec{m}}\times\hat{x})(1+m_x) - \hat{x}\,\vec{m}\cdot(\vec{m}\times\hat{x})\right] \sim \frac{1}{[1+m_x]^2}$

$\vec{n} = \hat{x}$, $\hat{x}\parallel$ ghost wire.

Сингулярно при $m_x = -1$ $\vec{m}$ ⟹ ⟸ $\to x$ в одном из доменов, далеко от BP-DW plane.

В синг. BP, $m_x \approx 0$ и $\vec{H}_{kin}(wire) = \frac{1}{\gamma}\left[(\dot{\vec{m}}\times\hat{x}) - \hat{x}\,\vec{m}\cdot(\dot{\vec{m}}\times\hat{x})\right]$ и сингулярность $\vec{H}_{kin}(wire)$ не важна.

$\vec{H}_{kin} = H^{\ell}_{kin}\hat{x} + \vec{H}^{t}_{kin}$, $H^{\ell}_{kin} = -\frac{1}{\gamma}\vec{m}\cdot(\dot{\vec{m}}\times\hat{x})$, $\vec{H}^{t}_{kin} = \frac{1}{\gamma}(\vec{m}\times\dot{\vec{m}}\times\hat{x})$

$H^2_{kin}(wire) = (H^{\ell}_{kin})^2 + (\vec{H}^{t}_{kin})^2$, $(\vec{H}^{t}_{kin})^2 = \frac{1}{\gamma^2}(\vec{m}\times\dot{\vec{m}}\times\hat{x})^2 = \frac{1}{\gamma^2}\left[(\dot{\vec{m}})^2 - (\dot{\vec{m}}\cdot\hat{x})^2\right]$

$H^2_{kin}\left(\frac{wire}{BP}\right) = \frac{1}{\gamma^2}\left[\vec{m}\cdot(\dot{\vec{m}}\times\hat{x})\right]^2 - \frac{1}{\gamma^2}(\dot{\vec{m}}\cdot\hat{x})^2 + \frac{1}{\gamma^2}(\dot{\vec{m}})^2$, $H_g^2 = \frac{1}{\gamma^2}(\dot{\vec{m}})^2$,

т.е. для wire с BP-DW H^2_{kin} (PRL '08) и H_g^2 (EPL '16) похожи, разница в члене $\frac{1}{\gamma^2}\left[\vec{m}\cdot(\dot{\vec{m}}\times\hat{x})\right] - \frac{1}{\gamma^2}(\dot{\vec{m}}\cdot\hat{x})^2$.

Видно, что для BP-DW in nanowire ($m_x \approx 0$) сингулярность $\vec{H}_{kin}$ не важна.

E-mails Оксане.

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(21-22/XII) Обсуждение с Глебом дел Vortex GM paper и Зарабатывание. Письмо Карпенчика с Grzegorz, meeting 3/I 2023, Monday, 9:00 AM

1. подарки в Киев: Дора 10 kg и Анна 10 kg — Дора.

(23/XII) с А. Ф., Lidar - Монтани.

BP stiffness

Magnetostatic (vol. charges) energy BP in cylindrical wire (Rhodaville angle)

$$W_m^{vol}(\beta,t) = \frac{1}{2}\beta t^4 \int_{-1/2}^{1/2} dz \int_{-1/2}^{1/2} dz' \int_0^\infty dk\, e^{-\beta k|z-z'|} I(\beta,t,k,z) I(\beta,t,k,z')$$

$$I(\beta,t,k,z) = \int_0^1 z\, J_0(kr) \frac{r^2}{(r^2+t^2)(\beta^2 z^2 + r^2)^{3/2}} dr.$$

Local approximation, $\beta \gg 1$: $\int_{-1/2}^{1/2} dz' e^{-\beta k|z-z'|} I(\beta,t,k,z) I(\beta,t,k,z') \approx$

$$\approx I^2(\beta,t,k,z) \left[\int_{-1/2}^{z} dz' e^{-\beta k(z-z')} + \int_{z}^{1/2} dz' e^{-\beta k(z'-z)} \right] = e^{-\beta k z} \int_{-1/2}^{z} dz' e^{\beta k z'} + e^{\beta k z} \int_{z}^{1/2} dz' e^{-\beta k z'} =$$

$$= e^{-\beta k z} \frac{1}{\beta k} e^{\beta k z'} \Big|_{-1/2}^{z} + e^{\beta k z} \left(-\frac{1}{\beta k}\right) e^{-\beta k z'} \Big|_{z}^{1/2} = \frac{1}{\beta k} e^{-\beta k z} \left[e^{\beta k z} - e^{-\frac{1}{2}\beta k} \right] -$$

$$- \frac{1}{\beta k} e^{\beta k z} \left[e^{-\frac{\beta k}{2}} - e^{-\beta k z} \right] = \frac{1}{\beta k} \left[1 - e^{-\frac{1}{2}\beta k} e^{-\beta k z} - e^{-\frac{\beta k}{2}} e^{\beta k z} + 1 \right] =$$

$$= \frac{2}{\beta k} \left[1 - e^{-\frac{\beta k}{2}} ch(\beta k z) \right] \equiv F(z) = F(\beta,k,z).$$

$$\int dz \int dz' e^{-\beta k|z-z'|} I(\beta,t,k,z) I(\beta,t,k,z') \approx \int dz\, F(z)\, I^2(\beta,t,k,z).$$

$$W_m^{vol}(\beta,t) = \frac{1}{2}\beta t^4 \int_0^\infty dk \int_{-1/2}^{1/2} dz\, F(\beta,k,z) I^2(\beta,t,k,z), \quad \beta = L/R, \ t = b/R \text{ - BP radius.}$$

MCad "Bloch-point energy-wire.mcdx" approx. local W_m^{vol} changes slightly, но заметно быстрее. Чем точнее расчет дает $t_{min} = 0{,}12$, с grid $t_{min} = 0{,}09$ $\beta = 4{,}16\pi$, $R = 60$ nm. // Numerical: W_m^{vol} (local) сильно завышена по сравнению с W_m^{vol} (exact). W_m^{vol} (local) не позволяет дать equilibrium радиус BP radius $t_{min} = 6_{min}/R$.

(24/XII) Интеграл $\int dk\, e^{-\beta k|z-z'|}(\dots)$ и вычисление через elliptic func. $K(k)$, 17/XII.

$$\int_0^\infty dk\, e^{-\beta k|z-z'|} I(\beta,t,k,z) I(\beta,t,k,z') = \int_0^1 dr \int_0^1 dr' \frac{r^3}{(r^2+t^2)} \frac{1}{(\beta^2 z^2 + r^2)^{3/2}} \frac{r'^3}{(r'^2+t^2)} \times$$

$$\times \frac{1}{(\beta^2 z'^2 + r'^2)^{3/2}} \int dk\, e^{-\beta k|z-z'|} J_0(kr) J_0(kr') = \int dr \int dr' \frac{r^3 r'^3}{(r^2+t^2)(r'^2+t^2)} \frac{1}{(\beta^2 z^2 + r^2)^{3/2}} \times$$

$$\times \frac{1}{(\beta^2 z'^2 + r'^2)^{3/2}} \frac{4}{\pi\sqrt{rr'}} K(k), \quad k^2 = \frac{4rr'}{(r+r')^2 + \beta^2 (z-z')^2}.$$

$$W_m^{vol} = \frac{1}{2}\beta t^4 \int_{-1/2}^{1/2} dz \int_{-1/2}^{1/2} dz' \int_0^1 dr \int_0^1 dr' \frac{r^3 r'^3}{(r^2+t^2)(r'^2+t^2)} \frac{1}{(\beta^2 z^2 + r^2)^{3/2}} \frac{1}{(\beta^2 z'^2 + r'^2)^{3/2}} \frac{4k}{\pi\sqrt{rr'}} K(k)$$

$$W_m^{vol} = \frac{1}{2}\beta t^4 \int_{-1/2}^{1/2} dz \int_{-1/2}^{1/2} dz' \int_0^1 dr \int_0^1 dr' \frac{1}{(\beta^2 z^2 + r^2)^{3/2}} \frac{1}{(\beta^2 z'^2 + r'^2)^{3/2}} G(z,z',r,r')$$

$$G(z,z',r,r') = \frac{r^3 r'^3}{(r^2+t^2)(r'^2+t^2)} \frac{4k}{\pi\sqrt{rr'}} K(k).$$ Вычислено на стадии моделирования через $\int dk\, e^{-\beta k|z-z'|}(\dots)$ метод.

Stiffness ∂_z: $W_m^{vol}(z) = \frac{1}{2}\beta t^4 \int_{-1/2-z}^{1/2-z} dz\, Q(z,z)$, $Q(z,z) = \int_{-1/2-z}^{1/2-z} dz' \int dk\, e^{-\beta k|z-z'|} I(k,z) I(k,z')$

$\frac{d}{dx}\int_{a(x)}^{b(x)} dz\, f(x,z) = f(x,b(x))\frac{db}{dx} - f(x,a(x))\frac{da}{dx} + \int_a^b \frac{\partial f(x,z)}{\partial x} dz$ — Leibniz integral rule.

in $\beta t^4/2$ units

$$\frac{\partial W_m^{vol}}{\partial z} = Q\left(\frac{1}{2}-z, z\right)(-1) - Q\left(-\frac{1}{2}-z, z\right)(-1) + \int_{-1/2-z}^{1/2-z} dz' \frac{\partial Q(z,z')}{\partial z}, \quad Q(z,z) = \int_{-1/2-z}^{1/2-z} dz' F(z,z',z^0)$$

$$F(z,z',z) = P(z,z') = \int dk\, e^{-\beta k|z-z'|} I(k,z) I(k,z'), \quad \frac{\partial Q}{\partial z} = P\left(z, \frac{1}{2}-z\right)(-1) - P\left(z, -\frac{1}{2}-z\right)(-1)$$

$a=\frac{1}{2}$, $\frac{\partial W_m^{vol}}{\partial Z} = Q(-a-Z,Z) - Q(a-Z,Z) + \int_{-a-Z}^{a-Z} dz \left[P(z,-a-Z) - P(z,a-Z) \right]$

$Q(z,Z) = \int_{-a-z}^{a-z} dz' P(z,z')$, $P(a,a) = \int dk\, e^{-\beta|k|(a-a)} I(k,a) I(k,a)$

$Q(z,Z) = Q(z,0) + \frac{\partial Q}{\partial Z}\Big|_{Z=0} Z + O(Z^2)$, $\frac{\partial Q}{\partial Z} = P(z,a-Z)(-1) - P(z,-a-Z)(+1)$

$\approx P(z,-a) - P(z,a)$, $Q(-a-Z,Z) \approx Q(-a-Z,0) + \left[P(-a-Z,-a) - P(-a-Z,a) \right] Z$

$\approx Q(-a-Z,0) + \left[P(-a,-a) - P(-a,a) \right] Z$, $Q(-a-Z,0) = Q(-a,0) + \frac{\partial Q(z,0)}{\partial z}\Big|_{z=-a} Z(-1)$

$= Q(-a,0) + \frac{\partial Q(z,0)}{\partial z}\Big|_{z=-a} Z(-1) = Q(-a,0) + \left[\frac{\partial Q(z,0)}{\partial z}\Big|_{z=-a} Z(-1) \right]$

$Q(z,0) = \int_{-a}^{a} dz' P(z,z')$, $\frac{\partial Q(z,0)}{\partial z} = \int_{-a}^{a} dz' \frac{\partial P(z,z')}{\partial z}\Big|_{z=-a}$

$Q(+a-Z,Z) = Q(a-Z,0) + \left[P(a,-a) - P(a,a) \right] Z = Q(a,0) + \frac{\partial Q(z,0)}{\partial z}\Big|_{z=a} Z(-1) + \left[\dots \right]$

$\frac{\partial W_m^{vol}}{\partial Z} = Q(-a,0) + \frac{\partial Q(z,0)}{\partial z}\Big|_{z=-a} Z(-1) + \left[P(-a,-a) - P(-a,a) \right] Z - \left\{ Q(a,0) + \right.$

$\left. + \frac{\partial Q(z,0)}{\partial z}\Big|_{z=a} (-1) Z + \left[P(a,-a) - P(a,a) \right] Z \right\} + \int_{-a-Z}^{a-Z} dz \left[P(z,-a-Z) - P(z,a-Z) \right] =$

$= P(-a,0) - Q(a,0) + \left[\frac{\partial Q(z,0)}{\partial z}\Big|_{z=a} - \frac{\partial Q(z,0)}{\partial z}\Big|_{z=-a} \right] Z + \left[P(-a,-a) - P(-a,a) - \right.$

$\left. - P(a,-a) + P(a,a) \right] Z + \int_{-a-Z}^{a-Z} dz \left[P(z,-a-Z) - P(z,a-Z) \right]$

$P(z,a-Z) = P(z,a) + \frac{\partial P(z,z')}{\partial z'}\Big|_{z'=a} Z(-1)$, $P(z,-a-Z) = P(z,-a) + \frac{\partial P(z,z')}{\partial z'}\Big|_{z'=-a} Z(-1)$

$\int_{-a-Z}^{a-Z} dz \left[P(z,-a-Z) - P(z,a-Z) \right] = \int_{-a-Z}^{a-Z} dz \left[P(z,-a) - P(z,a) + \left\{ \frac{\partial P(z,z')}{\partial z'}\Big|_{z'=-a} - \frac{\partial P(z,z')}{\partial z'}\Big|_{z'=a} \right\} Z \right]$

$= \int_{-a-Z}^{a-Z} dz \left[P(z,-a) - P(z,a) \right] + \int_{-a}^{a} dz \left[\frac{\partial P(z,z')}{\partial z'}\Big|_{z'=a} - \frac{\partial P(z,z')}{\partial z'}\Big|_{z'=-a} \right] Z = J(Z) + \left[\dots \right] Z$

$J(Z) = \int_{-a-Z}^{a-Z} dz \left[P(z,-a) - P(z,a) \right]$, $J(Z) = J(0) + \frac{\partial J(Z)}{\partial Z}\Big|_{Z=0} Z =$

$= \int_{-a}^{a} dz \left[P(z,-a) - P(z,a) \right] + Z \left\{ (+1) \left[P(a-Z,-a) - P(a-Z,a) \right] + (-1) \left[P(-a-Z,-a) - \right.\right.$

$\left.\left. - P(-a-Z,a) \right] \right\} \approx \int_{-a}^{a} dz \left[P(z,-a) - P(z,a) \right] + Z \left[-P(a,-a) + P(a,a) - \right.$

$\left. - P(-a,-a) + P(-a,a) \right]$

$P(z,z') = F(|z-z'|)$, поэтому $J(Z) = \int_{-a}^{a} dz \left[P(z,-a) - P(z,a) \right]$ не зависит от Z.

$\frac{\partial W_m^{vol}}{\partial Z} = \left[\frac{\partial Q(z,0)}{\partial z}\Big|_{z=a} - \frac{\partial Q(z,0)}{\partial z}\Big|_{z=-a} \right] Z + 2 \left[P(a,a) - P(a,-a) \right] Z +$

$+ \int_{-a}^{a} dz \left[\frac{\partial P(z,z')}{\partial z'}\Big|_{z'=a} - \frac{\partial P(z,z')}{\partial z'}\Big|_{z'=-a} \right] Z$, $Q(z,0) = \int_{-a}^{a} dz' P(z,z')$

$\frac{\partial Q(z,0)}{\partial z} = \int_{-a}^{a} dz' \frac{\partial P(z,z')}{\partial z}$, $\left(\frac{1}{Z} \frac{\partial W_m^{vol}}{\partial Z} = \left[\int_{-a}^{a} dz' \frac{\partial P(z,z')}{\partial z}\Big|_{z=a} - \int_{-a}^{a} dz' \frac{\partial P(z,z')}{\partial z}\Big|_{z=-a} \right] + \right.$

$\left. + 2 \left[P(a,a) - P(a,-a) \right] + \int_{-a}^{a} dz \left[\frac{\partial P(z,z')}{\partial z'}\Big|_{z'=a} - \frac{\partial P(z,z')}{\partial z'}\Big|_{z'=-a} \right] \right)$

$\int_{-a}^{a} dz' \frac{\partial P(z,z')}{\partial z'}\Big|_{z'=a} = \left| \begin{matrix} z \to z' \\ z' \to z \end{matrix} \right| = \int_{-a}^{a} dz \frac{\partial P(z',z)}{\partial z'}\Big|_{z'=a}$, $\frac{\partial}{\partial z'} P(z',z) = \frac{\partial}{\partial z'} P(|z'-z|) =$

$= \left| z'-z = \xi \right| = \frac{\partial}{\partial \xi} P(\xi)$, поэтому, будучи z и z' не зависимыми,

$\int_{-a}^{a} dz \left[\frac{\partial P(z,z')}{\partial z'}\Big|_{z'=a} + \frac{\partial P(z,z')}{\partial z'}\Big|_{z'=-a} \right]$? $P(z,z') = \int dk\, e^{-\beta|k|\,|z'-z|} I(k,z') I(k,z)$

$\int_{-a}^{a} dz' \frac{\partial P(z,z')}{\partial z'}\Big|_{z=a} = \frac{\partial}{\partial z} \int dz' P(z,z')\Big|_{z=a}$, $\int_{-a}^{a} \frac{\partial P(z,z')}{\partial z'}\Big|_{z'=a} dz = \frac{\partial}{\partial z'} \int dz P(z,z')\Big|_{z'=a}$

$\frac{\partial Q}{\partial z} = \frac{\partial}{\partial z} \int dz' P(z,z') = \frac{\partial}{\partial z} \int dz' P(|z-z'|)$, $\frac{\partial |z-z'|}{\partial z} = \frac{(z-z')}{|z-z'|} = \operatorname{sign}(z-z')$

$\frac{\partial Q}{\partial z} = \frac{\partial}{\partial z} \int dk\, e^{-\beta k|z-z'|} I(k,z') I(k,z)$, $\frac{\partial P(z,z')}{\partial z'} = \frac{\partial}{\partial z'} \int dk\, e^{-\beta k|z-z'|} I(k,z') I(k,z)$

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$$\frac{\partial P(z,z')}{\partial z} = \int dk \left[-\beta k \cdot s(z-z')\, e^{-\beta k|z-z'|} I(z) I(z') + e^{-\beta k|z-z'|} \frac{\partial I(z)}{\partial z} I(z') \right]$$

$$P(z,z') = \int dk\, e^{-\beta k|z-z'|} I(k,z') I(k,z) = P(z,z')$$

$$\frac{\partial P(z,z')}{\partial z'} = \int dk \left[-\beta k\, s(z'-z) e^{-\beta k|z-z'|} I(z) I(z') + e^{-\beta k|z-z'|} I(z) \frac{\partial I(z')}{\partial z'} \right]$$

$$\int_0^1 dz \left. \frac{\partial P(z,z')}{\partial z} \right|_{z=a} = \int_0^1 dz' \int dk \left[-\beta k\, s(a-z')\, e^{-\beta k|a-z'|} I(a) I(z') + e^{-(\dots)} \frac{\partial I(z)}{\partial z}\Big|_{z=a} I(z') \right]$$

$$W_m^{vol}(z) = \frac{1}{2} \varkappa_m z^2, \quad \frac{\partial W_m^{vol}}{\partial z} = \varkappa_m z, \quad \frac{1}{z} \frac{\partial W_m^{vol}}{\partial z} = \varkappa_m$$

$$\varkappa_m^{vol} = \frac{1}{a} \frac{\partial W_m^{vol}}{\partial z} = 2\left[P(a,a) - P(a,-a)\right] + \int_0^a dz' \left[\left. \frac{\partial P(z,z')}{\partial z} \right|_{z=a} - \left. \frac{\partial P(z,z')}{\partial z} \right|_{z=-a} \right] +$$

$$+ \int_0^a dz \left[\left. \frac{\partial P(z,z')}{\partial z'} \right|_{z'=a} - \left. \frac{\partial P(z,z')}{\partial z'} \right|_{z'=-a} \right], \quad a = L/2 \quad (a = L/2 \text{ in abs. units})$$

(in units of ρ^4)

$$I(k,z) = \int_0^1 dr\, r\, J_0(kr) \frac{r^2}{(r^2+t^2)\left[\beta^2 z^2 + r^2 + z^2\right]^{3/2}}, \quad \frac{\partial I(k,z)}{\partial z} = \int_0^1 dr\, J_0(kr) \frac{r^3}{(r^2+t^2)} \left(-\frac{3}{2}\right) \frac{\beta^2 \cdot 2z}{(\beta^2 z^2 + r^2)^{5/2}}$$

$$P(a,a) - P(a,-a) = \int dk \left[1 - e^{-\beta k}\right] I^2(k, 1/2) > 0.$$

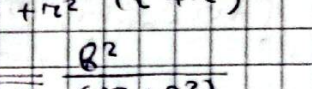
$$\gamma(\beta,t) = 2 \int_0^\infty dk \left[1 - e^{-\beta k}\right] I^2(\beta, t, k, 1/2)$$ – будущее значение $\varkappa_m^{vol}$.

MCad можно вычислить $\gamma(\beta,t)$. Интегралы с $\frac{\partial P}{\partial z}, \frac{\partial P}{\partial z'}$ дают сложное сокращение. Но $\varkappa_t^{vol}$ – total < 0 (!) $\varkappa_m^{vol}(t) < 0$ $t = [0.05 - 0.20]$

Найдем $\varkappa_m^{face}$. Можно ли вычислить $\varkappa_m$? (16/III) Попробуем $\varkappa_m^{vol}$ вычислить. Face magnetization

Eq. для face magnetostatics (16/III) сохранился, W_m^f. Но теперь сумма для BP position $z \neq 0$, где $m_z = \frac{\beta z}{\sqrt{\beta^2 z^2 + r^2}} \frac{t^2}{(t^2+r^2)}$, $\beta = L/R$, $t = \delta/R$, есть odd function on z.

WU new Eq. Kneb 300€ $m_z(z,r,Z) = \frac{\beta(z-Z)}{\sqrt{\beta^2(z-Z)^2 + r^2}} \frac{t^2}{(t^2+r^2)}$



В цил. переменных $m_z(z,\rho,Z) = \frac{(z-Z)}{\sqrt{(z-Z)^2+\rho^2}} \frac{\beta^2}{(\beta^2+\rho^2)}$

$$W_m^f = 4\pi^2 M_s^2 \int dk \left[K_+^2(k) - e^{-kL} K_+(k) K_-(k)\right], \quad K_\pm(k) = \int d\rho\, \rho\, m_z(\rho, z = \pm L/2) J_0(k\rho)$$

$$K_\pm(k) = R^2 \int dr\, r\, J_0(kRr)\, m_z(z,r,Z)\Big|_{z=\pm L/2}, \quad \frac{Z}{Z} \text{ in units } L, \; r \text{ in units } R$$

$\uparrow\vec{m} \uparrow\vec{m}$ $(\vec{m}\cdot\vec{n}) = +m_z$ при $z = \pm\frac{L}{2}$ $(\vec{m}\cdot\vec{n})_{z=-\frac{L}{2}} = (+m_z\hat{z})(-\hat{z}) = -m_z$, $z = -\frac{L}{2}$



$$(\vec{m}\cdot\vec{n})(\vec{m}'\cdot\vec{n}') = \begin{cases} m_z m_z', & z = L/2 \text{ и } z' = \frac{L}{2}, \; z = -\frac{L}{2}, z' = -\frac{L}{2} \\ m_z m_z'(-1), & z = L/2, z' = -L/2 \text{ и } z = -\frac{L}{2}, z' = \frac{L}{2} \end{cases} \Big| \Delta z = L$$

$$W_m^f = 4\pi^2 M_s^2 R^4 \int dk \left[\left\{ \int dr\, r\, J_0(kRr) m_z(z=\tfrac{L}{2}, r, Z) \right\}^2 - e^{-kL} \int dr\, r\, J_0(kRr)\, m_z(+\tfrac{L}{2}, r, Z) \cdot \right.$$

$$\left. \times \int dr\, r\, J_0(kRr)\, m_z(-\tfrac{L}{2}, r, Z) \right] = 4\pi^2 M_s^2 R^3 \beta^2 \int dx \left[\left\{ \int dr\, r\, J_0(xr) \underbrace{m_z(z=\tfrac{L}{2}, r, Z)}_{\beta} \right\}^2 - \right.$$

$$\left. - e^{-\beta x} \int dr\, r\, J_0(xr) \underbrace{m_z(+\tfrac{L}{2}, r, Z)}_{\beta} \int \underbrace{m_z(-\tfrac{L}{2}, r', Z)}_{\beta} dr'\, r'\, J_0(xr') \right], \quad \text{где } x = kR$$

25/VIII Two tail-to-tail domains: $m_z > 0$ $(z - Z > 0)$ и $m_z < 0$ $(z - Z < 0)$. BP position in the plane $z = Z$.

$4\pi^2 M_s^2 R^3 \beta^2 = 4\pi M_s^2 \pi R^3 \beta, \beta = 4\pi M_s^2 \beta V.$

$$W_m^f = 4\pi M_s^2 V \beta \int dx \left\{ \left[\int_{\bar{\beta}} dr\, r\, J_0(xr) m_z(z=\tfrac{1}{2}, r, Z) \right]^2 - e^{-\beta x} \int_{\bar{\beta}} dr\, r\, J_0(xr)\, m_z(+\tfrac{1}{2}, r, Z) \times \right.$$

$$\left. \times \int_{\bar{\beta}} dr' r' J_0(xr')\, m_z(-\tfrac{1}{2}, r', Z) \right\}$$

$$W_m^f = \frac{1}{2} M_s^2 \int_{z=\text{const}} \int_{z'=\text{const}} dS\, dS' \frac{(\vec{m}\cdot\vec{n})\cdot(\vec{m}'\cdot\vec{n}')}{|\vec{r} - \vec{r}'|}, \quad \vec{m} = \vec{m}(z,\rho,\varphi) = \vec{m}(z,\rho) \text{ – axial}$$

$$\vec{m}' = \vec{m}(z',\rho',\varphi') = \vec{m}(z',\rho') \text{ symmetry}$$

$$\frac{1}{|\vec{r} - \vec{r}'|} = \int dk\, e^{-k|z-z'|} \sum_n J_n(k\rho) J_n(k\rho') e^{in(\varphi - \varphi')}, \quad d\sigma = \rho\, d\rho\, d\varphi, \; dS' = \rho' d\rho' d\varphi'$$

$$W_m^f = \frac{1}{2} M_s^2 \int d\rho\, \rho\, d\varphi \int d\rho'\, \rho'\, d\varphi' \int dk\, e^{-k|z-z'|} \sum_n J_n(k\rho) J_n(k\rho') e^{in(\varphi - \varphi')} (\vec{m}\cdot\vec{n})(\vec{m}'\cdot\vec{n}')$$

Escaneado con CamScanner

$\vec{m}$ не зависит от φ. $\int_0^{2\pi} d\varphi e^{ik\varphi} = 2\pi\delta_{k0}$, $\int_0^{2\pi} d\varphi' e^{-ik\varphi'} = 2\pi\delta_{k0}$.

$$W_m^f = \frac{1}{2} M_s^2 \int d\rho\rho \int d\rho'\rho' (2\pi)^2 \int_0^\infty J_0(k\rho) J_0(k\rho') \int dk \exp(-k|z-z'|)(\vec{m}\cdot\vec{n})(\vec{m}'\cdot\vec{n}')$$

$e^{-k|z-z'|}(\vec{m}\cdot\vec{n})(\vec{m}'\cdot\vec{n}') = m_z m_z' + m_z m_z' - m_z m_z' e^{-kL} - m_z m_z' e^{-kL}$

$\vec{n} = \hat{z}$ – верхний домен, $\vec{n} = -\hat{z}$ – нижний домен. $\left(z=\frac{L}{2}, z'=\frac{L}{2}\right)$ $\left(z=-\frac{L}{2}, z'=-\frac{L}{2}\right)$ $\left(z=\frac{L}{2}, z'=-\frac{L}{2}\right)$ $\left(z=-\frac{L}{2}, z'=\frac{L}{2}\right)$

$$m_z = m_z(r,\varphi,Z) = \frac{(z-Z)}{\sqrt{(z-Z)^2+\rho^2}} \frac{R^2}{R^2+\rho^2}.$$

Planes $z=+\frac{L}{2}$ и $z=-\frac{L}{2}$ не эквивалентны $\forall m_z \neq 0$,

shifted BP along wire axis $\forall Z$.

$$W_m^f = 2\pi^2 M_s^2 \int d\rho\rho \int d\rho'\rho' \int dk\, J_0(k\rho) J_0(k\rho') \left[m_z\left(\tfrac{L}{2},\rho,Z\right) m_z\left(\tfrac{L}{2},\rho',Z\right) + m_z\left(-\tfrac{L}{2},\rho,Z\right) m_z\left(-\tfrac{L}{2},\rho'\right) - e^{-kL} m_z\left(\tfrac{L}{2},\rho,Z\right) m_z\left(-\tfrac{L}{2},\rho',Z\right) - e^{-kL} m_z\left(-\tfrac{L}{2},\rho,Z\right) m_z\left(\tfrac{L}{2},\rho',Z\right) \right]$$

$$K_\pm(k) = \int d\rho\rho\, J_0(k\rho)\, m_z\left(\pm\tfrac{L}{2},\rho,Z\right) = K_\pm(k,Z).$$

$$W_m^f = 2\pi^2 M_s^2 \int dk \left[K_+^2(k) + K_-^2(k) - e^{-kL} K_+(k) K_-(k) - e^{-kL} K_-(k) K_+(k) \right]$$

$$W_m^f = 2\pi^2 M_s^2 \int dk \left[K_+^2(k) + K_-^2(k) - 2e^{-kL} K_+(k) K_-(k) \right]$$

$r = \frac{\rho}{R}$, $t = \frac{\beta}{R}$, $z \to z/L$.

$$m_z(z,\rho,Z) = \frac{(z-Z)}{\sqrt{(z-Z)^2+\rho^2}} \frac{R^2}{(R^2+\rho^2)} = \frac{1}{R} \frac{(z-Z)}{\sqrt{\frac{1}{R^2}(z-Z)^2 + r^2}} \frac{t^2}{(t^2+r^2)} = \frac{\beta(z-Z)}{\sqrt{\beta^2(z-Z)^2 + r^2}} \frac{t^2}{(t^2+r^2)}$$

$$K_\pm(k) = R^2 \int dr r\, J_0(kRr)\, m_z\left(\pm\tfrac{1}{2}, r, Z\right), \quad z \text{ и } Z \text{ in units of } L \text{ – wire length.}$$

$kR = x$

$$W_m^f = 2\pi^2 M_s^2 R^4 \frac{1}{R} \int dx\, x \left[K_+^2(x) + K_-^2(x) - 2e^{-\beta x} K_+(x) K_-(x) \right],$$

$$K_\pm(x) = \int dr r\, J_0(xr)\, m_z\left(\pm\tfrac{1}{2}, r, Z\right) = \beta t^2 \int dr r\, J_0(xr) \frac{m_z}{\beta t^2}\left(\pm\tfrac{1}{2}, r, Z\right) - \text{удобно.}$$

$$\tilde{K}_\pm(x) = \int_0^\infty dr r\, J_0(xr) \frac{1}{\beta t^2} m_z\left(\pm\tfrac{1}{2}, r, Z\right), \quad 2\pi^2 M_s^2 R^3 \beta^2 = 2\pi M_s^2 V \beta.$$

$$W_m^f = 2\pi M_s^2 V \beta t^4 \int dk \left[\tilde{K}_+^2(k) + \tilde{K}_-^2(k) - 2e^{-\beta k} \tilde{K}_+(k) \tilde{K}_-(k) \right], \quad \tilde{K} = \tilde{K}(k,Z)$$

face magn. energy для BP – DW

$$\tilde{K}_\pm(k,Z) = \int_0^1 dr r\, J_0(kr) \frac{(z-Z)}{\sqrt{\beta^2(z-Z)^2 + r^2}} \frac{1}{(t^2+r^2)} \Big|_{z=\pm 1/2}.$$

MCad через спектр $W_m^f(Z)$ „Bloch_point_stability.mcdx",

(!) $W_m^f(Z)$ имеет с расчетом Z, $W_m^f \sim 10^{-4}$ при $\beta=10$, $t=0.1$ (in units $\mu_0 M_s^2 V$)

$\chi_z < 0$ Вывод: если $W_m^{vol} - \chi_m^{vol} < 0$ достаточно велико, то $\left(\chi_z = \chi_{ex} + \chi_m^{vol} + \chi_m^f < 0\right)$.

$W(\vec{X}) = \frac{1}{2}\chi(X^2+Y^2) + \frac{1}{2}\chi_z Z^2$, расчет χ – сложен даже для жесткого BP сдвига, т.к. системы теряют аксиальную симметрию.

(1) $\chi_z < 0$, $\chi < 0$ (case b) – BP осциллирует (∇) near energy max at $\vec{X}=0$

(2) $\chi_z < 0$, $\chi > 0$ (case c) – hyperbolic solutions eq Thiele – BP escapes through wire side surface during the time $t_{esc} \approx \Gamma^{-1}$, $\Gamma^2 = -\frac{\chi\chi_z}{G_0^2} > 0$ – instability

// Проверить χ_m^{vol} и аналитически найти χ_{ex} (дает χ_{ex}).

$\chi = \chi_{ex} + \chi_{in}^{vol} + \chi_m^f$, видно χ_m^f – отдельная, $\chi_{ex} < 0$, $\chi_m^{vol} > 0$, и $\chi = \chi_{ex} + \chi_m^{vol} > 0$ для достаточно больших wire radii R.

// В расчете BP dynamics б/пп надо добавить axial field H_z в Thiele eq.

$$W_H = -\int dV \vec{m}\cdot\vec{H} = -\int dV\, m_z H_z M_s, \quad m_z = m_z(z,\rho,Z) = \frac{(z-Z)}{\sqrt{(z-Z)^2+\rho^2}} \frac{R^2}{R^2+\rho^2}$$

Zeeman En.

$$W_H = -M_s H_z \int dV\, m_z(z,\rho,Z) = -M_s H_z 2\pi \int d\rho\rho\, dz\, m_z(z,\rho,Z)$$

$$J(z) = \int d\rho\rho \frac{1}{\sqrt{z^2+\rho^2}(b^2+\rho^2)} = \frac{1}{2}\int dx \frac{1}{\sqrt{z^2+x}(b^2+x)} = \frac{1}{2}\int dx \frac{1}{\sqrt{(a+x)(d+x)}} = \left|\begin{matrix} z^2=a \\ b^2=d \end{matrix}\right| =$$

$$= \frac{1}{2}\int \frac{dx}{\sqrt{t+x}}, \quad z = a+x, \quad t = d+x, \quad \text{Пр.-П. 2.245}, \quad \left(\int \frac{dx}{t\sqrt{z}} = (*)\right) \text{ не берется в элем. функциях.}$$

Escaneado con CamScanner

$$W_H(\bar{z}) = -2\pi M_s H_z \int_0^R d\rho\,\rho \int_{-\frac{L}{2}}^{L/2} dz \frac{(z-\bar{z})}{\sqrt{(z-\bar{z})^2+\rho^2}} \frac{b^2}{(b^2+\rho^2)}$$

$$J(\rho,\bar{z}) = \int_{-L/2}^{L/2} dz \frac{(z-\bar{z})}{\sqrt{(z-\bar{z})^2+\rho^2}} = \left| \begin{matrix} z-\bar{z}=u \\ dz=du \end{matrix} \right| = \int du \frac{u}{\sqrt{u^2+\rho^2}} = \frac{1}{2}\int dt \frac{1}{\sqrt{t+\rho^2}} =$$

$$= \sqrt{t+\rho^2}\Big|_{\dots} = \sqrt{u^2+\rho^2}\Big|_{\dots} = \sqrt{(z-\bar{z})^2+\rho^2}\Big|_{z=-L/2}^{z=L/2}.$$

$$W_H(\bar{z}) = -2\pi M_s H_z b^2 \int d\rho\,\rho \frac{1}{(b^2+\rho^2)} \int dz \frac{(z-\bar{z})}{\sqrt{(z-\bar{z})^2+\rho^2}} = -2\pi M_s H_z b^2 \int d\rho \frac{\rho}{(b^2+\rho^2)} \times$$

$$\times \sqrt{(z-\bar{z})^2+\rho^2}\Big|_{z=-L/2}^{z=L/2}$$

$$\int \frac{d\rho\,\rho}{(b^2+\rho^2)} \sqrt{z^2+\rho^2} = \left|\rho^2 = x\right| = \frac{1}{2}\int \frac{dx}{(b^2+x)} \sqrt{z^2+x} = \frac{1}{2}\int dx \frac{\sqrt{z}}{t} = \left| \begin{matrix} a=z^2 \\ \lambda=b^2 \\ b=1, \beta=1 \end{matrix} \right| =$$

$$= \frac{1}{2}\left[2\sqrt{z} + (z^2-b^2)\int \frac{dx}{t\sqrt{z}}, \text{ [p.+P. 2.248]} \right| \quad z = a+bx$$

— не берется в эл. функциях.

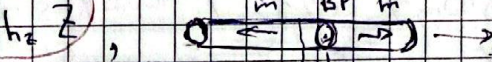
$$W_H(\bar{z}) = -2\pi M_s H_z b^2 \int_0^R \frac{d\rho\,\rho}{(b^2+\rho^2)} \sqrt{(z-\bar{z})^2+\rho^2}\Big|_{z=-\frac{L}{2}}^{z=+\frac{L}{2}} = -2\pi M_s H_z R^2 L \times$$

$$\times \int_0^1 dr\,r \frac{t^2}{(t^2+r^2)} \sqrt{(z-\bar{z})^2+\frac{1}{\beta^2}r^2}\Big|_{z=-1/2}^{z=1/2}, \quad 2\pi M_s H_z R^2 L = 2M_s H_z V = 8\pi M_s^2 H_z \frac{V}{(4\pi M_s)}$$

BP Zeeman

$$W_H(\bar{z}) = -8\pi M_s^2 h_z V t^2 \int_0^1 dr\,r \frac{1}{(t^2+r^2)} \sqrt{(z-\bar{z})^2+\frac{1}{\beta^2}r^2}\Big|_{z=-1/2}^{z=+1/2}, \quad h_z = \left(\frac{H_z}{4\pi M_s}\right)$$

При $z \ll 1$, $W_H(\bar{z}) = \mu h_z \bar{z}$,



$h_z < 0$ чтобы ДС была устойчива при $\bar{z} > 0$

В eq. Thiele для BP position $\vec{X}(X,Y,\bar{z})$ надо добавить $\frac{\partial W_H}{\partial \bar{z}} = \mu h_z$

$$\frac{\partial W}{\partial \bar{z}} = \left(\frac{\partial W}{\partial \vec{X}}\right)_z = \varkappa_z \bar{z}, \text{ значит } \varkappa_z \bar{z} \to \varkappa_z \bar{z} + \mu h_z$$

$$\begin{cases} -G_\rho \dot{\bar{z}} = \varkappa X_\varphi, \\ G_\rho X_\varphi = \varkappa_z \bar{z} + \mu h_z \end{cases} \to G_\rho \ddot{\bar{z}} = \varkappa \dot{X}_\varphi = \varkappa \frac{1}{G_\rho}(\varkappa_z \dot{\bar{z}} + \mu \dot{h}_z)$$

$$\ddot{\bar{z}} + \frac{\varkappa \varkappa_z}{G_\rho^2}\bar{z} + \frac{\varkappa}{G_\rho^2}\mu h_z = 0, \quad \ddot{\bar{z}} + \omega_G^2 \bar{z} = -\frac{\varkappa}{G_\rho^2}\mu \dot{h}_z, \quad \omega_G^2 = \varkappa \varkappa_z / G_\rho^2.$$

$$G_\rho \dot{X}_\varphi = \varkappa_z \dot{\bar{z}} + \mu \dot{h}_z, \quad \dot{X}_\varphi = \frac{1}{G_\rho}\left[\varkappa_z\left(-\frac{\varkappa}{G_\rho}\right)X_\varphi + \mu \dot{h}_z\right] = -\omega_G^2 X_\varphi + \frac{\mu}{G_\rho}\frac{dh_z}{dt}$$

$\bar{z}(t) = \bar{z}_0(t) - vt$ имеем в виду. $\ddot{\bar{z}}_0 + \omega_G^2(\bar{z}_0 - vt) = -\frac{\varkappa}{G_\rho^2}\mu \dot{h}_z$

Ищем $\bar{z}(t) = \bar{z}_0(t) + \bar{z}_h(t)$, $\bar{z}_0$ – oscillatory motion, $\bar{z}_h$ – field induced motion.

// Надо damping включить, чтоб было steady state ДС motion.

(26/XII) Definition of damping tensor – Gus' NanoRL 2014

$$D_{\alpha\beta} = -\frac{\alpha}{\gamma}\frac{M_s}{\gamma}\int dV \left(\partial_\alpha \vec{m} \cdot \partial_\beta \vec{m}\right), \quad \frac{\partial}{\partial X_\alpha} = \nabla_\alpha, \text{ для rigid Soliton model}$$

$\vec{m}(\vec{r},\vec{X}) = \vec{m}(\vec{r}-\vec{X})$, $\frac{\partial}{\partial X_\alpha} = -\frac{\partial}{\partial r_\alpha}$. Для cylindrical nanowire (ρ,φ,z)

$$\partial_\alpha \vec{m} = \frac{\partial \vec{m}}{\partial \alpha} = \nabla_\alpha \vec{m}, \quad \nabla = \hat{\rho}\frac{\partial}{\partial \rho} + \hat{\varphi}\frac{1}{\rho}\frac{\partial}{\partial \varphi} + \hat{z}\frac{\partial}{\partial z}.$$

$\vec{m}(\vec{r})$ of Bloch point – Thiaville ansatz для $\vec{X}=0$: $m_z(\rho,z) = \frac{z}{\sqrt{z^2+\rho^2}}\frac{b^2}{(b^2+\rho^2)}$

$m_\rho = 0$, $m_\varphi^2 = 1 - m_z^2$. $D_{\alpha\beta}\ (\alpha,\beta=\varphi) = 0$, т.к. $\frac{\partial \vec{m}}{\partial \varphi} = 0$. HET, 4/1123. $\vec{m} = \hat{\rho}m_\rho + \hat{z}m_z$

Остаются $D_{\rho\rho}$, $D_{\rho z} = D_{z\rho}$, D_{zz}, $D_{\rho\rho} = -\alpha J \int d\rho\,\rho \int d\varphi \int dz \left(\frac{\partial \vec{m}}{\partial \rho}\cdot\frac{\partial \vec{m}}{\partial \rho}\right)$

$$\left(\frac{\partial \vec{m}}{\partial \rho}\cdot\frac{\partial \vec{m}}{\partial \rho}\right) = \left(\frac{\partial \vec{m}}{\partial \rho}\right)^2 = \left(\frac{\partial m_\varphi}{\partial \rho}\right)^2 + \left(\frac{\partial m_z}{\partial \rho}\right)^2, \quad m_\varphi = \sqrt{1-m_z^2}, \quad \frac{\partial m_\varphi}{\partial \rho} = \frac{1}{2\sqrt{1-m_z^2}}(-2)m_z\frac{\partial m_z}{\partial \rho}$$

$$\left(\frac{\partial \vec{m}}{\partial \rho}\right)^2 = \frac{m_z^2}{m_\varphi^2}\left(\frac{\partial m_z}{\partial \rho}\right)^2 + \left(\frac{\partial m_z}{\partial \rho}\right)^2 = \frac{1}{m_\varphi^2}\left(\frac{\partial m_z}{\partial \rho}\right)^2, \quad D_{\rho z} = -\alpha J \int d\rho\,\rho \int d\varphi \int dz \left(\frac{\partial \vec{m}}{\partial \rho}\cdot\frac{\partial \vec{m}}{\partial z}\right)$$

$$\vec{m}(z,\rho)\cdot\left(\frac{\partial \vec{m}}{\partial \rho}\cdot\frac{\partial \vec{m}}{\partial z}\right) = \left(\left[\hat{\varphi}\frac{\partial m_\varphi}{\partial \rho} + \hat{z}\frac{\partial m_z}{\partial \rho}\right]\cdot\left[\hat{\varphi}\frac{\partial m_\varphi}{\partial z} + \hat{z}\frac{\partial m_z}{\partial z}\right]\right) = \frac{\partial m_\varphi}{\partial \rho}\frac{\partial m_\varphi}{\partial z} + \frac{\partial m_z}{\partial \rho}\frac{\partial m_z}{\partial z}$$

$$\frac{\partial m_\varphi}{\partial \rho}\frac{\partial m_\varphi}{\partial z} = \frac{(-2)m_z}{2\sqrt{1-m_z^2}}\frac{\partial m_z}{\partial \rho}\frac{(-2)m_z}{2\sqrt{1-m_z^2}}\frac{\partial m_z}{\partial z} = \frac{m_z^2}{(1-m_z^2)}\frac{\partial m_z}{\partial \rho}\frac{\partial m_z}{\partial z}$$

$$\left(\frac{\partial \vec{m}}{\partial \rho}\cdot\frac{\partial \vec{m}}{\partial \rho}\right) = \frac{m_z^2}{m_\varphi^2}\frac{\partial m_z}{\partial \rho}\frac{\partial m_z}{\partial z} + \frac{\partial m_z}{\partial \rho}\frac{\partial m_z}{\partial z} = \frac{1}{m_\varphi^2}\frac{\partial m_z}{\partial \rho}\frac{\partial m_z}{\partial z}; \quad \frac{\partial m_z}{\partial \rho} \text{ odd func. } z,$$

Escaneado con CamScanner

$\frac{\partial m_z}{\partial z}$ - even function of z. $\frac{\partial m_z}{\partial \varphi}\frac{\partial m_z}{\partial z}$ - odd function of z $\int_{-H_2}^{H_2} dz \frac{\partial m_z}{\partial \varphi}\frac{\partial m_z}{\partial z} = 0$.

$D_{z\varphi} = D_{\varphi z} = 0$. $D_{zz} = -dJ \int d\varphi \int d\varphi \int dz \left(\frac{\partial \vec{m}}{\partial z}\cdot\frac{\partial \vec{m}}{\partial z}\right)$

$\left(\frac{\partial \vec{m}}{\partial z}\cdot\frac{\partial \vec{m}}{\partial z}\right) = \left(\frac{\partial \vec{m}}{\partial z}\right)^2 = \left(\frac{\partial m_\rho}{\partial z}\right)^2 + \left(\frac{\partial m_z}{\partial z}\right)^2 = \frac{1}{m_\rho^2}\left(\frac{\partial m_z}{\partial z}\right)^2$.

Core 1 & 2: $W_{ex}(b) = A\int dV (\nabla \vec{m})^2 = A\int dV \left[\frac{1}{m_\rho^2}(\nabla m_z)^2 + \frac{m_\varphi^2}{\rho^2}\right]$, $m_\varphi^2 = 1 - m_z^2$

give Thirvilleansatz $m_z(z,\varphi)$, $m_\varphi = 0$.

$\hat{T}_z(\hat{D}) = -dJ \int dV \hat{T}_z\left(\frac{\partial \vec{m}}{\partial x_\alpha}\cdot\frac{\partial \vec{m}}{\partial x_\beta}\right) = -dJ\int dV \sum_\alpha \left(\frac{\partial \vec{m}}{\partial x_\alpha}\right)^2 = -dJ\int dV \left[\left(\frac{\partial \vec{m}}{\partial \varphi}\right)^2 + \left(\frac{\partial \vec{m}}{\partial z}\right)^2\right]$

$\alpha = \varphi, z$ $\left(\frac{\partial \vec{m}}{\partial x_\alpha}\right)^2 = \frac{1}{m_\rho^2}\left(\frac{\partial m_z}{\partial x_\alpha}\right)^2$, $\hat{T}_z(\hat{D}) = -dJ\int dV \left[\frac{1}{m_\rho^2}\left(\left(\frac{\partial m_z}{\partial \varphi}\right)^2 + \left(\frac{\partial m_z}{\partial z}\right)^2\right)\right]$

Domain $\hat{T}_z(\hat{D}) \sim W_{ex}$. However, compared by $\frac{1}{m_\rho^2}$ term? $\frac{\partial \hat{e}_\varphi}{\partial \varphi} = -\hat{\rho}$

Anyway, the BP in Thirville ansatz proves $D_{\varphi\varphi} \neq 0$ и $D_{zz} \neq 0$.

Eq. Thiele 6/VIII reads $\vec{G}\times\dot{\vec{X}} + \hat{D}\dot{\vec{X}} = \frac{\partial W}{\partial \vec{X}}$, $W = \frac{1}{2}\varkappa(X_\varphi^2 + Y_\varphi^2) + \frac{1}{2}\varkappa_z Z^2 + \int^\mu h_z Z$

$\vec{X}(t) = \hat{\varphi}X_\varphi(t) + \hat{\rho}X_\rho(t) + \hat{z}Z(t)$ $\vec{G}\times\dot{\vec{X}} = G_\varphi \dot{X}_\rho \hat{z} - G_\varphi \dot{Z}\hat{\rho}$, $G_\varphi \neq 0$.

$\frac{\partial W}{\partial \vec{X}} = \varkappa X_\varphi \hat{\varphi} + \varkappa X_\rho \hat{\rho} + (\varkappa_z Z + \int^\mu h_z)\hat{z}$. $\hat{D}\dot{\vec{X}} = \hat{D}\dot{\vec{X}} = \begin{pmatrix} D_{\varphi\varphi} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & D_{zz}\end{pmatrix}\begin{pmatrix}\dot{X}_\varphi \\ \dot{X}_\rho \\ \dot{X}_z\end{pmatrix} =$

$= \begin{pmatrix} D_{\varphi\varphi}\dot{X}_\varphi \\ 0 \\ D_{zz}\dot{Z}\end{pmatrix} = \hat{\varphi}D_{\varphi\varphi}\dot{X}_\varphi + \hat{z}D_{zz}\dot{Z}$ $G_\rho = 0$, $G_z = 0$.

$G_\varphi \dot{X}_\rho \hat{z} - G_\varphi \dot{Z}\hat{\rho} + \hat{\varphi}D_{\varphi\varphi}\dot{X}_\varphi + \hat{z}D_{zz}\dot{Z} = \varkappa X_\varphi \hat{\varphi} + \varkappa X_\rho \hat{\rho} + (\varkappa_z Z + \int^\mu h_z)\hat{z}$

$\hat{\varphi}$: $-G_\varphi \dot{Z} = \varkappa X_\rho$, $\hat{z}$: $G_\varphi \dot{X}_\rho + D_{zz}\dot{Z} = \varkappa_z Z + \int^\mu h_z$, $\hat{\rho}$: $D_{\varphi\varphi}\dot{X}_\varphi = \varkappa X_\varphi$

теперь $X_\varphi \neq 0$, но УД дает $X_\varphi(t)$ однозначно от УД дает $X_\rho(t)$, $Z(t)$.

$D_{\varphi\varphi} < 0$, $D_{zz} < 0$, $\dot{X}_\varphi = -\frac{\varkappa}{|D_{\varphi\varphi}|}X_\varphi$ dynamics зависит от $\varkappa$-sign $\begin{cases}\varkappa > 0, X_\varphi(t) \to 0 \\ \varkappa < 0, X_\varphi(t) \to \infty\end{cases}$

$X_\rho = -\frac{G_\varphi}{\varkappa}\dot{Z}$, $\dot{X}_\rho = -\frac{G_\varphi}{\varkappa}\ddot{Z}$, $G_\varphi\left(-\frac{G_\varphi}{\varkappa}\right)\ddot{Z} + D_{zz}\dot{Z} - \varkappa_z Z = \int^\mu h_z$. $h_z < 0$ дает $\dot{Z} > 0$.

$\ddot{Z} - \frac{\varkappa D_{zz}}{G_\varphi^2}\dot{Z} + \omega_G^2 Z = -\frac{\varkappa \int^\mu}{G_\varphi^2}h_z$, $\omega_G^2 = \left(\varkappa\varkappa_z/G_\varphi^2\right)$ может < 0.

Решение $\dot{Z} = 0$ не имеет смысла. Steady state motion $\ddot{Z} = \frac{dV_z}{dt} = 0$, если h_z?

Steady: $\dot{Z} = \frac{\varkappa_z}{D_{zz}}Z + \frac{\int^\mu}{D_{zz}}h_z$. При $h_z = 0$ $\dot{Z} = \frac{\varkappa_z}{D_{zz}}Z = \frac{|\varkappa_z|}{|D_{zz}|}Z$, $Z(t) \sim e^{rt}$, exp. рост $Z(t)$ и неустойчивость $Z = 0$ DW position.

$Z \ll 1$, $\dot{Z} = \frac{\int^\mu}{D_{zz}}h_z = \frac{\int^\mu}{|D_{zz}|}|h_z|$, $v_z(h_z) \sim h_z$ - rigid DW motion.

$\dot{Z} = aZ + b$, $Z(t) = Ce^{at} + Z_1(t)$ const $Cae^{at} = a[Ce^{at} + Z_1] + B$, $aZ_1 + b = 0$, $Z_1 = -\frac{b}{a}$

$Z(t) = Ce^{at} - \frac{b}{a}$, $\frac{b}{a} = \frac{\int^\mu h_z}{D_{zz}}$, $\frac{D_{zz}}{\varkappa_z} = \frac{\int^\mu h_z}{\varkappa_z}$. Не нам. Предъявим $Z(0) = 0$ нач. условие.

$Z(t) = Ce^{at} + Z_1$, $Z(0) = C + Z_1$, $Z_1 = -C$, $Z(t) = C[e^{at} - 1]$, $a = \frac{\varkappa_z}{D_{zz}} > 0$

Если $Z(0) = 0$ не предъявлять, то $Z(t) = Ce^{at} - \frac{\mu h_z}{\varkappa_z}$, $Z(t)$ растет с ростом $|h_z|$, $h_z < 0$.

$\dot{Z} = aZ + B = a\left(Z + \frac{b}{a}\right)$, $Z' = Z + b/a$, $\dot{Z} = \dot{Z}' = aZ'$, $Z'(t) = Ce^{at}$, $Z(t) = Ce^{at} - \frac{b}{a}$.

теперь в $Z(t) = Ce^{at} - \frac{b}{a}$ предъявим $Z(0) = 0$, $Z(0) = C - \frac{b}{a} = 0$, $C = \frac{b}{a}$ и

общее решение удобен. $Z(0) = 0$ если $Z(t) = \frac{b}{a}\left(e^{at} - 1\right)$ $\frac{b}{a} = \frac{\mu h_z}{\varkappa_z} = \mu\frac{|h_z|}{|\varkappa_z|} > 0$,

$h_z < 0$, $\varkappa_z < 0$. Steady motion solution $Z_s(t) = \frac{\mu h_z}{\varkappa_z}\left(e^{at} - 1\right)$, $a = \frac{\varkappa_z}{D_{zz}} > 0$, $Z(0) = 0$

$Z(t) \sim h_z$ и $Z(t)$ растет с $t \uparrow$.

Но $\dot{Z}_s \neq 0$, что противоречит steady solution definition, $\dot{Z} = 0$,

т.е. steady solution $Z = V(t) = \text{const}$ невозможно (при $\varkappa_z \neq 0$).

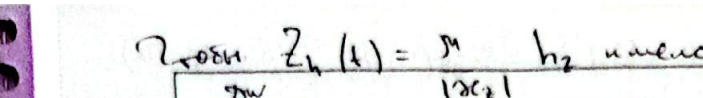
$\to Z(t) = Z_0(t) + Z_h(t)$, Z_0 - oscillatory part; $Z_h(t)$ steady part с $\dot{Z}_h = 0$.

$\ddot{Z}_0 - \frac{\varkappa D_{zz}}{G_\varphi^2}\dot{Z}_0 + \omega_G^2 Z_0 + \omega_G^2 Z_h = -\frac{\varkappa \int^\mu}{G_\varphi^2}h_z$, учтем $Z_0 - \frac{\varkappa D_{zz}}{G_\varphi^2}\dot{Z}_0 + \omega_G^2 Z_0(t) = 0$

и $\omega_G^2 Z_h = -\frac{\varkappa \int^\mu}{G_\varphi^2}h_z$, $Z_h(t) = \frac{G_\varphi^2}{\varkappa\varkappa_z}(-1)\frac{\varkappa \int^\mu}{G_\varphi^2}h_z = -\frac{\int^\mu}{\varkappa_z}h_z = \frac{\mu}{|\varkappa_z|}h_z$.

Escaneado con CamScanner

Force $Z_h(t) = \int^M h_z$ имеет смысл, Z_h росло с h_z росом чаго head-to head DW



ие есть "steady solution". Это new DW position at given $h_z = const$.

Существование "steady" solution $\dot{Z} = const$, $\ddot{Z} = 0$ означает $\omega_G^2 Z$ - член, если $\omega_G^2 = 0$, то steady motion $\dot{Z} = -\frac{G_S^2}{\varkappa D_{zz}}(-1)\frac{\varkappa \int^M}{G_S^2} h_z = \frac{\int^M}{D_{zz}} h_z$, $v_{DW} = \frac{\int^M}{D_{zz}} h_z$ наконец это Walker equation, mobility $\sim \frac{1}{D_{zz}}$, $\sim 1/\alpha_G$.

→ Выводы: 1) steady solution $\dot{Z} = 0$ для BP - DW position не существует;

2) $\varkappa_z < 0$ stiffness (и вероятно $\varkappa > 0$) задает hyperbolic gyrotropic BP dynamics

В каком D_{zz}? Когда $h_z = 0$ тогда $\omega_z = -\frac{\varkappa D_{zz}}{G_S^2} = \frac{\varkappa |D_{zz}|}{G_S^2}$, $\ddot{Z} + \omega_z \dot{Z} + \omega_G^2 Z(t) = 0$

$Z(t) \sim e^{i\omega t}$ тогда $-\omega^2 + i\omega\omega_z + \omega_G^2 = 0$, $\omega^2 - i\omega_z\omega - \omega_G^2 = 0$ кв. ур.

$2\omega_{1,2} = i\omega_z \pm \sqrt{-\omega_z^2 - 4(-\omega_G^2)} = i\omega_z \pm \sqrt{4\omega_G^2 - \omega_z^2}$. $\omega_z = \frac{\varkappa |D_{zz}|}{G_S^2}$, $\omega_G^2 = \frac{\varkappa \varkappa_z}{G_S^2}$.

1) $\omega_G^2 < 0$, $\varkappa_z < 0$, $\varkappa > 0$, $2\omega_{1,2} = i\omega_z \pm \sqrt{-4|\omega_G|^2 - \omega_z^2} = \pm i\omega_z \pm i\sqrt{4|\omega_G|^2 + \omega_z^2}$

$2\omega_{1,2} = i\left[\omega_z \pm \sqrt{4|\omega_G|^2 + \omega_z^2}\right]$, $\omega_1 = \frac{1}{2}i\left[\omega_z + \sqrt{4|\omega_G|^2 + \omega_z^2}\right] = i\delta_1$, $\delta_1 > 0$

$Z(t)e^{i\omega_1 t} = e^{i(i\delta_1)t} = e^{-\delta_1 t}$. $\omega_2 = \frac{1}{2}i\left[\omega_z - \sqrt{4|\omega_G|^2 + \omega_z^2}\right] = -i\delta_2$, $\delta_2 > 0$

и $Z_2(t) \sim e^{i\omega_2 t} = e^{i(-i\delta_2)t} = e^{\delta_2 t}$. $\omega_2 > 0$ - damped oscillator

2) $\omega_G^2 > 0$, $\varkappa_z < 0$, $\varkappa < 0$, бывает на ином кубике.

$\ddot{Z} + \omega_z \dot{Z} + \omega_G^2 Z(t) = 0$ имеет реш. $Z(t) = e^{-\frac{1}{2}\omega_z t}\cos(\omega_G t + \varphi)$

если $\omega_G^2 < 0$, $\omega_G = i\omega_0$, $\cos(\omega_G t + \varphi) = \cos(\omega_G t)\cos\varphi - \sin(\omega_G t)\sin\varphi =$

$= \cos(i\omega_0 t)\cos\varphi - \sin(i\omega_0 t)\sin\varphi = \operatorname{ch}(\omega_0 t)\cos\varphi - i\operatorname{sh}(\omega_0 t)\sin\varphi$, раскрывая

с t-росом гасло $\sim e^{\omega_0 t}$, $At\ t \to \infty$ $Z(t) \sim e^{-\frac{1}{2}\omega_z} e^{\omega_0 t} \sim e^{-(\frac{1}{2}\omega_z - \omega_0)t}$,

$Z(t), t \to \infty$ затухает при $\omega_z > 2\omega_0 = 2|\omega_G|$, т.е. hyperbolic regim BP

dynamics исчезает at large damping α, при $\omega_z > 2|\omega_G|$, $\frac{\varkappa|D_{zz}|}{G_S^2} > 2\frac{\varkappa|\varkappa_z|}{G_S^2}$,

$|D_{zz}| > 2|\varkappa_z|$, BP motion затухает равномерно как BP в обычном

до нанополя edge $S = R$.

Общее summary Осталось: $\varkappa_z < 0$, нет steady motion $\dot{Z} = const$ для BP.

(27/VII) УД для BP имеет $Z(t)$: $\ddot{Z} + \omega_z \dot{Z} + \omega_G^2 Z = 0$, $\omega_z = \frac{\varkappa|D_{zz}|}{G_S^2}$, $\omega_G^2 = \frac{\varkappa \varkappa_z}{G_S^2}$

Его решение $Z(t) = Ae^{-\frac{1}{2}\omega_z t}\cos(\omega_G t + \varphi_0)$. $Z(0) = A\cos\varphi_0$. Если нет разбега как

6/VIII, $\dot{Z}(0) = 0$, $Z(0) = 0$, то $\varphi_0 = \pi/2$. $Z(t) = Ae^{-\frac{1}{2}\omega_z t}\left[\cos(\omega_G t)\cos\varphi_0 - \sin(\omega_G t)\sin\varphi_0\right] =$

$= (\mp 1)Ae^{-\frac{1}{2}\omega_z t}\sin(\omega_G t)$. @ При $\varkappa_z < 0$ и $\varkappa > 0$, $\omega_G^2 < 0$, $\omega_G = i\omega_0$ pure imaginary

$Z(t) = \mp Ae^{-\frac{1}{2}\omega_z t}\sin(i\omega_0 t) = (\mp i)Ae^{-\frac{1}{2}\omega_z t}\operatorname{sh}(\omega_0 t)$, $\omega_0 = \Gamma$, $Z(t) = Ae^{-\frac{1}{2}\omega_z t}\operatorname{sh}(\Gamma t)$, это

обобщенное решение 6/VIII на случаи $\omega_z \neq 0$, $D_{zz} \neq 0$, non-zero damping $\hat{D}$ (hyperbolic solution)

⑥ Если $\varkappa < 0$ ($\varkappa_z < 0$), то $\omega_G^2 > 0$, ω_G is real. $Z(t) = Ae^{\frac{1}{2}|\omega_z|t}\sin(\omega_G t)$ - ОМ

$\omega_z < 0$

oscillation Вокруг $\max W(\vec{x})$ и $\vec{X} = 0$, с убыванием $Z(t)$, $X_0(t)$ аналогично y.

// Напом же, что это знак. $W(\vec{x}) = \frac{1}{2}\varkappa(X^2 + Y^2) + \frac{1}{2}\varkappa_z Z^2$.

Как найти $\vec{m}$ - components при $X \neq 0$ и/или $Y \neq 0$? См. 15/II. (S, φ, z) coord.

$m_x = m_S\cos\varphi - m_\varphi\sin\varphi$, $m_y = m_S\sin\varphi + m_\varphi\cos\varphi$. Для azimuthally symmetric

BP (Thiaville ansatz, наш случай), $m_S = 0$ и $m_x = -m_\varphi\sin\varphi$, $m_y = m_\varphi\cos\varphi$.

$\vec{m}(\theta, \Phi)$: $m_x + im_y = \sin\theta e^{i\Phi}$, $m_z = \cos\theta$, $\operatorname{tg}\Phi = \frac{m_y}{m_x}$. Если $m_S = 0$ - ansatz. то

$\operatorname{tg}\Phi = \frac{m_S\cos\varphi}{-m_S\sin\varphi} = -\frac{\cos\varphi}{\sin\varphi} = -\frac{x}{y}$, $\Phi(x,y,z) = \operatorname{arctg}(-\frac{x}{y})$, $(\nabla\Phi)^2 = \frac{1}{S^2}$ как для 2D

вортекса.

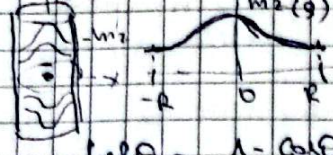
$\operatorname{arctg}\left(\frac{1}{u}\right) = \begin{cases} \pi/2 - \operatorname{arctg}(u), & u>0 \\ -\frac{\pi}{2} - \operatorname{arctg}(u), & u<0 \end{cases}$, $\operatorname{arctg}\left(\frac{1}{u}\right) = \operatorname{sign}(u)\cdot\frac{\pi}{2} - \operatorname{arctg}(u)$.

$\Phi(x,y) = \operatorname{arctg}\left(-\frac{x}{y}\right) = \left|-\frac{x}{y} = \frac{1}{u}\right| = \operatorname{sign}\left(\frac{y}{x}\right)\frac{\pi}{2} - \operatorname{arctg}\left(-\frac{y}{x}\right) = -\operatorname{sign}\left(\frac{y}{x}\right)\frac{\pi}{2} + \operatorname{arctg}\left(\frac{y}{x}\right)$

$\Phi(x,y) = \varphi - \operatorname{sign}\left(\frac{y}{x}\right)\frac{\pi}{2}$. $\operatorname{arctg}\left(\frac{y}{x}\right) = \operatorname{arctg}(\operatorname{tg}\varphi) = \varphi + \pi n$, $n\in\mathbb{Z}$, целое.

$\Phi(x,y) = \varphi + C\frac{\pi}{2}$, $C = \pm 1$ - chirality, т.е. ВР это 2D-Vortex for any $z=\text{const}$, совпадает $\Phi(x,y)$.

Но однако $m_z = \cos\theta = \frac{z}{\sqrt{z^2+s^2}}\frac{R^2}{(R^2+s^2)} = \text{odd } f(z)$, это не 2D vortex.



$m_z(s)$ - profile at $z=\text{const}$ ($\neq 0$), vortex, нужно $\sigma\neq 0$

$\zeta = x+iy$, $\vec{m}(\zeta,\bar{\zeta})$, $w(\zeta,\bar{\zeta}) = \operatorname{tg}\frac{\theta}{2}e^{i\Phi}$, $\theta = \theta(\zeta,\bar{\zeta})$, $\Phi = \Phi(\zeta)$

$\operatorname{tg}^2\frac{\theta}{2} = \frac{1-\cos\theta}{1+\cos\theta}$, Thiaville ansatz, $\cos\theta = \frac{z}{\sqrt{z^2+s^2}}f(s)$

$\operatorname{tg}^2\frac{\theta}{2} = \frac{1-\frac{z}{\sqrt{z^2+s^2}}f(s)}{1+\frac{z}{\sqrt{z^2+s^2}}f(s)} = F(z,s^2)$, $s^2 = |\zeta|^2 = x^2+y^2$, $w(\zeta,\bar{\zeta},z)$ не аналитическая.

$w(\zeta,\bar{\zeta},z) = \operatorname{tg}\frac{\theta}{2}(\zeta,\bar{\zeta},z)e^{i\Phi(\zeta)}$

$m_s + im_\varphi = (m_x+im_y)e^{-i\varphi} = (m_x+im_y)(\cos\varphi - i\sin\varphi) = m_x\cos\varphi + m_y\sin\varphi + i(m_y\cos\varphi - m_x\sin\varphi)$

$m_s + im_\varphi = \sin\theta e^{i(\Phi-\varphi)} = \sin\theta e^{iC\pi/2}$, $m_s = 0$, $m_\varphi = C\sin\theta$, для $\vec{X}=0$

Для $X,Y\neq 0$ $\varphi = \overset{\text{arc}}{\operatorname{tg}}\left(\frac{y}{x}\right) \to \operatorname{arctg}\left(\frac{y-Y}{x-X}\right)$, $m_s = \operatorname{Re}\sin\theta e^{i(\Phi-\varphi)}$

$\Phi-\varphi = \operatorname{arctg}\left(\frac{y-Y}{x-X}\right) - \operatorname{arctg}\left(\frac{y}{x}\right)$. Теперь для displaced ВР, $m_s\neq 0$, сохраняется side surface charges in nanowire. $\sin\theta$, $\theta = \theta(s,z)$, $s^2 = x^2+y^2$

надо $\to s'^2 = (x-X)^2 + (y-Y)^2$ within rigid model.

$m_s = \sin\theta(x,X,y,Y,z)\operatorname{Re}e^{i(\Phi-\varphi)}$ $\Phi-\varphi = F(x,y,X,Y) = \operatorname{arctg}\left(\frac{y-Y}{x-X}\right) - \operatorname{arctg}\left(\frac{y}{x}\right) + C\frac{\pi}{2}$

$m_\varphi = \sin\theta\operatorname{Im}e^{i(\Phi-\varphi)}$, $m_z = \cos\theta(x,y,X,Y)$

Вывод: $\vec{m}(s,z)$ of displaced ВР in xy-plane, (m_s, m_φ, m_z) находимо через, выражая через $\Theta(x,y,X,Y)$. Thiaville'93 использовали $\log\cos\theta(x,y)$ конформно, более чем через разумно. Rigid model $x\to(x-X)$, $y\to(y-Y)$ в $\vec{m}(x,y,z)$, или в $\theta_0(x,y,z)$, $\Phi_0(x,y)$. $\theta(x,y,X,Y) = \theta_0(x-X, y-Y)$, $\Phi_{(xy)} = \Phi_0\begin{pmatrix}x-X\\ y-Y\end{pmatrix}$

Все сводится к $\theta_0(x,y,z)$, т.к. $\Phi_0(xy) = \varphi + C\frac{\pi}{2}$ известна (из $m_\varphi = 0$).

$\theta_0(x,y,z) = \theta_0(s,z)$ axial symmetry.

Две работы: 1) ВР-cyl. wire statics (eq 4) 2) ВР-DW cyl wire dynamics $\vec{G}\neq 0$

Other BPLS project BP statics and dynamics.

Task 4. Classification of spin excitation spectra of 3D mag. topological solitons.

28/XII Новая аналит.: $\varkappa_z$ (exch) и $\varkappa_m^{face}$. Инвариант $\varkappa_z$ (volume magn.) $= \varkappa_m^{vol}$

Hubert-Schäfer: $(\nabla\vec{m})^2 = \left(\frac{\partial m_\varphi}{\partial\varrho}\right)^2 + \left(\frac{\partial m_z}{\partial s}\right)^2 + \left(\frac{\partial m_\varphi}{\partial z}\right)^2 + \left(\frac{\partial m_z}{\partial z}\right)^2 + \frac{m_s^2}{s^2}$ или

$\vec{m} = \vec{m}(s,z)$ - axial sym. and $m_s = 0$, $m_\varphi = \sqrt{1-m_z^2}$, $\left(\frac{\partial m_\varphi}{\partial s}\right)^2 + \left(\frac{\partial m_z}{\partial z}\right)^2 = \frac{1}{m_\varphi^2}\left(\frac{\partial m_z}{\partial s}\right)^2$

$\left(\frac{\partial m_\varphi}{\partial z}\right)^2 + \left(\frac{\partial m_z}{\partial z}\right)^2 = \frac{1}{m_\varphi^2}\left(\frac{\partial m_z}{\partial z}\right)^2$, см 26/XII. $(\nabla\vec{m})^2 = \frac{1}{m_\varphi^2}\left[\left(\frac{\partial m_z}{\partial s}\right)^2 + \left(\frac{\partial m_z}{\partial z}\right)^2\right] + \frac{m_\varphi^2}{s^2}$

$W_{ex} = A\int dV(\nabla\vec{m})^2 = A\int dV\left[\frac{1}{m_\varphi^2}(\nabla m_z)^2 + \frac{m_\varphi^2}{s^2}\right]$, т.к. все W_{ex} в конечном 16/II

в координатах $\vec{m}(\Theta,\Phi)$, $W_{ex} = A\int dV\left[(\nabla\Theta)^2 + \sin^2\Theta(\nabla\Phi)^2\right]$ и $\Phi = -\operatorname{arctg}\left(\frac{x}{y}\right)$, $(\nabla\Phi)^2 = \frac{1}{s^2}$.

$m_z = \cos\Theta$.

Пр. $\pi R_s^2 V$, $W_{ex}(Z) = \left(\frac{L_e}{R}\right)^2\int_{-a-Z}^{a-Z}dz\int_0^1 ds\, z\, W_{ex}(z,r)$, $W_{ex}(z,r) = \frac{1}{m_\varphi^2}(\nabla m_z)^2 + \frac{m_\varphi^2}{r^2}$

Пр. $E(Z) = \left(\frac{L_e}{R}\right)^2\int_0^1 ds\, z\, W_{ex}(z,r)$, $W(Z) = \int_{a-Z}^{a-Z}dz\, E(z)$, $a = Y_R$.

$\frac{\partial W_{ex}}{\partial Z} = (-1)E(a-Z) - (-1)E(-a-Z) = E(-a-Z) - E(a-Z)$. $E(-a-Z) = E(-a) + \frac{\partial E}{\partial z}\Big|_{z=-a}(-Z)$

$E(a-Z) = E(a) + \frac{\partial E}{\partial z}\Big|_{z=a}(-Z)$, $\frac{\partial W_{ex}}{\partial Z} = E(-a) - E(a) + Z\left[\frac{\partial E}{\partial z}\Big|_{z=a} - \frac{\partial E}{\partial z}\Big|_{z=-a}\right] = \varkappa_{ex}Z$

Escaneado con CamScanner

$\chi_{ex}^2 = \frac{\partial E(z)}{\partial z}\Big|_{z=a} - \frac{\partial E(z)}{\partial z}\Big|_{z=-a}$, $E(z) = \left(\frac{L\epsilon}{R}\right)^2 \int_0^1 dr\, r\, W_{ex}(z,r)$. $\frac{L\epsilon}{R} = l_\epsilon$

$W_{ex}(z,r) = \frac{1}{(1+m_z^2)^2}\left[\left(\frac{\partial m_z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial m_z}{\partial z}\right)^2\right] + \frac{(1-m_z^2)}{r^2}$, $m_z(z,\beta) = \frac{\beta z}{\sqrt{\beta^2 z^2 + r^2}}$, $t = \frac{r}{R}$, $\zeta = \frac{z}{R}$

$\frac{\partial E(z)}{\partial z} = l_\epsilon^2 \int_0^1 dr\, r \frac{\partial W_{ex}(z,r)}{\partial z}$, $\frac{\partial W_{ex}}{\partial z} = \frac{(-1)(-2)m_z}{(1-m_z^2)^2}\frac{\partial m_z}{\partial z}\left[(\nabla m_z)^2\right] + \frac{1}{(1-m_z^2)}\left[2\frac{\partial m_z}{\partial z}\frac{\partial m_z}{\partial r\partial z} + 2\frac{1}{r^2}\left(\frac{\partial m_z}{\partial r}\cdot\frac{\partial^2 m_z}{\partial z^2}\right)\right] + \frac{1}{r^2}(-2)m_z\frac{\partial m_z}{\partial z} = \frac{2m_z}{(1-m_z^2)^2}\frac{\partial m_z}{\partial z}(\nabla m_z)^2 + \frac{2}{(1-m_z^2)}\left[\frac{\partial m_z}{\partial r}\frac{\partial^2 m_z}{\partial r\partial z} + \frac{1}{r^2}\frac{\partial m_z}{\partial z}\frac{\partial^2 m_z}{\partial z^2}\right] - \frac{2m_z}{r^2}\frac{\partial m_z}{\partial z}$, $\frac{\partial E}{\partial z}\Big|_{z=a} - \frac{\partial E}{\partial z}\Big|_{z=-a} = \text{odd part}\left(\frac{\partial E}{\partial z}\right)\cdot 2\Big|_{z=a=1/2}$

MCad зачем считает χ_e^2. $\chi_e^2 < 0$ и $\chi_0^2 \sim 10^{-4}$ in reduced units. $Z = Z/L$, $W_{ex}/4\pi M_s^2 V$

$\chi_m^{vol} < 0$, $\chi_m^{vol} \sim -0{,}01$ для typical $t = 0{,}1$ и $\beta = 10$.

Аналитически найти даже самое простое χ_{ex}^2 сложно. Сравнить MCad-simulated.

29/XII) Перевел Арамбуру 32,70€ × 12 = 390,40€ за 12 мес 2022. Zaiman PP-DW.

Найден явно μ - coeff в $W_H(z) = \mu h_z z$ in cole. 25/XII.

$N\left(\frac{z}{Z}\right) = \int_0^1 dr\, r \frac{1}{(t^2+r^2)}\sqrt{(z-Z)^2 + \frac{1}{\beta^2}r^2}\Big]_{z=-1/2}^{z=1/2}$, $\sqrt{(z-Z)^2+x^2} = \sqrt{z^2 - 2zZ + Z^2 + x^2} =$

$= \sqrt{z^2+x^2}\sqrt{1 + \frac{1}{(z^2+x^2)}\left[-2zZ + Z^2\right]} = \sqrt{(\cdot)}\left[1 + \frac{1}{2}\frac{1}{(x^2+z^2)}(-2zZ+Z^2) - \frac{1}{8}\frac{1}{(x^2+z^2)^2}4z^2Z^2\right]$

$N(z) = \int_0^1 dr\, r \frac{1}{(t^2+r^2)}\sqrt{x^2+z^2}\left[1 + \frac{1}{2}\frac{1}{(x^2+z^2)}(-2zZ+Z^2) - \frac{1}{2}\frac{zZ^2}{(x^2+z^2)^2}\right]_{z=-1/2}^{z=1/2} =$

$= \int_0^1 dr\, r \frac{1}{(t^2+r^2)}\sqrt{x^2+z^2}(+1)\frac{zZ}{(x^2+z^2)}\Big|_{z=-1/2}^{z=1/2} = (-1)\int_0^1 dr\, r \frac{1}{(t^2+r^2)}\frac{1}{\sqrt{x^2+1/4}}Z$, $x^2 = r^2/\beta^2$.

$N(Z) = -Z\int_0^1 dr\, r \frac{1}{(t^2+r^2)}\frac{1}{\sqrt{1/4 + \frac{r^2}{\beta^2}}} = \Big|r^2 = u\Big| = -Z\frac{1}{2}\int_0^1 du \frac{1}{(t^2+u)}\frac{1}{\sqrt{\frac{1}{4}+\frac{1}{\beta^2}u}} =$

$= -\frac{1}{2}\beta Z\int_0^1 du \frac{1}{(t^2+u)}\frac{1}{\sqrt{\frac{1}{4}\beta^2+u}}$, и $u \le 1$, $\frac{4u}{\beta^2} \le \frac{1}{\beta^2} \ll 1$, т.к. $\beta = 10 \div 20$ и больше

$\frac{1}{\sqrt{\frac{1}{4}\beta^2+u}} = \frac{1}{(\beta/2)}\frac{1}{\sqrt{1+\frac{4u}{\beta^2}}} = \frac{2}{\beta}\left(1 - \frac{2u}{\beta^2} + O\left(\frac{1}{\beta^4}\right)\right)$

$N(Z) = -\frac{1}{2}\beta Z\int_0^1 du \frac{1}{(t^2+u)}\frac{2}{\beta}\left(1-\frac{2u}{\beta^2}\right) = -Z\int_0^1 du \frac{1}{(t^2+u)}\left(1-\frac{2u}{\beta^2}\right) = -Z\left[\int_0^1 du \frac{1}{t^2+u} - \frac{2}{\beta^2}\int du \frac{u}{t^2+u}\right] = -Z\left[\ln(t^2+u)\Big|_0^1 - \frac{2}{\beta^2}\int du \frac{u}{t^2+u}\right]$

$\int_0^1 du \frac{u}{t^2+u} = \int du \frac{t^2+u-t^2}{t^2+u} = \int du + t^2\int du \frac{1}{t^2+u} = \left[u - t^2\ln(t^2+u)\right]_0^1 =$

$= 1 - t^2\ln(t^2+1) + t^2\ln t^2 = 1 + t^2\ln\frac{t^2}{t^2+1} = 1 - t^2\ln\left(1+\frac{1}{t^2}\right)$

$N(Z) = -Z\left[\ln\left(1+\frac{1}{t^2}\right) - \frac{2}{\beta^2}\left\{1 - t^2\ln\left(1+\frac{1}{t^2}\right)\right\}\right] \approx -Z\ln\left(1+\frac{1}{t^2}\right)$

$W_H(Z) = -8\pi M_s^2 V h_z t^2 N(Z) = 8\pi M_s^2 V h_z t^2 \ln\left(1+\frac{1}{t^2}\right)Z = \mu h_z Z$, $h_z = \frac{H_z}{4\pi M_s}$

$\mu = 8\pi M_s^2 V t^2 \ln\left(1+\frac{1}{t^2}\right) > 0$, $\beta \gg 1$. Энергия tail-to-tail DW. Две head-to-head

DW $W_H(Z) = -\mu h_z Z$. $0 \to 0 \to 1 \to h_z$ $W_H(Z) = \mp\mu h_z Z$, (- head-to-head DW, + tail-to-tail DW)

Если бы было steady state DW motion то (26/XII) $\dot{Z} = \frac{\mu}{|D_{zz}|}h_z$.

$D_{zz} = -\alpha\frac{M_s}{\gamma}\int_0^R d\rho\,\rho\int_0^{2\pi}d\varphi\int_{-L/2}^{L/2}dz\left(\frac{\partial\vec{m}}{\partial z}\right)^2 = -\alpha\frac{M_s}{\gamma}R^2 2\pi\frac{1}{L}\int_{-L/2}^{L/2}dz\left(\frac{\partial\vec{m}}{\partial z}\right)^2 =$

$= -\alpha\frac{M_s}{\gamma}2V\frac{1}{L^2}\int_0^1 dr\, r\int_{-L/2}^{L/2}\left(\frac{\partial\vec{m}}{\partial z}\right)^2 \approx -\alpha\frac{M_s}{\gamma}\frac{2V}{L^2}\cdot\frac{2L}{L} = \frac{\mu}{|D_{zz}|}h_z = \frac{8\pi M_s V}{\alpha\frac{M_s}{\gamma}2\frac{V}{L^2}}h_z =$

$= \frac{8\pi}{2\alpha}\gamma M_s L^2 h_z$, $\dot{Z} = \frac{4\pi}{\alpha}\gamma M_s L$, $[\dot{Z}] = \frac{1}{s}L$ - линейно, $\dot{Z} = \frac{\omega_{in}}{\alpha} = 60\,GHz\cdot 2\pi\frac{1}{\alpha} =$

$= \frac{2\pi}{10^{-2}}60\cdot 10^9\frac{1}{s} = 3{,}6\cdot 10^9\frac{10^2}{10^{-2}} \approx 3{,}6\cdot 10^{13}\frac{1}{s}$, $v_{DW} \approx \dot{Z}L = 3{,}6\cdot 10^{13}\frac{1}{s}\cdot 1\,\mu m =$

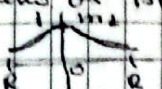
$= 3{,}6\cdot 10^{13}\frac{1}{s}\cdot 10^{-6}\,m = 3{,}6\cdot 10^7\,m/s$ - giant velocity, другая какая-то.

Нужен размер ячейки и D_{zz}-value.

Escaneado con CamScanner

30/VII Thiaville ansatz $m_z = \frac{z}{\sqrt{z^2+s^2}} \frac{b^2}{(b^2+s^2)}$, $m_s = 0$ не с нами для и хорошо.

При $|z| \gg s$ ($s \in [0, R]$), далеко от ВР - ДВ located at $z=0$ plane,

$m_z(z, s) = \frac{z}{|z|} \frac{b^2}{(b^2+s^2)}$  $m_z(s) = \frac{b^2}{(b^2+s^2)}$ - profile с $m_z(0) = 1$,

но $m_z(R) = \frac{b^2}{b^2+R^2} \ll 1$. Нужно это в domain с $m_z(r) = 1$? $m_z(s) = \frac{b^2}{(b^2+s^2)}$ Thiav.

28/VIII 20 Замечание, что $m_z = \cos\theta = p\cos\varphi = p\frac{z}{r}$, $\vec{r}(r, \varphi, \theta)$ - сферич. коорд.

Generalized Feldkeller's ansatz of BP $\Rightarrow \theta(r) = p\varphi + \frac{\pi}{2}(1-p)$, $\Phi(\vec{r}) = \frac{\pi}{2}\psi + \gamma(\vec{r})$

[$\vec{m}(\theta, \Phi)$, θ, Φ spherical angles]:

$m_s = \sin\theta\cos\gamma$, $m_\varphi = \sin\theta\sin\gamma$, $m_z = p\cos\varphi$, $\cos\theta = \frac{z}{r}$.

Thiaville 93 ansatz $m_s = 0$, $m_z = \frac{z}{r}\frac{b^2}{(b^2+s^2)}$ - 10/IX 21 г. с нами, что $m_z(\text{Feldkeller}) = p\frac{z}{\sqrt{z^2+s^2}}$ хорошо на $m_z(\text{Thiaville}) = \frac{z}{r}\frac{b^2}{(b^2+s^2)}$, $r^2 = z^2+s^2$.

Vortex BP by Feldkeller $\gamma = C\frac{\pi}{2}$, $m_s = 0$, $m_\varphi = C\sin\varphi$, $m_z = p\cos\varphi = p\frac{z}{r}$

Условие $m_s = 0$ хорошее для цилиндра, поэтому в частности $\gamma = C\frac{\pi}{2}$ будет

Feldkeller ansatz $\leftarrow$ $m_z = p\frac{z}{\sqrt{z^2+s^2}}$ Feldkeller, вообще говоря для цилиндрич. wire.

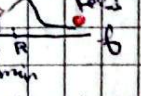
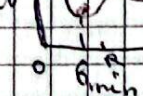
$m_z(\text{Feld})$ при $|z| \gg s$, $m_z(s, z) = p\frac{z}{|z|}$  $m_z(s) = \pm 1$, как

у домена обе in domains, far from $z=0$ BP - DW plane.

$m_z(\text{Thiaville})$ отличается от $m_z(\text{Feld})$ by "BP - radius" parameter b,

которая надо найти из $W(b) \to \min$ $b \approx 10$ nm при $R = 30$ nm wire.

Evident, $m_z(\text{Thiaville}) \to m_z(\text{Feld})$ at $b \to \infty$, поэтому надо сравнить

$W(b_{min})$ и $W(b \to \infty)$. W  или W  (?)

с ненулевой $W(b)$.

$\operatorname{div}\vec{m} = \frac{\partial m_z}{\partial z}$ дает m_s, $m_z(s, \varphi)$ any ansatz.

Feld $m_z = p\frac{z}{\sqrt{z^2+s^2}}$, $\frac{\partial m_z}{\partial z} = \frac{1}{\sqrt{z^2+s^2}} + z\frac{(-1)}{2(\sqrt{z^2+s^2})^3}2z = \frac{1}{(z^2+s^2)^{3/2}}\left(z^2+s^2-z^2\right) = \frac{s^2}{(z^2+s^2)^{3/2}}$

В расчете W_m^{vol} (16/II) надо $J(k, z) = \int_0^1 dr\, r\, J_0(kr)\frac{r^2}{(r^2+t^2)}\frac{1}{(\beta^2 z^2 + r^2)^{3/2}}$ при $\beta = 10$, $t = \frac{b}{R} \gg 1$

дает поправку к Feld-ansatz, $J(k, z) = \frac{1}{t^2}\int_0^1 dr\, r^3 J_0(kr)\frac{1}{(\beta^2 z^2 + r^2)^{3/2}} = \frac{1}{t^2}L(k, z)$

Feld - $W_m^{vol} = 2\pi M_s^2 V\beta \int_{-1/2}^{1/2} dz \int_{-1/2}^{1/2} dz' \int_0^\infty dk\, e^{-\beta k|z-z'|} L(k, z) L(k, z')$, $L(k, z) = \int_0^1 dr \frac{r^3 J_0(kr)}{(\beta^2 z^2 + r^2)^{3/2}}$

MathCad „Bloch point - energy - wise, medium" gap ($\beta = 10, t = 0.1$) $W_m^{vol}(\text{Thia}) = 2.28\cdot 10^{-4}$,

и $W_m^{vol}(\text{Feld}) = 0.049$, $W_{ex}(\text{Thiav}) = 0.0602$, $W_{ex}(\text{Feld}) = 0.0102$ при $R = 30$ nm.

Face m. energy (16/II): $W_m^f = 4\pi^2 M_s^2 \int dk\, K_+^2(k)(1+e^{-k})$, $K_+(k) = \int ds\, s\, J_0(ks) m_z(s)$ $z = 1/2$

$K_+(k) = R^2 \int_0^1 dr\, r\, J_0(kr) m_z(r, z)_{z=1/2}$, $m_z = \frac{z}{\sqrt{z^2+s^2}} = \frac{Lz}{\sqrt{L^2z^2 + z^2R^2}} = \frac{\beta z}{\sqrt{\beta^2 z^2 + r^2}}$ (Feld)

$W_m^f(\beta) = 4\pi^2 M_s^2 \frac{1}{R}\int dk\, R^4 \left[\int_0^1 dr\, r\, J_0(kr)\frac{\beta z}{\sqrt{\beta^2 z^2 + r^2}}\right]^2 (1+e^{-\beta k})$

$4\pi^2 M_s^2 R^3 \beta^2 = 4\pi M_s^2 V\beta$, $W_m^f(\beta) = 4\pi M_s^2 V\beta \int_0^\infty dk \left[\int_0^1 dr\, r\, J_0(kr)\frac{z}{\sqrt{\beta^2 z^2 + r^2}}\right]^2_{z=1/2}(1+e^{-\beta k})$.

$W_m^{face}(\text{Thiav}) = 1.21\cdot 10^{-4}$, $W_m^{face}(\text{Feld}) = 0.044$ при $\beta = 10$, $t = 0.1$

Вывод: W_m^{vol}, W_m^f - Thiaville $\lesssim W_m^{vol}, W_m^f$ - Feld, а $W_{ex}(\text{thiav}) \gg W_{ex}(\text{Feld})$,

т.е. для малых R more, $W_{tot}(\text{Feld}) < W_{tot}(\text{thiav})$, для больших R $W_{tot}(\text{Feld}) > W_{tot}(\text{Thiaville})$. Feldkeller ansatz is exchange dominated, он

будет хорош при малых particle sizes, малых R of nanowire.

Thiav $m_z = \frac{z}{\sqrt{z^2+s^2}}\frac{b^2}{(b^2+s^2)}$, Feldkeller $m_z = \frac{z}{\sqrt{z^2+s^2}}$, $W_m^{vol} \sim \left(\frac{dm_z}{dz}\right)^2$, $W_m^{face} \sim (m_s)^2$,

т.к. в m_s и $\partial m_z/\partial z$ у Thiaville меньше, то $W_m^{vol}, W_m^f < W_m^v, W_m^{face}$ - Feldkeller.

Escaneado con CamScanner

Наиден gyrovector чрез Feldkeller's ansatz $\Phi(\vec{r}) = q\varphi + \gamma(\vec{r})$, $\gamma(\vec{r}) = C\frac{\pi}{2}$.

Ch. 15/II, gyrovector density $\vec{g}^* = -\sin\theta \nabla\theta \times \nabla\Phi$ (in M_s/γ units), $\vec{g}^* = \nabla m_z \times \nabla\Phi$

$\nabla\Phi = \hat{s}\frac{\partial\Phi}{\partial s} + \frac{1}{s}\hat{\varphi}\frac{\partial\Phi}{\partial\varphi} + \hat{z}\frac{\partial\Phi}{\partial z} = \hat{\varphi}\frac{1}{s}q$, $\nabla m_z = \hat{s}\frac{\partial m_z}{\partial s} + \frac{1}{s}\hat{\varphi}\frac{\partial m_z}{\partial\varphi} + \hat{z}\frac{\partial m_z}{\partial z}$

$m_z = p\frac{z}{\sqrt{z^2+s^2}}$ (Feld), $\frac{\partial m_z}{\partial s} = z\left(-\frac{1}{2}\right)(z^2+s^2)^{-3/2}2s = -\frac{zs}{(z^2+s^2)^{3/2}}$, $\frac{\partial m_z}{\partial z} = \frac{s^2}{(z^2+s^2)^{3/2}}$, $\frac{\partial m_z}{\partial\varphi}=0$

$\vec{g}^* = \nabla m_z \times \nabla\Phi = \begin{vmatrix} \hat{s} & \hat{\varphi} & \hat{z} \\ a_s & 0 & a_z \\ 0 & q/s & 0 \end{vmatrix} = \hat{z}\,a_s\frac{q}{s} - \hat{s}\frac{q}{s}a_z$, $\nabla m_z = \vec{a}$

$\vec{G} = \int dV\,\vec{g}^*$, $G_z = \int ds\,s\int d\varphi\int_{-L/2}^{L/2} dz\,\frac{q}{s}a_s = q\int ds\,2\pi\int_{-L/2}^{L/2} dz\,(-1)\frac{zs}{(z^2+s^2)^{3/2}} = 0$, как иначе от чертеж.

$G_s = \int dV\,g_s = \int ds\,s\int d\varphi\int_{-L/2}^{L/2} dz\,(-q)\frac{1}{s}\frac{s^2}{(z^2+s^2)^{3/2}} = -q2\pi\int_0^R ds\,s^2\int_{-L/2}^{L/2} dz\,\frac{1}{(z^2+s^2)^{3/2}}$

$G_s = \int ds\,s\int d\varphi\int dz\,(-q)\frac{1}{s}\frac{\partial m_z}{\partial z} = -2\pi q\int ds\int_{-L/2}^{L/2}\frac{\partial m_z}{\partial z} = -2\pi q\int_0^R ds\left[m_z\left(z=\frac{L}{2}\right) - m_z\left(z=-\frac{L}{2}\right)\right] =$

$= -4\pi q\int_0^R ds\,m_z(z=L/2) = -4\pi qp\int_0^R ds\,\frac{L/2}{\sqrt{(L/2)^2+s^2}} = -4\pi pqL\int_0^R \frac{ds}{\sqrt{(L/2)^2+s^2}} =$

$= -4\pi pq\frac{\beta}{2}\int_0^1 dr\,\frac{1}{\sqrt{\beta^2/4+r^2}}R = -4\pi pq\frac{\beta}{2}\cdot\ln\left[r+\sqrt{\beta^2/4+r^2}\right]\Big|_0^1 R =$

$= -4\pi qp\frac{\beta}{2}R\left[\ln\left(1+\sqrt{\beta^2/4+1}\right) - \ln\frac{\beta}{2}\right]$, $G_s = -4\pi pq\frac{\beta}{2}\operatorname{Arsh}\left(\frac{z}{\beta/2}\right)\Big|_0^1 =$

$= -4\pi pq\frac{\beta}{2}\operatorname{Arsh}\left(\frac{2}{\beta}\right)R$, $G_s = -4\pi pqR\left(\frac{\beta}{2}\right)\operatorname{Arsh}\left(\frac{2}{\beta}\right)$. feld

и $G_s(\beta\gg1) = -4\pi pqR$. Такова же G_s (с точн. до знака) $= 4\pi R$ получим 15/II чрез Thiaville ansatz и $t = b/R \gg 1$. $G_s(\beta\gg1) = -4\pi\frac{M_s}{\gamma}pqR$ чрез Feld, ansatz наноwire.

$\vec{G}$ (thiaville) уточнен 3/IX и сменен знак.

$G_s(\text{thiav}) = -2\pi pL\left(\frac{M_s}{\gamma}\right)I_s(\beta,t)$, $I_s(\beta,t) = t^2\int_0^1 dr\,\frac{1}{\sqrt{\beta^2/4+r^2}}\frac{1}{(t^2+r^2)}$, $t = b/R$ BP size

$\beta\gg1$, $G_s(\text{thiav.}) = -4\pi\left(\frac{M_s}{\gamma}\right)\beta\arctg\left(\frac{1}{t}\right)$, если още и $t\gg1$, то $G_s = -4\pi M_s R$ - като в Feldkeller's ansatz $I_s(\beta,t) = \left|t\gg1\right| = \int_0^1 dr\,\frac{1}{\sqrt{\beta^2/4+r^2}} = \operatorname{Arsh}\left(\frac{2}{\beta}\right)$ γ

Важно: Feldkeller $m_z(z,s)$ дава $G_s\neq0$, $G_z=0$. $G_s(\text{thiav}) \xrightarrow[t\to\infty]{} G_s(\text{Feld})$, $t=\frac{b}{R}$

31/XII Спонсор "Cornerstone Market", Update CV Meer-Basque.

17/I **2023** BP gyrotropic dynamics in cylindrical nanowire.

Без работа transverse stiffness „xc" не обединяваме, за сравнение в ω_z, ω_c, h_z-зависимост. Сигнална 27/IX-22 г. чрез $X, Y \neq 0$ $m_s + im_\varphi = \sin\theta\,e^{i(\Phi-\varphi)}$, $m_z = \cos\theta$.

Within rigid BP model $\Phi - \varphi = \arctg\left(\frac{y-Y}{x-X}\right) - \arctg\left(\frac{y}{x}\right) + C\frac{\pi}{2}$.

$\Phi(x,y,X,Y) = \Phi_0(x-X, y-Y)$, $\Phi_0(x,y) = \varphi + C\frac{\pi}{2} = \arctg(y/x) + C\frac{\pi}{2}$

$\Theta(x,y,X,Y) = \Theta_0(x-X, y-Y)$, чрез $\theta_0(x,y,z) = \theta_0(s,z)$, $\Phi_0(x,y)$ - универсален axially symmetric BP ansatz в версии Thiaville или Feldkeller, etc.

$m_s = \sin\Theta\,\mathrm{Re}\,e^{i(\Phi-\varphi)} = \sin\theta\cos(\Phi-\varphi)$, $m_\varphi = \sin\theta\sin(\Phi-\varphi)$

$\cos(\Phi-\varphi) = \cos\left(\delta + C\frac{\pi}{2}\right) = \sin\delta\cdot\sin(C\pi/2)(-1) = -C\sin\delta$.

$\delta = \arctg\left(\frac{y-Y}{x-X}\right) - \arctg(y/x)$, чрез $X=0$, $W(\vec{x}) = \frac{1}{2}x(x^2+y^2)$, $x_x = x_y = x$

$\arctg\left(\frac{y-Y}{x}\right) = \arctg(y/x) + \frac{x}{x^2+(y-Y)^2}(-Y) + O(Y^2) = \varphi - Y\frac{1}{s}\cos\varphi + O(Y^2)$

$\delta = -\frac{Y}{s}\cos\varphi + O(Y^2)$ $\quad S$-side swing.

Side Surface magn. energy $W_m^S = \frac{M_s^2}{2}\int dS\int dS'\frac{(\vec{m}\cdot\vec{n})(\vec{m}'\cdot\vec{n}')}{|\vec{r}-\vec{r}'|} = \frac{1}{2}M_s^2\int dS\,dS'\frac{m_s m_s'}{|\vec{r}-\vec{r}'|}$

Escaneado con CamScanner

$m_\rho = m_\rho(\rho,\varphi,z)$, $m_\rho' = m_\rho(\rho',\varphi',z')$, $\int d\vec{S}(\dots) = R\int d\varphi \int dz (\dots)$ at $\rho = R$

$$W_m^s = \frac{1}{2} M_s^2 R^2 \cdot \int d\varphi \int dz \int d\varphi' \int dz' \frac{1}{|\vec{r}-\vec{r}'|} m_\rho(R,\varphi,z)\, m_\rho(R,\varphi',z') \sim y^2 \text{ t.k.}$$

$m_\rho = \sin\theta \cos(\Phi - \varphi) = -\zeta \sin\theta \sin\delta \approx -\zeta \sin\theta_0\, \delta$, $\delta = -\frac{y}{\rho}\cos\varphi + O(y^2)$

Dva W_m^s gaccatorno m_ρ ie linear no y approx., $m_\rho = \zeta \sin\theta_0 \frac{\cos\varphi}{\rho} y$, $\rho = R$.

$$W_m^s(y) = \frac{y^2}{2} M_s^2 \int_0^{2\pi} d\varphi \int_{-L/2}^{L/2} dz \int_0^{2\pi} d\varphi' \int_{-L/2}^{L/2} dz' \frac{\sin\theta_0 \sin\theta_0' \cos\varphi \cos\varphi'}{|\vec{r}-\vec{r}'|} = \frac{1}{2} \varkappa_m^s y^2$$

$$\underline{\varkappa_m^s} = M_s^2 \int d\varphi \int dz \int d\varphi' \int dz' \frac{\sin\theta_0 \sin\theta_0' \cos\varphi \cos\varphi'}{|\vec{r}-\vec{r}'|}, \quad \theta_0 = \theta_0(\rho,z)$$

$$J_\varphi = \int d\varphi \int d\varphi' \frac{\cos\varphi \cos\varphi'}{|\vec{r}-\vec{r}'|} = \int d\varphi \int d\varphi' \cos\varphi \cos\varphi' \int dk\, e^{-k|z-z'|} \sum_\mu J_\mu(k\rho) J_\mu(k\rho') e^{i\mu(\varphi-\varphi')}$$

$$\int_0^{2\pi} d\varphi \cos\varphi\, e^{i\mu\varphi} = \int d\varphi \frac{1}{2}\left(e^{i\varphi} + e^{-i\varphi}\right) e^{i\mu\varphi} = \frac{1}{2} 2\pi \left(\delta_{\mu,-1} + \delta_{\mu,1}\right)$$

$$J_\varphi = \int dk\, e^{-k|z-z'|} \sum_\mu J_\mu J_\mu' \pi^2 \left(\delta_{\mu,-1} + \delta_{\mu,1}\right)\left(\delta_{\mu,1} + \delta_{\mu,-1}\right) = \pi^2 \int dk\, e^{-k|z-z'|} \sum_\mu J_\mu J_\mu' \times$$

$$\times \left(\delta_{\mu,1}^2 + \delta_{\mu,-1}^2 + 2\delta_{\mu,1}\delta_{\mu,-1}\right) = \pi^2 \int dk\, e^{-k|z-z'|} \sum_\mu J_\mu J_\mu \left(\delta_{\mu,1} + \delta_{\mu,-1}\right) = 2\pi^2 \int dk\, e^{-k|z-z'|} J_1(k\rho) J_1(k\rho')$$

$\rho = \rho' = R$

$$\varkappa_m^s = M_s^2 \int dz \int dz' \sin\theta_0 \sin\theta_0' \cdot 2\pi^2 \int dk\, e^{-k|z-z'|} J_1(kR) J_1(kR)$$

$$\varkappa_m^s = 2\pi^2 M_s^2 \int dz \int dz' \sin\theta_0(R,z) \sin\theta_0(R,z') \int dk\, e^{-k|z-z'|} J_1^2(kR)$$

BP profile $\theta_0(\rho,z)$: $\cos\theta_0 = \frac{z}{\sqrt{z^2+\rho^2}}$ (Feld), $\cos\theta_0 = \frac{z}{\sqrt{z^2+\rho^2}} \frac{k^2}{(k^2+\rho^2)}$ (thiav)

$$\varkappa_m^s = 2\pi^2 M_s^2 L^2 \int_{-1/2}^{1/2} dz \int_{-1/2}^{1/2} dz' \sin\theta_0(1,z) \sin\theta_0(1,z') \frac{1}{R} \int dk\, e^{-\beta k|z-z'|} J_1^2(k)$$

$$\varkappa_m^s = 2\pi^2 M_s^2 L \beta \int dz \int dz' \sin\theta_0(1,z) \sin\theta_0(1,z') \int dk\, e^{-\beta k|z-z'|} J_1^2(k)$$

$\pi L = \pi L \frac{V}{\pi R^2 L} = \frac{V}{R^2}$, $W_m^s(y) = \frac{1}{2} \varkappa_m^s R^2 \left(\frac{y}{R}\right)^2$, если $\frac{y}{R} \to y$ (in units of R),

то $\varkappa_m^s R^2 \to \varkappa_m^s$, $\underline{\varkappa_m^s} = 2\pi M_s^2 V \beta \int dz \int dz' \sin\theta_0(1,z) \sin\theta_0(1,z') \int dk\, e^{-\beta k|z-z'|} J_1^2(k)$

$\cos\theta_0(r,z) = \frac{\beta z}{\sqrt{\beta^2 z^2 + r^2}}$ (Feld), $\cos\theta_0(r,z) = \frac{\beta z}{\sqrt{\beta^2 z^2 + r^2}} \frac{k^2}{(k^2+r^2)}$ (thiav), $r \to \frac{\rho}{R}$, $t = \frac{b}{R}$.

MatCad „Block_point-Stiffness.mcd" дал $\varkappa_m^s > 0$ (thiav / Feld) (!), $\varkappa_m^s$ (thiav) $= 0{,}457$ $\beta = 10$, $t = 0{,}1$

$\varkappa_m^s$ (Feld) $= 0{,}126$ $\beta = 10$

$\varkappa_z^{Vol} \sim -10^{-4}$, $\varkappa_z^{ef} \sim +10^{-4}$. поэтому $\varkappa_m^s > 0$ нужного знака по крайн. с $\varkappa_z < 0$.

(2/I) E-mails to Julian, Ruben, Oscana. Отдых дома 6 дней. New BP dyn. paper.

(3/I) Работа над Farkhad's manuscript (PRA subm.) BP/SV switching in nanowires. Writing new BP stability / gyrotropic dynamics paper.

УД для BP position $Z(t)$ along nanowire $\ddot{Z} + \omega_z \dot{Z} + \omega_G^2 Z = -\mu' h_z(t)$,

$\mu' = \varkappa\mu / G_s^2$ не имеет steady motion solution $\dot{Z} = \text{const}$. $\omega_G^2 Z$-term нужен.

Для $h_z(t) = h_0 e^{i\omega t}$, каким $Z(t)$ решение?

$\ddot{Z} + \omega_z \dot{Z} + \omega_G^2 Z = a e^{i\omega t}$, $a = -\mu' h_0$. Ищем $Z(t) = Z_0 e^{i\omega t}$ реш.

$-\omega^2 Z_0 + i\omega\omega_z Z + \omega_G^2 Z = a$. $Z(\omega) = \frac{1}{\left[-\omega^2 + i\omega\omega_z + \omega_G^2\right]} a$

GM resonance at $\omega \approx \omega_G$, если $\omega_G^2 > 0$. $\omega_G^2 = \varkappa\varkappa_z / G_s^2$ должно < 0,

т.к. $\varkappa_z < 0$, $\varkappa > 0$. $\omega_G^2 = -\Gamma^2$. $Z(\omega) = \frac{(-a)}{\omega^2 + \Gamma^2 - i\omega\omega_z}$. Надо $h_z(t)$ 

(4/I) Расчет damping $D_{\varphi\varphi}$, D_{zz} (26/III). Сл. лекция.

$D_{\varphi\varphi} = -dJ \int dV \left(\frac{\partial \vec{m}}{\partial \varphi} \cdot \frac{\partial \vec{m}}{\partial \varphi}\right)$, $D_{zz} = -dJ \int dV \left(\frac{\partial \vec{m}}{\partial z} \cdot \frac{\partial \vec{m}}{\partial z}\right)$, $J = M_s/\gamma$.

Cylindr. coordinates $\vec{m} = \hat{\rho} m_\rho + \hat{\varphi} m_\varphi + \hat{z} m_z$. $m_\rho = 0$ для Feld- и Thiaville ansatzes.

$\frac{\partial \hat{\rho}}{\partial \varphi} = \hat{\varphi}$, $\frac{\partial \hat{\varphi}}{\partial \varphi} = -\hat{\rho}$, $\frac{\partial \vec{m}}{\partial \varphi} = \frac{\partial}{\partial \varphi}\left[\hat{\varphi} m_\varphi + \hat{z} m_z\right] = \hat{\varphi} \frac{\partial m_\varphi}{\partial \varphi} + m_\varphi \frac{\partial \hat{\varphi}}{\partial \varphi} + \hat{z} \frac{\partial m_z}{\partial \varphi} =$

Escaneado con CamScanner

$= \hat{\varphi}\frac{\partial m_\varphi}{\partial s} + \hat{z}\frac{\partial m_z}{\partial s} + m_\varphi \frac{\partial \hat{\varphi}}{\partial s}$, $\frac{\partial \hat{\varphi}}{\partial s} = 0$? $\frac{\partial \vec{m}}{\partial \varphi} = \frac{\partial}{\partial \varphi}\left[\hat{\varphi} m_\varphi + \hat{z} m_z\right] = \hat{\varphi}\frac{\partial m_\varphi}{\partial \varphi}^{=0} +$

$+ m_\varphi \frac{\partial \hat{\varphi}}{\partial \varphi} + \hat{z}\frac{\partial m_z}{\partial \varphi}^{=0} = m_\varphi \frac{\partial \hat{\varphi}}{\partial \varphi} = -\hat{s}\, m_\varphi$ (!). $\left(\frac{\partial \vec{m}}{\partial \varphi}, \frac{\partial \vec{m}}{\partial \varphi}\right) = \left(-\hat{s} m_\varphi, (-\hat{s}) m_\varphi\right) =$

$= \hat{s}^2 m_\varphi^2 = m_\varphi^2$, tomee $\frac{\partial \vec{m}}{\partial x_\lambda}$, $\lambda = \varphi$, $\frac{\partial}{\partial x_\lambda} = \frac{1}{s}\frac{\partial}{\partial \varphi}$.

$\frac{1}{s}\frac{\partial m_\varphi}{\partial \varphi}(\hat{s} m_\varphi + \hat{z} m_z) = -\frac{1}{s}\hat{s}\, m_\varphi$, $\frac{1}{s}\frac{\partial \vec{m}}{\partial \varphi}\cdot\frac{1}{s}\frac{\partial \vec{m}}{\partial \varphi} = \frac{1}{s^2}\left(-\hat{s} m_\varphi \cdot (-\hat{s}) m_\varphi\right) = \frac{m_\varphi^2}{s^2}$

$D_{\varphi\varphi} = -\frac{dJ}{} \int dV \left(\frac{\partial \vec{m}}{\partial \varphi}\cdot\frac{\partial \vec{m}}{\partial \varphi}\right)\frac{1}{s^2} = -\frac{dJ}{}\int dV \frac{1}{s^2} m_\varphi^2 \neq 0$ (!)

$D_{s\varphi} = -\frac{dJ}{}\int dV \frac{\partial \vec{m}}{\partial s}\cdot\frac{1}{s}\frac{\partial \vec{m}}{\partial \varphi}$, $\frac{\partial \vec{m}}{\partial s}\cdot\frac{1}{s}\frac{\partial \vec{m}}{\partial \varphi} = \frac{1}{s}\left[\hat{s}\frac{\partial m_s}{\partial s} + \hat{z}\frac{\partial m_z}{\partial s} + \hat{\varphi}\frac{\partial m_\varphi}{\partial s}\right]\cdot$

$\cdot\left[\frac{\partial}{\partial \varphi}(\hat{s} m_s) + \frac{\partial m_z}{\partial \varphi}\hat{z}\right] = \frac{1}{s}\left[\hat{z}\frac{\partial m_z}{\partial s} + \hat{\varphi}\frac{\partial m_\varphi}{\partial s}\right]\cdot\left[\hat{\varphi}\frac{\partial m_\varphi}{\partial \varphi}^{=0} + m_\varphi\frac{\partial \hat{\varphi}}{\partial \varphi}\right] =$

$= \frac{1}{s}\left[\frac{\partial m_z}{\partial s}\hat{z} + \frac{\partial m_\varphi}{\partial s}\hat{\varphi}\right]\cdot\left[m_\varphi(-\hat{s})\right] = 0$, $D_{s\varphi} = 0$, $D_{\varphi s} = 0$.

$Tr(\hat{D}) = -\frac{dJ}{}\int dV\left[\left(\frac{\partial \vec{m}}{\partial s}\right)^2 + \frac{1}{s^2}m_\varphi^2 + \left(\frac{\partial \vec{m}}{\partial z}\right)^2\right]$, $\left(\frac{\partial \vec{m}}{\partial s}\right)^2 = \frac{1}{m_\varphi^2}\left(\frac{\partial m_\varphi}{\partial s}\right)^2$

$\left(\frac{\partial \vec{m}}{\partial z}\right)^2 = \left(\frac{\partial m_\varphi}{\partial z}\right)^2 + \left(\frac{\partial m_z}{\partial z}\right)^2 = \frac{m_\varphi^2}{m_\varphi^2}\left(\frac{\partial m_\varphi}{\partial z}\right)^2 + \left(\frac{\partial m_z}{\partial z}\right)^2 = \frac{1}{m_\varphi^2}\left(\frac{\partial m_\varphi}{\partial z}\right)^2$

$Tr(\hat{D}) = -\frac{dJ}{}\int dV\left[\frac{1}{m_\varphi^2}\left(\frac{\partial m_\varphi}{\partial s}\right)^2 + \frac{1}{m_\varphi^2}\left(\frac{\partial m_\varphi}{\partial z}\right)^2 + \frac{m_\varphi^2}{s^2}\right] = -\frac{dJ}{}\int dV\left[\frac{1}{m_\varphi^2}(\nabla m_\varphi)^2 + \frac{m_\varphi^2}{s^2}\right]$

$W_{ex} = A\int dV (\nabla \vec{m})^2 = A\int dV\left[\frac{1}{m_\varphi^2}(\nabla m_\varphi)^2 + \frac{m_\varphi^2}{s^2}\right]$, que $\vec{m}(s,z)$, $m_s = 0$ caso

Oralungo $Tr(\hat{D}) \sim W_{ex}$, icar u gousmo obrso.

$\rightarrow D_{ss} = -2\pi dJ\int ds\, s\int dz \frac{1}{m_\varphi^2}\left(\frac{\partial m_\varphi}{\partial s}\right)^2$, $D_{\varphi\varphi} = -2\pi dJ\int ds\, s\int dz \frac{m_\varphi^2}{s^2}$, $D_{zz} = -2\pi dJ\int ds\, s\, dz \cdot \frac{1}{m_\varphi^2}\left(\frac{\partial m_\varphi}{\partial z}\right)^2$

Ws $D_{\varphi\varphi} \neq 0$ thiele eq. a 26/11 ara. nago umlabuno. $\hat{G}\dot{\vec{X}} + \hat{D}\vec{X} = \frac{\partial W}{\partial \vec{X}}$.

$\hat{D}\dot{\vec{X}} = \begin{pmatrix} D_{ss} & 0 & 0 \\ 0 & D_{\varphi\varphi} & 0 \\ 0 & 0 & D_{zz}\end{pmatrix}\begin{pmatrix}\dot{X}_s \\ \dot{X}_\varphi \\ \dot{Z}\end{pmatrix} = \hat{s} D_{ss}\dot{X}_s + \hat{\varphi} D_{\varphi\varphi}\dot{X}_\varphi + \hat{z} D_{zz}\dot{Z}$.

$\hat{\varphi}: -G_s\dot{Z} + D_{\varphi\varphi}\dot{X}_\varphi = \varkappa X_\varphi$, $G_s\dot{X}_\varphi + D_{zz}\dot{Z} = \varkappa_z Z + \mu h_z$, $D_{ss}\dot{X}_s = \varkappa X_s$

$-G_s\dot{Z} + \frac{D_{\varphi\varphi}}{G_s}\left[\varkappa_z Z + \mu h_z - D_{zz}\dot{Z}\right] = \varkappa X_\varphi$, $-G_s\ddot{Z} + \frac{D_{\varphi\varphi}}{G_s}\left[\varkappa_z\dot{Z} + \mu\dot{h}_z - D_{zz}\ddot{Z}\right] =$

$= \varkappa\dot{X}_\varphi = \varkappa\frac{1}{G_s}\left[\varkappa_z Z + \mu h_z - D_{zz}\dot{Z}\right]$, $-\left(G_{ss} + \frac{D_{\varphi\varphi}D_{zz}}{G_s}\right)\ddot{Z} + \left(\frac{\varkappa_z D_{\varphi\varphi}}{G_s} + \frac{\varkappa D_{zz}}{G_s}\right)\dot{Z} +$

$+ \frac{D_{\varphi\varphi}}{G_s}\mu h_z - \frac{\varkappa\varkappa_z}{G_s}Z - \frac{\varkappa\mu}{G_s}h_z = 0$, $D_{\varphi\varphi}D_{zz}$ wyd² moreno wpuedpere.

$\left(1 + \frac{D_{\varphi\varphi}D_{zz}}{G_s^2}\right)\ddot{Z} - \frac{1}{G_s^2}\left(\varkappa_z D_{\varphi\varphi} + \varkappa D_{zz}\right)\dot{Z} - \frac{D_{\varphi\varphi}}{G_s^2}\mu\dot{h}_z + \frac{\varkappa\varkappa_z}{G_s^2}Z + \frac{\varkappa\mu}{G_s^2}h_z = 0$.

$\left(1 + \frac{D_{\varphi\varphi}D_{zz}}{G_s^2}\right)\ddot{Z} + \omega_\varkappa\dot{Z} + \omega_G^2 Z = \frac{D_{\varphi\varphi}}{G_s^2}\mu\dot{h}_z - \frac{\varkappa\mu}{G_s^2}h_z$, $\omega_\varkappa = -\frac{1}{G_s^2}\left(\varkappa_z D_{\varphi\varphi} + \varkappa D_{zz}\right)$

y rec $D_{\varphi\varphi} \neq 0$ ↑. znam $\omega_\varkappa$ ne ucen m.k. $-\frac{1}{G_s^2}\varkappa_z D_{\varphi\varphi} < 0$, $-\frac{1}{G_s^2}\varkappa D_{zz} > 0$.

$D^2/G_s^2 \ll 1$. $\ddot{Z} + \omega_\varkappa\dot{Z} + \omega_G^2 Z = \frac{\mu}{G_s^2}\left[D_{\varphi\varphi}\dot{h}_z - \varkappa h_z\right]$, $\omega_G^2 = \frac{\varkappa\varkappa_z}{G_s^2} < 0$

$\dot{h}_z(t) \sim \delta(t)$. $\varkappa h_z$ tuen moreno $\begin{cases} 0, & t<0 \\ h_0, & t\geq 0 \end{cases}$ u gegare nac. gcu. $\dot{Z}(0) = 0$.

$\ddot{Z} + \omega_\varkappa\dot{Z} + \omega_G^2 Z = a\delta(t)$, $\dot{h}_z = b\delta(t)$, $\frac{\mu}{G_s^2}D_{\varphi\varphi}\dot{h}_z = \frac{\mu D_{\varphi\varphi}}{G_s^2}b\delta(t) = a\delta(t)$

$Z(t)$ je Greenova funkcija za oscilatorsku jednačinu.

$\hat{L}G(t,t') = \delta(t-t')$, $\hat{L}$ je linearni dif. operator, $G(t,t') = G(t-t')$ ovaj $\hat{L}$ ima konst. koeficijente.

$\hat{L} = \frac{\partial^2}{\partial t^2} + 2\gamma\frac{\partial}{\partial t} + \omega_0^2$ ima Greenovu funkciju $G(t) = \theta(t)e^{-\gamma t}\frac{\sin\omega t}{\omega}$, kada $\gamma < \omega_0$

a $G(t) = \theta(t)e^{-\gamma t}\frac{\sinh(\omega t)}{\omega}$ kada $\gamma > \omega_0$, $\omega = \sqrt{\omega_0^2 - \gamma^2}$ ima $\omega = \sqrt{\gamma^2 + \omega_0^2}$ gde $\sinh(\omega t)$.

U naem $\hat{L} = \frac{\partial^2}{\partial t^2} + \omega_\varkappa\frac{\partial}{\partial t} + \omega_G^2$, $\omega_G^2 = -\Gamma^2 < 0$, $\omega = \sqrt{\omega_0^2 - \gamma^2} = \sqrt{-\Gamma^2 - \frac{1}{4}\omega_\varkappa^2} = i\Omega$

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$\sin(\omega t) = \sin(i\Omega t) = i\,\mathrm{sh}(\Omega t) = \mathrm{sh}(\Omega t)$, $\omega = \sqrt{-\Gamma^2 - \frac{1}{4}\omega_z^2} = i\Omega$

$G(t) = \theta(t)\, e^{-\frac{1}{2}\omega_z t}\, \frac{\mathrm{sh}(\Omega t)}{\Omega}$, $\Omega^2 = \frac{\varkappa|\varkappa_2|}{G_8^2} + \frac{1}{4}\frac{1}{G_8^4}(\varkappa_{zz}D_{ee} + \varkappa D_{zz})^2$ is real.

т.к. $D^2/G_8^2 \ll 1$, то $\Omega^2 \approx \Gamma^2$, $\Omega \approx \Gamma$.

$G(t) = \theta(t)\, e^{-\frac{1}{2}\omega_z t}\, \frac{1}{\Gamma}\mathrm{sh}(\Gamma t)$, $Z(t) = a\,G(t) = \theta(t)\,\frac{a}{\Gamma}\, e^{-\frac{1}{2}\omega_z t}\,\mathrm{sh}(\Gamma t)$

$\ddot{Z} + \omega_z \dot{Z} + \omega_G^2 Z = a\,\delta(t) - \frac{\varkappa J^M}{G_8^2} h_z$, $\omega_G^2 Z + \frac{\varkappa J^M}{G_8^2} h_z = \omega_G^2\left(Z + \frac{\varkappa J^M}{\omega_G^2 G_8^2} h_z\right)$

$Z' = Z + \frac{\varkappa J^M}{G_8^2 \omega_G^2} = Z + \frac{\varkappa J^M}{\varkappa \varkappa_2} h_z = Z + \frac{J^M}{\varkappa_2} h_z$, при $t>0$, $\dot{Z}' = \dot{Z}$, $\ddot{Z}' = \ddot{Z}$, $\dot{h}_z = 0$.

$\ddot{Z}' + \omega_z \dot{Z}' + \omega_G^2 Z'(t) = a\delta(t)$, $Z'(t) = \theta(t)\frac{a}{\Gamma} e^{-\frac{1}{2}\omega_z t}\,\mathrm{sh}(\Gamma t)$.

$Z(t) = Z'(t) - \frac{J^M}{\varkappa_2} h_z = \theta(t)\frac{a}{\Gamma} e^{-\frac{1}{2}\omega_z t}\,\mathrm{sh}(\Gamma t) - \frac{J^M}{\varkappa_2} h_z(t)$, $h_z(t) = \begin{cases} 0, & t \le 0 \\ h_0, & t > 0 \end{cases}$

$Z(0) = 0$ при скачке $h_z(t)$. $Z(t) = \theta(t)\frac{a}{\Gamma} e^{-\frac{1}{2}\omega_z t}\,\mathrm{sh}(\Gamma t) + \frac{J^M}{|\varkappa_2|} h_z$.

$\mathcal{E}_{\text{сам}} - \frac{1}{2}\omega_z + \Gamma < 0$, то $Z(t) = A e^{-\frac{1}{2}\omega_z t} e^{\Gamma t}$ затухание при $t \to \infty$ и остаточное $\mathrm{sh}\, Z_\infty(t) = \frac{J^M}{|\varkappa_2|} h_z$. Но $\omega_z \sim D$, $|\omega_z| \ll |\Gamma| = \frac{\varkappa|\varkappa_2|}{G_8^2} = |\omega_G|$ — область $Z(t) \neq 0$.

$Z(t)$ as above is formal solution of eq. for $Z(t)$, $a = J^M \frac{D_{ee}}{G_8^2}$ в, где

для $\dot{h}_z = b\,\delta(t)$, $h_z(t)$ — h_0 на t, $\delta(t) = \frac{d}{dt}\theta(t)$ — $\theta(t)$ — Heaviside step func.

$h_z(t) = h_0\theta(t)$, $\dot{h}_z(t) = h_0 \frac{d\theta(t)}{dt} = h_0\delta(t)$ и $b = h_0$.

$\frac{a}{\Gamma} = \frac{J^M D_{ee} h_0}{G_8^2} \frac{1}{(-\varkappa\varkappa_2/G_8^2)^{1/2}} = -\frac{J^M D_{ee} h_0}{\varkappa \varkappa_2}$. Знак подбирает $Z(t)$ расчет для $\sigma \to 0$ $\frac{1}{0}$ $\frac{1}{2}$ h_z head-to-head DW, $h_z > 0$

$p = -1$ для head-to-head, $p = +1$ для tail-to-tail DW. $D_{ee} < 0$, $\varkappa_2 < 0$.

$Z'(t) = -p\frac{J^M |D_{ee}| h_0}{G_8\sqrt{\varkappa|\varkappa_2|}}\theta(t) e^{-\frac{1}{2}\omega_z t}\,\mathrm{sh}(\Gamma t) + \frac{J^M}{|\varkappa_2|} h_z$ — решение Thiele eq. for BP-DW in gyl. nanowire.

$Z(t) \sim h_z$, $h_z(t)$ возбуждает DW motion $\perp Oz$-wise axis.

D_{ee} is principally important (!). Магнитостатика $D_{\alpha\beta}$ и $\varkappa > 0$.

$D/G \approx 2$, $[D] = [G]$, $G \sim \frac{M_S}{\gamma} L \sim \frac{\omega_M}{\gamma} L \sim HL$. $h_z(t) = h_0\theta(t)$.

При $\Gamma t \ll 1$, $Z^*(t) \approx at + \frac{J^M h_z(t)}{|\varkappa_2|}$, $a = \frac{J^M D_{ee} h_0}{G_8^2}$, $\frac{dZ^*}{dt} = a = V(t)$.

а — имеет смысл initial DW velocity at $\Gamma t \ll 1$ (точное решение решение $Z^*(t)$).

$D_{ee} = -2\pi L\left(\frac{M_S}{\gamma}\right)^2 \int dg\, g \int dz \frac{1}{S^2} m_e^2$, $m_z(\rho, z) = \frac{\beta^2 z}{\sqrt{\beta^2 z^2 + r^2}} \frac{t^2}{(t^2 + r^2)}$ в reduced units $\beta = \frac{L}{R}$, $r = \frac{\rho}{R}$, $t = \frac{\beta}{R}$

$J_z = \int dg\, g \frac{1}{g^2}(m_z^2\, dz) = L\int_0^\infty dr\, r \int_{-1/2}^{1/2} dz \frac{\beta^2 z^2}{(\beta^2 z^2 + r^2)} \frac{t^4}{(t^2 + r^2)^2} = L\int_0^\infty dr\, r \int_{-1/2}^{1/2} dz \frac{\beta^2 z^2 + r^2 - r^2}{(\beta^2 z^2 + r^2)} \frac{t^4}{(t^2+r^2)^2} =$

$= L\int_0^\infty dr\, r \left(\int_{-1/2}^{1/2} dz \left[1 - \frac{r^2}{(\beta^2 z^2 + r^2)}\right]\right) \frac{t^4}{(t^2+r^2)^2} = \int_{-1/2}^{1/2} dz \frac{1}{(\beta^2 z^2 + r^2)} = \frac{1}{\beta^2}\int dz \frac{1}{\left(z^2 + \frac{r^2}{\beta^2}\right)} =$

$= \frac{1}{\beta^2}\left(\frac{\beta^2}{r^2}\right)^{1/2} \mathrm{arctg}\left(\frac{r^{-1} z}{\beta^{-1}}\right) = \frac{1}{\beta r}\beta\,\mathrm{arctg}\left(\frac{\beta}{r} z\right)\Big|_{-1/2}^{1/2} = \frac{1}{\beta r} 2\,\mathrm{arctg}\left(\frac{\beta}{2r}\right) = |\beta \gg 1| = \frac{2}{\beta r}\frac{\pi}{2} = \frac{\pi}{\beta r}$

$J_z = L\int dr\, r\left[1 - r^2 \frac{\pi}{\beta r}\right]\frac{t^4}{(t^2+r^2)^2} = Lt^4 \int dr\, r \frac{1}{(t^2+r^2)^3} = Lt^4 \frac{1}{2}\int dx \frac{1}{(t^2+x)^2} =$

$= \frac{1}{2} Lt^4 (-1)\frac{1}{t^2 + x}\Big|_0^1 = -\frac{1}{2} Lt^4 \left[\frac{1}{t^2+1} - \frac{1}{t^2}\right] = -\frac{1}{2} Lt^4 \frac{(-1)}{t^2(t^2+1)} = \frac{1}{2} L \frac{t^4}{t^2(t^2+1)} = J_z$

$J_e = \int dg\, g \frac{1}{g^2}\int dz\, m_e^2 = \int dg\, g \frac{1}{\beta^2} \beta z \cdot 1 - J_z = \int \frac{dg}{\beta} - J_z$, $\left(\int_0^\pi \frac{dg}{\beta} = \int_0^1 \frac{dr}{r}\right)$ — расходится.

$m_e^2 = 1 - m_z^2$, $m_z^2 = \frac{\beta^2 z^2}{(\beta^2 z^2 + r^2)} \frac{t^4}{(t^2+r^2)^2} = \left[1 - \frac{r^2}{(\beta^2 z^2 + r^2)}\right]\frac{t^4}{(t^2+r^2)^2}$. При $\beta \to 0$ и $t \gg 1$ (RBL)

$m_z^2 = 1 - \frac{r^2}{(\beta^2 z^2 + r^2)}$, $m_e^2 = \frac{r^2}{(\beta^2 z^2 + r^2)}$, $J_e = \int dg\, g \frac{1}{g^2}\int dz\, m_e^2 = L\int dz \int dr\, r \frac{r^2}{(\beta^2 z^2 + r^2)} \frac{1}{r^2}$

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$$J_\rho = L\int dr r^3 \int dz \frac{1}{(\beta^2+z^2+r^2)} = L\int dr r^3 \frac{1}{\rho^2} \operatorname{arctg}\left(\frac{\beta}{\rho} z\right)\Big|_{-1/2}^{1/2} = L\frac{2}{\beta r}\int dr r^3 \operatorname{arctg}\left(\frac{\beta}{2r}\right) =$$

$$= L\frac{2}{\beta}\int dr r^2 \operatorname{arctg}(\beta/2r) = L\frac{2}{\beta}\int_0^1 dr r^2 \frac{\pi}{2} = \frac{\pi}{\beta} L \int dr r^2 = \frac{1}{3}\frac{\pi}{\beta} L. \quad (L \gg 1,\ \ell = \ell(R))$$

(2) $D_{eff} = -2\pi\alpha \frac{M_s}{\gamma} J_\rho = -2\pi\alpha \frac{M_s}{\gamma}\frac{1}{3}\frac{\pi}{\beta}L = -\frac{2}{3}\pi^2\alpha\frac{M_s}{\gamma}R$ — неверно, см. 14/I.

$a = V(\text{small } t) = \frac{\mu D_{eff} h_z}{G_0^2}$. $G_0(\beta\gg1, t\gg1) = -4\pi\left(\frac{M_s}{\gamma}\right)R$, $h_z = H_z/4\pi M_s$

$V(0) = \frac{2}{3}\pi^2\alpha(M_s/\gamma)R \frac{1\cdot\mu h_z}{16\pi^2(M_s/\gamma)^2R^2} = \frac{1}{24}\alpha\frac{\gamma}{M_s}\frac{1}{R}\mu h_z$, $\mu \approx 8\pi M_s^2 V$

$V(0) = \frac{1}{24}\alpha\frac{\gamma}{M_s}\frac{h_z}{R} 8\pi M_s^2 V = \frac{1}{24}\alpha\gamma M_s\frac{h_z}{R}\cdot 8\pi\cdot\pi R^2 L = \frac{1}{12}\alpha\omega_M h_z R L$ — разумно?

Thiele $W_H = \mu h_z Z$, $\frac{\partial W_H}{\partial Z} = \mu h_z$, Z here is in absolute units. $[\mu] = erg/cm$

$W_H = \mu h_z Z$ из W_H — работа 25/XII 22г., Z in L-units, $\mu = 8\pi M_s^2 V$

т.е. надо в ур. Thiele $\mu = 8\pi M_s^2 V\frac{1}{L}$, если x, x_t in erg/cm^2, а Z in abs units

$V(0) = \left|\mu \to \frac{\mu}{L}\right| = \frac{1}{12}\alpha\omega_M h_z R$, $V(0) \approx \frac{1}{12}\cdot 0{,}01\cdot 2\pi\cdot 60\,GHz\cdot\frac{20\,mT}{}\cdot 30\,nm =$

$= \frac{1}{12}\cdot 10^{-2}\cdot 360\cdot 10^9\frac{1}{s}\cdot 10^{-2}\,30\cdot 10^{-9}\,m = \frac{1}{12}3{,}6\cdot 3\cdot 10^2\cdot 10^2\cdot 10^9\cdot 10^{-2}\cdot 10^{-3}\frac{m}{s} = 0{,}9\cdot 10^1\frac{m}{s}$ —

— симулятор давал $V(0)$ около 1 $H_z = 20$ mT. Нужен ответ.

Симулятор давал $V(0) \sim 1.5$–2 km/s при $H_z = 20$ mT — нет, $Z^*(t)$ —

(5/I) — это частное решение Л.Н.Д.У. (linear. diff. equation), $\sim h_z$.

Л.Н.Д.У. $\ddot{Z} + \omega_r\dot{Z} + \omega_G^2 Z(t) = F(t)$, $F(t) = \frac{\mu}{G_0^2}[D_{eff}\dot{h}_z - \varkappa h_z]$, $h_z = h_z(t)$

$V(t) = \frac{dZ_*(t)}{dt}$, $V(t\to 0) \sim 0{,}1$ m/s очень мало. Но это нельзя не учитывать.

$\Gamma t \ll 1$, $t \ll \Gamma^{-1}$, если $\Gamma \sim \omega_G$, то $t \ll \frac{1}{\omega_G} = \frac{1}{2\pi f_G} = \frac{T_G}{2\pi}$, $T_G \sim 1$ ns

$\Gamma t \ll 1$, $t \ll \frac{(\sim 1\ ns)}{2\pi} \approx 0{,}1$ ns. $\omega_G/2\pi \sim GHz$

а) $V(t) = dZ^*(t)/dt$ не есть начальная DW velocity, т.к. есть вклад Л.Н.Д.У

б) $t \ll 0.1$ ns нереалистично, у Оксаны typical t is ~ 1–2 ns.

Общее решение Л.Н.Д.У = Общее решение Л.О.Д.У. + частное реш. Л.Н.Д.У

$Z(t) = Z_0(t) + Z^*(t)$, $Z^*(t) \sim h_z$ частное решение.

$Z_0(t)$ из Л.О.Д.У. $\ddot{Z} + \omega_r\dot{Z} + \omega_G^2 Z(t) = 0$, см. 26/XII – 27/XII 22г.

$Z_0(t) = Ae^{-\frac{1}{2}\omega_r t}\cos(\omega_G t + \varphi_0)$, $\omega_G^2 = -\Gamma^2$, $\omega_G = i\Gamma$.

$\cos(\omega_G t + \varphi_0) = \cos(\omega_G t + \varphi_0)\cos\varphi_0 - \sin(\omega_G t)\sin\varphi_0 = ch\Gamma t\cdot\cos\varphi_0 - ish\Gamma t\cdot\sin\varphi_0$

Можно преобразовать $Z(0) = 0$ $Z(0) = A\cos\varphi_0 + \frac{\mu}{|\varkappa_x|}h_z(0) = 0$, это определяет φ_0.

// $V(t)$ anyway is variable, $V(t) = dZ(t)/dt$ физ. смысл имеет только ~ 1 km/s

при $t = 1$–2 ns.

(6/I) Уточнить где ссылка. Розенбаум. Вернуть. Revision Nutation paper. Responses to referees. Написание ВР Gyrotropic dynamics paper.

(7/I) Revision Vortex nutation paper (Nanomaterials). Начать писать $W(\delta)$

δ	0,1	0,15	0,20	0,30	0,35	0,40	0,45	0,50
$W(\delta)$	0,060	0,054	0,049	0,045	0,042	0,042	0,042	0,042

$R = 30$ nm, $\beta = 20$

$L_e = 3{,}96$ nm (Re)

$W(\text{Feld}) = 0{,}052$

Thiaville ansatz: $m_z = \frac{\beta z}{\sqrt{\beta^2 z^2 + r^2}}\frac{t^2}{(t^2+r^2)}$, $z = s/R \in [0,1]$, $\beta = L/R$, в работе значение $\beta \gg 1$

$\frac{1}{\sqrt{\beta^2 z^2 + r^2}} = \frac{1}{\beta}\frac{1}{\sqrt{z^2 + r^2/\beta^2}} \approx \frac{1}{\beta|z|}\frac{1}{\sqrt{1 + r^2/\beta^2 z^2}}$, но $\frac{1}{\beta}$ нельзя разложить в ряд т.к. $z = 0$.

b	0,08	0,10	0,12	0,14	0,16	0,18	0,20	0,22	0,24	0,26	0,28	0,30
W(b)	0,023	0,022	0,021	0,020	0,019	0,019	0,019	0,018	0,0181	0,0180	0,0181	

0,185 0,183

$\beta=20$, $R=50$ nm

$W(\text{Feld})=0,048$

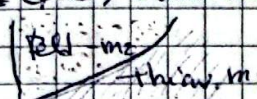
b	0,32	0,34	0,36	0,38	0,40	0,42	0,45	0,50
W(b)	0,182	0,183	0,185	0,188	0,190	0,193	0,198	0,208

(8/I) Invited speakers SolStyMag 2023 (nepemecka)

$W(\text{thian}) < W(\text{Feld})$ gune $\beta=20$, $R=30,50$nm. С увеличением β и уменьшением R W_{min} $W(b)$-Thian становится все более полным и смещается к большим "b". $m_z(\text{thian})$ работает при относительно малых β и больших R, когда cyl. wire → cylindrical dot.

$m_z(\text{thian})$ описывает BP-DW energy vortex domains ($z>0$ и $z<0$), small β, large R.

$m_z(\text{Feld})$ описывает BP-DW energy saturated domains, large β, small R.



Есть crit. line $\beta_c(R)$, где $W(\text{Feld})=W(\text{thian})$.



Печатает: $\beta=20$, $R=30$nm

b	0,35	0,40	0,45	0,50	0,55	060	0,65	0,70	0,75	080	0,85	0,90	1,0
W(b)	432	0,424	0,419	0,417	0,417	0,419	421	424	428	432	435	0,439	0,0446

b	0,1	0,2	0,28	0,30	0,32	0,35	0,40	0,50	0,60
W(b) ×10	0,606	0,512	0,489	0,487	0,487	0,489	0,496	0,524	0,562

$W(\text{Feld})=0,104$

$\beta=10$, $R=30$nm

Tables above are calculations (mathCad) of the minimum BP-DW energy $W(b)$ with respect to Bloch point size "b".

(8/I) Selfstyмag-2023 nepemecka. Nutation Nanomat.MS

(9/I) Meeting with G. Gestela (Poznan) Его темы: Elastic media – b. conditions on the interface и DW in coupled spheroidal dots with surface anisotropy

В Cafeteria с Мурмаи. перемена о Ricardo's jury.

(10/I) E-mails to Chichkin, Arantxa, Paule – formation Equipo de Investigacion new Ministerio proposal. В рез. "Gaceta Cubide" с jury members.

(11/I) Уточнение Задания Ph.D. thesis By Ricardo. рез. "Lukes" – jury. Supervisors

(12/I) Skype обсуждение с Оксаной (Ministerio proposal + BP-DW paper) + Paymag (revision of PoP manuscript Ph. Rev. Appl.)

(13/I) Edited and sent to Oksana review "Curvilinear magnetism" – new proposal.

(14/I) Iverbayeva report за 2022. Editing Nutation MS, Nanomaterials

$$D_{ex} = -2\pi d\left(\frac{M_s}{\gamma}\right)\int d\varphi\int dz\,\frac{m_\varphi^2}{8^2} = -2\pi d\left(\frac{M_s}{\gamma}\right)J_e,\quad J_e=\int\frac{d\varphi}{8}\int dz\,m_\varphi^2$$

BP-DW damping

$$m_z = p\frac{z}{\sqrt{8^2+z^2}}\left(\frac{\beta^2}{\beta^2+8^2}\right),\ z\to zL,\ \rho\to\rho R,\quad m_z=\rho\frac{\beta z}{\sqrt{\beta^2z^2+r^2}}\frac{t^2}{(t^2+r^2)}\quad\text{(Thiaville ansatz)}$$

$t=\beta/R$

$$m_\varphi^2=1-m_z^2,\quad m_z^2=\frac{\beta^2z^2}{(\beta^2z^2+r^2)}\frac{t^4}{(t^2+r^2)^2}=\left[1-\frac{r^2}{(\beta^2z^2+r^2)}\right]\frac{t^4}{(t^2+r^2)^2}$$

При $t\gg1$, Feld ansatz, $m_\varphi^2=\frac{r^2}{(\beta^2z^2+r^2)}$, $m_z^2=1-\frac{r^2}{(\beta^2z^2+r^2)}$

$$J_e=L\int_{-z_2}^{z_2}dz\int_0^1\frac{dr}{r}m_\varphi^2=L\int dz\int\frac{dr}{r}\frac{r^2}{(\beta^2z^2+r^2)}=\left|z^2=u\right|=L\int dz\,\frac{1}{2}\int\frac{du}{(\beta^2z^2+u)}$$

$$\int dz\,\frac{1}{(\beta^2z^2+u)}=\frac{1}{\beta\sqrt{u}}\arctg\left(\frac{\beta}{\sqrt{u}}z\right)\Big|_{-z_2}^{z_2}=\frac{2}{\beta\sqrt{u}}\arctg\left(\frac{\beta}{\sqrt{2u}}\right)$$

$$J_e=L\int dz\int\frac{dr}{r}\frac{1}{(\beta^2z^2+r^2)}=L\int dz\,\frac{2}{\beta z}\arctg\left(\frac{\beta}{2z}\right)=\frac{2L}{\beta}\int dz\,\arctg\left(\frac{\beta}{2z}\right)$$

Если $\beta\gg1$, $\arctg(\beta/2z)\approx\pi/2-2z/\beta\approx\pi/2$ и $J_e\cong\frac{2L}{\beta}\cdot\int_0^1dz\,\frac{\pi}{2}\cong\pi L/\beta$.

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4/I я ошибочно перед 2πn $J_e = \frac{1}{3}\frac{\pi L}{\beta}$ (из-за неопределённости $\frac{1}{\infty^2}$). Надо $J_e = \frac{\pi L}{\beta}$

$D_{ee} = -2\pi d\left(\frac{M_s}{\gamma}\right) J_e = -2\pi d \frac{M_s}{\gamma} \pi L \cdot \frac{R}{L} = -2\pi^2 d \frac{M_s}{\gamma} R$ (Это $t \to \infty$, $\beta \to \infty$) Feld Kelvin

$\operatorname{arctg}\left(\frac{1}{x}\right) = \operatorname{sign}(x)\frac{\pi}{2} - \operatorname{arctg}(x)$, $\operatorname{arctg}\left(\frac{\beta}{2\pi}\right) = \frac{\pi}{2} - \operatorname{arctg}\left(\frac{2\pi}{\beta}\right)$

$J_e = \frac{2L}{\beta}\int_0^1 dz \operatorname{arctg}\left(\frac{\beta}{2\pi}\right) = \frac{2L}{\beta}\int dz\left[\frac{\pi}{2} - \operatorname{arctg}\left(\frac{2\pi}{\beta}\right)\right] = \frac{2L}{\beta}\left[\frac{\pi}{2}z - z\operatorname{arctg}\left(\frac{2\pi}{\beta}\right) + \frac{1}{2}\frac{\beta}{2}\ln\left(\beta^2/4 + \pi^2\right)\right]_0^1 = \frac{2L}{\beta}\left[\frac{\pi}{2} - \operatorname{arctg}\frac{2}{\beta} + \frac{\beta}{4}\ln\left(\frac{\beta^2}{4}+1\right) - \frac{\beta}{4}\ln\left(\frac{\beta^2}{4}\right)\right] =$

$= \frac{2L}{\beta}\left[\frac{\pi}{2} - \operatorname{arctg}\frac{2}{\beta} + \frac{1}{4}\beta\ln\left(1 + \frac{4}{\beta^2}\right)\right]$, $J_e = 2R\left[\frac{\pi}{2} - \operatorname{arctg}\frac{2}{\beta} + \frac{1}{4}\beta\ln\left(1+\frac{4}{\beta^2}\right)\right]$

$D_{ee} = -2\pi d\left(\frac{M_s}{\gamma}\right) J_e = -4\pi d R\left(\frac{M_s}{\gamma}\right)\left[\frac{\pi}{2} - \operatorname{arctg}\frac{2}{\beta} + \frac{1}{4}\beta\ln\left(1+\frac{4}{\beta^2}\right)\right]$, $\frac{L}{R} \gg 1$.

15/I Я получил частное решение LP-DW motion $Z_x(t)$ для step driving field $h_z(t) = h_0\theta(t)$, $\dot{h}_z = h_0\delta(t)$. А решение для $X_e(t)$?

Сом. ур.: $-G_s\dot{z} + D_{ee}\dot{X}_e = \varkappa X_e$, $G_s\dot{X}_e + D_{zz}\dot{z} = \varkappa_z Z + \mu h_z$

$\dot{z} = \frac{1}{G_s}\left(D_{ee}\dot{X}_e - \varkappa X_e\right)$, $G_s\dot{X}_e + \frac{D_{zz}}{G_s}\left(D_{ee}\dot{X}_e - \varkappa X_e\right) = \varkappa_z \dot{z} + \mu h_z$

$G_s\ddot{X}_e + \frac{D_{zz}}{G_s}\left(D_{ee}\ddot{X}_e - \varkappa\dot{X}_e\right) = \varkappa_z\dot{z} + \mu\dot{h}_z = \frac{\varkappa_z}{G_s}\left(D_{ee}\dot{X}_e - \varkappa X_e\right) + \mu\dot{h}_z$

$\left(G_s + \frac{D_{zz}D_{ee}}{G_s}\right)\ddot{X}_e + \dot{X}_e\left[-\varkappa\frac{D_{zz}}{G_s} - \varkappa_z\frac{D_{ee}}{G_s}\right] + \frac{\varkappa\varkappa_z}{G_s}X_e = \mu\dot{h}_z$

$\left(1 + \frac{D_{zz}D_{ee}}{G_s^2}\right)\ddot{X}_e - \frac{1}{G_s^2}\left(\varkappa D_{zz} + \varkappa_z D_{ee}\right)\dot{X}_e + \frac{\varkappa\varkappa_z}{G_s^2}X_e = \frac{\mu}{G_s}\dot{h}_z$

$D_{zz}D_{ee}/G_s^2 \ll 1$ выполним, $\omega_z = -\frac{1}{G_s^2}\left(\varkappa_z D_{ee} + \varkappa D_{zz}\right)$, $\omega_G^2 = \frac{\varkappa\varkappa_z}{G_s^2}$

$\ddot{X}_e + \omega_z\dot{X}_e + \omega_G^2 X_e = \frac{\mu}{G_s}\dot{h}_z$. Было $\ddot{Z} + \omega_0\dot{Z} + \omega_G^2 Z = \frac{\mu}{G_s^2}\left(D_{ee}\dot{h}_z - \varkappa h_z\right)$

Правая часть УР для $X_e(t)$ содержит только $\dot{h}_z$, и не содержит $h_z(t)$.

$\ddot{X}_e + \omega_z\dot{X}_e + \omega_G^2 X_e(t) = a_e\delta(t)$, $\dot{h}_z = h_0\delta(t)$, $a_e = \frac{\mu h_0}{G_s}$. $X_e(t) = a_e G(t)$

где Green function из $\ddot{G} + \omega_z\dot{G} + \omega_G^2 G = \delta(t)$, $\hat{L}G(t) = \delta(t)$.

$\hat{L} = \frac{\partial^2}{\partial t^2} + \omega_z\frac{\partial}{\partial t} + \omega_G^2$. $2\gamma = \omega_z$, $\omega_G^2 = \omega_0^2 = -\Gamma^2$. $G(t) = \theta(t)e^{-\gamma t}\frac{\sin(\omega t)}{\omega}$

$\omega = \sqrt{\omega_0^2 - \gamma^2} = \sqrt{\omega_G^2 - \frac{1}{4}\omega_z^2} = \sqrt{-\Gamma^2 - \frac{1}{4}\omega_z^2} = i\Omega$, $\Omega = \sqrt{\Gamma^2 + \frac{1}{4}\omega_z^2}$ is real.

$\frac{\sin(\omega t)}{\omega} = \frac{\sin(i\Omega t)}{i\Omega} = \frac{i\operatorname{sh}(\Omega t)}{i\Omega} = \frac{1}{\Omega}\operatorname{sh}(\Omega t)$. $G(t) = \theta(t)e^{-\omega_z t/2}\frac{\operatorname{sh}(\Omega t)}{\Omega}$

$X_e(t) = \frac{\mu h_0}{G_s}\theta(t)e^{-\frac{1}{2}\omega_z t}\frac{\operatorname{sh}(\Omega t)}{\Omega}$, $\Omega \approx \Gamma$, $X_e(t) = \frac{\mu h_0}{G_s\Gamma}e^{-\frac{1}{2}\omega_z t}\operatorname{sh}(\Gamma t)$ - частное решение $\sim h_z$.

$Z_x(t) = \frac{P\mu D_{ee}h_0}{G_s^2\Gamma}e^{-\frac{1}{2}\omega_z t}\operatorname{sh}(\Gamma t) + \frac{\mu}{|\varkappa_z|}h_z$, $t > 0$.

$\frac{\text{Amplitude}\left[Z_x(t)\right]}{\text{Amplitude}\left[X_e(t)\right]} = \frac{\mu D_{ee}h_0}{G_s^2\Gamma}\frac{G_s\Gamma}{\mu h_0} = \frac{D_{ee}}{G_s} \ll 1$, т.к. $D_{ee} \sim d$.

Для long wire и $t = b/R \gg 1$ (field angle): $G_s = -4\pi R\frac{M_s}{\gamma}$, $D_{ee} = -2\pi^2 dR\frac{M_s}{\gamma}$

$D_{ee}/G_s = \frac{-2\pi^2 dR(M_s/\gamma)}{-4\pi R(M_s/\gamma)} = \frac{\pi}{2}d \ll 1$.

Вывод: displacement (displacement) of field-h_z driven BP motion $\perp$ to transverse e-direction $\Rightarrow$ displacement along nanowire axis of BP looking at the wire side surface $\rho = R$. Вектор тем значительно сдвинется вдоль оси. Но он это для малых Z_x/L и X_e/R, когда $W(\vec{X}) = \frac{1}{2}\varkappa(x^2+y^2) + \frac{1}{2}\varkappa_z z^2$ - вертик

Escaneado con CamScanner

Quench BP escape time t_e $X_e(t_e)=R$ $X_e(t)=\frac{\mu h_0}{G_s\Gamma}e^{-\frac{1}{2}\omega_z t}\sinh(\Gamma t)$.

$\frac{RG_s\Gamma}{\mu h_0}=\sinh(\Gamma t_e)$ when $\sinh(\Gamma t)=0$. $t_e=\Gamma^{-1}\operatorname{arcsinh}\left(\frac{RG_s\Gamma}{\mu h_0}\right)$, $h_0=\frac{H_z}{4\pi M_s}$

$\operatorname{arcsinh}(x)=\ln\left(x+\sqrt{x^2+1}\right)$, $\operatorname{arcsinh}(x)\approx\ln(2x)$, $x\gg1$.

$x=\frac{RG_s\Gamma}{\mu h_0}$, $\mu=8\pi M_s^2\frac{V}{L}$ at $t=\beta/\alpha\gg1$. $x=\frac{R\cdot 4\pi R(M_s/\gamma)\Gamma}{8\pi M_s^2(V/L)h_0}=$

$=\frac{M_s}{\gamma}\Gamma\frac{4\pi R^2}{2\pi M_s^2\cdot\pi R^2 h_0}=\frac{\Gamma}{2\gamma M_s h_0}=2\pi^2\frac{\Gamma}{\omega_M h_0}$, $x=\frac{2\pi\Gamma}{h_0\omega_M}$, $\omega_M=\gamma\cdot 4\pi M_s$

$x(R)=\frac{\Gamma}{h_0(\omega_M/2\pi)}\approx\frac{\Gamma}{h_0}\frac{1}{60\,GHz}$, $\Gamma\approx|\omega_G|=2\pi f_G=2\pi\cdot 1\,GHz$ – муст 56 (?).

$x(R)\simeq\frac{2\pi\cdot 1\,GHz}{60\,GHz}\frac{1}{h_0}=\frac{\pi}{30}\frac{1}{h_0}$, $h_0=\frac{H_z}{4\pi M_s}=\frac{H_z}{2\,T}=\frac{(10+20)\,mT}{2\,T}\sim 0{,}01$

$x(R)=\frac{1}{10}\frac{1}{h_0}\simeq\frac{1}{10}\frac{1}{10^{-2}}\simeq 10$, $x\gg1$ when $H_z=20\,mT$.

$t_e=\frac{1}{\Gamma}\operatorname{arcsinh}(x)\approx\Gamma^{-1}\ln(2x)$, $t_e(x=10)=\Gamma^{-1}\ln(20)=3\frac{1}{\Gamma}=$

$=3\frac{1}{\omega_G}=3\frac{1}{2\pi f_G}\approx 0.5\frac{1}{f_G}=0.5\frac{1}{GHz}=0.5\,ns$ when $\frac{\Gamma}{2\pi}\approx\frac{\omega_G}{2\pi}\simeq 1\,GHz$.

$t_e=\Gamma^{-1}\operatorname{arcsinh}\left(\frac{RG_s\Gamma}{\mu h_0}\right)\approx\Gamma^{-1}\ln\left(\frac{2RG_s\Gamma}{\mu h_0}\right)=\Gamma^{-1}\ln\left(\frac{4\pi\Gamma}{\omega_M h_0}\right)$

$t_e(h_0)=\Gamma^{-1}\operatorname{arcsinh}\left(\frac{2\pi\Gamma}{\omega_M h_0}\right)$ t_e h_0 $t_e(h_0\gg1)\cong\Gamma^{-1}\frac{2\pi\Gamma}{\omega_M h_0}=\frac{2\pi}{\omega_M}\frac{1}{h_0}$

$t_e(h_0\to 0)\cong\Gamma^{-1}\ln\left(\frac{4\pi\Gamma}{\omega_M h_0}\right)$ singular

Calculate Γ, $\Gamma^2=\frac{\alpha|x_2|}{G_s^2}$ when $x>0$.

$\operatorname{arcsinh}\left(\frac{1}{x}\right)=\ln\left(\frac{1}{x}+\sqrt{\left(\frac{1}{x}\right)^2+1}\right)=\ln\left[\frac{1}{x}\left(1+\sqrt{1+x^2}\right)\right]=\ln\left[\frac{1}{x^2}\left(x+x\sqrt{1+x^2}\right)\right]$

Long version Γ, i.e. solve x, $x\ll1$ and $\beta/\alpha\gg1$, Feldkeller ansatz.

$\dot{X}_s=\frac{\alpha}{D_{ss}}\dot{X}_s=-\omega_z'X_s$, $X_s(t)=C_1e^{-\omega_z' t}+C_2=C_1e^{-\omega_z' t}$, $X_s(0)=C_1$

$X_s(t)=X_s(0)e^{-\omega_z' t}$. Uses when $t=0$, $X_s(0)$, so $X_s(t)=0$, we solve $h_z(t)$

so $X_s(t)$ no longer.

(16/I) Порядок нумерации revised text of PRA – Block point M_S – responses to Referees. C.A. 6 "Correspondent Author". Оставим 2 amplitudes new text MICEWW – project new.

Our Hopfion project BP dynamics calculations of nanowire $4\pi M_s=2\,T$

– $A=25\,pJ/m$, two gaps $l_{ex}=\sqrt{A/2\pi M_s^2}=3{,}96\,nm$ – used gaps $W(\beta)$, π, x_2 calc.

(w/ Fairhead) BP nanosoliton, RCoCu wires) Порядок

& Overcome BP – DW dynamics M_S, Fe-nanowires, $M_S=1700\,G$, $A=21\,pJ/m$, $l_{ex}=3{,}40\,nm$ Overcome

M_S (Fairhead) $=20\,kOe/\mu_0=1592\,G$. $\frac{\omega_M}{2\pi}=\frac{\beta}{2\pi}4\pi M_S\doteq 3\,\frac{GHz}{kOe}\cdot 20\,kOe=60\,GHz$

$\frac{1}{2\pi}\omega_M$ (Fairhead) $=60\,GHz$.

$t_e(h_0)=\Gamma^{-1}\operatorname{arcsinh}\left(\frac{2\pi}{h_0}\frac{\Gamma}{\omega_M}\right)$, $\gamma=\frac{\Gamma}{\omega_M}$ typical $\sim 0{,}01-0{,}02$, $\Gamma=\frac{\Gamma}{2\pi}\sim\frac{\omega_G}{2\pi}\sim 1\,GHz$.

$\Gamma^{-1}=\frac{1}{\Gamma}=\omega_M^{-1}\frac{\omega_M}{\Gamma}=\frac{1}{\omega_M}\frac{1}{\gamma}$, in abs. units $t_e(h_0)=\frac{1}{\omega_M}\frac{1}{\gamma}\operatorname{arcsinh}\left(\frac{2\pi}{h_0}\gamma\right)$

$\frac{1}{\omega_M}=\frac{1}{2\pi f_M}=\frac{1}{2\pi\,60\,GHz}=\frac{1}{2\pi\cdot 60}\,ns=2{,}653\cdot10^{-3}\,ns$

Example $t_e(h_0,\gamma)=2{,}653\cdot10^{-3}\,ns\,\frac{1}{\gamma}\operatorname{arcsinh}\left(\frac{2\pi}{h_0}\gamma\right)$, $\gamma=0{,}01-0{,}02$, t_e in ns

$\Gamma^2=-\alpha x_2/G_s^2$. Оценим Γ. Use $R=30\,nm$, $\beta=20$, solve M_S min $W(\beta)$ $\beta=0.5$.

$t=0.5$ in all variables. Can fake, Block-point stiffness needed "in passing" 24/II/2022

$x_2^{vol}(\beta=20,t=0.5)=-4.97\cdot10^{-5}$, $W(z)=\frac{1}{2}x_2z^2$, x_2 in $(4\pi M_S^2V)$ units each

z in L units. $W(z)=\frac{1}{2}(x_2L)^2(z/L)^2$, i.e. $x_2^{vol}L^2=-4.97\cdot10^{-5}$ – HLP

$\beta=20$, $R=30\,nm$, $L=\beta R=20\cdot30=600\,nm$, Absolute $x_2^{vol}=-4.97\cdot10^{-5}\cdot(600\,nm)^2=$

$=-5\cdot10^{-3}\cdot36\cdot10^{-14}\,nm^2$, Herrera, $\frac{W}{4\pi M_S^2V}=\frac{1}{2}x_2z^2$, x_2 is dimensionless

MCd x

$\Gamma=$

$\frac{|x_2|}{|G_s|}=$

$\frac{\Gamma}{\omega_M}=$

$x=\frac{RG_s\Gamma}{\mu h_0}=$

$\mu(\beta)=8\pi M$

$x=\frac{RG_s\Gamma}{\mu h_0}=$

β/α $x\gg1$

$f(\beta)\sim\frac{1}{\beta}$

$\beta=0.5$

$x=\frac{2\Gamma}{h_0}$

(17/I) Revision

(18/I) meeting

in plane

PRA 2021

Lim-Kettemann

face localiz

eq. хорошо

(19/I) Paper

(20/I) Meeting

21–22/I

(23/I) E-mails

Bonn Univ

UAM/EHU

(24/I) Chair

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$[\tilde{\varkappa}_4] = erg/cm^2$ или $W(Z) = \frac{1}{2}\tilde{\varkappa}_4 Z^2$, все в абсолютных единицах.

MCad $\varkappa_4 \quad \frac{W}{4\pi M_s^2 V} = \frac{1}{2}\varkappa_2 \left(\frac{Z}{L}\right)^2 = \frac{1}{2}\frac{\varkappa_2}{L^2} Z^2$, $W(Z) = 4\pi M_s^2 V \cdot \frac{1}{2}\frac{\varkappa_2}{L^2} Z^2$

$\tilde{\varkappa}_4 = \frac{4\pi M_s^2 V}{L^2}\varkappa_2$, $G_s(\beta \gg 1) = -4\pi R\, b\left(\frac{M_s}{\gamma}\right) \operatorname{arctg}(1/b)$, $b \to \frac{b}{R} = t$.

$\Gamma = \sqrt{-\varkappa \varkappa_4}\,\frac{1}{|G_s|} = \sqrt{\frac{\varkappa}{\varkappa_2}}\,\frac{|\varkappa_2|}{|G_s|}$, $\frac{|\varkappa_2|}{|G_s|} = (\text{здесь } \tilde{\varkappa}_4) = \frac{4\pi M_s^2 V \varkappa_2}{L^2 \cdot 4\pi R\, b \frac{M_s}{\gamma} \operatorname{arctg}(\frac{1}{b})}$

$\frac{|\varkappa_2|}{|G_s|} = \gamma M_s \frac{4\pi \cdot \pi R^2 L}{L^2 \cdot 4\pi R \operatorname{barctg}(1/b)} = \pi\gamma M_s \frac{1}{\beta}\frac{1}{\operatorname{barctg}(1/b)} = \omega_M \frac{1}{4\beta}\frac{1}{\operatorname{barctg}(1/b)}$

т.к. $\varkappa \propto |\varkappa_2|$, тогда $\Gamma \propto \frac{|\varkappa_2|}{|G_s|} = \omega_M \frac{1}{4\beta}\frac{1}{b \operatorname{arctg}(1/b)}$, $\operatorname{arctg}(\frac{1}{b}) = 0{,}554$ при $b = 0{,}5$.

$\frac{\Gamma}{\omega_M} = \frac{1}{4\beta}\frac{1}{0{,}554} = |\beta = 20| = \frac{1}{80}\frac{1}{0{,}554} = 0{,}023$. Для $\gamma = \Gamma/\omega_M = 0{,}01 \div 0{,}02$.

Но нужно знать $\varkappa$, чтобы $\Gamma^2 = +\varkappa|\varkappa_2|/G_s^2$ найти точно.

Для $\varkappa = \frac{2\pi}{h_0}\frac{\Gamma}{\omega_M}$, $k_e = \Gamma^{-1}\operatorname{arcsinh}(\varkappa)$ только при $t = b/R \gg 1$. Для $\beta = 20$, $R = 30$ nm, $\frac{b}{R} = 0{,}5$

$\varkappa = \frac{RG_s\Gamma}{\mu h_0}$, можно $\beta \gg 1$, но $b/R \gg$ ненужд. применяется within Thiaville ansatz.

$\mu(\beta \underset{b \gg 1}{\to} \infty) = 8\pi M_s^2 V/L$, $G_s(\beta \gg 1) = -4\pi R\, b\left(\frac{M_s}{\gamma}\right)\operatorname{arctg}\left(\frac{1}{b}\right)$

$\frac{RG_s}{\mu} = \frac{R \cdot 4\pi R \operatorname{barctg}(1/b) M_s/\gamma}{8\pi M_s^2 \pi R^2} = \frac{1}{2\pi}\frac{1}{\gamma M_s}\operatorname{barctg}\left(\frac{1}{b}\right) = \frac{2}{\omega_M}\, b \operatorname{arctg}\left(\frac{1}{b}\right)$.

$\mu(b) = 8\pi M_s^2\left(\frac{V}{L}\right) b^2 \ln\left(1 + 1/b^2\right)$, $\frac{RG_s}{\mu} = \frac{4\pi R^2 \operatorname{barctg}(\frac{1}{b}) M_s}{8\pi M_s^2 \pi R^2 b^2 \ln(1+1/b^2)} = \frac{2}{\omega_M}\cdot\frac{\operatorname{barctg}(1/b)}{b^2 \ln(1+1/b^2)}$

$\varkappa = \frac{RG_s\Gamma}{\mu h_0} = \frac{2}{h_0}\left(\frac{\Gamma}{\omega_M}\right)\frac{\operatorname{barctg}(1/b)}{b^2 \ln(1+1/b^2)}$ within Thiaville ansatz. при $b \gg 1$, $\varkappa = \frac{RG_s\Gamma}{\mu h_0} = \frac{2}{h_0}\frac{\Gamma}{\omega_M}$ Tetsuji

Thiav

13/I я использовал $\varkappa = \frac{2\pi\Gamma}{h_0\omega_M}$, π — мистика. Надо field- $\varkappa = \frac{2\Gamma}{h_0\omega_M}$.

(Tetsuji)

$\varkappa$-Thiav ansatz отличается от $\varkappa$-field множителем $f(b) = \frac{b \operatorname{arctg}(1/b)}{b^2 \ln(1+1/b^2)}$; $f(b)$ ~ $\frac{1}{b}$ при $b \ll 1$, расходится, $f(b \ll 1) \gg 1$.



$b = 0.5$ для $\beta = 20$, $R = 30$ nm. Но с увеличением β и/или с увеличением R может быть мало, $b = 0{,}1 - 0{,}2$ (область Thiav. ansatz). $f(0{,}5) = 1{,}376$, $f(0{,}1) = 3{,}188$

$\varkappa = \frac{2\Gamma}{h_0\omega_M} f(b) = \varkappa(b)$, но и само $\Gamma = \Gamma(b)$, поэтому вид $\varkappa(b)$ — не ясен.

17/I Revision Casso-BP manuscript (Ph. Rev. Appl.) и cover lett. ответы referees

Подготовил Report задачи POLS project и послал Лене и Мацею вместе с Rachmuck

18/I meeting с G. Costals в IMN/EHU. M. elastic b.c. — SW in square dot array; сказала Hanen experiments и simulations square dot array in m_y field // in plane, у него $\vec{M}$ is out-plane + surface anisotropy. Hanen Lim-Kletterson PRA-2021 о ровере SW in cylindrical wires (long cylinders).

Lim-Kletterson ответ: ① the lowest SW (symmetric or anti-sym.) is face localized like vortex GM (Guy Magnetism 2022). ② Herring-Kittel eq. хорошо работает для "bulk" modes (the lowest m. is $n_z = 0$, $m = 0$).

19/I Работа IWA-Augsburg до 2022. Min. proposal с Oxc.

20/I Минцберг proposal с Oxc.: описание IMN/EHU group и ее возможности.

21-22/I Написание Минцберг "curvilinear magnetism" proposal.

23/I E-mails to Maciej, Gutstein, Chyzhyk, Ruben, Оксана, Vicerrectora.

Вопрос только про я могу быть PI of the project, только если будет со-PI from IMN/EHU. Зеленский разреш. быть PI в случае с Iberbasque - IMN/EHU agreement

24/I Ответ свой sub-project и вопрос как member Res. team. Оксанин про ICMM

project. Сек/решила Vicerrector представила мне upgrade my Status to Investigador Distinguido. Prof. Ibon PI de UPV/EHU. Нужна от Ikerbasque authorization for the project as Res. Team member.

25/I Сек/решила (Patricia Julian) прислала мне ускоренную application to Investigador Distinguido. Felipe submitted „BP dyn" to Com. Phys.

26/I Нодро-е. в ноябре to V. Gros recom. lett., to A. Thiaville for EPMA prize. Обсужд. с Гнатом Artem's paper: because Badescu tag Cover Lett. UPV/EHU разрешил мне Ekar PI of Ministerio project, при single Nat. Ph. условии что его proposal is accepted. Я договорился project registred as distinguished researcher.

27/I Нерешимся с Зухрой о Conf. 2023 и slides „Nutation" talk.

30/I Нерешимся с Оксаной Фарраж, Зухра (abs, HMM 2023).

V II → Сек. сбрал группу в AMU Poznan „Vortex nutations".

2-3/II Подготовил slides для Poznan „Nutation" talk.

4-16/II поездка в Poznan с П. Invited talk „Nutation excitations in the gyrotropic vortex dynamics" 3 ноября. Inst. Spintronics and Quantum Information. Беседы с Леной Т., Матеуш (ring over dot/film), K. Sobucki (Hopfion stab. eqs. data), P. Gruszecki (Hopfion dynamics by $\vec{DE}$), J. Kos (magn.-elastic bound conditions, SW in square dot arrays), Katarina/Mateusz (SW in stripe with Skyrm. single dot), Jarek, Sobucki, Pawel (3-magnon scattering – нужд. Tu Sepsebura $\vec{S}$), Лена, Дарья (Hopfion gyrovector). Прибыл 1.600 €. Получил 2.200 PLN – суб.

План встреч с Maciej group: 1) Hopfion gyrovector; 2) Skyrmion under ring (Матеуш) 3) SW in arrays of square dots (Klos, Gostek).

Maciej представил мне подать на AMU travel grant 12 kPLN для 2-week visit ~ June 25 – July 8, 2023.

16/II Hopfion gyrovector $\vec{G} = \int dV \vec{B}$, $\vec{B}$ - gyrocoupling density or emergent mag. field.

$B_i = \frac{1}{2}\varepsilon_{ijk}\vec{m}\cdot(\partial_j\vec{m}\times\partial_k\vec{m})$.

Gladikowski 1997 (см. 11/II 2023): $m_x + im_y = \sqrt{1-w^2(\eta,\beta)}\,e^{i[N\varphi+\nu(\eta,\beta)]}$, $m_z = w(\eta,\beta)$

(η,β,φ) - toroidal coordinats, φ - azimuthal angle in xy-plane.

$\nu(\eta,\beta) = M\beta + \nu_0(\eta,\beta)$, $H = -\frac{1}{(2\pi)^2}\int dV \vec{B}\cdot\vec{A}$, при $\nu_0 = const$, $H = NM$ - integer Hopf index.

Emergent field $\vec{B} = (B_\eta, B_\beta, B_\varphi)$, $B_\eta = \frac{2}{a^2}\frac{\tau^2}{\sin\eta}N\frac{\partial w}{\partial\beta}$, $B_\beta = -\frac{2}{a^2}\frac{\tau^2}{\sin\eta}N\frac{\partial w}{\partial\eta}$, $B_\varphi = \frac{2\tau^2}{a^2 N}\frac{\partial w}{\partial\eta}$

при $\nu_0 = const$ $dV = \frac{a^3}{\tau^3}\sin\eta\,d\eta\,d\beta\,d\varphi$, $\tau = \cosh\eta - \cos\beta$, a - hopfion radius

Hamilton $w(\eta,\beta) = m_z$. Hietarinta 1999: $m_z = 1 - \frac{2g^2}{(1+g^2+\bar{z}^2)^2}$, g, z in units of a.

m_z (Hietarinta) подставил сюда $w(\eta,\beta) = m_z = 1 - 2H^2_{2N}$, удобно. $w(0,\beta) = 1$, $w(\infty,\beta) = -1$

$\frac{\partial w}{\partial\beta} = 0 \to G_\eta = \int dV B_\eta = 0$. $\vec{G} = (G_\eta, G_\beta, G_\varphi) = 16\pi^2 a\left(0, \frac{\pi}{2}N, -M\right)$ Gladikowski.

Hietarinta 2000: $m_x + im_y = \frac{2u}{1+|u|^2}$, $m_z = \frac{|u|^2-1}{|u|^2+1}$, $u(\eta,\beta,\varphi) = f(\eta)e^{i(N\beta+M\varphi)}$, $f(\eta) = 1/\sinh\eta$. см. 11/IX 2023.

deVega 78: $\vec{\gamma} = \frac{1}{4\pi c^2}\frac{\tau^2}{\sinh\eta}\sin\Theta\frac{\partial\Theta}{\partial\eta}\left[\hat{e}_\beta + \sinh\eta\,\hat{e}_\varphi\right]$, где $m_z = \cos\Theta(\eta) = 1 - 2\frac{1}{\cosh^2\eta}$, $m_z(0) = -1$, $m_z(\infty) = 1$

$\vec{\gamma} = -\frac{1}{m_z}(\nabla m_x \times \nabla m_y)$ gyrocoupl. density. $\vec{G} = \int dV\vec{\gamma} = -2\pi c\left[\frac{\pi}{2}\hat{e}_\beta + \hat{e}_\varphi\right]$, c = a hopf radius.

$\vec{G}$ (Gladikowski) $= 16\pi^2 a\left(\frac{\pi}{2}N\hat{e}_\beta - M\hat{e}_\varphi\right)$ совпадает с $\vec{G}$ (deVega) с точностью до переименования при $N = 1$, $M = -1$, $Q_H = -1$.

Сфер. Wiki: $\hat{e}_\eta = \cos\beta\cos\varphi\,\hat{x} + \cos\beta\sin\varphi\,\hat{y} + \sin\beta\hat{z}$, $\hat{e}_\varphi = -\sin\varphi\,\hat{x} + \cos\varphi\,\hat{y}$

$\hat{e}_\beta = -\sin\beta\cos\varphi\,\hat{x} - \sin\beta\sin\varphi\,\hat{y} + \cos\beta\hat{z}$, $\hat{e}_\varphi = \cos\varphi\,\hat{x} + \sin\varphi\,\hat{y}$

$\hat{e}_\eta = \cos\beta\,\hat{e}_\beta + \sin\beta\hat{z}$, $\hat{e}_\beta = -\sin\beta\,\hat{e}_\eta + \cos\beta\hat{z}$.

$\vec{G}$ (Glad) $= 16\pi^2 a\left[\frac{\pi}{2}\hat{e}_\beta + \hat{e}_\varphi\right] = 16\pi^2 a\left[\frac{\pi}{2}(-\sin\beta\,\hat{e}_\eta + \cos\beta\hat{z}) + \hat{e}_\varphi\right]$. $\vec{G}$ (Glad) $\neq 0$!

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В цилиндр. координатах $\vec{G}$(Glad): $G_\rho = -\frac{\pi}{2}\sin\beta$, $G_\varphi = 1$, $G_z = \frac{\pi}{2}\cos\beta(\hat{z})$ in units $16\pi^2 a$.

Gladikowski'97 и/ли helicity $\nu_0(\eta,\beta) = \text{const}$:

$m_x + i m_y = \sqrt{1-w^2(\eta,\beta)}\, e^{i[N\varphi + M\beta + \nu_0]}$, $m_z = w(\eta,\beta)$, $Q_{II} = NM$ целое и/ли $\nu_0 = \text{const}$

Hietarinta'2000: $m_x + i m_y = \frac{2u}{1+|u|^2}$, $m_z = \frac{|u|^2-1}{|u|^2+1}$, $u(\eta,\beta,\varphi) = f(\eta) e^{i(m\beta + n\varphi)}$

и/ли $\vec{m}(\theta,\Phi)$, $W = \operatorname{tg}\frac{\theta}{2} e^{i\Phi}$, $m_x + i m_y = \frac{2W}{1+|W|^2}$, $m_z = \frac{1-|W|^2}{1+|W|^2}$. Standard, кам в 2D soliton theory.

$m_z = \frac{u\bar{u}-1}{u\bar{u}+1}$, $u = \frac{1}{w} = \frac{1/|W|^2 - 1}{1/|W|^2+1} = \frac{1-|W|^2}{1+|W|^2}$, u, W - eqs. эквивалентны.

и/ли $m_z = \cos\Theta$, Glad'97 $m_x + i m_y = \sin\Theta\, e^{i[N\varphi + M\beta + \nu_0]}$, $m_z = \cos\Theta = w(\eta,\beta)$.

Нетривиальность [Glad.] $\Phi(\beta,\varphi) = N\varphi + M\beta + \nu_0$ - Wang'2019, $\Theta = \Theta(\rho,z)$, $\Phi(\vec{r}) = q\varphi + \gamma(\rho,z)$.

$m_z = \cos\Theta$, $\Theta = \Theta(\eta)$.

Соотношение м/у Φ - Gladkowski / Hietarinta с Φ - Wang-Braatas'19?

Связь цил. (ρ,φ,z) и тороид. координат (η,β,φ): $\rho = a\frac{\sinh\eta}{\tau}$, $z = \frac{a\sin\beta}{\tau}$, $\varphi = \varphi$, $\tau = \cosh\eta - \cos\beta$.

$z + i\rho = ia\coth\frac{1}{2}(\eta + i\beta)$, $\eta + i\beta = \ln\frac{z + i(\rho+a)}{z + i(\rho-a)}$, $\beta = \operatorname{sign}(z)\arccos\frac{r^2 - a^2}{\sqrt{(r^2-a^2)^2 + 4a^2z^2}}$, $r^2 = \rho^2 + z^2$

$\eta = \operatorname{Re}\ln\frac{z + i(\rho+a)}{z + i(\rho-a)} = \eta(z,\rho)$, $\Theta(\eta) = \Theta(\rho,z)$ - получено.

Тогда $N=1$, $M=1$, $\Phi(\beta,\varphi) = \varphi + \beta + \nu_0$ совпад. с Wang'19 $\Phi(\vec{r}) = \varphi + \gamma(\rho,z)$

$\beta = \beta(\rho,z)$, $\beta(\rho,z) + \nu_0 = \gamma(\rho,z)$ - больше получено, чем ожидали. $\gamma(\rho,z)$.

Вывод: Φ-angle of $\vec{m}$ by Gladkowski (тороид. коорд) и Φ-Wang'19 (цил. координаты) совпадают, т.к. $\Theta = \Theta(z,\rho)$ и $\beta = \beta(\rho,z)$, η и β тороид. коорд. не зависят от φ.

(17/II) Gladkowski'97 и Hietarinta'2000 дают $W(\eta,\beta) = m_z(\eta,\beta)$ исчезающие на бесконечности boundary conditions ① $W(\eta=0,\beta) = 1$, $m_z = 1$ на $z=0$ оси. $\eta = 0 \to \rho = 0$, z-any, φ - any. Θz ось

② $W(\infty,\beta) = -1$, $\forall\beta$ $m_z = -1$ на окружности $\rho = a$ в xOy плоскости, $\rho(\eta=\infty) = a$, $z = 0$, any φ

Именно Glad'97 и Hiet'2000 работают с $\rho \to \infty$ (at finite z) limit.

$\eta(\rho\to\infty) = \operatorname{Re}\ln\left[\frac{z + i(\rho+a)}{z + i(\rho-a)}\right] = \operatorname{Re}\ln\frac{i\rho}{i\rho} = \operatorname{Re}\ln 1 = 0$, $\eta(\rho\gg a) = \operatorname{Re}\ln\left[\frac{z+i\rho}{z+i\rho}\right] = 0$, any z

$\beta(\rho\gg a) = \operatorname{Im}\ln\left[\frac{z + i(\rho+a)}{z + i(\rho-a)}\right] = \operatorname{Im}\ln\left[\frac{z+i\rho}{z+i\rho}\right] = \operatorname{Im}\ln 1 = 0$, $\beta(\rho\gg a) = 0$.

$z = a\frac{\sin\beta}{\cosh\eta - \cos\beta} = \left|\rho\gg a,\ \eta = 0,\ \beta \to 0\right| = a\frac{\sin\beta}{1-\cos\beta} = a\frac{\sin\beta}{2\sin^2\frac{\beta}{2}} = a\frac{2}{\beta} \to \infty$, $\beta \to 0$.

③ $\rho \gg a$, $\eta = 0$, $w(\rho,\beta) = m_z(\rho,\beta) = 1$ ген. в conditions by Wang-Braatas'19

// В $\vec{m}$ - Hietarinta'1999, в m_x, m_y не нуж. общего решения, см. 14/$\bar{V}$ 2020.

Wang-Braatas'2019: $\Theta = \Theta(\rho,z)$, $\Phi(\vec{r}) = \Delta\Phi(\rho,z) + n\varphi = n\varphi + \gamma(\rho,z)$ - доказано.

Emergent mag. field $\vec{B} = \sin\Theta\,\nabla\Theta\times\nabla\Phi$ и/ли $\vec{m} = \vec{m}(\Theta,\Phi)$.

Cylindr. coordinates $\nabla f = \frac{\partial f}{\partial\rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial\varphi}\hat{\varphi} + \frac{\partial f}{\partial z}\hat{z}$. $\nabla\Theta(\rho,z) = \frac{\partial\Theta}{\partial\rho}\hat{\rho} + \frac{\partial\Theta}{\partial z}\hat{z}$

$\nabla\Phi(\vec{r}) = \frac{\partial\gamma}{\partial\rho}\hat{\rho} + \frac{n}{\rho}\hat{\varphi} + \frac{\partial\gamma}{\partial z}\hat{z}$

$\nabla\Theta\times\nabla\Phi = \begin{pmatrix} \hat{\rho} & \hat{\varphi} & \hat{z} \\ \partial\Theta/\partial\rho & 0 & \partial\Theta/\partial z \\ \partial\gamma/\partial\rho & n/\rho & \partial\gamma/\partial z \end{pmatrix} = \hat{\varphi}\left[\frac{\partial\Theta}{\partial z}\frac{\partial\gamma}{\partial\rho} - \frac{\partial\Theta}{\partial\rho}\frac{\partial\gamma}{\partial z}\right] + \hat{\rho}\left[-\frac{n}{\rho}\frac{\partial\Theta}{\partial z}\right] + \hat{z}\left[\frac{n}{\rho}\frac{\partial\Theta}{\partial\rho}\right]$

$B_\rho = \sin\Theta\frac{(-n)}{\rho}\frac{\partial\Theta}{\partial z}$, $B_\varphi = \sin\Theta\,\ldots = \sin\Theta\left[\frac{\partial\Theta}{\partial z}\frac{\partial\gamma}{\partial\rho} - \frac{\partial\Theta}{\partial\rho}\frac{\partial\gamma}{\partial z}\right]$, $B_z = \sin\Theta\frac{n}{\rho}\frac{\partial\Theta}{\partial\rho}$

Это $\vec{B}$ (цил.) $= \sin\Theta(\nabla\Theta\times\nabla\Phi)$ совпадает с B_ρ, B_φ, B_z у Wang-Braatas'2019

$G_\rho = \int dV B_\rho = \int d\rho\int d\varphi\int dz\,\frac{(-n)}{\rho}\sin\Theta\frac{\partial\Theta}{\partial z} = -2\pi n\int d\rho\int dz\,\sin\Theta\frac{\partial\Theta}{\partial z} = 2\pi n\int d\rho\int dz\,\frac{\partial\cos\Theta}{\partial z}$

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$G_\varphi = 2\pi n \int_0^R d\varrho \int_{-L/2}^{L/2} dz \frac{\partial m_z(\varrho,z)}{\partial z} = 2\pi n \int_0^R d\varrho \left[m_z(\varrho,\frac{L}{2}) - m_z(\varrho,-\frac{L}{2}) \right] = 0$ т.к. $m_z(\varrho,z)$ even по z

$G_\varphi = \int d\varrho\varrho \int d\varphi \int dz \sin\theta \left[\frac{\partial\theta}{\partial z}\frac{\partial\gamma}{\partial\varrho} - \frac{\partial\theta}{\partial\varrho}\frac{\partial\gamma}{\partial z} \right] = 2\pi \int d\varrho\varrho \int dz \sin\theta \left[\frac{\partial\theta}{\partial z}\frac{\partial\gamma}{\partial\varrho} - \frac{\partial\theta}{\partial\varrho}\frac{\partial\gamma}{\partial z} \right]$

$\gamma(\varrho,z) = \beta(\varrho,z)$ при $v_0 = 0$. $\beta(\varrho,z) = \mathrm{sign}(z) \arccos \frac{(r^2 - a^2)}{\sqrt{(r^2-a^2)^2 + 4a^2z^2}}$ из

$r^2 = \varrho^2 + z^2$

odd функция от "z".

$\int_{-L/2}^{L/2} dz \sin\theta \frac{\partial\theta}{\partial z}\frac{\partial\gamma}{\partial\varrho} = -\int_{-L/2}^{L/2} dz \frac{\partial m_z(\varrho,z)}{\partial z} \frac{\partial\gamma(\varrho,z)}{\partial\varrho} \neq 0$ как $\int_{-L/2}^{L/2} dz$ (even $\cdot$ odd) от z

$\int_{-L/2}^{L/2} dz \sin\theta \frac{\partial\theta}{\partial\varrho}\frac{\partial\gamma}{\partial z} \neq 0$, т.к. $\frac{\partial\gamma(\varrho,z)}{\partial z}$ is even f. от z. $\int d\varrho\varrho \int dz \frac{\partial m_z}{\partial\varrho}\frac{\partial\gamma}{\partial z} \cdot \frac{\partial\gamma}{\partial z} = \frac{\partial\beta}{\partial z}$

$G_\varphi \neq 0$ в общем случае для тороидальной конфигурации $\theta(\varrho,z)$, $\Phi = n\varphi + \gamma(\varrho z)$

$G_z = \int d\varrho\varrho \int d\varphi \int dz \frac{n}{\varrho} \sin\theta \frac{\partial\theta}{\partial\varrho} = -2\pi n \int_0^R d\varrho \int_{-L/2}^{L/2} dz \frac{\partial}{\partial\varrho}(\cos\theta) = -2\pi n \int_{-L/2}^{L/2} dz \left[m_z(0,z) - m_z(R,z) \right]$

при $R \to \infty$ $G_z(R) \to 0$, т.к. $m_z(0,z) = m_z(\infty,z) = 1$. $m_z(\gamma=0, z) = 1$ и

$\gamma = 0$ при $\varrho = 0$ и $\varrho \to \infty$. $G_z = 0$ at Rinf.

Вывод: для тор. конфигурации $\theta(\varrho,z)$, $\Phi = n\varphi + \gamma(\varrho,z)$, $G_\varrho = 0$, $G_z = 0$, но $G_\varphi \neq 0$ в общем случае. $v_0 = 0$ не существенно, т.к. входят $\frac{\partial\gamma}{\partial\varrho}, \frac{\partial\gamma}{\partial z}$ в G_φ-деф.

Wang-Brooks'19: $A_\varrho = -(1+\cos\theta)\frac{\partial\gamma}{\partial\varrho}$, $A_\varphi = \frac{n}{\varrho}(1-\cos\theta)$, $A_z = -(1+\cos\theta)\frac{\partial\gamma}{\partial z}$

$\mathrm{rot}\vec{A} = \left(\frac{1}{\varrho}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z} \right)\hat{\varrho} + \left(\frac{\partial A_\varrho}{\partial z} - \frac{\partial A_z}{\partial\varrho} \right)\hat{\varphi} + \frac{1}{\varrho}\left[\frac{\partial(\varrho A_\varphi)}{\partial\varrho} - \frac{\partial A_\varrho}{\partial\varphi} \right]\hat{z}$. $\vec{A} = \vec{A}(\varrho,z)$

$(\mathrm{rot}\vec{A})_\varrho = \frac{1}{\varrho}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z} = -\frac{\partial A_\varphi}{\partial z} = -\frac{\partial}{\partial z}\frac{n}{\varrho}(1-\cos\theta) = \frac{n}{\varrho}\frac{\partial}{\partial z}\cos\theta = -\frac{n}{\varrho}\sin\theta\frac{\partial\theta}{\partial z} = B_\varrho$ - верно.

$(\mathrm{rot}\vec{A})_\varphi = \frac{\partial A_\varrho}{\partial z} - \frac{\partial A_z}{\partial\varrho} = \frac{\partial}{\partial z}(-1)(1+\cos\theta)\frac{\partial\gamma}{\partial\varrho} - \frac{\partial}{\partial\varrho}(-1)(1+\cos\theta)\frac{\partial\gamma}{\partial z} =$

$= \sin\theta\frac{\partial\theta}{\partial z}\frac{\partial\gamma}{\partial\varrho} + (-1)(1+\cos\theta)\frac{\partial^2\gamma}{\partial z\partial\varrho} - \sin\theta\frac{\partial\theta}{\partial\varrho}\frac{\partial\gamma}{\partial z} - (-1)(1+\cos\theta)\frac{\partial^2\gamma}{\partial\varrho\partial z} =$

$= \sin\theta\left(\frac{\partial\theta}{\partial z}\frac{\partial\gamma}{\partial\varrho} - \frac{\partial\theta}{\partial\varrho}\frac{\partial\gamma}{\partial z} \right) = B_\varphi$ - верно.

$(\mathrm{rot}\vec{A})_z = \frac{1}{\varrho}\frac{\partial(\varrho A_\varphi)}{\partial\varrho} = \frac{1}{\varrho}\frac{\partial}{\partial\varrho} n(1-\cos\theta) = \frac{n}{\varrho}\sin\theta\frac{\partial\theta}{\partial\varrho} = B_z$ - верно.

$\vec{A}$ - Wang'2019 записан верно, в системе $\mathrm{rot}\vec{A} = \vec{B}$.

$\int dV \vec{A}\cdot\vec{B} = \int dV \left[+(1+\cos\theta)\frac{\partial\gamma}{\partial\varrho}\sin\theta\frac{n}{\varrho}\frac{\partial\theta}{\partial z} + \frac{n}{\varrho}(1-\cos\theta)\sin\theta\left(\frac{\partial\theta}{\partial z}\frac{\partial\gamma}{\partial\varrho} - \frac{\partial\theta}{\partial\varrho}\frac{\partial\gamma}{\partial z} \right) + \right.$

$\left. + \frac{n}{\varrho}\sin\theta\frac{\partial\theta}{\partial\varrho}(-1)(1+\cos\theta)\frac{\partial\gamma}{\partial z} \right] = \int dV \frac{n}{\varrho}\sin\theta\left[(1+\cos\theta)\frac{\partial\gamma}{\partial\varrho}\frac{\partial\theta}{\partial z} + (1-\cos\theta)\left(\frac{\partial\theta}{\partial z}\frac{\partial\gamma}{\partial\varrho} - \frac{\partial\theta}{\partial\varrho}\frac{\partial\gamma}{\partial z} \right) \right.$

$\left. + (-1)(1+\cos\theta)\frac{\partial\theta}{\partial\varrho}\frac{\partial\gamma}{\partial z} \right] = \int dV \frac{n}{\varrho}\sin\theta\left[2\left(\frac{\partial\gamma}{\partial\varrho}\frac{\partial\theta}{\partial z} - \frac{\partial\theta}{\partial\varrho}\frac{\partial\gamma}{\partial z} \right) \right] = 4\pi n \int d\varrho \int dz \sin\theta \left[\frac{\partial\gamma}{\partial\varrho}\frac{\partial\theta}{\partial z} - \frac{\partial\theta}{\partial\varrho}\frac{\partial\gamma}{\partial z} \right]$ +

- обнуляется с Eq. (4.10) Wang'2019.

$G_\varphi = 2\pi \int d\varrho\varrho \int dz \sin\theta \left[\frac{\partial\theta}{\partial z}\frac{\partial\gamma}{\partial\varrho} - \frac{\partial\theta}{\partial\varrho}\frac{\partial\gamma}{\partial z} \right]$, $\int dV \vec{A}\cdot\vec{B} = 4\pi n \int d\varrho \int dz \sin\theta \left[\frac{\partial\theta}{\partial z}\frac{\partial\gamma}{\partial\varrho} - \frac{\partial\theta}{\partial\varrho}\frac{\partial\gamma}{\partial z} \right]$

$G_\varphi \sim \int dV \vec{A}\cdot\vec{B}$ содержат похожие выражения, отлич. лишь ϱ-множителем.

$\int dV \vec{A}\cdot\vec{B} \neq 0$, поэтому и $G_\varphi \neq 0$ $G_\varphi = 2\pi \int_0^R d\varrho\varrho \int_{-\frac{L}{2}}^{\frac{L}{2}} dz \left[\frac{\partial m_z}{\partial\varrho}\frac{\partial\gamma}{\partial z} - \frac{\partial\gamma}{\partial\varrho}\frac{\partial m_z}{\partial z} \right]$

Note: 16/II. В начале обсуждали единичные векторы тороидальной и сферической систем координат: (ϱ,β,φ) (ϱ,φ,z)

$\hat{e}_\beta = -\sin\beta\hat{e}_\varrho + \cos\beta\hat{z}$. Но $\beta = \beta(\varrho,z) = \mathrm{sign}(z)\arccos\frac{(r^2-a^2)}{\sqrt{(r^2-a^2)^2+4a^2z^2}}$ $r^2 = \varrho^2 + z^2$

Переход от $\vec{G}(\gamma\beta,\varphi)$ к $\vec{G}(\varrho\varphi z)$ is coordinate-dependent.

$G_\varphi = 1$, но $G_\varrho(\varrho,z) = -\frac{\pi}{2}\sin\beta(\varrho,z)$, $G_z = \frac{\pi}{2}\cos\beta(\varrho,z)$ in units $16\pi^2 a$.

Вторичное поле должно описываться переходом $(B_\gamma, B_\beta, B_\varphi)$ к $(B_\varrho, B_\varphi, B_z)$ а потом и $\vec{G} = \int dV \vec{B}$ уже привычно по теореме.

Поэтому $\int dV \vec{B} = \vec{G} = \oint_S d\vec{S} \times \vec{A}$ - kind of Gauss theorem, $\vec{B} = \mathrm{rot}\vec{A}$.

$\vec{B} = B_\gamma\hat{e}_\gamma + B_\beta\hat{e}_\beta + B_\varphi\hat{e}_\varphi = B_\gamma(\cos\beta\hat{e}_\varrho + \sin\beta\hat{e}_z) + B_\beta(-\sin\beta\hat{e}_\varrho + \cos\beta\hat{e}_z) + B_\varphi\hat{e}_\varphi$ — 2

$B_\varrho = B_\gamma\cos\beta - B_\beta\sin\beta$ Нет

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(18/II) Связь toroidal и Cartesian (cylindrical) coordinates я начинаю
(y,β,φ) (x,y,z) (ρ,φ,z)

16/II Будучи отвлечься к standard toroidal/poloidal coordinates,

где $x=(R_0+y\cos\beta)\cos\varphi$, $y=s_\varphi(R_0+y\sin\beta)\sin\varphi$, $z=s_\beta y\sin\beta$,
$s_\varphi=\pm1$, $s_\beta=\pm1$ [wiki, toroidal and poloidal coordinates]

A toroidal coordinates: $x=a\frac{\sinh y\cos\varphi}{\tau}$, $y=a\frac{\sinh y\sin\varphi}{\tau}$, $z=a\frac{\sin\beta}{\tau}$, $\tau=\operatorname{ch}y-\cos\beta$
(y,β,φ)

и ли $\rho=a\frac{\operatorname{sh}y}{\tau}$, $z=a\frac{\sin\beta}{\tau}$, $\varphi=\varphi$, $\tau=\operatorname{ch}y-\cos\beta$ как в Gladkowski 1997.

Корн, гл. 6: toroidal coordinates: $\beta=\frac{i}{2}\ln\left[\frac{\rho^2+(z-ia)^2}{\rho^2+(z+ia)^2}\right]$, $y=\frac{1}{2}\ln\left[\frac{z^2+(\rho+a)^2}{z^2+(\rho-a)^2}\right]$
(y,β,φ)

Note: $\beta=\beta(\rho,z)$, $y=y(\rho,z)$ и не забудьте ось φ, (ρ,φ,z) cyl. coord.
$\beta(\rho,z)=\operatorname{Im}\ln\left[\frac{z+i(\rho+a)}{z+i(\rho-a)}\right]$ - из Wiki.

Morse-Feshbach, ch. 1: Vector in orthogonal curvilinear coordinate system $\vec F=\sum_m F_m\vec a_m$, $F_m=\vec F\cdot\vec a_m$, $F_m=\gamma_{m1}F_x+\gamma_{m2}F_y+\gamma_{m3}F_z$, $\vec F(F_x,F_y,F_z)$

$\gamma_{n1}=\frac{1}{h_n}\frac{\partial x}{\partial\xi_n}=h_n\frac{\partial\xi_n}{\partial x}$, $\gamma_{n2}=\frac{1}{h_n}\frac{\partial y}{\partial\xi_n}=h_n\frac{\partial\xi_n}{\partial y}$, $\gamma_{n3}=\frac{1}{h_n}\frac{\partial z}{\partial\xi_n}=h_n\frac{\partial\xi_n}{\partial z}$

$ds^2=dx^2+dy^2+dz^2=\sum_n h_n^2 d\xi_n^2$, $h_n^2=g_{nn}$, g_{ik} - tensor is diagonal

toroidal coordinates: $x=\frac{a}{\tau}\operatorname{sh}y\cos\varphi$, $y=\frac{a}{\tau}\operatorname{sh}y\sin\varphi$, $z=\frac{a}{\tau}\sin\beta$, $\tau=\operatorname{ch}y-\cos\beta$
(yβφ)

$(\xi_1,\xi_2,\xi_3)=(y\beta\varphi)$. Я хотел сравнить не с Cartesian coord. (xyz), а с cylindrical $(\rho\varphi z)$ coordinates: $\rho=a\frac{\operatorname{sh}y}{\tau}$, $\varphi=\varphi$, $z=a\frac{\sin\beta}{\tau}$

$F_m=\gamma_{m\rho}F_\rho+\gamma_{m\varphi}F_\varphi+\gamma_{mz}F_z$, $\vec F(F_\rho,F_\varphi,F_z)=\vec F(F_y,F_\beta,F_\varphi)$

$\gamma_{m\rho}=\frac{1}{h_m}\frac{\partial\rho}{\partial\xi_m}$, $\gamma_{m\varphi}=\frac{1}{h_m}\frac{\partial\varphi}{\partial\xi_m}$, $\gamma_{mz}=\frac{1}{h_m}\frac{\partial z}{\partial\xi_m}$, $(\xi_1,\xi_2,\xi_3)=(y\beta\varphi)$

$\gamma_{y\rho}=\frac{1}{h_y}\frac{\partial\rho}{\partial y}=\frac{1}{h_y}\frac{\partial}{\partial y}a\frac{\operatorname{sh}y}{\tau}=\frac{a}{h_y}\frac{\operatorname{ch}y(\operatorname{ch}y-\cos\beta)-\operatorname{sh}y\cdot\operatorname{sh}y}{\tau^2}$ ξ_n: $n=1-y$, $n=2-\beta$, $n=3-\varphi$

$\gamma_{y\rho}=\frac{a}{h_y}\frac{1-\operatorname{ch}y\cos\beta}{\tau^2}$, $\gamma_{\beta\rho}=\frac{1}{h_\beta}\frac{\partial\rho}{\partial\beta}=\frac{a}{h_\beta}\frac{\partial}{\partial\beta}\frac{\operatorname{sh}y}{\tau}=\frac{a\operatorname{sh}y}{h_\beta}\left(-\frac{1}{\tau^2}\right)\sin\beta$ Cyl. $\left(\begin{matrix}h_\rho=h_z=1\\h_\varphi=\rho\end{matrix}\right)$

$\gamma_{\varphi\rho}=\frac{1}{h_\varphi}\frac{\partial\rho}{\partial\varphi}=0$, $h_y=h_\beta=\frac{a}{\tau}$, $h_\varphi=\frac{a\operatorname{sh}y}{\tau}$, $\tau=\operatorname{ch}y-\cos\beta$.

$\gamma_{y\rho}=\frac{a}{a}\frac{a^2\tau(1-\operatorname{ch}y\cos\beta)}{\tau^2}=\frac{1}{\tau}(1-\operatorname{ch}y\cos\beta)$, $\gamma_{\beta\rho}=\frac{a}{a}\tau\operatorname{sh}y\frac{(-1)}{\tau^2}\sin\beta=-\frac{1}{\tau}\operatorname{sh}y\sin\beta$, $\gamma_{\varphi\rho}=0$

$F_y=\gamma_{y\rho}F_\rho+\gamma_{y\varphi}F_\varphi+\gamma_{yz}F_z$ с учетом $\gamma_{y\rho}$, $\gamma_{\beta\rho}$, $\gamma_{\varphi\rho}$. $(xyz)\to(y\beta\varphi)$

Я меня задача (F_y,F_β,F_φ) и надо перейти к (F_ρ,F_φ,F_z) $n=(\rho\varphi z)$

$F_n=\gamma_{ny}F_y+\gamma_{n\beta}F_\beta+\gamma_{n\varphi}F_\varphi$. $\gamma_{ny}=h_n\frac{\partial\xi_n}{\partial y}$, $\gamma_{n\beta}=h_n\frac{\partial\xi_n}{\partial\beta}$, $\gamma_{n\varphi}=h_n\frac{\partial\xi_n}{\partial\varphi}$ ξ_n

$\gamma_{\rho y}=h_\rho\frac{\partial\rho}{\partial y}=a\operatorname{sh}y\left(\frac{1-\operatorname{ch}y\cos\beta}{\tau^2}\right)$, $\gamma_{\varphi y}=h_\varphi\frac{\partial\varphi}{\partial y}=0$, $\gamma_{zy}=h_z\frac{\partial z}{\partial y}$. Нужно найти $\frac{\partial\xi_n}{\partial y}$.

$\frac{\partial z}{\partial y}=\frac{\partial}{\partial y}a\frac{\sin\beta}{\tau}=a\sin\beta\frac{(-1)}{\tau^2}\operatorname{sh}y$, $\gamma_{zy}=h_z\frac{(-1)}{\tau^2}a\operatorname{sh}y\sin\beta$. $h_\rho=h_z=1$, $h_\varphi=\rho$.

$F_\rho=\gamma_{\rho y}F_y+\gamma_{\rho\beta}F_\beta+\gamma_{\rho\varphi}F_\varphi=\frac{a}{\tau^2}(1-\operatorname{ch}y\cos\beta)F_y+\gamma_{\rho\beta}F_\beta+\gamma_{\rho\varphi}F_\varphi$

$\gamma_{\rho\beta}=h_\rho\frac{\partial\rho}{\partial\beta}=h_\rho a\operatorname{sh}y\frac{\partial}{\partial\beta}\left(\frac{1}{\tau}\right)=h_\rho a\operatorname{sh}y\left(\frac{1}{\tau^2}\right)\sin\beta$, $\gamma_{\rho\varphi}=h_\rho\frac{\partial\rho}{\partial\varphi}=0$.

$F_\rho=\frac{a}{\tau^2}(1-\operatorname{ch}y\cos\beta)F_y-\frac{a}{\tau^2}\operatorname{sh}y\sin\beta F_\beta$. Применим к $\vec B(B_y,B_\beta,B_\varphi)$ Glad.

$B_\rho=\frac{a}{\tau^2}(1-\operatorname{ch}y\cos\beta)\frac{2}{a^2}\frac{\tau^2}{\operatorname{sh}y}N\frac{\partial w}{\partial\beta}-\frac{a}{\tau^2}\operatorname{sh}y\sin\beta\left(-\frac{2}{a^2}\right)\frac{\tau^2}{\operatorname{sh}y}N\frac{\partial w}{\partial y}=$

$=\frac{2}{a}(1-\operatorname{ch}y\cos\beta)N\frac{\partial w}{\partial\beta}\frac{1}{\operatorname{sh}y}+\frac{2}{a}\sin\beta N\frac{\partial w}{\partial y}$, $w(y)=1-2\operatorname{th}^2y$, $\partial w/\partial y=-4\operatorname{th}y/\operatorname{ch}^2y$

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$$G_\beta = \int dV B_\beta = \int_0^\infty dy \int_{-\pi}^{\pi} d\beta \int_0^{2\pi} d\varphi \frac{a^3}{\tau^3} \sinh y \frac{2}{a} N \frac{\partial w}{\partial y} \left[\frac{(1 - \cosh y \cos\beta)}{\sinh y} + \sin\beta \right] =$$

$$= 2\pi \cdot 2a^2 N \int_0^\infty dy \int_{-\pi}^{\pi} d\beta \frac{\sinh y}{(\cosh y - \cos\beta)^3} (-4) \frac{\tanh y}{\cosh^2 y} \left[\frac{(1 + \cosh y \cos\beta)}{\sinh y} + \sin\beta \right] =$$

$$= 4\pi a^2 N (-4) \int_0^\infty dy \int_{-\pi}^{\pi} d\beta \frac{\sinh^4 y}{(\cosh y - \cos\beta)^3} \frac{1}{\cosh^3 y} \left[1 - \cosh y \cos\beta + \sinh y \sin\beta \right]$$

$$G_\beta = -16\pi a^2 N \int_0^\infty dy \int_{-\pi}^{\pi} d\beta \frac{1}{(\cosh y - \cos\beta)^3} \frac{\tanh y}{\cosh^2 y} \left[1 - \cosh y \cos\beta + \sinh y \sin\beta \right]$$

$$J_\beta(y) = \int_{-\pi}^{\pi} d\beta \frac{1}{(\cosh y - \cos\beta)^3} \left[1 - \cosh y \cos\beta + \sinh y \sin\beta \right] = 0?$$ как $\int$ от нечет. ф-ции по нечет. T.

$$\int_{-\pi}^{\pi} d\beta \sin\beta \frac{1}{(\cosh y - \cos\beta)^3} = \left| \cos\beta = x \right| = -\int_{-1}^{-1} dx \frac{1}{(\cosh y - x)^3} \equiv 0$$ как $\int$ от нечет. ф-ции

$$\int_{-\pi}^{\pi} d\beta \frac{(1 - \cosh y \cos\beta)}{(\cosh y - \cos\beta)^3} = \int_{-\pi}^{\pi} \frac{d\beta \sin\beta}{\sqrt{1 - \cos^2\beta}} \frac{(1 - \cosh y \cos\beta)}{(\cosh y - \cos\beta)^3} = -\int_{-1}^{-1} dx \frac{(1 - \cosh y \cdot x)}{\sqrt{1 - x^2}(\cosh y - x)^3} = 0$$

$$J_\beta(y) = \int_{-\pi}^{\pi} d\beta \frac{(1 - \cos\beta \cdot \cosh y)}{(\cosh y - \cos\beta)^3}$$ MathCad дал $J_\beta(y \to 0) = -\infty$, $J_\beta(y > 3) = 0$.

11/IV 2020. $J(y) = + \int_{-\pi}^{\pi} d\beta \frac{1}{(\cosh y - \cos\beta)} = \frac{2\pi}{\sinh y}$, также сингулар. $y \to 0$.

$$J_\beta(y) = J_1(y) + J_2(y), \quad J_1(y) = \int_{-\pi}^{\pi} d\beta \frac{1}{(\cosh y - \cos\beta)^3} = \left| a = \frac{1}{\cosh y} \right| =$$

$$= 2a^3 \int_0^{\pi} d\beta \frac{1}{(1 + a\cos\beta)^3} \cdot \quad G_\beta = -16\pi a^2 N \int_0^\infty dy \int_{-\pi}^{\pi} d\beta \frac{1}{(\cosh y - \cos\beta)^3} \frac{\tanh y}{\cosh^2 y} (1 - \cosh y \cdot \cos\beta)$$

MathCad дал расходимость G_β при $y \to 0$ из-за $J_\beta(y) \to -\infty$ при $y \to 0$.

Проверим G_β - calc. Найдем G_z.

$$\gamma_{zy} = -\frac{a}{\tau^2} \sinh y \sin\beta, \quad \gamma_{z\beta} = h_2 \frac{\partial z}{\partial \beta} = \frac{\partial}{\partial \beta} a \frac{\sin\beta}{\tau} = \frac{a}{\tau^2} \left[\cos\beta (\cosh y - \cos\beta) + \sin\beta \sin\beta \right] =$$

$$= \frac{a}{\tau^2} (\cosh y \cos\beta - 1), \quad \gamma_{z\varphi} = h_2 \frac{\partial z}{\partial \varphi} = 0.$$

$$B_z = \gamma_{zy} B_y + \gamma_{z\beta} B_\beta + \gamma_{z\varphi} B_\varphi = \frac{a}{\tau^2} (\cosh y \cos\beta - 1) \frac{2N}{a^2} \frac{\tau^2}{\sinh y} \frac{\partial w}{\partial \beta} =$$

$$= -\frac{a}{\tau^2} \sinh y \sin\beta \frac{2N}{a^2} \frac{\tau^2}{\sinh y} \frac{\partial w}{\partial \beta}_{=0} + \frac{a}{\tau^2} (\cosh y \cos\beta - 1) \frac{(-1)}{a^2} N \frac{\tau^2}{\sinh y} \frac{\partial w}{\partial y} = -\frac{2N}{a} (\cos\beta \cosh y - 1) \frac{1}{\sinh y} \frac{\partial w}{\partial y}$$

$$B_z = -\frac{2N}{a} (\cosh y \cos\beta - 1) \frac{1}{\sinh y} (-4) \tanh y \frac{1}{\cosh^2 y} = \frac{8N}{a} (\cosh y \cos\beta - 1) \frac{1}{\cosh^3 y}$$

$$G_z = \int dV B_z = \int dy \int d\beta \int d\varphi \frac{a^3}{\tau^3} \sinh y \frac{8N}{a} (\cosh y \cos\beta - 1) \frac{1}{\cosh^3 y} = 16\pi a^2 N \int_0^\infty dy \int_{-\pi}^{\pi} d\beta \frac{(\cosh y \cos\beta - 1)}{(\cosh y - \cos\beta)^3} \frac{\tanh y}{\cosh^2 y}$$

Ошибка в G_β. $\frac{\partial w}{\partial \beta} = 0$, $B_\beta = \frac{2N}{a} \sin\beta \frac{\partial w}{\partial y}$.

$$G_\beta = \int dV B_\beta = \int dy \int d\beta \int d\varphi \frac{a^3}{\tau^3} \sinh y \frac{2N}{a} \frac{\partial w}{\partial y} \sin\beta \sim \int_{-\pi}^{\pi} d\beta \frac{1}{(\cosh\cosh y - \cos\beta)^3} \sin\beta \equiv 0 \ (!)$$

как $\int$ от нечет. функции в сим. пределах

Показали $G_\beta = 0$ (!). $B_\beta = \frac{2N}{a} \sin\beta \frac{\partial w}{\partial y}$ (Glad) должно быть равно B_β (Wang)

$$B_\beta(\text{Wang}) = -\frac{n}{\rho} \sin\theta \frac{\partial \theta}{\partial z} = \frac{n}{\rho} \frac{\partial (\cos\theta)}{\partial z} = \frac{n}{\rho} \frac{\partial m_z}{\partial z}, \quad m_z = w(y) = 1 - 2\tanh^2 y$$

$$B_\beta(\text{Glad}) = \frac{2N}{a} \sin\beta (-4) \frac{\tanh y}{\cosh^2 y} = \frac{2N}{a} \sin\beta \frac{\partial m_z}{\partial y}, \quad z = \frac{a}{\tau} \sin\beta, \quad dz = \frac{a \sin\beta}{(-\tau^2)} d\tau$$

$$\rho = \frac{\sinh y}{\tau} a \cdot d\tau = \frac{\sinh y \, dy}{\frac{dy}{d\tau} = -\frac{\tau^2}{a \sin\beta \sinh y}}$$

$$B_\beta(\text{Wang}) = n \frac{\tau}{a \sinh y} \frac{\partial m_z}{\partial y} \frac{\partial y}{\partial z} = n \frac{\tau}{a \sinh y} \frac{\partial m_z}{\partial y} \frac{(-\tau^2)}{a \sin\beta \sinh y}$$ (?)

Ничего не сходится

Исправлено (см. 11/IV 2020).

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19/II Наиден B_φ и $G_\varphi = \int dV B_\varphi$. $\gamma(\rho,z) = \Delta\Phi(\rho,z) = \beta(\rho,z) + \nu_0$ (helicity) Wang'19

Wiki: $\beta(\rho,z) = \mathrm{Im}\ln\left[\frac{z+i(\rho+a)}{z+i(\rho-a)}\right]$, Корн: $\beta(\rho,z) = \frac{i}{2}\ln\left[\frac{\rho^2+(z-ia)^2}{\rho^2+(z+ia)^2}\right]$

$\beta(\rho,z) = \mathrm{sign}(z)\arccos\left[\frac{(r^2-a^2)}{\sqrt{(r^2-a^2)^2+4a^2z^2}}\right]$, $r^2=\rho^2+z^2$. $\beta(\rho,z)$ – odd func on z

$B_\varphi = \sin\theta\left[\frac{\partial\theta}{\partial z}\frac{\partial\gamma}{\partial\rho} - \frac{\partial\theta}{\partial\rho}\frac{\partial\gamma}{\partial z}\right] = \frac{\partial m_z}{\partial\rho}\frac{\partial\gamma}{\partial z} - \frac{\partial m_z}{\partial z}\frac{\partial\gamma}{\partial\rho}$, $\gamma(\rho,z) = \beta(\rho,z) + \alpha\rho$

$m_z(\rho,z) = 1 - \frac{8\rho^2a^2}{(a^2+\rho^2+z^2)^2}$, $\frac{\partial\beta}{\partial z} = \mathrm{Im}\left[\frac{z+i(\rho-a)}{z+i(\rho+a)}\right]\frac{(z+i(\rho-a))-(z+i(\rho+a))\cdot 1}{[z+i(\rho-a)]^2} =$

$= \mathrm{Im}\frac{-2ia}{[z+i(\rho+a)][z+i(\rho-a)]} = \mathrm{Im}(-2ia)\frac{1}{(\xi+ia)(\xi-ia)}$, $\xi = z+i\rho$, $|\xi|^2 = r^2$

$\beta(\rho,z) = \beta(\xi) = \mathrm{Im}\ln\left(\frac{\xi+ia}{\xi-ia}\right)$, $\frac{\partial\beta(\xi)}{\partial\xi} = \mathrm{Im}\left(\frac{\xi-ia}{\xi+ia}\right)\frac{1\cdot(\xi-ia)-(\xi+ia)\cdot 1}{(\xi-ia)^2} =$

$= \mathrm{Im}\frac{(-2ia)}{(\xi+ia)(\xi-ia)} = \mathrm{Im}(-2ia)\frac{(\bar\xi-ia)(\bar\xi+ia)}{|\xi+ia|^2|\xi-ia|^2} = -\frac{2a}{|\xi+ia|^2|\xi-ia|^2}\mathrm{Im}\,i\left[\bar\xi^2+a^2\right] =$

$= -\frac{2a}{|\xi+ia|^2|\xi-ia|^2}\mathrm{Im}\,i\left[z^2-\rho^2-2iz\rho+a^2\right] = -\frac{2a}{|\xi+ia|^2|\xi-ia|^2}\left[z^2-\rho^2+a^2\right]$

$\frac{\partial\beta}{\partial\xi} = -\frac{2a}{|\xi+ia|^2|\xi-ia|^2}(z^2-\rho^2+a^2)$

$m_z = m_z(y)$, $\frac{\partial m_z}{\partial\rho} = \frac{\partial m_z}{\partial y}\frac{\partial y}{\partial\rho} = \left|_{y=\mathrm{Re}\ln\left(\frac{\xi+ia}{\xi-ia}\right)}\right.$, $\frac{\partial m_z}{\partial z} = \frac{\partial m_z}{\partial y}\frac{\partial y}{\partial z}$

$B_\varphi = \frac{\partial m_z}{\partial y}\frac{\partial y}{\partial\rho}\frac{\partial\beta}{\partial z} - \frac{\partial m_z}{\partial y}\frac{\partial y}{\partial z}\frac{\partial\beta}{\partial\rho} = \frac{\partial m_z}{\partial y}\left[\frac{\partial y}{\partial\rho}\frac{\partial\beta}{\partial z} - \frac{\partial y}{\partial z}\frac{\partial\beta}{\partial\rho}\right]$, $y=y(\xi)$, $\beta=\beta(\xi)$

$\frac{\partial y}{\partial\rho} = \frac{\partial y}{\partial\xi}\frac{\partial\xi}{\partial\rho} = i\frac{\partial y}{\partial\xi}$, $\frac{\partial y}{\partial z} = \frac{\partial y}{\partial\xi}\frac{\partial\xi}{\partial z} = \frac{\partial y}{\partial\xi}$, $\frac{\partial\beta}{\partial z} = \frac{\partial\beta}{\partial\xi}\frac{\partial\xi}{\partial z} - \frac{\partial\beta}{\partial\bar\xi}\frac{\partial\bar\xi}{\partial z}$, $\frac{\partial\beta}{\partial\rho} = \frac{\partial\beta}{\partial\xi}\frac{\partial\xi}{\partial\rho} = i\frac{\partial\beta}{\partial\xi}$

$B_\varphi = \frac{\partial m_z}{\partial y}i\left[\frac{\partial y}{\partial\xi}\frac{\partial\beta}{\partial\xi} - \frac{\partial y}{\partial\xi}\frac{\partial\beta}{\partial\bar\xi}\right] = 0$ (?). Винаги можем $y=y(\xi)$, $\beta(\xi)$.

$\beta(\rho,z) = \mathrm{sign}(z)\arccos\left[\frac{|\xi|^2-a^2}{\sqrt{(|\xi|^2-a^2)^2+4a^2z^2}}\right]$ Вече не е ясно $\beta(\xi)$ – "аналитична" "z" отделно

$Z(\rho,z) = y+i\beta = \ln\left(\frac{\xi+ia}{\xi-ia}\right) = Z(\xi)$, $\xi = z+i\rho$, $Z = y+i\beta$. $m_z = m_z(y)$ можем.

$B_\varphi = \frac{\partial m_z}{\partial y}\left[\frac{\partial y}{\partial\rho}\frac{\partial\beta}{\partial z} - \frac{\partial y}{\partial z}\frac{\partial\beta}{\partial\rho}\right]$. $y = \mathrm{Re}\,Z$, $\beta = \mathrm{Im}\,Z$. $m_z(y) = 1-2\mathrm{th}^2 y$.

$\frac{\partial\beta}{\partial z} = \mathrm{Im}(-2ia)\frac{1}{(\xi+ia)(\xi-ia)} = -2a\,\mathrm{Im}\,i\frac{(\bar\xi-ia)(\bar\xi+ia)}{|\xi+ia|^2|\xi-ia|^2} = -\frac{2a(z^2-\rho^2+a^2)}{|\xi+ia|^2|\xi-ia|^2}$

$\frac{\partial\beta}{\partial\rho} = \frac{\partial}{\partial\rho}\mathrm{Im}\ln\left[\frac{z+i(\rho+a)}{z+i(\rho-a)}\right] = -\mathrm{Im}\frac{z+i(\rho-a)}{z+i(\rho+a)}\frac{i(z+i(\rho-a))-(z+i(\rho+a))i}{[z+i(\rho-a)]^2} =$

$= \mathrm{Im}\frac{1}{[z+i(\rho+a)][z+i(\rho-a)]}(2a) = 2a\,\mathrm{Im}\frac{(\bar\xi-ia)(\bar\xi+ia)}{|\xi+ia|^2|\xi-ia|^2} = \frac{2a}{|+|^2|-|^2}\mathrm{Im}\left(\bar\xi^2+a^2\right) =$

$= \frac{2a}{|+|^2|-|^2}\mathrm{Im}\left[z^2-\rho^2-2iz\rho+a^2\right] = \frac{2a(-2z\rho)}{|\xi+ia|^2|\xi-ia|^2}$, $\frac{\partial\beta}{\partial\rho} = -\frac{4z\rho a}{|\xi+ia|^2|\xi-ia|^2}$

$\frac{\partial m_z}{\partial\rho} = \frac{\partial}{\partial\rho}\left[1-\frac{8\rho^2a^2}{(a^2+\rho^2+z^2)^2}\right] = -8a^2\frac{2\rho(a^2+z^2+\rho^2)^2 - \rho^2 2(a^2+z^2+\rho^2)2\rho}{(a^2+z^2+\rho^2)^4} =$

$= -16a^2\rho\frac{(a^2+z^2+\rho^2)-2\rho^2}{(a^2+z^2+\rho^2)^3} = -16a^2\rho\frac{(a^2+z^2-\rho^2)}{(a^2+z^2+\rho^2)^3}$

$\frac{\partial m_z}{\partial z} = \frac{\partial}{\partial z}\left[1-\frac{8\rho^2a^2}{(a^2+\rho^2+z^2)^2}\right] = -8\rho^2a^2\frac{\partial}{\partial z}\frac{1}{(a^2+\rho^2+z^2)^2} = -8\rho^2a^2\frac{(-2)}{(a^2+\rho^2+z^2)^3}\cdot 2z$

$B_\varphi = \frac{\partial m_z}{\partial\rho}\frac{\partial\beta}{\partial z} - \frac{\partial m_z}{\partial z}\frac{\partial\beta}{\partial\rho} = -16a^2\rho\frac{(a^2+z^2-\rho^2)}{(a^2+z^2+\rho^2)^3}\frac{(-2a)(z^2+\rho^2+a^2)}{|\xi+ia|^2|\xi-ia|^2} - \frac{32\rho^2a^2z}{(a^2+z^2+\rho^2)^3}(-1)\frac{4z\rho a}{|\xi+ia|^2|\xi-ia|^2} =$

$= \frac{32a^3\rho}{(a^2+z^2+\rho^2)^3|\xi+ia|^2|\xi-ia|^2}\left[(a^2+z^2-\rho^2)^2+4\rho^2z^2\right]$

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$|\zeta + ia|^2 = (\zeta + ia)(\bar\zeta - ia) = |\zeta|^2 + ia\bar\zeta - ia\zeta + a^2 = z^2+s^2+a^2 + ia\left[z - is - (z+is)\right] =$

$= z^2+s^2+a^2 + ia(-2is) = z^2+s^2+a^2+2as = z^2+(s+a)^2$, $\zeta = z+is$, $|\zeta - ia|^2 = z^2+(s-a)^2$

$$B_\varphi = \frac{32a^3 s}{(a^2+z^2+s^2)^3\left[z^2+(s+a)^2\right]\left[z^2+(s-a)^2\right]}\left[(a^2+z^2-s^2)^2 + 4z^2s^2\right]$$

$(a^2+z^2-s^2)^2 + 4z^2s^2 = a^4 + 2a^2(z^2-s^2) + (z^2-s^2)^2 + 4z^2s^2 = a^4 + 2a^2(z^2+s^2) + (z^2+s^2)^2$

$G_\varphi = \int dV\, B_\varphi(s,z) = 2\pi \int ds\, s \int_{-L/2}^{L/2} dz\, B_\varphi(s,z)$. Можно $R\to\infty$, $L\to\infty$, infinite sample.

$s/a \to s$, $z/a \to z$.

$$B_\varphi(s,z) = \frac{1}{a^2}\,\frac{32\,s}{(1+z^2+s^2)^3\left[z^2+(s+1)^2\right]\left[z^2+(s-1)^2\right]}\left[(1+z^2-s^2)^2 + 4z^2s^2\right]$$

$$G_\varphi = 2\pi a^3 \int_0^\infty ds\, s \int_{-\infty}^{\infty} dz\, B_\varphi(s,z) = 64\pi a \int_0^\infty ds\, s^2 \int_{-\infty}^{\infty} dz\, \frac{(1+z^2-s^2)^2 + 4z^2s^2}{(1+z^2+s^2)^3\left[z^2+(s+1)^2\right]\left[z^2+(s-1)^2\right]}$$

$B_\varphi(s,z)$ при $s \gg 1$, $B_\varphi(s\gg1) \sim \frac{s}{s^{10}} s^4 \sim \frac{1}{s^5}$, $\int_0^\infty ds\, s\, B_\varphi(s) \sim \int_0^\infty ds\, \frac{1}{s^4}$ сходится при $s\to\infty$.

Намагниченность $\beta(s,z)$.

G_φ – конечно $\Rightarrow$ поле – градиентное, магн.

$$B_z = \frac{n}{s}\sin\theta\frac{\partial\theta}{\partial s} = -\frac{n}{s}\frac{\partial(\cos\theta)}{\partial s} = -\frac{n}{s}\frac{\partial m_z}{\partial s},\quad \frac{\partial m_z}{\partial s} = -16a^2 s\,\frac{(a^2+z^2-s^2)}{(a^2+z^2+s^2)^3}$$

i/ii: $G_z = -2\pi n \int_{-L/2}^{L/2} dz\,\left[m_z(0,z) - m_z(R,z)\right]$, $m_z(s,z) = 1 - \frac{8s^2a^2}{(a^2+s^2+z^2)^2}$

$$G_z = -2\pi n \int_{-L/2}^{L/2} dz\left[1 - \left(1 - \frac{8R^2a^2}{(a^2+R^2+z^2)^2}\right)\right] = -2\pi n\, 8R^2a^2 \int_{-L/2}^{L/2} \frac{dz}{(a^2+R^2+z^2)^2} = |n=1| =$$

$$= -16\pi R^2a^2 \int dz\,\frac{1}{(a^2+R^2+z^2)^2} = -16\pi R^2a^2\left[\frac{z}{2(a^2+R^2)(a^2+R^2+z^2)} + \frac{1}{2(a^2+R^2)^{3/2}}\operatorname{arctg}\frac{z}{\sqrt{a^2+R^2}}\right]_{-\frac{L}{2}}^{L/2} =$$

$$= -16\pi R^2a^2\left[\frac{L}{2(a^2+R^2)(a^2+R^2+L^2/4)} + \frac{1}{(a^2+R^2)^{3/2}}\operatorname{arctg}\frac{L}{2\sqrt{a^2+R^2}}\right],\quad R, L \gg a$$

$$G_z(R,L\gg a) = -16\pi R^2a^2\left[\frac{L}{2R^2(R^2+\frac{L^2}{4})} + \frac{1}{R^3}\operatorname{arctg}\frac{L}{2R}\right] = -16\pi a^2\left[\frac{L}{2(R^2+\frac{L^2}{4})} + \frac{1}{R}\operatorname{arctg}\frac{L}{2R}\right]$$

$$G_z(R\gg a) = -16\pi a^2\left[\frac{\beta}{2(1+\beta^2/4)}\frac{1}{R} + \frac{1}{R}\operatorname{arctg}\frac{\beta}{2}\right] = -16\pi a\left[\frac{\beta}{2(1+\beta^2/4)} + \operatorname{arctg}\frac{\beta}{2}\right]\left(\frac{a}{R}\right)$$

$G_z(R\gg a) \sim \left(\frac{a}{R}\right)^n \to 0$ при $R/a \to \infty$. $16\pi a$ – безразмерная порядка $|G_z|$.

Т. Пуассона/применение $\gamma + \delta$:

Тороид. координаты (γ, β, φ)

$s = \frac{a\sinh\gamma}{\tau}$, $z = \frac{a\sin\beta}{\tau}$, $\tau = \cosh\gamma - \cos\beta$, $\varphi = \varphi$

(s, φ, z) – цилиндр. с.

$z + is = ia\operatorname{cth}\frac{1}{2}(\gamma + i\beta)$ – конф. в-ть

$\gamma = \operatorname{arth}\left(\frac{2as}{s^2+z^2+a^2}\right)$, $\beta = \operatorname{arctg}\left(\frac{2az}{s^2+z^2-a^2}\right)$

$\operatorname{tg}\beta = x$, $\cos\beta = \frac{1}{\sqrt{1+\operatorname{tg}^2\beta}} = \frac{1}{\sqrt{1+\frac{4a^2z^2}{(r^2-a^2)^2}}} = \frac{(r^2-a^2)}{\sqrt{(r^2-a^2)^2+4a^2z^2}}$ – верно.

$\beta(s,z) = \operatorname{sign}(z)\arccos\frac{(r^2-a^2)}{\sqrt{(r^2-a^2)^2+4z^2a^2}}$, $r^2 = z^2+s^2$.

$\frac{\partial\beta}{\partial s} = \frac{1}{1+\frac{4a^2z^2}{(s^2+z^2-a^2)^2}}\frac{\partial}{\partial s}\left(\frac{2az}{s^2+z^2-a^2}\right)$ – см. выше

$\operatorname{cth}\frac{1}{2}(\gamma+i\beta) = \frac{\sinh\gamma - i\sin\beta}{\cosh\gamma - \cos\beta}$, $z + is = ia\frac{1}{\tau}(\sinh\gamma - i\sin\beta)$, $z = \frac{a}{\tau}\sin\beta$, $s = \frac{a}{\tau}\sinh\gamma$

$\frac{1}{ia}(z+is) = \frac{1}{a}(s - iz) = \operatorname{cth}\frac{1}{2}(\gamma+i\beta)$, $\frac{1}{2}(\gamma+i\beta) = \operatorname{arcth}\frac{1}{a}(s-iz) = \frac{1}{2}\ln\frac{(z+i(s+a))}{(z+i(s-a))}$

$F_x = \sum_{m=1,2,3}\gamma_{mx}F_m$, $F_y = \sum_m \gamma_{my}F_m$, $F_z = \sum_m \gamma_{mz}F_m$, $F_m = (F_\gamma, F_\beta, F_\varphi)$ тороидальные комп.

$$\begin{pmatrix} \gamma_{\gamma x} = K\cos\varphi, & \gamma_{\gamma y} = K\sin\varphi, & \gamma_{\gamma z} = L \\ \gamma_{\beta x} = L\cos\varphi, & \gamma_{\beta y} = L\sin\varphi, & \gamma_{\beta z} = -K \\ \gamma_{\varphi x} = -\sin\varphi, & \gamma_{\varphi y} = \cos\varphi, & \gamma_{\varphi z} = 0 \end{pmatrix}$$

$K = \frac{1}{\tau}(1 - \operatorname{ch}\gamma\cos\beta)$

$L = -\frac{1}{\tau}\operatorname{sh}\gamma\sin\beta$

$F_m \to F_x, F_y, F_z$ через сферич. комп.

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$F_x = \gamma_{xx} F_\varphi + \gamma_{x\beta} F_\beta + \gamma_{\varphi x} F_\varphi = K\cos\varphi F_\varphi + L\cos\varphi F_\beta - \sin\varphi F_\varphi$

$F_y = \gamma_{yy} F_\varphi + \gamma_{\beta y} F_\beta + \gamma_{\varphi y} F_\varphi = K\sin\varphi F_\varphi + L\sin\varphi F_\beta + \cos\varphi F_\varphi$

$A_\beta \hat\beta + A_\varphi \hat\varphi = A_x \hat x + A_y \hat y = A_x \hat x + A_y \hat y = A_x(\cos\varphi\hat\beta - \sin\varphi\hat\varphi) + A_y(\sin\varphi\hat\beta + \cos\varphi\hat\varphi)$

$A_\beta = A_x\cos\varphi + A_y\sin\varphi, \quad A_\varphi = -A_x\sin\varphi + A_y\cos\varphi$

$F_\beta = K\cos^2\varphi F_\varphi + L\cos^2\varphi F_\beta - \sin\varphi\cos\varphi F_\varphi + K\sin^2\varphi F_\varphi + L\sin^2\varphi F_\beta + \sin\varphi\cos\varphi F_\varphi$

$F_\beta = KF_\varphi + LF_\beta, \quad B_\beta = KB_\varphi + LB_\varphi = |B_\varphi = 0| = LB_\varphi, \quad B_\beta = -\frac{\sh\gamma\sin\beta}{\tau}\frac{z^2}{a^2}\frac{\partial w}{\partial\gamma}$

$B_\beta = -\frac{2}{a^2}\tau(\sh\gamma\sin\beta)\frac{\partial w}{\partial\gamma}$, ОР-method gaer $B_\beta = \frac{1}{\beta}\frac{\partial m_z}{\partial z}$, $m_z = w = 1 - 2\th^2\gamma$

$B_\beta = \frac{\tau}{a\sh\gamma}\frac{\partial m_z}{\partial\gamma}\frac{\partial\gamma}{\partial z}$, $z = \frac{a}{\tau}\sin\beta$, $dz = -\frac{1}{\tau^2}a\sin\beta\,d\tau = -\frac{1}{\tau^2}a\sin\beta\sh\gamma\,d\gamma$

$\frac{\partial\gamma}{\partial z} = -\frac{\tau^2}{a\sin\beta\sh\gamma}$, $B_\beta = \frac{\tau}{a\sh\gamma}\frac{\partial w}{\partial\gamma}\frac{(-\tau^3)}{a\sin\beta\sh\gamma}$ - не го.

$\frac{\partial m_z}{\partial z} = \frac{32\rho^2 a^2 z}{(a^2+\rho^2+z^2)^3}$, $\rho^2 + z^2 + a^2 = a^2 + \frac{a^2}{\tau}(\ch\gamma + \cos\beta) = \frac{a^2}{\tau}[\ch\gamma - \cos\beta + \ch\gamma + \cos\beta] =$

$= \frac{a^2}{\tau}2\ch\gamma$, $\frac{\partial m_z}{\partial z} = 32a^2\frac{a^2\sh^2\gamma}{\tau^2}\frac{a}{\tau}\sin\beta\frac{1}{\left[\frac{2a^2}{\tau}\ch\gamma\right]^3} = 32\tau^3\sh^2\gamma\sin\beta\frac{1}{8a^6\ch^3\gamma}$

$\frac{\partial m_z}{\partial z} = \frac{4}{a^4}\sin\beta\frac{\sh^2\gamma}{\ch^3\gamma}$, $B_\beta = \frac{1}{\beta}\frac{\partial m_z}{\partial z} = \frac{\tau}{a\sh\gamma}\frac{4}{a^4}\sin\beta\frac{\sh^2\gamma}{\ch^3\gamma} = \frac{4}{a^2}\tau\sin\beta\frac{\sh\gamma}{\ch^3\gamma}$ -

- нямае $B_\beta(\gamma,\beta)$ in toroidal coordinates.

Методът пулсиращи балони $B_\beta = -\frac{2\tau}{a^2}\sh\gamma\sin\beta\frac{\partial w}{\partial\gamma} = -\frac{2\tau}{a^2}\sh\gamma\sin\beta(-4)\frac{\th\gamma}{\ch^2\gamma} =$

$= \frac{8\tau}{a^2}\sin\beta\frac{\sh^2\gamma}{\ch^3\gamma}$ - нещо сетнагаен с ОР - B_β.

(20.27/III) Бъдеща Рибка (нощувки ниво и П. - 120 €). Artem's manuscript „6M thick dots" Subm. to Nat Commun. (20/III). Отдих, нощ и П. ферм P. Zoom с O. Klein, Vort. dynamics (21/III). E-mails to Paweł и Maciej о travel grant Poznan. переписка с Gonzalo Muga, ETH Quantum Center workshop June 12, 2023 in Laguardia, Riga. Кореспонденция SolSkyMag переписка с Окс. разходка MICINN project. Снег в Гоцата (27/III).

(28/II) Среща с Ronni - нещо „Hopfion gyrovector" paper.

Начална и крайна точка „free World line" Hopfion gyrovector" $\vec C = (0, G_\varphi, 0)$ to Poznań to Paweł plan of visit to Poznan June 25 - July 8 for travel grant appl.

(1/III) SB abstracts SolSkyMag. Reminders to invited speakers to send titles/abstr. Нещо дневник работа: 1) Разходка проект Окс.; 2) SolSkyMag'23 program; 3) UPV/EHU status „Distinguished scientist"; 4) реф. Barabash-EPJP и Müller-JMMM; 5) travel reports Poznań и Гоце Делчев. 6) Gyrovector Hopfion paper. 7) Financial report PID2019 project до 6/III 23г.

(2/III) Fin. report, time tables PID2019 project за 2022. переписка SolSkyMag'2023.

(3/III)

(4/III) Travel reports Poznań и Kakatei - Donostia.

(5/III) Hopfion gyrovector paper: introduction.

D. Sugic et al. Nat. Com. 12, 6785 (2021). Particle-like topologies in light.

Light beam travelling in z-direction $\vec E = E_R\hat e_R + E_L\hat e_L$, $\hat e_R = \frac{1}{\sqrt2}(\hat x + i\hat y)$, $\hat e_L = \frac{1}{\sqrt2}(\hat x - i\hat y)$

$E_R = \cos\frac{\beta}{2}e^{i\frac{1}{2}(\gamma-\alpha)}$, $E_L = \sin\frac{\beta}{2}e^{i(\gamma+\alpha)/2}$

$0 \le \beta \le \pi$, $-\pi \le \alpha \le \pi$, $-2\pi \le \gamma \le 2\pi$, бързане скирмион с Euler angles (α,β,γ). Не „Skyrme density (Jacobian $(\alpha,\beta,\gamma)\to(x,y,z)$) : $\Omega = \frac{1}{(4\pi)^2}\nabla\gamma\cdot(\nabla\cos\beta\times\nabla\alpha)$ for 3D

Skyrme density 2D : $\Omega(2D) = \frac{1}{4\pi}\hat z\,(\nabla\cos\beta\times\nabla\alpha)$

Escaneado con CamScanner

Suppl. Mat. toroidal coordinates (τ, β, φ)

$\varphi = \arctan\left(\frac{y}{x}\right) = \frac{1}{2}(\gamma+\alpha)$, $\beta = \arctan\left(\frac{x^2+y^2+z^2-1}{2z}\right) = \frac{1}{2}(\gamma-\alpha)$, $m=n=1$.

$\cosh\tau = \frac{r^2+1}{\sqrt{r^4+1-2(\beta^2-r^2)}} = \sec\frac{1}{2}\beta = \sqrt{\frac{2}{1+\cos\beta}}$, $r^2 = x^2+y^2+z^2$, $\beta^2 = x^2+y^2$

3-sphere volume $dV = \frac{1}{(4\pi)^2}\, d\alpha\, d(\cos\beta)\, d\gamma$.

2D skyrmion density $\Omega(2D)\, dx\, dy = \frac{1}{4\pi}\, d\alpha\, d(\cos\beta)$

3D skyrmion density $\Omega(3D)\, dx\, dy\, dz = \frac{1}{(4\pi)^2}\, d\alpha\, d(\cos\beta)\, d\gamma$. Jacobian

$\Omega = \frac{1}{(4\pi)^2}\nabla\gamma\cdot(\nabla\cos\beta\times\nabla\alpha)$, $\Omega = \vec{J}\cdot\vec{\omega}$, $\vec{J} = \frac{1}{4\pi}(\nabla\gamma - \cos\beta\nabla\alpha)$, $\vec{\omega} = \nabla\times\vec{J}$

$\vec{\omega} = \frac{1}{4\pi}\nabla\alpha\times\nabla(\cos\beta)$ – vorticity.

Ideal "skyrmion" $E_R = \mathrm{sech}^n\tau\, e^{in\beta}$, $E_I = \mathrm{th}^m\tau\cdot e^{im\varphi}$

$\gamma = m\varphi + n\beta$, $\alpha = m\varphi - n\beta$, $\cos\beta = \frac{\mathrm{sech}^{2n}\tau - \mathrm{th}^{2n}\tau}{\mathrm{sech}^{2n}\tau + \mathrm{th}^{2n}\tau}$, $\mathrm{sech}\tau = \frac{1}{ch\tau}$,

$\nabla\gamma = (\cosh\tau - \cos\beta)\left[\frac{m}{\sinh\tau}\hat{\varphi} + n\hat{\beta}\right]$

$\nabla\alpha = (\cosh\tau - \cos\beta)\left[\frac{m}{\sinh\tau}\hat{\varphi} - n\hat{\beta}\right]$, $\nabla\cos\beta = -4(ch\tau - \cos\beta)\,\mathrm{csch}\tau\cdot\mathrm{sech}^{2m+1}\tau \times \frac{\mathrm{th}^{2n}\tau}{\left[\mathrm{sech}^{2n}\tau + \mathrm{th}^{2n}\tau\right]}\hat{\tau}$

$\int dV\,\Omega = -mn = Q_H$ (Hopf index)

$dV = \frac{\sinh\tau}{(\cosh\tau - \cos\beta)^3}\, d\alpha\, d\beta\, d\varphi$. $Q_H = \int dV\,\Omega(3D)$ (!), т.е. $\Omega \sim (\vec{B}\cdot\vec{A})$.

Для $n=1$ $\mathrm{sech}^2\tau + \mathrm{th}^2\tau = 1$.

Обычно $ch\tau = \sqrt{\frac{2}{1+\cos\beta}}$ даёт $\cos\beta = 1 - 2\,\mathrm{th}^2\tau$, т.е. $\cos\beta = m_z$ $(\tau \to \gamma)$.

$\beta = \theta$, $\vec{m}(\theta,\Phi)$.

$\nabla\cos\beta = -4(ch\tau - \cos\beta)\frac{1}{\sinh\tau}\frac{1}{ch^2\tau}\,\mathrm{th}^2\tau\,\hat{\tau}$, $n=1, m=1$.

Stokes parameters (S_1, S_2, S_3): $\alpha = \arctan\left(\frac{S_1}{S_2}\right)$, $\beta = \arccos S_3$

Y. Shen et al. Adv. Photonics 5, 01 (2023)

$\vec{S}(x,y,z) = (S_x, S_y, S_z)$ unit spin vector $\vec{S}^2 = 1$

Hopf map hypersphere S^3 ($\vec{\gamma} = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$) to real space R^3 (x,y,z)

$\vec{S}(x,y,z) = \langle\xi|\hat{\vec{\sigma}}|\xi\rangle$, $|\xi\rangle = (\gamma_4 + i\gamma_3,\ \gamma_1 + i\gamma_2)^T$ spinor

$\vec{S}$ corresponds to Stokes vector (S_1, S_2, S_3) in optics

Hopf index $Q_H = pq$, p, q are skyrmion numbers in x-z and x-y planes.

$\alpha(\tau) = m\varphi + \alpha_0$ counter. $\Phi = m\varphi + \Phi_0$, $\vec{m}(\theta,\Phi)$ $\beta = \theta$.

$\alpha = \Phi$, $\beta = \theta$ give $\vec{m}(\theta,\Phi)$ parametrization by spherical angles θ, Φ.

In mem $\Phi(\beta,\varphi) = N\varphi + M\beta + \text{const}$, toroidal coord. (τ,β,φ). $\beta \to \beta$, $\gamma \to \tau$. (тор. Hopf'ом)

$\nabla\alpha = \nabla\Phi = N\nabla\varphi + M\nabla\beta$, $\nabla\alpha$ (Singic) $= (ch\tau - \cos\beta)\left[\frac{m}{\sinh\tau}\hat{\varphi} - n\hat{\beta}\right]$

$\nabla\Phi$ (toroidal coord) $= \frac{1}{\alpha}(\cosh\tau - \cos\beta)\left[(\hat{\gamma})\hat{\tau}\frac{\partial\Phi}{\partial\tau} + \hat{\beta}\frac{\partial\Phi}{\partial\beta} + \hat{\varphi}\frac{1}{\sinh\tau}\frac{\partial\Phi}{\partial\varphi}\right] =$

$\equiv \frac{1}{\alpha}[ch\tau - \cos\beta]\left[\hat{\varphi}\frac{1}{\sinh\tau}N + \hat{\beta}M\right]$. Тогда $m = N$, $M = -n$ совпадение с Singic 2021.

(6-7/III) Переписка с invited speakers SolSkyMag'23, поиск оргкомитета PID 2019 за 2022 to Cristina. Books Hotel Arbesø "18-23/IX" для Jette A. Fort.

(2/III) Meeting с Cristela. Вопросы по fig' JMMM 2021 – Green functions

Оценки кол-во проекторов – Zyšoč 1,980 €. Program SolSkyMag. Trav. Reports orgen Eli.

9/III Successful program SolSkyMag. Received 6 emails 55 €; N.D. 12.5 kg + one 10 kg.

10/III CP. "Carrefour", "Lidl" Oliveira. Program SolSkyMag.

11/III Работа в copy. Chess - Elena.

12/III Hopf index via Euler angles (α,β,γ). Rot. matrix $R = e^{i\gamma J_z} e^{i\beta J_y} e^{i\alpha J_z}$ $\mathcal{D}(\beta)$

Gus'EPL'2016: $\hat{D}_\mu = \partial_\mu - \hat{A}_\mu$, pure gauge potential $\hat{A}_\mu = \partial_\mu R\, R^{-1}$ - 3×3 matrix.

$\hat{A}_\mu = (\vec{A}_\mu \cdot \vec{G}) = A^a_\mu G_a$, $G_a = i J_a$ $SO(3)$ generators, $J=1$, $\mu = t, x, y, z$. $(0,1,2,3)$.

Emergent field vector potential $A^e_\mu = (\vec{A}_\mu, \vec{m})$.

Emergent field tensor $F^e_{\mu\nu} = \vec{m}\cdot[\partial_\mu \vec{m} \times \partial_\nu \vec{m}]$. $F^e_{\mu\nu} = \partial_\mu A^e_\nu - \partial_\nu A^e_\mu$.

$F^e_{\mu\nu} = \sin\theta\,(\partial_\mu\theta\partial_\nu\varphi - \partial_\mu\varphi\partial_\nu\theta)$, $\vec{m} = \vec{m}(\theta,\varphi)$. $A^e_\mu = (1-\cos\theta)\partial_\mu\varphi$,

также A^e_μ при $\gamma = \pi - \varphi$, $R = R(\alpha,\beta,\gamma)$. Если $R = e^{i\omega \vec{n}\vec{J}}$, то $\vec{A}_\mu = 2\partial_\mu \vec{n} \times \vec{n}$.

Hopf density $\mathcal{Q}(3D) = \frac{1}{(4\pi)^2} d\gamma \cdot (\nabla\cos\beta \times \nabla\alpha)$, Hopf index $Q_H = \int dV\, \mathcal{Q}(3D)$

В Gus'EPL'16: $\gamma = \pi - \varphi$, $d\gamma = -d\varphi$ и $\mathcal{Q}(3D) = \frac{1}{(4\pi)^2}(-1)\, d\varphi \cdot (\nabla\cos\theta \times \nabla\varphi) \equiv 0$,

если $\alpha = \varphi$, $\beta = \theta$, $\vec{m} = \vec{m}(\theta,\varphi)$. Нужно $\mathcal{Q}(3D) \equiv 0$?

Sugic'2021 дает α,β,γ интервалы: $\beta \in [0,\pi]$, $\alpha \in [-\pi,\pi]$, $\gamma \in [-2\pi, 2\pi]$

Euler angles (α,β,γ): $\alpha \in [0,2\pi]$, $\beta \in [0,\pi]$, $\gamma \in [0,2\pi]$.

$\gamma = \gamma$(Sugic) имеет в 2 раза большие интервалы, чем γ - Euler.

Sugic'2021: $Q_H = \int dV\, \mathcal{Q} = -mn$ (integer), $\mathcal{Q} = \frac{1}{(4\pi)^2} d\gamma \cdot (\nabla\cos\beta \times \nabla\alpha)$

Varshalovich, D.A.: $dV = \sin\beta\, d\alpha\, d\beta\, d\gamma = 4\sin^2\frac{\omega}{2}\, d\omega \sin\theta\, d\theta\, d\varphi$, $\vec{n}(\theta,\varphi)$ определяет

$\int dV = \int \sin\beta\, d\alpha\, d\beta\, d\gamma = \int_0^{2\pi} d\alpha \int_0^{\pi} d\beta \sin\beta \int_0^{2\pi} d\gamma = 8\pi^2 = \frac{1}{2}(4\pi)^2$.

$dV = \mathcal{Q}\, dx\, dy\, dz = \frac{1}{(4\pi)^2} d\alpha\, d(\cos\beta)\, d\gamma$, $\int dV$(Sugic) $= \frac{1}{(4\pi)^2}\int d\alpha\, d(\cos\beta)\, d\gamma = (\text{integer})(-1)$

$\frac{1}{(4\pi)^2}\int dV$(Varsh) $= \frac{1}{(4\pi)^2}\int d\alpha \int \sin\beta\, d\beta \int d\gamma = -\frac{1}{(4\pi)^2}\int d\alpha\, d(\cos\beta)\, d\gamma = -\frac{1}{2}$.

Результат $\int dV$(Sugic) и $\int dV$(Vars) в 2 раза из-за разных интервалов угла γ.

$\int dV$(Sugic) $= \frac{1}{(4\pi)^2}\int d\alpha\, d(\cos\beta)\, d\gamma = \int dx\, dy\, dz \frac{1}{(4\pi)^2} d\gamma \cdot (\nabla\cos\beta \times \nabla\alpha) = -mn$.

Вопрос, можно считать $\alpha = \varphi$, $\beta = \theta$, $\vec{m}(\theta,\varphi)$ в (α,β,γ) - Sugic, но тогда γ(Sugic)? Gus'EPL'16 дает $R(\alpha,\beta,\gamma)$, $\alpha = \varphi$, $\beta = \theta$, γ - Euler angles, $\gamma = \pi - \varphi$.

γ(Sugic) отличается от γ(Gus) не только интервалом.

- а) Если принять что (α,β,γ) есть Euler angles of $\vec{m}(\vec{r})$, то $\int d\alpha\, d(\cos\beta)\, d\gamma = -8\pi^2$ [Varshalovich] и $Q_H = \int dx\, dy\, dz\, \mathcal{Q}(3D) = \frac{1}{(4\pi)^2}\int d\alpha\, d(\cos\beta)\, d\gamma = -\frac{8\pi^2}{16\pi^2} = -\frac{1}{2}$,

т.е. $Q_H = -1/2$ для любой 3D конфигурации $\vec{m}(\vec{r})$, что неверно.

- б) если принять $(\alpha,\beta,\gamma) = (\varphi,\theta,\pi-\varphi)$ - Euler angles как в Gus'EPL'2016, то $\mathcal{Q}(3D) = \frac{1}{4\pi^2} d\gamma \cdot (\nabla\cos\beta \times \nabla\alpha) = -\frac{1}{4\pi^2} d\varphi \cdot (\nabla\cos\theta \times \nabla\varphi) = 0$ и $Q_H = 0$, $\vec{m}(\theta,\varphi)$ $Q_H = 0$ неверно.

Вывод: (α,β,γ) Sugic'2021 нельзя считать Euler angles of $\vec{m}(\vec{r})$, $\alpha = \varphi$, $\beta = \theta$ - допущение, но γ не есть Euler angle. Dr. Gu. 17/III

13/III Program SolSkyMag. Changed prorroga appl. (Oksana). Informacion Ruben, SolSkyMag

14/III Program SolSkyMag. Corrected Viajes, horarios 2022 и послал Gustavo. Informacion Ruben Giraldo Quantum Project. 89 talks - conf.

15/III Обзор papers Shnir, Subcliffe по hopfions. Prorroga PID-2019

Escaneado con CamScanner

Y. Shnir. Topol. Solitons, 2018. Ch. 8. Coordinates (x_1, x_2, x_3, x_4) in R^4

natural mapping S_3 - hypersphere $\sum_{i=1}^{4} x_i^2 = 1$, angles $\theta \in [0,\pi]$, $\alpha, \varphi \in [0, 2\pi]$

$x_1 = \cos\frac{1}{2}\theta \cos\alpha$, $x_2 = \cos\frac{1}{2}\theta \sin\alpha$, $x_3 = \sin\frac{1}{2}\theta \cos(\varphi+\alpha)$, $x_4 = \sin\frac{1}{2}\theta \sin(\varphi+\alpha)$

The Hopf map is $S_3(\theta,\varphi,\alpha) \to S_2(\theta,\varphi)$. $S_3 \sim R_3 \times S^1$

$z_1 = x_1 + i x_2 = \cos\frac{\theta}{2} e^{i\alpha}$, $z_2 = x_3 + i x_4 = \sin\frac{\theta}{2} e^{i(\varphi+\alpha)}$

Spinor $z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} e^{i\alpha} \\ \sin\frac{\theta}{2} e^{i(\varphi+\alpha)} \end{pmatrix}$, $z^+ z = 1$, Hopf map $S_3 \to S_2$ $m_k = z^+ \sigma_k z$, $k=1,2,3$

$\vec{m} = z^+ \vec{\sigma} z$

$m_1 = \sin\theta\cos\varphi$, $m_2 = \sin\theta\sin\varphi$, $m_3 = \cos\theta$, $\vec{m} = (m_1, m_2, m_3)$, $\vec{m}^2 = 1$

T.e. $\theta = \Theta$, $\varphi = \Phi$ имеют смысл spherical angles $\vec{m}(\Theta,\Phi)$. Map $z(\theta,\varphi,\alpha) \to (\theta,\varphi)$ $S_3 \to S_2$

SU(2) matrix $\hat{U}(\theta,\varphi,\alpha) = \begin{pmatrix} z_1^* & z_2^* \\ -z_2 & z_1 \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} e^{-i\alpha} & \sin\frac{\theta}{2} e^{-i(\varphi+\alpha)} \\ -\sin\frac{\theta}{2} e^{i(\varphi+\alpha)} & \cos\frac{\theta}{2} e^{i\alpha} \end{pmatrix}$

$U U^+ = 1$

$(\vec{\sigma}\cdot\vec{m}) = \hat{U}^{-1} \sigma_3 \hat{U}$

Gus'EPL'2016: $U^+ (\vec{m}\cdot\vec{\sigma}) U = \sigma_z$, $SU(2)$ $U = -iR(\vec{n},\pi)$, $R(\vec{n},\pi) = e^{i\pi\vec{n}\cdot\vec{J}}$

$R(\vec{n},\omega) = e^{i\omega\vec{n}\vec{J}}$

$\vec{n}$ - ось вращения coord. system на угол π от oz к оси $\vec{m}(r)$.

$R(\alpha,\beta,\gamma) = e^{i\gamma J_z} e^{i\beta J_y} e^{i\alpha J_z}$, $\gamma = \pi - \Phi$ comp. $\vec{n} = (\sin\frac{\Theta}{2}\cos\Phi, \sin\frac{\Theta}{2}\sin\Phi, \cos\frac{\Theta}{2})$

Также $\vec{n}$ есть в Tatarra Phys. Rep. 2008, где $U = \vec{n}\cdot\vec{\sigma}$.

$U = -iR(\vec{n},\pi) = e^{i\pi\vec{n}\vec{J}}(-i) = -i\, e^{i\pi\vec{n}\vec{\sigma}/2} = i(-1)\left[I\cos\frac{\pi}{2} + i(\vec{n}\vec{\sigma})\sin\frac{\pi}{2}\right] =$

$= -i\cdot i(\vec{n}\vec{\sigma}) = (\vec{n}\vec{\sigma})$. $\gamma = \pi - \Phi$ мы берем. $\vec{m}(\Theta,\Phi)$ - magn. unit vector.

В x'y'z' $\subset$ oz' $\parallel \vec{m}$, $\vec{m}' = (0,0,1)$.

$U = (\vec{n}\cdot\vec{\sigma}) = \begin{pmatrix} n_z & n_x - i n_y \\ n_x + i n_y & -n_z \end{pmatrix} = \begin{pmatrix} \cos\frac{\Theta}{2} & \sin\frac{\Theta}{2} e^{-i\Phi} \\ \sin\frac{\Theta}{2} e^{i\Phi} & -\cos\frac{\Theta}{2} \end{pmatrix}$ $SU(2)$ - matrix из Tatarra '08, Gus'EPL'2016.

$\hat{U}(\Theta,\Phi,\alpha)$-Shnir, не совпадает с $U(\Theta,\Phi)$ - Tatarra '08. $U(\Theta,\Phi) = -iR(\vec{n},\pi)$

SU(2) matrix (Varshalovich) $D^{1/2}(-\gamma,-\beta,-\alpha) = e^{i\gamma S_z} e^{i\beta S_y} e^{i\alpha S_z} = \begin{pmatrix} \cos\frac{\beta}{2} e^{\frac{i}{2}(\alpha+\gamma)} & \sin\frac{\beta}{2} e^{\frac{i}{2}(\alpha-\gamma)} \\ -\sin\frac{\beta}{2} e^{\frac{i}{2}(\gamma-\alpha)} & \cos\frac{\beta}{2} e^{-\frac{i}{2}(\alpha+\gamma)} \end{pmatrix}$, $s = 1/2$.

$D^{1/2}(-\gamma,-\beta,-\alpha)$ совпад. мою $R(\alpha,\beta,\gamma) = e^{i\gamma J_z} e^{i\beta J_y} e^{i\alpha J_z}$ с $\vec{J} = \frac{1}{2}\vec{\sigma}$, $J = 1/2$.

Shnir $\hat{U}(\theta,\varphi,\delta) = \begin{pmatrix} \cos\frac{\Theta}{2} e^{-i\delta} & \sin\frac{\Theta}{2} e^{-i(\varphi+\delta)} \\ -\sin\frac{\Theta}{2} e^{i(\varphi+\delta)} & \cos\frac{\Theta}{2} e^{i\delta} \end{pmatrix}$, $D^{1/2}(-\gamma,-\beta,-\alpha) = R(\alpha,\beta,\gamma) =$

$= R(\Phi,\Theta,\pi-\Phi)$

Сравнивая $\hat{U}(\theta,\varphi,\delta)$ с $R(\alpha,\beta,\gamma)$ видим $\beta = \Theta$, $-\delta = \frac{1}{2}(\alpha+\gamma)$, $\alpha = \Phi$.

$e^{-i(\varphi+\delta)} = e^{\frac{i}{2}(\alpha-\gamma)}$, $\varphi + \delta = \alpha - \gamma$, $\Phi + \delta = \Phi - \gamma$, $\delta = -\gamma$, $\delta = \frac{1}{2}(\Phi+\gamma)$

$-\gamma = -\frac{1}{2}\Phi = \frac{1}{2}\gamma$, $\Phi = \gamma$, что говорит, будто $\gamma = \pi - \Phi$.

$U^+ (\vec{m}\cdot\vec{\sigma}) U = \sigma_z$. Tatarra, Gus $\to$ $(\vec{m}\cdot\vec{\sigma}) = \hat{U}^{-1}\sigma_3\hat{U}$, $\hat{U}(\vec{m}\cdot\vec{\sigma})\hat{U}^{-1} = \sigma_3$

Поскольку $U^+ = \hat{U}$, это не так. $U = U(\Theta,\Phi)$, $\hat{U}$ (Shnir) $= \hat{U}(\Theta,\Phi,\delta)$

$U^{-1} = U^+$ for unitar matrix. Как Shnir параметр δ (?) обобщает Euler angle γ? Никак

Shnir 23 $\vec{S} = \langle \chi | \vec{\sigma} | \chi \rangle$, $|\chi\rangle = (\chi_4 + i\chi_3, \chi_1 + i\chi_2)^T$ comp. $\vec{m} = z^+\vec{\sigma}z$, Shnir $\vec{m}^2 = 1$, $\vec{m} = \vec{m}(x,y,z)$

$\vec{S}^2 = 1$, $\vec{S}(x,y,z)$

Sugic 21 дает Stokes $\vec{S}$-vector components via $F_R = \cos\frac{\beta}{2} e^{i\left(\frac{\gamma-\alpha}{2}\right)}$, $F_L = \sin\frac{\beta}{2} e^{i\left(\frac{\gamma+\alpha}{2}\right)}$

$S_1 = 2\left[\cos\frac{\beta}{2}\sin\frac{\beta}{2}\cos\left(\frac{\gamma-\alpha}{2}\right)\cos\left(\frac{\gamma+\alpha}{2}\right) + \cos\frac{\beta}{2}\sin\frac{\beta}{2}\sin\left(\frac{\gamma-\alpha}{2}\right)\sin\left(\frac{\gamma+\alpha}{2}\right)\right] =$

$= 2\cos\frac{\beta}{2}\sin\frac{\beta}{2}\cos\left[\frac{\gamma-\alpha}{2} - \frac{(\gamma+\alpha)}{2}\right] = \sin\beta\cos\alpha$, $S_2 = \sin\beta\sin\alpha$, $S_3 = \cos\beta$

$\vec{S} = (\sin\beta\cos\alpha, \sin\beta\sin\alpha, \cos\beta)$, $\alpha = \Phi$, $\beta = \Theta$ и $\vec{S}(\Theta,\Phi)$, $\vec{S}$ от γ-Sugic не зависит!

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Видимо γ-Зудзі означають $\delta(\alpha)$ - shniz - parameter of hypersphere S^3, а начального обозначения к Euler angle γ, $R(\alpha,\beta,\gamma)$ не имеет.

→ $\delta(\alpha)$ Shniz не связаны с Euler angle γ.

Библиография: 1) γ (Sugie-2021) и δ (α-Shniz 2018) angles не связаны с γ-Euler angle in (α,β,γ) Euler parameterization. Это групповые параметризации of $SU(2)$ matrix.

2) Hopf index density Sugie $\delta L(3D) = \frac{1}{(4\pi)^2}\nabla\gamma\cdot(\nabla\cos\beta\times\nabla\alpha)$ может и формально верна, но $\nabla\gamma$ не имеет градиента, т.к. angle γ Sugie is undefined.

(16/III) E-mails Auben (Gipuzkoa project), Окс (provincia), Лена (gyroscopes).

Sugie'21 выражены $\vec{J}$ и vorticity $\vec{\omega}$, $\mathcal{L}(3D) = \vec{J}\cdot\vec{\omega}$, $\vec{J} = \vec{A}$, $\vec{\omega} = \vec{B}$ - em. mag. fields.

$\vec{B} = \frac{1}{4\pi}\nabla\alpha\times\nabla(\cos\beta) = \frac{1}{4\pi}\nabla\Phi\times\nabla\cos\Theta = -\frac{1}{4\pi}\nabla\Phi\times\nabla\Theta\sin\Theta$.

$\vec{A} = \frac{1}{4\pi}(\nabla\gamma - \cos\beta\nabla\alpha)$, $\vec{B} = \mathrm{rot}\vec{A}$, $\vec{B} = \nabla\times(\nabla\gamma - \cos\beta\nabla\alpha)\frac{1}{4\pi}$, rot grad $\varphi = \nabla\times\nabla\varphi = 0$

$\vec{B} = -\frac{1}{4\pi}\nabla\times(\cos\beta\nabla\alpha) = -\frac{1}{4\pi}[\cos\beta\cdot\nabla\times\nabla\alpha + \nabla\cos\beta\times\nabla\alpha] = \frac{1}{4\pi}\nabla\alpha\times\nabla\cos\beta$

Gus EPL'2016: energy $A^e_\mu = (\vec{A}_\mu, \vec{m})$, $A^e_\mu = (A^e_0, \vec{A}^e)$, $\vec{B} = \mathrm{rot}\vec{A}^e$

$A^e_\mu = (1-\cos\Theta)\partial_\mu\Phi$, $\vec{A}^e = (1-\cos\Theta)\nabla\Phi$, $\mathrm{rot}\vec{A}^e = \nabla(1-\cos\Theta\nabla\Phi) = \nabla\alpha\nabla\cos\beta = \sin\Theta\nabla\Theta\times\nabla\Phi$

$\vec{B} = \sin\Theta\nabla\Theta\times\nabla\Phi$ как в Tejo et al. Sci. Rep. 2021.

$4\pi\vec{A} = \nabla\gamma - \cos\beta\nabla\alpha = \nabla\gamma - \cos\Theta\nabla\Phi$ и $\vec{A}^e = (1-\cos\Theta)\nabla\Phi$, можно ли считать $4\pi\vec{A} = \vec{A}^e$?

Если да, то $\nabla\gamma = \nabla\Phi$. $\mathcal{L}(3D) = \frac{1}{(4\pi)^2}\nabla\gamma\cdot(\nabla\cos\beta\times\nabla\alpha) = \frac{1}{(4\pi)^2}\nabla\Phi\cdot(\nabla\cos\Theta\times\nabla\Phi) = 0$

$\mathcal{L}(3D) = 0$ for any $\vec{m}(\vec{r})$? Неверно.

Вид $\vec{A} = \vec{A}_0 - \cos\Theta\nabla\Phi$ - разумен, т.к. $\vec{B} = \mathrm{rot}\vec{A} = \mathrm{rot}\vec{A}_0 + \sin\Theta\nabla\Theta\times\nabla\Phi$

Берем $\mathrm{rot}\vec{A}_0 = 0$ - gauge. $\vec{A}_0 = \nabla\Gamma$, rot grad $\Gamma \equiv 0 \Rightarrow \vec{B} = \sin\Theta\nabla\Theta\times\nabla\Phi$

$\vec{A} = \nabla\Gamma - \cos\Theta\nabla\Phi$, $\vec{m}(\Theta,\Phi)$ и $\vec{A}\cdot\vec{B} = [\nabla\Gamma - \cos\Theta\nabla\Phi]\cdot\sin\Theta\nabla\Theta\times\nabla\Phi = \sin\Theta\nabla\Gamma\cdot(\nabla\Theta\times\nabla\Phi)$

Чтобы $Q_H \sim \int dV\vec{A}\cdot\vec{B} \neq 0$, надо $\Gamma \neq f(\Theta)$ только или $\Gamma \neq f(\Phi)$, т.е. $\Gamma = \Gamma(\vec{m})$, $\Gamma = \Gamma(\Theta,\Phi)$

$\nabla\Gamma = \nabla\Gamma(\Theta,\Phi) = \frac{\partial\Gamma}{\partial\Theta}\nabla\Theta + \frac{\partial\Gamma}{\partial\Phi}\nabla\Phi$ и $\vec{A}\cdot\vec{B} = \sin\Theta[\Gamma_\Theta\nabla\Theta + \Gamma_\Phi\nabla\Phi]\nabla\Theta\times\nabla\Phi \equiv 0$

т.е. нужно $\Gamma = \Gamma(\vec{m})$. Что же такое $\vec{\Gamma}$ и $\vec{A} = \nabla\Gamma - \cos\Theta\nabla\Phi$? $\Gamma = \Gamma(\vec{r})$?

(17/III) $\vec{G} = \int dV\vec{B} = \int dV\,\mathrm{rot}\vec{A} = \oint_S dS\,\vec{n}\times\vec{A}$, $\vec{n}$ - outward-pointing unit normal vector to S surface

$\vec{G} = \oint_S dS\,\vec{n}\times\vec{A} = \oint_S dS\,\vec{n}\times[\nabla\Gamma - \cos\Theta\nabla\Phi]$, дерем S as a cylinder, height L, radius A.

$\oint_S d\vec{S}\,\vec{n}\times\vec{A} = \int_{z=L/2} dS_1\,\hat{z}\times\vec{A} + \int_{z=-L/2} dS_2\,(-\hat{z})\times\vec{A} + \int_{\rho=R} dS_3\,\hat{\rho}\times\vec{A}$, $\vec{A} = A_\rho\hat{\rho} + A_\varphi\hat{\varphi} + A_z\hat{z}$



$\hat{z}\times\vec{A} = A_\rho\hat{z}\times\hat{\rho} + A_\varphi\hat{z}\times\hat{\varphi} = A_\rho\hat{\varphi} - A_\varphi\hat{\rho}$, $\hat{\rho}\times\vec{A} = A_\varphi\hat{\rho}\times\hat{\varphi} + A_z\hat{\rho}\times\hat{z} = A_\varphi\hat{z} - A_z\hat{\varphi}$

$\vec{G} = \int_{z=\frac{L}{2}} dS_1(A_\rho\hat{\varphi} - A_\varphi\hat{\rho}) - \int_{z=-\frac{L}{2}} dS_2(A_\rho\hat{\varphi} - A_\varphi\hat{\rho}) + \int_{\rho=R} dS_3(A_\varphi\hat{z} - A_z\hat{\varphi})$.

$G_\rho = -\int_{z=L/2} dS_1 A_\varphi + \int_{z=-L/2} dS_2 A_\varphi$, $G_\varphi = \int dS_1 A_\rho - \int dS_2 A_\rho - \int_{\rho=R} dS_3 A_z$, $G_z = \int dS_3 A_\varphi$

Gladivevski: $\vec{A}$ в тороидальных координатах (γ,β,φ), φ-угол такой же как в цилиндр. координатах. $A_\varphi = \frac{2N}{a}\frac{\tau}{\mathrm{sh}\gamma}[w(\gamma,\beta) - 1]$, $\tau = \mathrm{ch}\gamma - \cos\beta$, $w(\gamma,\beta) = m_z$.

$\rho = a\frac{\mathrm{sh}\gamma}{\tau}$, $z = a\frac{\sin\beta}{\tau}$, $\varphi = \varphi$ цил. координаты. $A_\varphi = \frac{2N}{a}\frac{a}{\rho}[m_z - 1] = \frac{2N}{\rho}(m_z - 1)$, $N = 1$

$G_z = \int_0^{2\pi} d\varphi\,\rho\int_{-L/2}^{L/2} dz\,(\dots) = |\rho = R| = 2\pi R\int_{-L/2}^{L/2} dz\,\frac{2}{R}(m_z - 1) = 4\pi\int_{-L/2}^{L/2} dz\,[m_z(R,z) - 1]$, $\vec{m}(\rho,z)$

$G_z = 4\pi \int_{-L/2}^{L/2} dz \left[m_z(R,z) - 1 \right]$, 17/II через $\Theta\Phi$ и можно $2\pi n$ $G_z = -2\pi n \int dz \left[m_z(\rho,z) - m_z(R,z) \right]$
тогда Сайса, $n=1$, $m_z(\rho,z) = 1$, т.к. $\rho_{\infty} \to \infty$.

$G_\rho = \int dS_1 A_\varphi + \int dS_2 A_\varphi = -\int d\rho\rho d\varphi \left[A_\varphi(\rho, z=\frac{L}{2}) - A_\varphi(\rho, z=-\frac{L}{2}) \right]$, $A_\varphi = \frac{2}{\rho}(m_z - 1)$

$G_\rho = -2\pi \int d\rho\rho \left[\frac{2}{\rho} \left(m_z(\rho, \frac{L}{2}) - 1 \right) - \frac{2}{\rho} \left(m_z(\rho, -\frac{L}{2}) \right) \right] = -4\pi \int_0^R d\rho \left[m_z(\rho, \frac{L}{2}) - m_z(\rho, -\frac{L}{2}) \right]$

$G_\rho \equiv 0$ если $m_z(\rho, z)$ is even function z и как у Нистеренко '19. $G_\rho = 0$ for any L, R - dot.

Gladkowski '17: $A_\varphi = \frac{2N}{\rho}(m_z - 1) = -\frac{2N}{\rho}(1 - \cos\Theta)$

Wang-Bradacs '19: $A_\varphi = \frac{n}{\rho}(1 - \cos\Theta)$, $A_\rho = -(1+\cos\Theta)\frac{\partial\gamma}{\partial\rho}$, $A_z = -(1+\cos\Theta)\frac{\partial\gamma}{\partial z}$,

где γ из $\Phi(\vec{r}) = n\varphi + \gamma(\rho, z)$. $\vec{A}(\text{Wang})$ дает вершие $\vec{B} = \text{rot}\,\vec{A}$. См. I-4/II, $\vec{B}(\rho, z)$

Приведем $\vec{B}(\text{wang})$ из $\vec{B} = \sin\Theta\, \nabla\Theta \times \nabla\Phi$, $\nabla\Theta(\rho, z) = \frac{\partial\Theta}{\partial\rho}\nabla\rho + \frac{\partial\Theta}{\partial z}\nabla z = \frac{\partial\Theta}{\partial\rho}\hat{\rho} + \frac{\partial\Theta}{\partial z}\hat{z}$. $\vec{A}(\rho, z)$

$\vec{B} = \sin\Theta \left[\Theta_\rho \hat{\rho} + \Theta_z \hat{z} \right] \times \left[\frac{\partial\gamma}{\partial\rho}\hat{\rho} + \frac{\partial\gamma}{\partial z}\hat{z} + \frac{\hat{\varphi}}{\rho} n \right] = \sin\Theta \left[\Theta_\rho \gamma_z \hat{\rho}\times\hat{z} + \Theta_\rho \frac{n}{\rho}\hat{\rho}\times\hat{\varphi} + \Theta_z \frac{\partial\gamma}{\partial\rho}\hat{z}\times\hat{\rho} + \right.$

$\left. + \Theta_z \frac{n}{\rho}\hat{z}\times\hat{\varphi} \right] = \sin\Theta \left[\Theta_\rho \gamma_z (-\hat{\varphi}) + \frac{n}{\rho}\Theta_\rho \hat{z} + \Theta_z \frac{\partial\gamma}{\partial\rho}\hat{\varphi} + \Theta_z \frac{n}{\rho}(-\hat{\rho}) \right]$

$B_\rho = \sin\Theta \left[-\frac{n}{\rho}\frac{\partial\Theta}{\partial z} \right]$, $B_\varphi = \sin\Theta \left[\frac{\partial\Theta}{\partial z}\frac{\partial\gamma}{\partial\rho} - \frac{\partial\Theta}{\partial\rho}\frac{\partial\gamma}{\partial z} \right]$, $B_z = \sin\Theta \left[\frac{n}{\rho}\frac{\partial\Theta}{\partial\rho} \right]$ - как в Wang-Bz' 2019

Выход: $\vec{A}$ в Gladkowski '17 и Wang-Bradacs '19 отличается от $\vec{A}^e$ Gus'EPL'16,
т.к. $\vec{B} = \text{rot}\,\vec{A}$ (Wang-Bz) $= \sin\Theta\, \nabla\Theta \times \nabla\Phi$ - верное. $\vec{A}^e = (1-\cos\Theta)\nabla\varphi$

$A^e_\varphi = (1-\cos\Theta)\frac{\partial\Phi}{\partial\varphi}\frac{1}{\rho} = (1-\cos\Theta)\frac{n}{\rho}$ совп. с A_φ (Wang).

$A^e_\rho = (1-\cos\Theta)\frac{\partial\Phi}{\partial\rho} = (1-\cos\Theta)\frac{\partial\gamma}{\partial\rho}$, $A^e_z = (1-\cos\Theta)\frac{\partial\gamma}{\partial z}$ не совпадают с A_ρ(wang), A_z(wang)

A_φ(Glad) $= -\frac{2N}{\rho}(1-\cos\Theta)$ отличается от $A^e_\varphi = A_\varphi$(Wang) $= \frac{n}{\rho}(1-\cos\Theta)$ в 2 раза.

Как $\vec{A}^e$ - Gus'EPL'16 и $\vec{A}$(Wang) различны, оба дают верное $\vec{B} = \text{rot}\,\vec{A} = \sin\Theta\nabla\Theta\times\nabla\Phi$

$\vec{A}^e = \nabla\Phi - \cos\Theta\nabla\Phi$, $\vec{A}$(wang) $= \frac{n}{\rho}(1-\cos\Theta)\hat{\varphi} - (1+\cos\Theta)\frac{\partial\gamma}{\partial\rho}\hat{\rho} - (1+\cos\Theta)\frac{\partial\gamma}{\partial z}\hat{z} =$

$= (\nabla\Phi)_\varphi\hat{\varphi} - \cos\Theta(\nabla\Phi)_\varphi\hat{\varphi} - \frac{\partial\Phi}{\partial\rho}\hat{\rho} - \cos\Theta\frac{\partial\Phi}{\partial\rho}\hat{\rho} - \frac{\partial\Phi}{\partial z}\hat{z} - \cos\Theta\frac{\partial\Phi}{\partial z}\hat{z} =$

$= (\nabla\Phi)_\varphi\hat{\varphi} - \frac{\partial\Phi}{\partial\rho}\hat{\rho} - \frac{\partial\Phi}{\partial z}\hat{z} - \cos\Theta\left[(\nabla\Phi)_\varphi\hat{\varphi} + \frac{\partial\Phi}{\partial\rho}\hat{\rho} + \frac{\partial\Phi}{\partial z}\hat{z} \right] = (\nabla\Phi)_\varphi\hat{\varphi} - \frac{\partial\Phi}{\partial\rho}\hat{\rho} - \frac{\partial\Phi}{\partial z}\hat{z} - \cos\Theta\nabla\Phi$

$\vec{A}$(wang) $= \hat{\varphi}(\nabla\Phi)_\varphi - \frac{\partial\Phi}{\partial\rho}\hat{\rho} - \frac{\partial\Phi}{\partial z}\hat{z} - \cos\Theta\nabla\Phi$, мой $\vec{A}^e = (1-\cos\Theta)\nabla\Phi = \nabla\Phi - \cos\Theta\nabla\Phi$

$\vec{A}$(wang) $= -2\frac{\partial\Phi}{\partial\rho}\hat{\rho} - 2\frac{\partial\Phi}{\partial z}\hat{z} + \nabla\Phi - \cos\Theta\nabla\Phi = \vec{A}_w + \nabla\Phi - \cos\Theta\nabla\Phi$, $\vec{A}_w(A^w_\rho, 0, A^w_z)$

$\text{rot}\,\vec{A}_w = \left(\frac{\partial A^w_\rho}{\partial z} - \frac{\partial A^w_z}{\partial\rho} \right)\hat{\varphi}$, $\vec{A}_w = -2\left(\frac{\partial\Phi}{\partial\rho}, 0, \frac{\partial\Phi}{\partial z} \right)$ $\vec{A}_w = \vec{A}_w(\rho, z)$

$\text{rot}\,\vec{A}_w = \left[\frac{\partial}{\partial z}(-2)\frac{\partial\Phi}{\partial\rho} - \frac{\partial}{\partial\rho}(-2)\frac{\partial\Phi}{\partial z} \right] = (-2)\left[\frac{\partial^2\Phi}{\partial z\partial\rho} - \frac{\partial^2\Phi}{\partial\rho\partial z} \right] = 0$, если $\Phi(\rho, z)$, $\gamma(\rho, z)$ плавная.

$\text{rot}\,\vec{A}_w = 0$, т.е. к мерному gauge potential $\vec{A}^e$ добавили $\vec{A}_w$ с $\text{rot}\,\vec{A}_w = 0$. continuous

$\vec{A}_w = -2\left(\frac{\partial\gamma}{\partial\rho}, 0, \frac{\partial\gamma}{\partial z} \right) = -2\nabla\gamma$, $\gamma = \gamma(\rho, z)$. $\Phi = n\varphi + \gamma$.

$\vec{A}$(wang) $= \vec{A}_w + \vec{A}^e = -2\nabla\gamma + \vec{A}^e$, - choice of gauge of $\vec{A}^e$ potential.

$\vec{A}\vec{B} = (-2\nabla\gamma + \vec{A}^e)\cdot\sin\Theta\left[\nabla\Theta\times\nabla\Phi\right] = -2\sin\Theta\cdot\nabla\gamma\cdot\left[\nabla\Theta\times\nabla\Phi\right]$, т.к. $\vec{A}^e \perp \vec{B} = 0$.

$\vec{A}$(Sugic) $= \nabla\gamma(S) - \cos\Theta\nabla\Phi$ см. (9)I. Hopf density Sugic $\mathcal{H}(3\vartheta) = \vec{A}\vec{B} = \nabla\gamma(S)\cdot\sin\Theta\nabla\Theta\times\nabla\Phi$

Сравнивая $\vec{A}\cdot\vec{B}$ (Wang) с $\vec{A}\cdot\vec{B}$(Sugic), находим $-2\nabla\gamma = \nabla\gamma$(Sugic), т.е.

можно γ(Sugic) $= -2\gamma$ (Wang) или $\gamma(S) = 2\gamma(W)$, $\gamma(W) = \gamma(\rho, z) = \beta(\rho, z) + \text{const}$.

где $\beta(\rho, z)$ poloidal angle in (γ, β, φ) toroidal coordinates.

т.е. γ(Sugic) определено, $\gamma(S) = \pm 2\gamma(\text{Wang}) = \pm 2\beta(\rho, z)$ имеет такого вида.

связь $\gamma(S)$ от $\vec{m}(\vec{r})$ не забываем.

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19/II $\beta(\rho,z) = \mathrm{Im}\ln\left[\frac{z+i(\rho+a)}{z+i(\rho-a)}\right]$, $\beta(\rho,z) = \mathrm{sign}(z)\arccos\left[\frac{(r^2-a^2)}{\sqrt{(r^2-a^2)^2+4a^2z^2}}\right]$, $r^2=z^2+\rho^2$

Ценното $\beta\gamma = v\beta \neq 0$ дава $Q_{11} \neq 0$.

$G_\varphi \equiv 0$, G_z decreasing function of R, $G_z(R) \sim \frac{a}{R}$, $R \gg a$, остават G_ρ и G_z.

$\vec{A} = \vec{A}(\rho,z)$ помага $G_\varphi = \int_{z=L/2} dS_1 A_\varphi - \int_{z=-L/2} dS_2 A_\varphi - \oint_{\rho=R} dS_3 A_\varphi$.

$\int dS_1 A_\varphi - \int dS_2 A_\varphi = \int_0^R d\rho\rho \int_0^{2\pi} d\varphi \left[A_\varphi(\rho,z=\frac{L}{2}) - A_\varphi(\rho,z=-\frac{L}{2})\right]$, $A_\varphi = -(1+m_2)\frac{\partial\beta}{\partial\rho}$

$\beta(\rho,z)$ is odd function of z. $A_\varphi(\rho,z) = -[1+m_2(\rho,z)]\frac{\partial}{\partial\rho}\beta(\rho,z)$ is odd func. of z

$\int dS_3 A_\varphi = \int_0^{2\pi} d\varphi R \int_{-L/2}^{L/2} dz A_z(R,z) = 2\pi R \int_{-L/2}^{L/2} dz (-)\left[1+m_2(R,z)\right]\frac{\partial\beta}{\partial z}$, $\frac{\partial\beta}{\partial z}$ is even func. of z

$G_\varphi = 4\pi \int_0^R d\rho\rho\, A_\varphi(\rho,\frac{L}{2}) + 2\pi R \int_{-\frac{L}{2}}^{L/2} dz \left[1+m_2(R,z)\right]\frac{\partial\beta}{\partial z}$, $A_\varphi = -(1+m_2)\frac{\partial\beta}{\partial\rho}$

$m_2(\rho,z) = 1 - \frac{2\rho^2a^2}{(a^2+\rho^2+z^2)^2}$, $m_2(\rho,\frac{L}{2}) \underset{L\gg a}{\simeq} 1 - \frac{a^2}{L^2}(\cdot)$, $m_2(R,z) \underset{R\gg a}{\simeq} 1 - \frac{a^2}{R^2}(\cdot)$, $\frac{L}{R} = \mathrm{const}$

При $L \gg a$, $R \gg a$ заменяме $m_2(\rho,z) \to 1$.

$G_\varphi = -4\pi \int_0^R d\rho\rho \frac{\partial\beta(\rho,L/2)}{\partial\rho} + 2\pi R \int_{-L/2}^{L/2} dz \frac{\partial\beta(R,z)}{\partial z}$, $\int_{-\frac{L}{2}}^{L/2} dz \frac{\partial\beta}{\partial z} = \beta(R,z)\Big|_{-L/2}^{L/2} =$

$= \beta(R,\frac{L}{2}) - \beta(R,-\frac{L}{2}) = 2\beta(R,\frac{L}{2}) = 2\arccos\left[\frac{L^2/4+R^2-a^2}{\sqrt{(\frac{L^2}{4}+R^2-a^2)^2+4a^2L^2}}\right] =$

$= 2\arccos\left[\frac{L^2/4+R^2}{\sqrt{(\frac{L^2}{4}+R^2)^2}}\right] = 2\arccos 1 = 2\cdot 0 = 0$ при $L, R \gg a$.

$\beta(\rho,z) = \frac{i}{2}\ln\left[\frac{\rho^2+(z-ia)^2}{\rho^2+(z+ia)^2}\right]$ или $\beta(\rho,z) = \mathrm{Im}\ln\left[\frac{z+i(\rho+a)}{z+i(\rho-a)}\right]$

Видно $\beta(\rho,\frac{L}{2}) \to 0$ и $\beta(R,z) \underset{R/a\gg 1}{\to} 0$. Искаме $A_\rho, A_z \to 0$ и $G_\varphi \to 0$.

$G_\varphi \underset{R}{\to} 0$ не очевидно, в $\int_0^R d\rho\rho \frac{\partial\beta(\rho,L/2)}{\partial\rho}$ интеграл.

$\int_0^R d\rho\rho \frac{\partial\beta}{\partial\rho} = \int d\rho \frac{\partial\beta}{\partial\rho}\rho = \left|\begin{matrix} u=\rho, & dv=\frac{\partial\beta}{\partial\rho}d\rho \\ du=d\rho, & v=\beta \end{matrix}\right| = \rho\beta\Big|_0^R - \int_0^R \beta d\rho$, $\rho\beta(\rho,\frac{L}{2}) \underset{\rho\to R}{\to} 0$?

$\ln[z+i(\rho+a)] = \ln(z+i\rho(1+\frac{a}{\rho})) = \ln[z+i\rho+ia] = \ln(z+i\rho)[1+\frac{ia}{z+i\rho}] =$

$= \ln(z+i\rho) + \ln[1+\frac{ia}{z+i\rho}] = \ln(z+i\rho) + \frac{ia}{z+i\rho}$ при $z,\rho \gg a$.

$\ln\left[\frac{z+i(\rho+a)}{z+i(\rho-a)}\right] = \ln(z+i(\rho+a)) - \ln(z+i(\rho-a)) = \ln(z+i\rho) + \frac{ia}{(z+i\rho)} -$

$-\ln(z+i\rho) + \frac{ia}{(z+i\rho)} = 2i\frac{a}{z+i\rho} = 2ia\frac{z-i\rho}{z^2+\rho^2} = 2a\frac{\rho+iz}{(z^2+\rho^2)}$, $\beta = \mathrm{Im}\ln(\cdot) = \frac{2az}{\rho^2+z^2}$

$\beta(\rho,z) = \frac{2az}{\rho^2+z^2}$ ако $\rho, z \gg a$. $\frac{\partial\beta}{\partial\rho} = \frac{-2az\cdot 2\rho}{(\rho^2+z^2)^2} = -4a\frac{z\rho}{(\rho^2+z^2)^2}$

$\partial\beta/\partial z = \frac{2a(\rho^2+z^2)-2az\cdot 2z}{(\rho^2+z^2)^2}$, $\frac{\partial\beta}{\partial z} = \frac{(\rho^2-z^2)}{(\rho^2+z^2)^2}2a$.

$G_\varphi = -4\pi\int_0^R d\rho\rho(-4a)\frac{z\rho}{(\rho^2+z^2)^2} = |z=\frac{L}{2}| = 16\pi a z\int_0^R \frac{d\rho\rho^2}{(\rho^2+z^2)^2} = 16\pi a z\left[\int_0^R \frac{d\rho}{(\rho^2+z^2)} - \right.$

$\left. - z^2\int_0^R \frac{d\rho}{(\rho^2+z^2)^2}\right] = 16\pi a z\left[\frac{1}{z}\mathrm{arctg}\frac{\rho}{z} - \frac{z^2\rho}{2z^2(\rho^2+z^2)} + \frac{1}{2z^3}\mathrm{arctg}\frac{\rho}{z}\right]_0^R =$

$= 16\pi a z\left[\frac{1}{2z}\mathrm{arctg}\frac{\rho}{z} - \frac{\rho}{2(z^2+\rho^2)}\right]_0^R = 16\pi a z\left[\frac{1}{2z}\mathrm{arctg}\frac{R}{z} - \frac{R}{2(z^2+R^2)}\right]$

$J_1 = -4\pi\int_0^R d\rho\rho\frac{\partial\beta(\rho,z)}{\partial\rho} = 8\pi a z\left[\frac{1}{z}\mathrm{arctg}\frac{R}{z} - \frac{R}{(z^2+R^2)}\right]_{z=\frac{L}{2}} \approx 8\pi a\cdot\mathrm{arctg}\left(\frac{2R}{L}\right)$

$J_2 = 2\pi R\int dz\frac{\partial\beta(R,z)}{\partial z} = 2\pi R\int dz\cdot 2a\frac{(\rho^2+z^2)}{(\rho^2+z^2)^2} = 4\pi aR\left[\rho^2\int dz\frac{1}{(\rho^2+z^2)^2} - \int dz(\dots)\right] =$

$= 4\pi aR\int dz\frac{(\rho^2+z^2-2z^2)}{(\rho^2+z^2)^2} = 4\pi aR\left[\int dz\frac{1}{(\rho^2+z^2)} - 2\int dz\frac{z^2}{(\rho^2+z^2)^2}\right] = 4\pi aR\left[\frac{1}{\rho}\mathrm{arctg}\frac{z}{\rho} - \right.$

$\left. -2\left\{-\frac{1}{2}\frac{z}{(z^2+\rho^2)} + \frac{1}{2\rho}\mathrm{arctg}\frac{z}{\rho}\right\}\right]_{-L/2}^{L/2} = 4\pi aR\frac{z}{(z^2+\rho^2)}\Big|_{-L/2}^{L/2} = 8\pi aR\frac{z}{(z^2+R^2)}$, $z = L/2$

Escaneado con CamScanner

$G_\varphi = J_1 + J_2 = 8\pi a \operatorname{arctg}\left(\frac{2R}{L}\right) + 8\pi a \frac{R}{L} \frac{1}{(L^2/4 + R^2)} = 2\pi a \left[\operatorname{arctg}\left(\frac{2}{\beta}\right) + \frac{1}{2} \frac{\beta}{1+\beta^2/4} \right]$

$G_\varphi(\beta) = 2\pi a \left[\operatorname{arctg}\left(\frac{2}{\beta}\right) + \frac{1}{2} \frac{\beta}{1+\beta^2/4} \right]$, $\beta = L/R$ - dot aspect ratio,

$L/R \to \infty$ при $\beta = \text{const}$.

19/II в матлабе $G_\varphi \neq 0$ считаю, MathCad. при $R, L \gg a$, $\frac{R}{a} \to \infty$, $\frac{L}{a} \to \infty$.

File «Wpd-gyrovector.mcd» Сравнить $G_\varphi(\beta)$ и G_φ расч. не вводя $\beta = \frac{L}{R}$.

Флаг индекс $\vec{A} \cdot \vec{B} = -2 \sin\theta \nabla\gamma \cdot [\nabla\theta \times \nabla\Phi]$, $\Phi(\vec{r}) = m\varphi + \gamma(s,z)$ - helicity

$\nabla\Phi = \hat{s}\frac{\partial\gamma}{\partial s} + \hat{\varphi}\frac{m}{s} + \hat{z}\frac{\partial\gamma}{\partial z} = \hat{\varphi}\frac{m}{s} + \nabla\gamma$, $\nabla\gamma \cdot \left[\nabla\theta \times \left\{\hat{\varphi}\frac{m}{s} + \nabla\gamma\right\}\right] = \nabla\gamma \cdot \left[\nabla\theta \times \hat{\varphi}\frac{m}{s}\right]$

$\nabla\theta \times \hat{\varphi} = \left[\frac{\partial\theta}{\partial s}\hat{s} + \frac{\partial\theta}{\partial z}\hat{z}\right] \times \hat{\varphi} = \frac{\partial\theta}{\partial s}\hat{s} \times \hat{\varphi} + \frac{\partial\theta}{\partial z}\hat{z} \times \hat{\varphi} = \frac{\partial\theta}{\partial s}\hat{z} + \frac{\partial\theta}{\partial z}(-\hat{s})$

$\nabla\gamma \cdot [\nabla\theta \times \nabla\Phi] = \frac{m}{s}\nabla\gamma \cdot [\nabla\theta \times \hat{\varphi}] = \frac{m}{s}\left[\frac{\partial\gamma}{\partial s}\hat{s} + \frac{\partial\gamma}{\partial z}\hat{z}\right] \cdot \left[\frac{\partial\theta}{\partial s}\hat{z} - \frac{\partial\theta}{\partial z}\hat{s}\right] =$

$= \frac{m}{s}\left[\frac{\partial\gamma}{\partial z}\frac{\partial\theta}{\partial s} - \frac{\partial\gamma}{\partial s}\frac{\partial\theta}{\partial z}\right]$, $\vec{A} \cdot \vec{B} = -2\sin\theta \frac{m}{s}\left[\frac{\partial\gamma}{\partial z}\frac{\partial\theta}{\partial s} - \frac{\partial\gamma}{\partial s}\frac{\partial\theta}{\partial z}\right]$

$Q_H = -\frac{1}{(4\pi)^2}\int dV \vec{A}\cdot\vec{B} = +\frac{2m}{(4\pi)^2}\int ds\int d\varphi\int dz \frac{\sin\theta}{s}[\ldots] = \frac{m}{4\pi}\int ds\int dz \sin\theta\left[\frac{\partial\gamma}{\partial z}\frac{\partial\theta}{\partial s} - \frac{\partial\gamma}{\partial s}\frac{\partial\theta}{\partial z}\right]$,

Далее с инв. по Вангу (небольшой зазор Wang'ing) Сравнить с $Q_H(\text{Wang})$, $E_2(s,0)$

$n=1$, $Q_H = \frac{1}{4\pi}\int_0^R ds \int_{-\frac{L}{2}}^{\frac{L}{2}} dz \left[\frac{\partial m_z}{\partial z}\frac{\partial\gamma}{\partial s} - \frac{\partial\gamma}{\partial z}\frac{\partial m_z}{\partial s}\right]$, $\frac{\partial\gamma}{\partial s} = -4a\frac{sz}{(s^2+z^2)^2}$, $\frac{\partial\gamma}{\partial z} = 2a\frac{(s^2-z^2)}{(s^2+z^2)^2}$

т.к. $s \in [0,R]$, $z \in [-\frac{L}{2}, \frac{L}{2}]$, $\frac{z}{a} \gg 1$, $\frac{s}{a} \gg 1$ как при $s, z \gg a$

где $\frac{\partial m_z}{\partial s}(\ldots)$ интегрируемая функция. $m_z(s,z) = 1 - \frac{2s^2a^2}{(a^2+s^2+z^2)^2}$

$\gamma(s,z) = \operatorname{Im}\left[\ln\{z+i(s+a)\} - \ln\{z+i(s-a)\}\right]$

$\frac{\partial\gamma}{\partial s} = \operatorname{Im}\left[\frac{i}{z+i(s+a)} - \frac{i}{z+i(s-a)}\right] = \operatorname{Im} i\left[\frac{z-i(s+a)}{z^2+(s+a)^2} - \frac{z-i(s-a)}{z^2+(s-a)^2}\right] =$

$= iz\left[\frac{1}{z^2+(s+a)^2} - \frac{1}{z^2+(s-a)^2}\right]$, $\frac{\partial\gamma}{\partial z} = \operatorname{Im}\left[\frac{1}{z+i(s+a)} - \frac{1}{z+i(s-a)}\right] =$

$= \operatorname{Im}\left[\frac{z-i(s+a)}{z^2+(s+a)^2} - \frac{z-i(s-a)}{z^2+(s-a)^2}\right] = \frac{(s-a)}{z^2+(s-a)^2} - \frac{(s+a)}{z^2+(s+a)^2}$, см. $\frac{\partial\beta}{\partial s}, \frac{\partial\beta}{\partial z}$ - 19/II

Сравн. 19/II $\frac{\partial m_z}{\partial z} = 32\frac{s^2a^2z}{(a^2+s^2+z^2)^3}$, $\frac{\partial m_z}{\partial s} = -16a^2s\frac{(a^2+z^2-s^2)}{(a^2+z^2+s^2)^3}$

$B_s = \frac{m}{s}\frac{\partial m_z}{\partial z} = 32\pi a^2\frac{sz}{(a^2+s^2+z^2)^3}$, Gauge invariance of $Q_H = \frac{1}{(4\pi)^2}\int dV \vec{A}\cdot\vec{B}$ (Wang'ing)

$\vec{A}' = \vec{A} + \nabla\chi$, $\vec{B}' = \vec{B}$, $H' = Q_H' = \frac{1}{(4\pi)^2}\int dV \vec{A}'\vec{B} = \frac{1}{(4\pi)^2}\int dV \vec{A}\cdot\vec{B} = Q_H + \frac{1}{(4\pi)^2}\int \vec{B}\cdot\nabla\chi\, dV$

$\int dV \vec{B}\cdot\nabla\chi = \int dV(\chi\vec{B}) - \int dV \chi \operatorname{div}\vec{B} = \oint_S \chi\vec{B}\cdot d\vec{S}$ должно $=0$ для g.invariance.

$Q_H' = Q_H + \frac{1}{(4\pi)^2}\oint_S \chi\vec{B}\, d\vec{S}$ $\int_{S=R} \chi\vec{B}\cdot\hat{s}\, dS = \int dS \chi B_s = 0$ $\forall\chi$?

$dS = R d\varphi dz$, $\int_{S=R} dS \chi B_s = R\int d\varphi\int dz \chi B_s = 2\pi R\int dz \chi B_s = 2\pi R^2 \cdot 32a^2\int dz \chi\frac{z}{(a^2+R^2+z^2)^3}$

$\int_{S=R} dS \chi B_s = 64\pi R^2a^2\int dz \chi(\vec{r})\frac{z}{(R^2+z^2)^3} \sim R^2\int d\varphi \chi(\vec{r})\frac{s}{(R^2+s^2)^3} \sim \frac{R^3}{R^6}\chi(R) = \frac{\chi(R)}{R^3}$

$R \gg a$

$\int_{S=R} dS \chi B_s = 0$ при $\frac{\chi(R)}{R^3} \to 0$, $R\to\infty$, т.е. $\chi(R) \sim R^{3-\delta}$, $\chi(R)$ растёт $R\uparrow$ медленнее R^3

$Q_H' = Q_H$ верно пот для произвольной $\chi(\vec{r})$, а лишь расходящихся медленнее R^3, $R\to\infty$

$\vec{A} = -2\nabla\chi + \vec{A}^e$, $\vec{A}^e = (1-\cos\theta)\nabla\Phi$, $\chi = \chi(s,z) = \beta(s,z) + \text{const}$, β - poloidal angle.

Целое число $\vec{A}$ дает Q_H = integer and gauge $\vec{A}' = \vec{A} + \nabla\chi$ invariant.

Калибровка $\chi(s,z)$ is fixed в ее задании от $\vec{r}(\vec{r})$. Use $\vec{G}$ независимо на $\beta(s,z)$ angle.

Какая калибровка $\vec{A}$ (Gladkowski) и как он ее выбрал? $\beta = \operatorname{arctg}\left(\frac{2a^2z}{s^2+z^2-a^2}\right)$

V. potential

(18/III) Gladikowski'97: $A_\alpha = \frac{2N}{a}\frac{\tau}{\sinh\gamma}(m_z-1)$, $A_\beta = \frac{2M}{a}\tau(m_z+1)$, $A_\gamma = 0$

$B_\alpha = \frac{2M}{a^2}\tau^2\frac{\partial w}{\partial\gamma}$, $B_\beta = -\frac{2N}{a^2}\frac{\tau^2}{\sinh\gamma}\frac{\partial w}{\partial\gamma}$, $B_\gamma = \frac{2N}{a^2}\frac{\tau^2}{\sinh\gamma}\frac{\partial w}{\partial\beta}$, $w = m_z(\gamma,\beta)$, $v_0 = \text{const}$

toroidal coord. (γ,β,α), $\alpha=\varphi$, $\rho = \frac{a}{\tau}\sinh\gamma$, $z = \frac{a}{\tau}\sin\beta$ - cyl. coordinates. $\tau = \cosh\gamma - \cos\beta$. The gauge chosen: $A_\alpha(\gamma=0)=0$, $A_\beta(\gamma=\infty)=0$ (Gauge)

$m_z(\gamma=0,\beta)=1$, $m_z(\gamma=\infty,\beta)=-1$. $\gamma=0$ - two of z-axis, $\gamma=\infty$ - circle $\rho=a$ in xOy-plane.

Boundary Conditions for $\vec m$ в координатах тора. $\gamma = \operatorname{arcth}\left(\frac{2a\rho}{\rho^2+z^2+a^2}\right)$

$A_\alpha = A_\varphi = 0$ на оси oz. $m_z(\gamma) = 1-2\,\mathrm{th}^2\gamma$, $m_z(\rho,z) = 1 - \frac{8a^2\rho^2}{(\rho^2+z^2+a^2)^2}$

$m_z+1 = -2\,\mathrm{th}^2\gamma$, $A_\alpha = \frac{2N}{a}\tau(-2)\,\mathrm{th}^2\gamma \to 0$ when $\gamma\to 0$.

$A_\beta = \frac{2M}{a}\tau(2-2\,\mathrm{th}^2\gamma) = \frac{4M}{a}\frac{\tau}{\cosh^2\gamma} = \frac{4M}{a}\frac{\sinh\gamma}{\cosh^2\gamma}(\cosh\gamma-\cos\beta) \to 0$ when $\gamma\to\infty$.

$A_\alpha = A_\varphi = -\frac{4N}{a}\tau\frac{\sinh\gamma}{\cosh^2\gamma}$, $A_\varphi = \frac{2N}{a}\frac{a}{\rho}(m_z-1) = \frac{2N}{\rho}(\cos\theta-1)$. $\vec A$(Glad), $\vec B$(Gl) в сферах больше,

т.к. $B_i = \varepsilon_{ijk}\vec m\cdot[\partial_j\vec m\times\partial_k\vec m]$ не содержит $(1/2)$.

$m_x+im_y = \sqrt{1-m_z^2}\,e^{i(N\alpha+M\beta+\nu_0)}$, $m_z(\gamma,\beta)$.

Shmiz'18: $W = \frac{m_x+im_y}{1+m_z} = \frac{2(x-iy)a}{(a^2-r^2-2iza)}$ дано $n=m=1$ вторичная, $m_x+im_y = \sqrt{(\cdot)}\,e^{i(n\varphi-m\beta)}$ с единицей из Gladkowski'97

Найти трансформ. от $(A_\gamma, A_\beta, A_\alpha)$ toroidal components to (A_ρ, A_φ, A_z) cyl. components.

См. 18/III. $\vec F = (F_\gamma, F_\beta, F_\alpha) = (F_\rho, F_\varphi, F_z)$. $\rho = a\frac{\sinh\gamma}{\tau}$, $z = a\frac{\sin\beta}{\tau}$, $\varphi=\alpha$,

$F_n = \gamma_{n\gamma}F_\gamma + \gamma_{n\beta}F_\beta + \gamma_{n\varphi}F_\varphi$. $\gamma_{\rho\gamma} = a h_\rho\frac{(1-\cosh\gamma\cos\beta)}{\tau^2}$, $\gamma_{\rho\beta} = -h_\rho a\frac{\sinh\gamma\sin\beta}{\tau^2}$, $\gamma_{\rho\varphi}=0$

$B_\rho = \frac{2N}{a}\sin\beta\frac{\partial w}{\partial\gamma}$, $h_\rho = h_z = 1$, $h_\varphi = \rho$. $\gamma_{z\gamma} = -\frac{a}{\tau^2}\sinh\gamma\sin\beta$, $\gamma_{z\beta} = \frac{a}{\tau^2}(\cosh\gamma\cos\beta-1)$

$\gamma_{\varphi\gamma} = h_\varphi\frac{\partial\varphi}{\partial\gamma}=0$, $\gamma_{\varphi\beta} = h_\varphi\frac{\partial\varphi}{\partial\beta}=0$. A_φ, B_φ можно не надо, они одинаковые в $\gamma_{\varphi\varphi}=0$.

обеих coordinate systems $(\alpha=\varphi)$. $B_\varphi = B_\alpha = \frac{2M}{a^2}\tau^2\frac{\partial w}{\partial\gamma}$.

$G_\varphi = \int dV B_\varphi = \int_0^\infty d\gamma\int_{-\pi}^{\pi}d\beta\int_0^{2\pi}d\varphi\,\frac{a^3}{\tau^3}\frac{2M}{a^2}\tau^2\frac{\partial w}{\partial\gamma} = 2\pi\cdot 2Ma\int_0^\infty d\gamma\int_{-\pi}^{\pi}d\beta\,\frac{1}{(\cosh\gamma-\cos\beta)}(-1)\frac{\sinh\gamma}{\cosh^2\gamma} =$

$= -16\pi aM\int_0^\infty d\gamma\int_{-\pi}^{\pi}d\beta\,\frac{\sinh\gamma}{\cosh^2\gamma(\cosh\gamma-\cos\beta)}$, $J(\gamma) = \int_{-\pi}^{\pi}d\beta\frac{1}{(\cosh\gamma-\cos\beta)} = \frac{4\pi}{\sqrt{\Lambda_{\cosh^2\gamma}-1}}\cdot\frac{1}{\cosh\gamma}$.

$J(\gamma) = \frac{2}{\cosh\gamma}\int_0^\pi d\beta\frac{1}{1-\frac{1}{\cosh\gamma}\cos\beta} = \frac{2}{\cosh\gamma}\frac{\pi}{\sqrt{1-1/\cosh^2\gamma}} = \frac{2\pi}{\cosh\gamma}\frac{\cosh\gamma}{\sqrt{\cosh^2\gamma-1}} = \frac{2\pi}{\sinh\gamma}$

$G_\varphi(\infty) = -16\pi aM\int_0^\infty d\gamma\frac{\sinh\gamma}{\cosh^2\gamma}J(\gamma) = -32\pi^2aM\int_0^\infty d\gamma\frac{\sinh\gamma}{\cosh^2\gamma}\frac{1}{\sinh\gamma} = -32\pi^2aM\int_0^\infty\frac{d\gamma}{\cosh^3\gamma}$

$\int_0^\infty\frac{d\gamma}{\cosh^3\gamma} = |\text{Fp.-P. 3.512}| = \frac{1}{2}B\left(\frac{1}{2},\frac{3}{2}\right) = \frac{1}{2}\frac{\Gamma(1/2)\Gamma(3/2)}{\Gamma(2)} = \frac{1}{2}\sqrt{\pi}\frac{\sqrt{\pi}}{2}1!! = \frac{\pi}{4}$

$G_\varphi(\infty) = -32\pi^2aM\cdot\frac{\pi}{4} = -8\pi^3aM$ $(M=\frac{N}{2})$ - это для бесконечного образца.

$G_\varphi(b) = -8\pi a\left[\operatorname{arctg}\left(\frac{2}{b}\right)+\frac{1}{2}\frac{b}{1+b^2/4}\right]$ для (L,R) цилиндрического слоя при $L,R\gg a$, $b=\frac{L}{R}$.

19/II дано $G_\varphi(\infty) = 6\pi\cdot 3\pi a\cdot 0{,}393$ - численно. $\frac{1}{a}G_\varphi^{num}(\infty) = 79{,}02$, $\frac{1}{a}G_\varphi^{an} = 8\pi^3 = 248{,}05$ -

сделал в 3,14 раз больше, в π раз больше см. 21/III $(0{,}393\approx\pi/8)$

$A_\beta = \gamma_{\beta\gamma}A_\gamma + \gamma_{\beta\beta}A_\beta + \gamma_{\beta\varphi}A_\varphi = \gamma_{\beta\beta}A_\beta = -\frac{a}{\tau}e\sinh\gamma\sin\beta\frac{2M}{a}\tau(m_z+1) = -2M\frac{\sinh\gamma\sin\beta}{\tau}(m_z+1)$

$A_\beta = -2M\frac{\sinh\gamma\sin\beta}{\tau}(m_z+1)$ сравнить с $A_\beta(\text{Wang}) = -(1+\cos\theta)\frac{\partial\beta}{\partial\rho}$, $\frac{\partial\beta}{\partial\rho} = -\frac{4az\rho}{(\rho^2+z^2)^2}$

→ Проверить $2)\int d\gamma\,d(\cos\theta)\,d\varphi\,\frac{1}{(\pi\pi)^2} = \int dx\,dy\,dz\,2\nu_0\cdot(\nabla\Phi\times\nabla\Phi)\sin\theta = \int dx\,dy\,dz\,J(\gamma,\Phi_1,\Phi_2)$ Якобиан

(19/III) Verne Volkov et al. 3D magn. Nature Comm. 14, 1491 (2023). Skyrm. с полем в кубике

проект и G-vector manuscript. Ей проверить G_φ и посмотреть Figs. $G_\varphi z(R)$.

20/III Изучение работы ppt PSD-2019 и поиск Оксане. переписка с Ruben и Gonzalo о Gyrotron Quantum project.

21/III Проверка hopfion gyrovector G_e - computation.

① 18/II G_e any L,R using (35)-eq. Wang'19 give B_e и $\Delta\Phi = \gamma(\rho,z) = \beta(\rho,z)$ "Hopf-gyrovector.mcd" $G_e(\infty) = 8\pi^2 a$. $G_e(R)$ part. $= R\int_0^{\frac{1}{R/a}} \frac{dt}{t} = \text{const}$

② 17/III G_e из $\int dV\vec{B} = \oint_S dS\,\vec{n}\times\vec{A}$, use eq. $\beta(\rho,z)$ при $\rho \gg a$, $z \gg a$, т.к. это биполярное на поверхности large at $\frac{R}{a} \gg 1$ и $\frac{L}{a} \gg 1$

$G_e(\beta) = 8\pi a\left[\arctg\frac{2}{b} + \frac{1}{2}\frac{b}{1+b^2/4}\right]$, $b = L/R$, будучи надо $\times 2$ для согласования с Ranci

③ Расчет $B_e(\infty)$ по eq. (3) Gladikowski, инт. в тороидальных координатах (γ,β,φ)

18/III $G_e(\infty) = -8\pi^3 a M$, $(M \pm 1)$. Это $G_e(\infty)$ в 2 раза больше $G_e(\infty)$ - 8/II.

В работе $G_e(\infty)$ 18/III ошибка. Надо $dV = \frac{a^3}{\tau^3}\sh\gamma\,d\gamma d\beta d\varphi$

$$G_e = \int dV\cdot B_e = \int_0^\infty d\gamma\int_{-\pi}^{\pi}d\beta\int_0^{2\pi}d\varphi\,\frac{a^3}{\tau^3}\sh\gamma\cdot\frac{2M}{a^2}\frac{\partial w}{\partial\gamma}\tau^2,\quad \tau = \ch\gamma - \cos\beta,\quad \frac{\partial w}{\partial\gamma} = -4\frac{\th\gamma}{\ch^2\gamma}$$

$$G_e = 2\pi\,2Ma(-4)\int_0^\infty d\gamma\int_{-\pi}^{\pi}d\beta\,\frac{\sh\gamma}{(\ch\gamma-\cos\beta)}\frac{\th\gamma}{\ch^2\gamma} = -16\pi Ma\int_0^\infty d\gamma\,\sh\gamma\frac{\th\gamma}{\ch^2\gamma}\int_{-\pi}^{\pi}\frac{d\beta}{(\ch\gamma-\cos\beta)} =$$

$$= -16\pi Ma\int_0^\infty d\gamma\,\sh\gamma\frac{\th\gamma}{\ch^2\gamma}\cdot\frac{2\pi}{\sh\gamma} = -32\pi^2 Ma\int_0^\infty d\gamma\frac{\th\gamma}{\ch^2\gamma} = \left|\begin{matrix}\th\gamma = x\\ \frac{d\gamma}{\ch^2\gamma} = dx\end{matrix}\right| = -32\pi^2 Ma\int_0^1 dx\,x =$$

$$= -32\pi^2 Ma\,\frac{x^2}{2}\Big|_0^1 = -16\pi^2 Ma,\quad G_e(\infty) = 16\pi^2 a\,(\mp 1),\ \text{где } M = \pm 1.$$

Это если брать Gladikowski'97 $B_e = B_\varphi = \frac{2M\tau^2}{a^3}\frac{\partial w}{\partial\gamma}$, но $\vec{B}(\text{Glad}) = -2\vec{B}(\text{Wang'19})$ т.к. $\vec{B}(\text{Glad})$ из $B_i = -\varepsilon_{ijk}\,\vec{m}\,(\partial_j\vec{m}\times\partial_k\vec{m})$ eq. (12).

③ Corrected $G_e(\infty) = \frac{1}{2}\cdot(16\pi^2 a)(\mp 1) = 8\pi^2 a\,(\mp 1)$ где $M = \pm 1$, это совпадает с работой ① $G_e(\infty) = 8\pi^2 a$. $\vec{G}(\text{Glad}) = (G_\gamma, G_\beta, G_\varphi) = 16\pi^2 a\left(0, \frac{\pi}{2}N, -M\right)$ я получил еще 11/XI 2020 (!). 14/XI 20г. сделанное compleдy $W(x+iy)\to\vec{m}$ у Lin'2020 и Shnir'2018. у Shnir непонятно.

Метод ③ (γ,β,φ) toroidal coordinates нормируют γ на ∞-сферу $(\gamma\in[0,\infty])$.

Cylindr. coordinates $\rho = \frac{a}{\tau}\sh\gamma$, $z = \frac{a}{\tau}\sin\beta$, $\varphi = \varphi$, $\tau = \ch\gamma - \cos\beta$.

$$B_\varphi = \frac{M}{a^2}\tau^2\frac{\partial w}{\partial\gamma},\quad \frac{\partial w}{\partial\gamma} = -4\frac{\th\gamma}{\ch^2\gamma}\cdot\quad \gamma + i\beta = \ln\left(\frac{z+i(\rho+a)}{z+i(\rho-a)}\right),\quad \beta = \arctg\left(\frac{2az}{\rho^2+z^2-a^2}\right)$$

$$\frac{\partial\beta}{\partial\rho} = z\left[\frac{1}{z^2+(\rho+a)^2} - \frac{1}{z^2+(\rho-a)^2}\right],\quad \frac{\partial\beta}{\partial z} = \frac{(\rho-a)}{z^2+(\rho+a)^2} - \frac{(\rho+a)}{z^2+(\rho-a)^2},\quad \gamma = \frac{1}{2}\ln\left[\frac{z^2+(\rho+a)^2}{z^2+(\rho-a)^2}\right]$$

$\gamma = \frac{1}{2}\ln x$, $\sh\gamma = \frac{1}{2}\left[e^\gamma - e^{-\gamma}\right] = \frac{1}{2}\left[e^{\frac{1}{2}\ln x} - e^{-\frac{1}{2}\ln x}\right] = \frac{1}{2}\left[\sqrt{x} - \frac{1}{\sqrt{x}}\right]$.

$\ch\gamma = \frac{1}{2}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$, $\th\gamma = \frac{\sqrt{x}-\frac{1}{\sqrt{x}}}{\sqrt{x}+\frac{1}{\sqrt{x}}} = \frac{x-1}{x+1}$, $x = \frac{z^2+(\rho+a)^2}{z^2+(\rho-a)^2}$.

$\th^2\gamma = 1 - 2\th^2\gamma$, $m_z = 1 - \frac{8\rho^2a^2}{(a^2+\rho^2+z^2)^2}$, $\th^2\gamma = \frac{4\rho^2a^2}{(a^2+z^2+\rho^2)^2}$, $\th\gamma = \frac{2\rho a}{(z^2+\rho^2+a^2)}$

$\frac{\partial w}{\partial\gamma} = (-4)\frac{\th\gamma}{\ch^2\gamma} = (-4)\th\gamma\left(1-\th^2\gamma\right)$. $\tg\beta = \frac{2az}{(\rho^2+z^2-a^2)}$, $\cos\beta = \frac{1}{\sqrt{1+\tg^2\beta}}$.

$$B_e(\rho,z) = \frac{M}{a^2}\left(\frac{x+1}{2\sqrt{x}} - \frac{1}{\sqrt{1+\frac{z^2}{\rho^2}}\rho}\right)^2(-4)\th\gamma\left(1-\th^2\gamma\right) - \text{что-то здесь больше } \int_0^{\beta_{max}}\int_0^{B_e}(\ldots)$$

Сам. Wang'19: $B_\varphi = \sin\beta\left[\frac{\partial\gamma}{\partial\rho}\frac{\partial\beta}{\partial z} - \frac{\partial\gamma}{\partial z}\frac{\partial\beta}{\partial\rho}\right] = \frac{\partial m_z}{\partial\rho}\frac{\partial\beta}{\partial z} - \frac{\partial m_z}{\partial z}\frac{\partial\beta}{\partial\rho}$, $m_z = 1 - 8\frac{\rho^2a^2}{(a^2+\rho^2+z^2)^2}$

Сам. 19/II $\frac{\partial m_z}{\partial\rho} = -16a^2\rho\frac{(a^2+z^2-\rho^2)}{(a^2+z^2+\rho^2)^3}$, $\frac{\partial m_z}{\partial z} = 32\frac{\rho^2a^2z}{(a^2+\rho^2+z^2)^3}$

$$B_\varphi(\rho,z) = -16a^2\rho\frac{(a^2+z^2-\rho^2)}{(a^2+z^2+\rho^2)^3}z\left[\frac{1}{z^2+(\rho+a)^2} - \frac{1}{z^2+(\rho-a)^2}\right] - 32\frac{\rho^2a^2z}{(a^2+\rho^2+z^2)^3}[\ldots] - \text{нет.}$$

$$\frac{\partial\beta}{\partial\rho} = \frac{z}{\left[z^2+(\rho+a)^2\right]\left[z^2+(\rho-a)^2\right]}\left[z^2+(\rho-a)^2 - z^2-(\rho+a)^2\right] = -4\frac{\rho a z}{\left[z^2+(\rho+a)^2\right]\left[z^2+(\rho-a)^2\right]}$$



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$$\frac{\partial\beta}{\partial z} = \frac{1}{[z^2+(\varrho+a)^2][z^2+(\varrho-a)^2]}\left[(\varrho-a)\left\{z^2+(\varrho+a)^2\right\} - (\varrho+a)\left\{z^2+(\varrho-a)^2\right\}\right] =$$

$$= \frac{1}{[\dots][\dots]}\left[\varrho\cdot 4\varrho a - a\left\{z^2+(\varrho+a)^2+z^2+(\varrho-a)^2\right\}\right] = 2a\,\frac{2\varrho^2-z^2-\varrho^2-a^2}{[z^2+(\varrho+a)^2][z^2+(\varrho-a)^2]}$$

$$B_\varrho(\varrho,z) = -16a^2\varrho\,\frac{(a^2+z^2-\varrho^2)}{(a^2+z^2)^3}\cdot 2a\,\frac{(\varrho^2-z^2-a^2)}{[z^2+(\varrho+a)^2][z^2+(\varrho-a)^2]} -$$

$$- 32\,\frac{\varrho^2a^2z}{(a^2+z^2)^3}(-4)\,\frac{\varrho a z}{[z^2+(\varrho+a)^2][z^2+(\varrho-a)^2]} = \frac{32a^3\varrho}{(a^2+z^2)^3[z^2+(\varrho+a)^2][z^2+(\varrho-a)^2]} \times$$

$$\times\left[(a^2+z^2-\varrho^2)^2 + 4\varrho^2z^2\right], \quad \underline{B_\varrho(\varrho,z) = \frac{32a^3\varrho\left[(a^2+z^2-\varrho^2)^2+4\varrho^2z^2\right]}{(a^2+z^2)^3[z^2+(\varrho+a)^2][z^2+(\varrho-a)^2]}}$$

Это $B_\varrho(\varrho,z)$ с учётом $\varrho\to\varrho/a$, $z\to z/a$ совпадает с $B_\varrho(\varrho,z)$ из III, т.е. оба они верны.

$$[z^2+(\varrho+a)^2][z^2+(\varrho-a)^2] = z^4+z^2\left\{(\varrho+a)^2+(\varrho-a)^2\right\}+(\varrho+a)^2(\varrho-a)^2 =$$

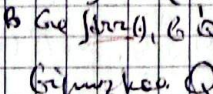
$$= z^4+2z^2(\varrho^2+a^2)+(\varrho^2-a^2)^2, \quad [\text{знаменатель}] = (a^2-\varrho^2)^2+z^4+2z^2(a^2-\varrho^2)+4\varrho^2z^2$$

Упрощение $\underline{B_\varrho(\varrho,z) = \frac{32a^3\varrho}{(a^2+z^2+\varrho^2)^3}}$ – emergent mag. field of toroidal hopfion with $Q_H=1$.

$$G_{(L,R)} = \int_{tot} dV\, B_\varrho(\varrho,z) = 64\pi a^3\int_0^R d\varrho\,\varrho\int_{-L/2}^{L/2} dz\,\frac{\varrho}{(a^2+z^2+\varrho^2)^3} = 64\pi a\int_0^{R/a}d\varrho\,\varrho\int_{\frac{1}{2a}}^{L/2a}dz\,\frac{\varrho}{(1+z^2+\varrho^2)^3}$$

$G_\varrho(\infty) = 8\pi^2 a$.

Note: Hopf index Q_H is proportional to $G_\varrho = \int dV B_\varrho$, $Q_H(R)\to 1$ т.к. $Q_H\neq 0$, G_ϱ не может $=0$ (!). Нет B G_ϱ растёт, а Q_H стремится к 1.



(22/III) подготовка к новым applications на Eurasian Quantum Project (с Putkem)
Подам с Ант. application на firma digital.

(23/III) Ананян 1408€ deposit 25% сена Londres. переписка с Рабе Стрижка Quantum. работа в саду.

(24/III) Event «Tecnologias quanticas (Cuanticas)» в Tabakalera, Donostia. Meeting с Julian o Sobsky Mag-2023.

Hopf index $Q_H = \frac{1}{(4\pi)^2}\int dV\,\vec A\cdot\vec B = \frac{n}{4\pi}\int_0^R\int_{-L/2}^{L/2} d\varrho\int dz\,\sin\theta\left[\frac{\partial\beta}{\partial z}\frac{\partial\gamma}{\partial\varrho} - \frac{\partial\beta}{\partial\varrho}\frac{\partial\gamma}{\partial z}\right] =$

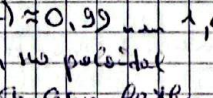
$\gamma(\varrho,z) = \beta(\varrho,z) + \text{const}$ · helicity.

$$= \frac{n}{4\pi}\int_0^R d\varrho\int_{-L/2}^{L/2} B_\varrho(\varrho,z) = Q_H(R,L)$$

$Q_H(R)$ насыщается быстрее чем $G_\varrho(R)$.
(assuming $\beta=L/R=\text{const}$)

Hopfion radius $a = a(R,L)$ из $\min E(R,L,\dots)$.
A given magnetic and geometric nonparametric $a_{eq}(R,L,\dots)$ надо сначала минимизировать, что $R/a_{eq}\gg 1$ и $Q_H(R,L)\approx 0,99$ или $1,00$.

При $R/a=10$, $\beta=1$. $G_\varrho(10,1)=0,973$, $Q_H(10,1)=0,996$
$G_\varrho(5,1)=0,901$, $Q_H(5,1)=0,943$.



$\beta(\varrho,z) = \arctg\left(\frac{2az}{\varrho^2+z^2-a^2}\right)$, т.е. $\beta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, но полоидальный угол $\beta\in[-\pi,\pi]$ или $[0,2\pi]$. Надо сделать ген. фейк

$\beta(\varrho,z) = \operatorname{Im}\ln\left[\frac{z+i(\varrho+a)}{z+i(\varrho-a)}\right]$. $\ln(x+iy) = \ln\left(\sqrt{x^2+y^2}\right) + i(\varphi+2\pi k)$, $k=0,\pm1,\pm2,\dots$ $\arctg(i\infty)$.

$x+iy = |x+iy|e^{i\varphi}$, $\varphi = \arctg\left(\frac{y}{x}\right)$

(25/III) Работа в саду. Огород дома. Бумаги.

(26/III) M.A. Berger et al. J. Fluid Mech. 147, 133-148 (1984). The topological properties of magnetic helicity. Helicity (Hopf index) $H = \int d^3x\,\vec A\cdot\vec B$, $\vec B = \nabla\times\vec A$
Coulomb gauge $\vec A(\vec r) = \frac{1}{c}\int d^3x'\frac{\vec j(\vec x')}{\vec r}$, $\vec r = \vec x - \vec x'$, $\vec j = \frac{c}{4\pi}\operatorname{rot}\vec B \to \vec A(\vec x) = -\frac{1}{4\pi}\int d^3x'\,\frac{\vec r}{r^3}\times\vec B(\vec x')$,
Вижу ещё Hopf index $H = -\frac{1}{4\pi}\int d^3x\,d^3x'\,\vec B(\vec x)\cdot\frac{\vec r}{r^3}\times\vec B(\vec x')$, есть у Kundu '82.

Coulomb: $\operatorname{div}\vec A = 0$

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V.I. Arnold, B.A. Khesin. topological methods in hydrodynamics. Springer 1998.

Ch. III, p. 121. Helicity (topol. invariant) $H(\vec{\xi}) = \int dV\, \vec{v}\cdot \mathrm{curl}^{-1}\vec{\xi}$, $\vec{A} = \mathrm{curl}^{-1}\vec{\xi}$ a divergence free vector potential of field $\vec{\xi}$, $\mathrm{rot}\vec{A} = \mathrm{curl}\vec{A} = \vec{\xi}$, $\mathrm{div}\vec{A} = 0$

Gladkowski 97: toroidal coordinates (γ, β, α), $r_0 = \mathrm{const}$, Cu. 11/12 2020.

$$A_\alpha = \frac{2M}{a}\frac{\tau}{\mathrm{sh}\gamma}(m_z - 1),\quad A_\beta = \frac{2M}{a}\tau(m_z+1),\quad A_\gamma = 0,\quad (\alpha = \varphi \text{ cylindr.})$$

$$\mathrm{div}\vec{A} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial\beta}\left(\frac{a^2}{\tau^2}\mathrm{sh}\gamma A_\beta\right) + \frac{\partial}{\partial\gamma}\left(\frac{a^2}{\tau^2}\mathrm{sh}\gamma A_\gamma\right) + \frac{\partial}{\partial\alpha}\left(\frac{a^2}{\tau^2}A_\alpha\right)\right] =$$

$$= \frac{\tau^3}{a^3\mathrm{sh}\gamma}\left[\frac{\partial}{\partial\beta}\left(\frac{a^2}{\tau^2}\mathrm{sh}\gamma\frac{2M}{a}\tau(m_z+1)\right) + \frac{\partial}{\partial\alpha}\left(\frac{a^2}{\tau^2}\frac{2M}{a}\frac{\tau}{\mathrm{sh}\gamma}(m_z-1)\right)\right],\quad \tau = \mathrm{ch}\gamma - \cos\beta,\quad m_z = \psi(\gamma)$$

$\frac{\partial}{\partial\alpha}(\dots) = 0$, i.e. $(\dots)$ не зависит от α. $\mathrm{div}\vec{A} = \frac{\tau^3}{a^3\mathrm{sh}\gamma}a^2\mathrm{sh}\gamma\frac{2M}{a}(m_z+1)\frac{\partial}{\partial\beta}\left(\frac{1}{\tau}\right) =$

$$= \frac{2M}{a^2}\tau^3(m_z+1)\frac{\partial}{\partial\beta}\frac{1}{(\mathrm{ch}\gamma - \cos\beta)} = \frac{2M}{a^2}\tau^3(m_z+1)(-1)\frac{\sin\beta}{(\mathrm{ch}\gamma-\cos\beta)^2} \neq 0,\quad \mathrm{div}\vec{A}(\mathrm{Glad}) \neq 0.$$

Wang-Breaking: $A_\beta = -(1+\cos\theta)\frac{\partial\chi}{\partial\varphi}$, $A_\varphi = \frac{n}{\beta}(1-\cos\theta)$, $A_z = -(1+\cos\theta)\frac{\partial\chi}{\partial z}$, (β, φ, z) cyl.

$$\mathrm{div}\vec{A} = \frac{1}{\beta}\frac{\partial}{\partial\beta}(\beta A_\beta) + \frac{1}{\beta}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z} = \frac{1}{\beta}\frac{\partial}{\partial\beta}\left[-\beta(1+\cos\theta)\frac{\partial\chi}{\partial\beta}\right] + \frac{\partial}{\partial z}(-1)(1+\cos\theta)\frac{\partial\chi}{\partial z},\quad \theta(\beta, z),\ \chi(\beta, z).$$

$$\mathrm{div}\vec{A} = -\frac{1}{\beta}\left[(1+\cos\theta)\frac{\partial\chi}{\partial\beta} + \beta\frac{\partial}{\partial\beta}\left((1+\cos\theta)\frac{\partial\chi}{\partial\beta}\right)\right] - \frac{\partial}{\partial z}(1+\cos\theta)\frac{\partial\chi}{\partial z},\quad \theta(\beta, z),\ \chi(\beta, z).$$

$$\mathrm{div}\vec{A}(\mathrm{Glad}) = -\frac{2M}{a^2}\tau^3(m_z+1)\frac{\sin\beta}{\tau^2} = \left|\sin\beta = \frac{\tau z}{a}\right| = -\frac{2M}{a^2}\tau^3(m_z+1)\frac{1}{\tau^2}\frac{\tau z}{a}$$

$$\mathrm{div}\vec{A}(\mathrm{Glad}) = -\frac{2M}{a^3}\cdot z\tau^2(1+m_z) \neq 0$$

Видно $\mathrm{div}\vec{A}(\mathrm{Glad}) \neq 0$, $\mathrm{div}\vec{A}(\mathrm{Wang}) \neq 0$, хотя Arnold'98, Berger'84 ($\vec{A}$ — hydrodynamics) $\mathrm{div}\vec{A} = 0$ Coulomb gauge.

Почему $\mathrm{div}\vec{A}$ тороидальное поле и $\mathrm{div}\vec{A}$ (Arnold) $= 0$ разумно? Разные $S_3 \to S_2$ maps?

A.M. Kamchatnov. Topological solitons in magnetohydrodynamics. Sov. Phys. JETP 55, 69 (1982).

Явный вид $S_3 \to S_2$ mapping for Hopf index $Q_H = 1$. S_3 $(u_i, i=1..4, \sum u_i^2 = 1) \to S_2$ $(\xi_i, i=1,2,3, \sum \xi_i^2 = 1)$

$$\xi_1 = 2(u_1u_3 + u_2u_4),\quad \xi_2 = 2(u_2u_3 - u_1u_4),\quad \xi_3 = u_1^2 + u_2^2 - u_3^2 - u_4^2$$

Сам $u_1 + u_2 i = a\cos(\varphi+\varphi_1)$, $u_2 = a\sin(\varphi+\varphi_1)$, $u_1 + u_2 i = ae^{i(\varphi+\varphi_1)}$

и $u_3 + iu_4 = be^{i(\varphi+\varphi_0)}$, $a^2 + b^2 = 1$, dv — like element на S_3, то $(x_1, x_2, x_3) \in S_2$

$$x_1 + ix_2 = \frac{a}{\Sigma}e^{i(\varphi+\varphi_1)},\quad x_3 = \frac{b}{\Sigma}\sin(\varphi+\varphi_0),\quad \Sigma = 1 + b\cos(\varphi+\varphi_0)$$

$\vec{A} = \frac{1}{2\pi}(-u_2, u_1, u_4, -u_3)$, $\vec{A}$ из $A_i = (1+u_4)\tilde{A}_i - u_i\tilde{A}_4$, $i = 1,2,3$.

$\tilde{A}_\mu$ из условия $\tilde{A}_\mu u_\mu = 0$, $\vec{A}$ is tangent to the sphere S_3. vector potential $\vec{B} = \mathrm{rot}\vec{A}$, $H = \int d^3x\, \vec{A}\cdot\vec{B} = 1$ (Hopf index)

S.J. van Enk. J. Phys. A: Math. Theor. 46, 175204 (2013). Hopf — Rañada solutions.

$\vec{B} = \nabla\alpha\times\nabla\beta$, α, β — Euler potentials, почему нельзя $\vec{A} = -\beta\nabla\alpha$? $Q_H = 0$

$\vec{B} = \mathrm{rot}\vec{A}$, описание $\vec{A}$ через зон. угол ψ.

Y. Liu, H. Watanabe, N. Nagaosa. Emergent magnetic multipoles and nonlinear responses of magnetic hopfion. PRL 129, 267201 (2022).

Утверждение: это тороидальный хопфион с $Q_H = 1$, $\vec{B}^e = (0, B_\varphi^e, 0)$ ext. field is circulating around Oz, поэтому $B_\varphi^e \neq 0$ и $\langle\vec{B}^e\rangle_V = \vec{G} = 0$ (zero gyrovector).

$$B_\varphi(\beta, z) = \frac{32a^3\beta}{(a^2+z^2+\beta^2)^3},\quad B_\beta(\beta, z) = \frac{1}{\beta}\frac{\partial m_z}{\partial z} = \frac{32a^2\beta z}{(a^2+z^2+\beta^2)^3},\quad B_z = -\frac{1}{\beta}\frac{\partial m_z}{\partial\beta} = \frac{16a^2(a^2+z^2-\beta^2)}{(a^2+z^2+\beta^2)^3}$$

Только $\lim z=0$, $\beta = a$, на оси будет $B_\varphi(a,0) = \frac{4}{a^2}$, $B_\beta(a,0) = 0$, $B_z(a,0) = 0$

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$G_\varphi = 0$, т.к. $B_\varphi(\rho,z)$ is odd on "z" $\forall L,R$. $G_z \sim \frac{a}{R}$, $R/a \gg 1$, т.к $B_z(\rho,z)$ changes sign, $G_\varphi \neq 0$ т.к. $B_\varphi(\rho,z) \geq 0$ $\forall \rho,z$ + имеет постоянный знак, $B_\varphi \geq 0$.

→ Hopfion gyrovector $\vec{G} \neq 0$, поэтому eq-s of motion Liu-Nagaosa, a'22 с $\vec{G}=0$ - неверна.

Ivanov-Kosevich 'Spin' 2013, $B_z = \frac{\hbar}{2e} \vec{m}\cdot(\partial_x\vec{m} \times \partial_y \vec{m}) = \frac{\hbar}{2e} \sin\theta\,\nabla\theta \times \nabla\varphi$

Superconducting flux quantum $\Phi_0 = \frac{h}{2e}$. $B_z = \frac{1}{2\pi}\Phi_0 \sin\theta\,\nabla\theta\times\nabla\varphi$

$\int dS_z \sin\theta\,\nabla\theta(\rho)\times\nabla\varphi(\varphi) = \int dS \sin\theta \frac{\partial\theta}{\partial\rho}\frac{\partial\varphi}{\partial\varphi}\hat{z}\cdot\hat{z}\frac{1}{\rho} = \int d\rho \sin\theta \frac{\partial\theta}{\partial\rho}\int d\varphi \frac{1}{\rho} =$

$= |\Phi = q\varphi + \Psi_0| = q\int_0^R d\rho \int_0^{2\pi} d\varphi \left(-\frac{\partial m_z}{\partial\rho}\right)\frac{1}{\rho} = -2\pi q \int d\rho \frac{\partial m_z}{\partial\rho} = -2\pi q\,[m_z(R) - m_z(0)]$

Дно вортекса $m_z(R)=0$ $\int dS \sin\theta\,\nabla\theta\times\nabla\varphi = 2\pi q p$, $p = m_z(0)$ - скачок. по знаку

Flux $\int d\vec{S}\vec{B} = \int dS_z B_z = \frac{\Phi_0}{2\pi}\int dS_z \sin\theta\nabla\theta\times\nabla\varphi = \frac{\Phi_0}{2\pi}\cdot 2\pi pq = \Phi_0 \cdot pq$, $\Phi_0 = \frac{h}{2e}$.

$\int d\vec{S}\cdot\vec{B} = pq\Phi_0$, $\Phi_0 = h/2e$. Может надо — flux quantum deg. $\Phi_0 = h/e$?

Nagaosh-Tokura Nat Nanot '2013: $\Phi_0 = h/e$ - unit flux associated with each skyrmion

для скирмиона $\int d\vec{S}\cdot\sin\theta\nabla\theta\times\nabla\varphi = 4\pi pq$

$\int d\vec{S}\cdot\vec{B} = \int d\vec{S}_z B_z = \int dS_z B_z = \int dS_z \frac{h}{4\pi e}\sin\theta\nabla\theta\nabla\varphi = \frac{h}{4\pi e}\cdot 4\pi pq = \frac{h}{e}pq$ и

$\Phi_0 = h/e$ is flux quantum.

(27/III) B_φ и $G_\varphi = \int dV B_\varphi$ считал по Wang-Braatas-PRL'19 на основе $\beta(\rho,z)$.

$\Delta\Phi = \gamma(\rho,z) = \beta(\rho,z) + const$, β - poloidal angle toroidal coord. (η,β,α).

$G_\varphi(\omega)$ для ∞-sample можно найти directly по $B_\varphi(Glad) = f(\eta,\beta)$.

$\vec{G}(Glad) = (G_\eta, G_\beta, G_\alpha) = 16\pi^2 a\,(0, \frac{\pi}{2}N, -M)$ для ∞-Sample.

21/III $B_\varphi(\rho,z) = -\frac{4}{a^2}\left(\frac{x+1}{2\sqrt{x}} - \frac{1}{\sqrt{1+4q^2\rho}}\right)^2 th\eta\,(1-th^2\eta)$, $x = \frac{z^2+(\rho+a)^2}{z^2+(\rho-a)^2}$, $th\eta = \frac{2\rho a}{(z^2+\rho^2+a^2)}$

$\frac{x+1}{2\sqrt{x}} - \frac{1}{\sqrt{1+4q^2\rho}} = \frac{1}{2}\frac{z^2+(\rho+a)^2+z^2+(\rho-a)^2}{\left[z^2+(\rho-a)^2\right]\sqrt{\frac{z^2+(\rho+a)^2}{z^2+(\rho-a)^2}}} - \frac{1}{\sqrt{1+\frac{4a^2z^2}{(\rho^2+z^2-a^2)^2}}}$, $tg\beta = \frac{2az}{(\rho^2+z^2-a^2)}$

$= \frac{1}{2}\frac{2z^2+2\rho^2+2a^2}{\sqrt{\left[z^2+(\rho-a)^2\right]\left[z^2+(\rho+a)^2\right]}} - \frac{(\rho^2+z^2-a^2)}{\sqrt{(\rho^2+z^2-a^2)^2+4a^2z^2}}$

$(\rho^2+z^2-a^2)^2 + 4a^2z^2 = (\rho^2-a^2)^2 + z^4 + 2z^2(\rho^2-a^2) + 4a^2z^2 = (\rho^2-a^2)^2 + z^4 + 2z^2\rho^2 + 2z^2a^2 =$

$= \left(z^2+(\rho+a)^2\right)\left[z^2+(\rho-a)^2\right]$, $\frac{x+1}{2\sqrt{x}} - \frac{1}{\sqrt{4q^2\rho+1}} = \frac{z^2+\rho^2+a^2-(\rho^2+z^2-a^2)}{\sqrt{\left[z^2+(\rho-a)^2\right]\left[z^2+(\rho+a)^2\right]}} =$

$= \frac{2a^2}{\sqrt{\left[z^2+(\rho-a)^2\right]\left[z^2+(\rho+a)^2\right]}}$, $B_\varphi(\rho,z) = -\frac{4}{a^2}\left[\frac{2a^2}{\sqrt{(+)}\sqrt{(-)}}\right]^2 \frac{2\rho a}{(z^2+\rho^2+a^2)}\left[1-\frac{4\rho^2a^2}{(z^2+\rho^2+a^2)^2}\right]$

$= -8\frac{1}{\sqrt{\left[z^2+(\rho-a)^2\right]\left[z^2+(\rho+a)^2\right]}}\frac{\rho a}{(z^2+\rho^2+a^2)^3}\left[(z^2+\rho^2+a^2)^2 - 4\rho^2a^2\right]$ — надо что-то, надо (...)^2.

$\left[(z^2+\rho^2+a^2)^2 - 4\rho^2a^2\right] = (z^2+a^2)^2 + \rho^4 + 2\rho^2(z^2+a^2) - 4\rho^2a^2 = (z^2+a^2)^2 + \rho^4 + 2\rho^2(z^2-a^2) =$

$= z^4 + 2a^2z^2 + a^4 + \rho^4 + 2\rho^2z^2 - 2\rho^2a^2 = (\rho^2-a^2)^2 + z^4 + 2z^2(a^2+\rho^2) =$

$= \left[z^2+(\rho+a)^2\right]\left[z^2+(\rho-a)^2\right]$.

$B_\varphi(\rho,z) = -\frac{4}{a^2}\frac{4a^4}{\left[z^2+(\rho+a)^2\right]\left[z^2+(\rho-a)^2\right]}\frac{2a\rho}{(z^2+\rho^2+a^2)^3}\left[(z^2+\rho^2+a^2)^2 - 4\rho^2a^2\right] = -32\frac{a^3\rho}{(z^2+\rho^2+a^2)^3}$

- модуль $B_\varphi(\rho,z)$ как 21/III, знак (-1) из-за $M=+1$, надо $M=-1$?

$B_\varphi = \frac{M}{a^2}\hat{\tau}^2\frac{\partial w}{\partial\eta}$ Gladikowski // т.е. $B_\varphi(\rho,z) = \frac{32a^3\rho}{(z^2+\rho^2+a^2)^3}$ могу ↑ из $B_\varphi(Glad)$

Заменить (η,β,α) на (ρ,φ,z) и не использовать $\gamma = \varphi + \beta(\rho,z)$, $\Delta\Phi = \beta(\rho,z)$ в B_φ (Wang'19). $B_\varphi(\rho,z) = \frac{32a^3\rho}{(z^2+\rho^2+a^2)^3}$ - верно.

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Поканена на PM-2023 Poznan talk Mag. vortex string gyrotropic dynamics in thick cylindrical nanodots"

Поканен на NAP-2023 invited "Nutation excitations ..."

SMM-2023, Prague, 4-7 Sept. 2023 NAP-2023, Bratislava, Sep. 10-15, 2023

Поканен comments to Nagaosa [Lin PRL 2022], 0 как ли gyrovector $\vec{G} \neq 0$ ($G_\varphi \neq 0$).

SolSkyMag'23 пеpомиска. 31 May - 2 June 2023, La Guardia Portugal — Quantum meeting ЕКЦ.

Lin-Nagaosa PRL'2022: $\int dV \vec{A}^e = \frac{1}{2}\int dV\, \vec{r}\times\vec{B}^e = \vec{T}^e$ - emergent toroidal moment. $\vec{A}^e, \vec{B}^e$ - emergent vector potential, $(B^e)_i = \frac{1}{2}\varepsilon_{ijk}\vec{m}\cdot(\partial_j\vec{m}\times\partial_k\vec{m})$

$\vec{B}^e = \mathrm{rot}\vec{A}^e$, $\vec{A}^e = \frac{1}{2}(1-\cos\theta)\,\nabla\varphi$ [Skyrmion], φ по перпендикулярна

Comm. 19/III $B_z(\rho,z) = -\frac{n}{\rho}\frac{\partial m_z}{\partial\rho} = -\frac{n}{\rho}(-16)a^2\rho\frac{(a^2+z^2-\rho^2)}{(a^2+\rho^2+z^2)^3} = 16a^2\frac{(a^2+z^2-\rho^2)}{(a^2+\rho^2+z^2)^3}$, Сн. 26/III

$\mathrm{div}\vec{B} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho B_\rho) + \frac{1}{\rho}\frac{\partial B_\varphi}{\partial\varphi} + \frac{\partial B_z}{\partial z} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\frac{32a^2z\rho^2}{Z^3} + \frac{\partial}{\partial z}\frac{16a^2(a^2+z^2-\rho^2)}{Z}$ $Z = a^2+z^2+\rho^2$

$\mathrm{div}\vec{B} = \frac{32a^2z}{\rho}\frac{\partial}{\partial\rho}\frac{\rho^2}{Z^3} + 16a^2\frac{(a^2+z^2-\rho^2)}{Z^3} = \frac{32a^2z}{\rho}\frac{2\rho(a^2+z^2+\rho^2)-\rho^2\cdot 2\rho}{Z^6}$ - нет.

$\mathrm{div}\vec{B} = \frac{32a^2z}{\rho}\frac{1}{Z^6}\left[2\rho Z^3 - \rho^2 3Z^2\cdot 2\rho\right] + 16a^2\frac{1}{Z^6}\left[2zZ^3 - (a^2+z^2-\rho^2)3Z^2\cdot 2z\right] =$

$= \frac{32a^2z}{Z^4}\left[2\rho^2(a^2+z^2+\rho^2) - 6\rho^2 + (a^2+z^2+\rho^2) - 3(a^2+z^2-\rho^2)\right] = \frac{32a^2z}{Z^4}\cdot 0 = 0$

$\mathrm{div}\vec{B} = 0$, $\vec{B} = \mathrm{rot}\vec{A}$ - Бяг Ковчеги (Dirac strings) конф. Тръго '2021.

23/III Writing skyrmion gyrovector"ms. (Л. в Carrefour market"

29/III SolSkyMag пеpомиска и пеpедачка конф. програм. E-mail Орсане о $\vec{G}$(hopfion) $\neq 0$ и необходим the papers Wang-Brataas PRL'19 и Lin-Nagaosa PRL'20. Poquet Figs to hopfion $\vec{G}$" ms.

30/III meeting с G. Consolo: обяснение formation SW in single dot. Черу, чи ещи идеи $w_n, m_n(x)$ stripes, Gus PRB'02. Черодеи сруже pdf-ковете. С А. в „slide" Hermann.

L. Gambin report, 2019. "toroidal coordinates (η,β,φ), $\vec{B}(B_\eta, B_\beta, B_\varphi)$

$B_x = \frac{1}{\tau}(1-\mathrm{ch}\eta\cos\beta)\cos\varphi B_\eta - \frac{1}{\tau}\mathrm{sh}\eta\sin\beta\cos\varphi B_\beta - \sin\varphi B_\varphi$

$B_y = \frac{1}{\tau}(1-\mathrm{ch}\eta\cos\beta)\sin\varphi B_\eta - \frac{1}{\tau}\mathrm{sh}\eta\sin\beta\sin\varphi B_\beta + \cos\varphi B_\varphi$

$B_z = -\frac{1}{\tau}\mathrm{sh}\eta\sin\beta B_\eta + \frac{1}{\tau}(\mathrm{ch}\eta\cos\beta - 1)B_\beta$

$\vec{B} = B_\rho\hat{\rho} + B_\varphi\hat{\varphi} + B_z\hat{z} = B_\rho(\cos\varphi\hat{x} + \sin\varphi\hat{y}) + B_\varphi(-\sin\varphi\hat{x} + \cos\varphi\hat{y}) + B_z\hat{z}$

$\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z} = B_x(\cos\varphi\hat{\rho} - \sin\varphi\hat{\varphi}) + B_y(\sin\varphi\hat{\rho} + \cos\varphi\hat{\varphi}) + B_z\hat{z} =$

$= B_\rho\hat{\rho} + B_\varphi\hat{\varphi} + B_z\hat{z}$, $B_\rho = B_x\cos\varphi + B_y\sin\varphi$, $B_\varphi = -B_x\sin\varphi + B_y\cos\varphi$, $B_z = B_z$

$\rightarrow B_\rho = \frac{1}{\tau}(1-\mathrm{ch}\eta\cos\beta)B_\eta - \frac{1}{\tau}\mathrm{sh}\eta\sin\beta B_\beta$, $\tau = \mathrm{ch}\eta - \cos\beta$

$B_\varphi = B_\varphi$, $B_z = -\frac{1}{\tau}\mathrm{sh}\eta\sin\beta B_\eta + \frac{1}{\tau}(\mathrm{ch}\eta\cos\beta - 1)B_\beta$

Gladkowski ansatz $m_z = w(y)$, $\frac{\partial w}{\partial\beta} = 0$, $B_\eta = 0 \Rightarrow B_\rho = -\frac{1}{\tau}\mathrm{sh}\eta\sin\beta B_\beta$, $B_z = \frac{1}{\tau}(\mathrm{ch}\eta\cos\beta - 1)B_\beta$

$\rightarrow B_\rho = -\frac{1}{\tau}\mathrm{sh}\eta\sin\beta\left(-\frac{N}{a^2}\frac{\tau^2}{\mathrm{sh}\eta}\frac{\partial w}{\partial\eta}\right)$, $B_z = \frac{1}{\tau}(\mathrm{ch}\eta\cos\beta - 1)\left(-\frac{N}{a^2}\frac{\tau^2}{\mathrm{sh}\eta}\frac{\partial w}{\partial\eta}\right)$, $B_\varphi = B_\varphi$

$B_\rho = \frac{N}{a^2}\tau\sin\beta\frac{\partial w}{\partial\eta}$, $B_z = -\frac{N}{a^2}\tau\frac{(\mathrm{ch}\eta\cos\beta - 1)}{\mathrm{sh}\eta}\frac{\partial w}{\partial\eta}$, $w(\eta) = 1 - 2\mathrm{th}^2\eta$, $\mathrm{th}\eta = \frac{2a\rho}{(\rho^2+z^2+a^2)}$

$\rho = \frac{a}{\tau}\mathrm{sh}\eta$, $z = \frac{a}{\tau}\sin\beta$, $\mathrm{tg}\beta = \frac{2az}{(\rho^2+z^2-a^2)}$, $B_\varphi = \frac{M}{a^2}\tau^2\frac{\partial w}{\partial\eta}$, $\frac{\partial w}{\partial\eta} = -4\frac{\mathrm{th}\eta}{\mathrm{ch}^2\eta}$, $\tau = \mathrm{ch}\eta - \cos\beta$

$dV = a^3\frac{\mathrm{sh}\eta}{\tau^3}d\eta d\beta d\varphi$, $G_\rho = \int dV B_\rho = \int d\eta d\beta d\varphi\frac{a^3}{\tau^3}\mathrm{sh}\eta\frac{N}{a^2}\tau\sin\beta\frac{\partial w}{\partial\eta} = N a 2\pi\int d\eta d\beta\frac{1}{\tau^2}\mathrm{sh}\eta\sin\beta\frac{\partial w}{\partial\eta}$

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$\rho_z = \frac{a^2}{\tau^2} \mathrm{sh}\gamma \sin\beta$, $G_\varphi = 2\pi N a \int d\gamma d\beta \frac{\rho_z}{a^2} \frac{\partial w}{\partial \gamma}$, $B_\rho(\gamma,\beta)$, $B_\varphi(\gamma,\beta)$, $B_z(\gamma,\beta)$ и
переходим к $B_{\rho,\varphi,z}(\rho,z)$.

Кривая "бубликов" получается при $\gamma = \mathrm{const}$, $m_z = \mathrm{const}$:

$z^2 + (\rho + a\coth\gamma)^2 = \frac{a^2}{\mathrm{sh}^2\gamma}$, бублик центрирован на к/уре $\rho = a\coth\gamma$
и имеет радиус $a/\mathrm{sh}\gamma$. $m_z = 1 - 2\mathrm{th}^2\gamma$, $\gamma = \mathrm{arcth}\left(\frac{1-m_z}{2}\right)^{1/2}$

при $\gamma \to \infty$, $m_z = -1$, тор соприкасается на к/уре $\rho = a$ с радиусом бублика $= 0$.

$\rho \to a$ $\gamma \to R$. Чтобы кривая начала в газе, надо $R > a$.

$G_\varphi \neq 0$ газ $\vec{G}$. Хотя $\vec{G} = (0, G_\varphi, 0) \neq 0$, но вследствие спину $\langle \vec{G} \rangle = 0$.

31/III Имеем (18/III, 18/II) детектор-сферы $\gamma_{\rho\gamma} = \frac{a}{\tau^2}(1 - \mathrm{ch}\gamma\cos\beta)$, $\gamma_{\rho\beta} = -\frac{a}{\tau^2}\mathrm{sh}\gamma\sin\beta$, $\gamma_{\rho\varphi} = 0$

$\gamma_{z\gamma} = -\frac{a}{\tau^2}\mathrm{sh}\gamma\sin\beta$, $\gamma_{z\beta} = \frac{a}{\tau^2}(\mathrm{ch}\gamma\cos\beta - 1)$, $\gamma_{z\varphi} = 0$

На совпадение с Gambini'17: $\gamma_{\rho\gamma} = \frac{1}{\tau}(1 - \cos\beta\,\mathrm{ch}\gamma)$, $\gamma_{\gamma\beta} = -\frac{1}{\tau}\mathrm{sh}\gamma\sin\beta$, $\gamma_{z\beta} = +\frac{1}{\tau}(\mathrm{ch}\gamma\cos\beta - 1)$

$\gamma_{z\gamma} = -\frac{1}{\tau}\mathrm{sh}\gamma\sin\beta$ совпадает с Gambini?

Имеем (18/III) $\gamma_{\gamma\rho} = \frac{1}{\tau}(1 - \mathrm{ch}\gamma\cos\beta)$, $\gamma_{\beta\rho} = -\frac{1}{\tau}\mathrm{sh}\gamma\sin\beta$, $\gamma_{\varphi\rho} = 0$

Gambini: $B_\gamma = \frac{1}{\tau}(1 - \cos\beta\,\mathrm{ch}\gamma)B_\rho + \frac{1}{\tau}(1 - \mathrm{ch}\gamma\cos\beta)\sin\varphi\, B_\gamma - \frac{1}{\tau}\mathrm{sh}\gamma\sin\beta B_z =$

$= \frac{1}{\tau}(1 - \mathrm{ch}\gamma\cos\beta)B_\rho - \frac{1}{\tau}\mathrm{sh}\gamma\sin\beta B_z = \gamma_{\gamma\rho}B_\rho + \gamma_{\gamma z}B_z$, $\gamma_{\gamma\rho} = \frac{1}{\tau}(1 - \mathrm{ch}\gamma\cos\beta)$

$\gamma_{\gamma z} = -\frac{1}{\tau}\mathrm{sh}\gamma\sin\beta$

Имеем $\gamma_{n\gamma}, \gamma_{n\beta}$ (18/III, 18/II) ковариантные (?) надо $\gamma_{n\gamma} = h_n \frac{\partial \xi_n}{\partial \gamma}$, $\gamma_{n\beta} = h_n \frac{\partial \xi_n}{\partial \beta}$, где
$\xi_n = \rho, \varphi, z$. Вид $\rho(\gamma,\beta,\varphi)$, $z(\gamma,\beta,\varphi)$ известен. Недавно, можно ли $\gamma_{n\gamma} = h_n \frac{\partial \xi_n}{\partial \gamma}$,
$\gamma_{n\beta} = h_n \frac{\partial \xi_n}{\partial \beta}$ из-за со-, contravariant components.

Лучше вычислить $\gamma_{n\rho} = \frac{1}{h_n}\frac{\partial \rho}{\partial \xi_n}$, $\gamma_{n\varphi} = \frac{1}{h_n}\frac{\partial \varphi}{\partial \xi_n}$, $\gamma_{nz} = \frac{\partial z}{\partial \xi_n}\frac{1}{h_n}$, где $\xi_n = (\gamma, \beta, \varphi)$ тороидальные.

h_n: $h_\gamma = h_\beta = \frac{a}{\tau}$, $h_\varphi = \frac{a}{\tau}\mathrm{sh}\gamma$, $\rho = \frac{a}{\tau}\mathrm{sh}\gamma$, $\varphi = \varphi$, $z = \frac{a}{\tau}\sin\beta$ как 18/II.

$\gamma_{\gamma\varphi} = \frac{1}{h_\gamma}\frac{\partial \varphi}{\partial \gamma} = 0$, $\gamma_{\beta\varphi} = \frac{1}{h_\beta}\frac{\partial \varphi}{\partial \beta} = 0$, $\gamma_{\varphi\varphi} = \frac{1}{h_\varphi}\frac{\partial \varphi}{\partial \varphi} = \frac{1}{h_\varphi}$. $h_\varphi = \rho = \frac{a}{\tau}\mathrm{sh}\gamma$

$\gamma_{\gamma z} = \frac{1}{h_\gamma}\frac{\partial z}{\partial \gamma} = \frac{\tau}{a}\frac{\partial}{\partial \gamma}\left(\frac{a}{\tau}\sin\beta\right) = \tau\sin\beta\left(-\frac{1}{\tau^2}\right)\frac{\partial \tau}{\partial \beta} = -\frac{1}{\tau}\mathrm{sh}\gamma\sin\beta$, $\gamma_{\varphi z} = \frac{1}{h_\varphi}\frac{\partial z}{\partial \varphi} = 0$

Gambini: $B_\beta = \frac{1}{\tau}\mathrm{sh}\gamma\cos\varphi\sin\beta(-1)B_x - \frac{1}{\tau}\mathrm{sh}\gamma\sin\beta\sin\varphi B_y + \frac{1}{\tau}(\mathrm{ch}\gamma\cos\beta - 1)B_z =$

$= -\frac{1}{\tau}\mathrm{sh}\gamma\sin\beta B_\rho + \frac{1}{\tau}(\mathrm{ch}\gamma\cos\beta - 1)B_z = \gamma_{\beta\rho}B_\rho + \gamma_{\beta z}B_z$, $\gamma_{\beta\rho} = -\frac{1}{\tau}\mathrm{sh}\gamma\sin\beta$, $\gamma_{\beta z} = \frac{(\mathrm{ch}\gamma\cos\beta - 1)}{\tau}$

$\gamma_{\beta\varphi} = 0$

Имеем $\gamma_{\gamma\rho} = \frac{1}{\tau}(1 - \mathrm{ch}\gamma\cos\beta)$, $\gamma_{\beta\rho} = -\frac{1}{\tau}\mathrm{sh}\gamma\sin\beta$, $\gamma_{\beta z} = -\frac{1}{\tau}\mathrm{sh}\gamma\sin\beta$, $\gamma_{\beta z} = \frac{1}{h_\beta}\frac{\partial z}{\partial \beta} =$

$= \frac{\tau}{a}\frac{\partial}{\partial \beta}\left(a\frac{\sin\beta}{\tau}\right) = \tau\frac{\partial}{\partial \beta}\left(\frac{\sin\beta}{\tau}\right) = \tau\left(+\frac{1}{\tau^2}\right)\left[\cos\beta(\mathrm{ch}\gamma - \cos\beta) - \sin\beta\sin\beta\right] = +\frac{1}{\tau}(\cos\beta\,\mathrm{ch}\gamma - 1)$

Совпадение с $\gamma_{\gamma\rho}, \gamma_{\beta\rho}, \gamma_{\gamma z}, (\gamma_{\beta z})$ - Gambini. $\gamma_{\beta z}$ (наш) $= +\gamma_{\beta z}$ (Gambini).

$B_\gamma = \gamma_{\gamma\rho}B_\rho + \gamma_{\gamma z}B_z$, $B_\beta = \gamma_{\beta\rho}B_\rho + \gamma_{\beta z}B_z$, $B_\varphi = B_\varphi$ перевод $(B_\gamma, B_\varphi, B_z)$
перевод $(B_\gamma, B_\beta, B_\varphi)$ находим. (B_ρ, B_φ, B_z) и $(B_\gamma, B_\beta, B_\varphi)$

$\begin{pmatrix} B_\rho \\ B_z \end{pmatrix} = \begin{pmatrix} \gamma_{\gamma\rho} & \gamma_{\gamma z} \\ \gamma_{\beta\rho} & \gamma_{\beta z} \end{pmatrix}^{-1} \begin{pmatrix} B_\gamma \\ B_\beta \end{pmatrix} = \frac{1}{\det\Gamma}\begin{pmatrix} \gamma_{\beta z} & -\gamma_{\gamma z} \\ -\gamma_{\beta\rho} & \gamma_{\gamma\rho} \end{pmatrix}\begin{pmatrix} B_\gamma \\ B_\beta \end{pmatrix}$, $\det\Gamma = \gamma_{\gamma\rho}\gamma_{\beta z} - \gamma_{\gamma z}\gamma_{\beta\rho} =$

$\begin{pmatrix} \gamma_{\gamma\rho} & \gamma_{\gamma z} \\ \gamma_{\beta\rho} & \gamma_{\beta z} \end{pmatrix}\begin{pmatrix} B_\rho \\ B_z \end{pmatrix} = \begin{pmatrix} B_\gamma \\ B_\beta \end{pmatrix}$

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$= \frac{1}{\tau}(1-\mathrm{ch}\gamma\cos\beta)\frac{1}{\tau}(\mathrm{ch}\gamma\cos\beta-1) - \left(-\frac{1}{\tau}\right)\mathrm{sh}\gamma\sin\beta\left(-\frac{1}{\tau}\right)\mathrm{sh}\gamma\sin\beta =$

$= \frac{1}{\tau^2}\left[-(1-\mathrm{ch}\gamma\cos\beta)^2 - \mathrm{sh}^2\gamma\sin^2\beta\right] = \frac{1}{\tau^2}\left[-1+2\mathrm{ch}\gamma\cos\beta+\mathrm{ch}^2\gamma\cos^2\beta-\sin^2\beta\,\mathrm{sh}^2\gamma\right] =$

$= \frac{1}{\tau^2}\left[-1+2\mathrm{ch}\gamma\cos\beta+\mathrm{ch}^2\gamma\cos^2\beta-\mathrm{sh}^2\gamma(1-\cos^2\beta)\right] = \frac{1}{\tau^2}\left[-1+2\mathrm{ch}\gamma\cos\beta+\right.$

$\left.+\mathrm{ch}^2\gamma\cos^2\beta-(1-\cos^2\beta)(\mathrm{ch}^2\gamma-1)\right] = \frac{1}{\tau^2}\left[-1+2\mathrm{ch}\gamma\cos\beta+2\mathrm{ch}^2\gamma\cos^2\beta+1-\cos^2\beta-\mathrm{ch}^2\gamma\right] =$

$= \frac{1}{\tau^2}\left[2\mathrm{ch}\gamma\cos\beta+\mathrm{ch}^2\gamma\cos^2\beta-\cos^2\beta-\mathrm{ch}^2\gamma\right]$, $\tau^2 = (\cos\beta-\mathrm{ch}\gamma)^2 = \cos^2\beta+\mathrm{ch}^2\gamma-2\mathrm{ch}\gamma\cos\beta$

$\det\Gamma = \frac{1}{\tau^2}\left[\mathrm{ch}^2\gamma\cos^2\beta-\tau^2\right]$? Whole upper green $\gamma_{\beta\beta}$. Green $\gamma_{\beta\beta}$ Before.

Hago $\det\Gamma = \frac{1}{\tau^2}\left[-1+2\mathrm{ch}\gamma\cos\beta-\mathrm{ch}^2\gamma\cos^2\beta-\sin^2\beta\,\mathrm{sh}^2\gamma\right] =$

$= \frac{1}{\tau^2}\left[-1+2\mathrm{ch}\gamma\cos\beta-\mathrm{ch}^2\gamma\cos^2\beta-(1-\cos^2\beta)(\mathrm{ch}^2\gamma-1)\right] = \frac{1}{\tau^2}\left[-1+2\mathrm{ch}\gamma\cos\beta-\right.$

$\left.-\mathrm{ch}^2\gamma\cos^2\beta+\cos^2\beta\mathrm{ch}^2\gamma+1-\cos^2\beta-\mathrm{ch}^2\gamma\right] = \frac{1}{\tau^2}\left[2\mathrm{ch}\gamma\cos\beta-\cos^2\beta-\mathrm{ch}^2\gamma\right] = \frac{-\tau^2}{\tau^2} = -1$

$\det\Gamma = -1$. $\begin{pmatrix}B_\beta\\B_\varphi\end{pmatrix} = (-1)\begin{pmatrix}\gamma_{\beta\varphi} & -\gamma_{\gamma\varphi}\\ -\gamma_{\beta\beta} & \gamma_{\gamma\beta}\end{pmatrix}\begin{pmatrix}B_\gamma\\B_\beta\end{pmatrix}$, $B_\beta = -\gamma_{\beta\varphi}B_\gamma + \gamma_{\gamma\varphi}B_\beta$

$B_\varphi = \gamma_{\beta\beta}B_\gamma - \gamma_{\gamma\beta}B_\beta$

$B_\beta = -\frac{1}{\tau}(\mathrm{ch}\gamma\cos\beta-1)B_\gamma - \frac{1}{\tau}\mathrm{sh}\gamma\sin\beta\,B_\beta$, $B_\varphi = -\frac{1}{\tau}\sin\beta\,\mathrm{sh}\gamma\,B_\gamma - \frac{1}{\tau}(1-\cos\beta\,\mathrm{ch}\gamma)B_\beta$

Cambio: $B_\beta = \frac{1}{\tau}(1-\cos\beta\,\mathrm{ch}\gamma)B_\gamma - \frac{1}{\tau}\sin\beta\,\mathrm{sh}\gamma\,B_\beta$, $B_\varphi = -\frac{1}{\tau}\sin\beta\,\mathrm{sh}\gamma\,B_\gamma + \frac{1}{\tau}(\cos\beta\,\mathrm{ch}\gamma-1)B_\beta$

T.e. mon B_β, B_φ cobregzwor $-B_\beta, B_\varphi$ - Cambio.

// Mon $\gamma_{\beta\beta}$ pacmuraho 12/11 no $\gamma_{\alpha\beta} = \frac{1}{h_\alpha}\frac{\partial\beta}{\partial\xi_\alpha}$, $\gamma_{\alpha\varphi} = \frac{1}{h_\alpha}\frac{\partial\varphi}{\partial\xi_\alpha}$, $(\xi_1,\xi_2,\xi_3) = (\gamma,\beta,\varphi)$ - toroidal coord.

$h_\gamma = h_\beta = \frac{a}{\tau}$, $h_\varphi = \frac{a}{\tau}\mathrm{sh}\gamma$

Toroidal Hopfion $\vec{G}$ manuscript u vocuan to Lena u Dania.

(1/IV) Kundu 1982: $\vec{B} = \frac{1}{2\pi m_z}(\nabla m_x \times \nabla m_y)$, $\vec{m}^2 = 1$, $m_z = W(r,\varphi)$ $\Phi = m_d + \nu(r,\varphi)$ Spherical coord. $\vec{r}(r,\varphi,\alpha)$

Hopfion.com: $\vec{B} = -\nabla\cos\theta\times\nabla\Phi$, $\vec{m}(\theta,\Phi)$, where $\theta=\theta(s,z)$, $\Phi(\vec{r}) = Q\varphi + \chi(s,z)$ axially sym. hopfion in $\vec{r}(s,\varphi,z)$ cylindrical coordinates.

$\vec{A} = \left(-\frac{Q}{s}(1-\cos\theta)\sin\varphi + 2\cos^2\frac{\theta}{2}\cos\varphi\frac{\partial\chi}{\partial s}, \frac{Q}{s}(1-\cos\theta)\cos\varphi - 2\cos^2\frac{\theta}{2}\sin\varphi\frac{\partial\chi}{\partial s}, -(1+\cos\theta)\frac{\partial\chi}{\partial z}\right)$

- vector potential Gladikowski 1997. $Q_H = \frac{Q}{4\pi}\int_0^\infty ds\int_{-\infty}^\infty dz\,\sin\theta\left[\frac{\partial\theta}{\partial s}\frac{\partial\chi}{\partial z}-\frac{\partial\theta}{\partial z}\frac{\partial\chi}{\partial s}\right]$

$A_\beta = A_x\cos\varphi + A_y\sin\varphi = +2\cos^2\frac{\theta}{2}\frac{\partial\chi}{\partial s} = -(1+\cos\theta)\frac{\partial\chi}{\partial s}$

$A_\varphi = -A_x\sin\varphi + A_y\cos\varphi = \frac{Q}{s}(1-\cos\theta)$, $A_z = -(1+\cos\theta)\frac{\partial\chi}{\partial z}$,

$\vec{A}(A_s, A_\varphi, A_z)$ rot & wang-B rotacas 19 in $(s,\varphi z)$ cyl. coordinates.

L. Faddeev, A. J. Niemi. Toroidal configurations as stable solitons (arXiv)

P.R. Battye, P.M. Sutcliffe. Solitons, Links and Knots, Proc. Roy Soc. London A455 4305(1999)

Hopf maps parameterization $\vec{m} = Z^+\vec{\sigma}Z$, $SU(2)U = e^{i\xi\vec{v}\cdot\vec{\sigma}}$, $\vec{v}(v_1,v_2,v_3)$, $\vec{v}^2=1$

$\vec{m}_1 = 2(v_3v_1\sin\xi - v_2\cos\xi)\sin\xi$, $m_3 = 1-2(1-v_3^2)\sin^2\xi$ $\vec{r}(r,\theta,\varphi)$ - spherical coord

$m_2 = 2(v_3v_2\sin\xi + v_1\cos\xi)\sin\xi$, $\xi = f(\theta)$ hopfion profile.

- A.P. Protogenov. Knots and links... Usp. Fiz. Nauk 176, 689 - HS (2006)

§2.1. $\vec{m} = \vec{f}\vec{\sigma}\vec{f}$, $\vec{a} = i\left[f_1\nabla f_1^* - c.c. + f_2\nabla f_2^* - c.c.\right]$, $F_{ik} = \vec{m}\left[\partial_i\vec{m}\times\partial_k\vec{m}\right] = \partial_i a_k - \partial_k a_i$

Mermin - Ho relation $B_i = (\mathrm{rot}\vec{a})_i = \frac{1}{2}\varepsilon_{ijk}F_{jk}$, $|\partial f_1|^2 + |\partial f_2|^2 = \frac{1}{4}\vec{a}^2 + \frac{1}{4}(\partial\vec{m})^2$

$f = (f_1, f_2)$ spinor. §2.5. Gauge potential $C_\mu = ie^+\partial_\mu e + (\cos^2\theta\partial_\mu\varphi)$, $F_{\mu\nu} = i(\vec{e}_\mu\vec{e}_\nu^+ - \vec{e}_\nu\vec{e}_\mu^+)$

§4.1. Toroidal hopfions. $Q_H = 1$ $\vec{m}\vec{\sigma} = \psi^+\partial_3\psi$, $\psi = (1+i\vec{r}\vec{\sigma})/(1+i\vec{r}\vec{\sigma})$

In toroidal coordinates (u,β,α), $\mathrm{tg}(m_1/m_2) = +\alpha-\beta$, $m_3 = 1-2\mathrm{th}^2u$ (?) th2u gonzano.

Escaneado con CamScanner

Spinor $\psi(\vec{r})$ gne $Q_H = 1$ bzer uz "E.A. Kuznetsov, A.V. Mikhailov, Phys. Lett. 74A, 37 (1980)

"J. Jäykkä (PhD thesis) On topological solitons in the Faddeev-Skyrme model" 2009.

Hopf map $\vec{m} = \vec{h}(y) = y^+ \vec{\sigma} y = \left(2(y_1y_3 + y_2y_4),\ 2(y_2y_3 - y_1y_4),\ y_1^2 + y_2^2 - y_3^2 - y_4^2\right)$

$y = (y_1, y_2, y_3, y_4)^T$ - 4-x vector, $y^2 = 1$ i $\vec{m}^2 = 1$. Map of $S_3(y)$ to $S_2(\vec{m})$

$$\vec{m} = \begin{pmatrix} 2g\sqrt{1-g^2}\cos(p\beta + q\varphi) \\ 2g\sqrt{1-g^2}\sin(p\beta + q\varphi) \\ 2g^2 - 1 \end{pmatrix}, \quad m_x + i m_y = 2g\sqrt{1-g^2}\, e^{i(p\beta + q\varphi)}, \quad Q_H(\vec{m}) = pq$$

$g = g(y)$, toroidal coordinates (y, β, φ).

h/m $p = q = 1$ [Hopf 1931], $g(y) = g(x, y, z) = \sqrt{\dfrac{(r^2+1)^2 - 4z^2}{(r^2+1)^2}}$, $\vec{r} = (\xi, z)$, $\mathrm{th}\,\gamma = \dfrac{2a\xi}{(r^2+a^2)}$ (wg Gladikowski)

Rozwiązanie $\vec{B}$ (Gladikowski) wg $\vec{m}$ in toroidal coordinates

$m_+ = m_x + i m_y = \sqrt{1-w^2}\, e^{i(N\varphi + M\beta + \nu_0)}$, $m_z = w(y)$. (y, β, φ)

$\vec{B} = \dfrac{\nabla m_x \times \nabla m_y}{m_z}$ czyli $\vec{B} = \sin\Theta \nabla\Theta \times \nabla\Phi = -\nabla(\cos\Theta) \times \nabla\Phi = -\nabla m_z \times \nabla\Phi$, $\Phi = N\varphi + M\beta$

$$\nabla\Phi = \frac{\tau}{a}\left[\frac{\partial\Phi}{\partial y}\hat{e}_y + \frac{\partial\Phi}{\partial\beta}\hat{e}_\beta + \frac{1}{\sinh y}\frac{\partial\Phi}{\partial\varphi}\hat{e}_\varphi\right] = \frac{\tau}{a}\left[M\hat{e}_\beta + \frac{1}{\sin\beta}N\hat{e}_\varphi\right], \quad \nabla m_z = \frac{\tau}{a}\frac{\partial m_z}{\partial y}\hat{e}_y$$

$$\nabla m_z \times \nabla\Phi = \begin{vmatrix} \hat{e}_y & \hat{e}_\beta & \hat{e}_\varphi \\ \frac{\tau}{a}\frac{\partial m_z}{\partial y} & 0 & 0 \\ 0 & \frac{\tau M}{a} & \frac{\tau N}{a\sin\beta} \end{vmatrix} = \hat{e}_\varphi \frac{\tau^2}{a^2} M \frac{\partial m_z}{\partial y} - \frac{\tau^2}{a^2}\frac{N}{\sin\beta}\frac{\partial m_z}{\partial y}\hat{e}_\beta, \quad \vec{B} = -\nabla m_z \times \nabla\Phi$$

$B_y = 0$, $B_\beta = \dfrac{\tau^2}{a^2}\dfrac{N}{\sin\beta}\dfrac{\partial m_z}{\partial y}$, $B_\varphi = -\dfrac{\tau^2}{a^2} M \dfrac{\partial m_z}{\partial y}$. Hago $\sin\beta \to \mathrm{sh}\,\beta$ i $\tau\Phi$.

Gladikowski '97: $B_y = 0$, $B_\beta = -\dfrac{2N}{a^2}\dfrac{\tau^2}{\mathrm{sh}\,y}\dfrac{\partial w}{\partial y}$, $B_\varphi = \dfrac{2M}{a^2}\tau^2\dfrac{\partial w}{\partial y}$.

2° y Gladikowski y sposób (i $\vec{B}$ - definition). Czy można go zgrabnie $\vec{B} = -\nabla m_z \times \nabla\Phi$ zobaczyć z $\vec{B}$ defined by Eq. (9) Gladikowski '97.

$$\nabla\Phi = \frac{\hat{e}_y}{h_y}\frac{\partial\Phi}{\partial y} + \frac{\hat{e}_\beta}{h_\beta}\frac{\partial\Phi}{\partial\beta} + \frac{\hat{e}_\varphi}{h_\varphi}\frac{\partial\Phi}{\partial\varphi} = \left|h_y = h_\beta = \frac{a}{\tau},\ h_\varphi = \frac{a\sinh y}{\tau}\right| = \frac{\tau}{a}\left[\frac{\partial\Phi}{\partial y}\hat{e}_y + \frac{\partial\Phi}{\partial\beta}\hat{e}_\beta + \frac{1}{\sinh y}\frac{\partial\Phi}{\partial\varphi}\hat{e}_\varphi\right]$$

(2/IV) Definitions of gyrovector $\vec{G} = \int dV \vec{B}$ (1) i $G_\alpha = \int dV B_\alpha$ (2) - przyjmę. Czy zgodzisz się z $\alpha = x, y, z$ Cartesian vector components.

$$\vec{G} = \int dV \vec{B} = \int dV \sum_\alpha B_\alpha \vec{e}_\alpha(\vec{r}), \quad G_\alpha = (\vec{G}\cdot\vec{e}_\alpha) = \vec{e}_\alpha \cdot \int dV \sum_\beta B_\beta \vec{e}_\beta(\vec{r}) \quad (1)$$

$G_\alpha = (\vec{G}\cdot\vec{e}_\alpha)$, (1) $G_\alpha(\vec{r}) = \vec{G}\cdot\vec{e}_\alpha(\vec{r}) = \int dV' \sum_\beta B_\beta(\vec{r}')\left(\vec{e}_\alpha(\vec{r})\cdot\vec{e}_\beta(\vec{r}')\right)$

(2) $G_\alpha = \int dV' B_\alpha(\vec{r}') = \mathrm{const}(\vec{r}')$.

I zgodzisz $G_\alpha = \int dV B_\alpha$ (2) definition z y Wang-Brataas '2019, $\alpha = \rho, \varphi, z$.

G zależeć od: a) $\vec{m}(\vec{r})$, $\vec{B}(\vec{r})$ symmetry; b) sample shape (bo $V \in \int dV(\ldots)$)

$m_x + i m_y = (m_\xi + i m_\eta)e^{i\varphi}$, $m_\xi + i m_\eta = m_x\cos\varphi + m_y\sin\varphi + i\left[-m_x\sin\varphi + m_y\cos\varphi\right] =$

$= m_x e^{-i\varphi} + m_y\sin\varphi + i m_y\cos\varphi = m_x e^{-i\varphi} + i m_y(\cos\varphi - i\sin\varphi) = (m_x + i m_y)e^{-i\varphi}$

$m_\xi + i m_\eta = (m_x + i m_y)e^{-i\varphi}$ for any vector $\vec{m}$, (ρ, φ) - cylindrical coord.

Gladikowski '97 hopfion $(m_x + i m_y) = \sqrt{1-w^2}\, e^{i(N\varphi + M\beta + \nu_0)} = F(y, \beta)\, e^{iN\varphi}$ in toroidal coord. (y, β, φ):

$m_\xi + i m_\eta = (m_x + i m_y)e^{-i\varphi} = F(y, \beta)\, e^{i(N-1)\varphi}$ nie zależeć od φ tylko $N = 1$.

$B_\xi + i B_\eta$ nie zależeć od φ też hopfion tylko $N = 1$?

$\vec{B}(\mathrm{Glad}) = (B_y, B_\beta, B_\varphi)$ nie zależeć od φ, $B_y(y, \beta)$, $B_\beta(y, \beta)$, $B_\varphi(y, \beta)$ for $\forall N$

$m_z(y, \beta) = w(y, \beta)$ (Glad). Bo oczywiście $\Phi(y, \beta, \varphi) = N\varphi + M\beta + \nu_0$ i $\vec{B} = \nabla m_z \times \nabla\Phi$

$B_\xi(y, \beta) = \frac{1}{c}(1 - \mathrm{ch}\,y\cos\beta)B_y - \frac{1}{c}\sinh y\sin\beta B_\beta$, $B_\eta(y, \beta)$, $B_\varphi(y, \beta) = -\frac{1}{c}\mathrm{sh}\,y\sin\beta B_y + \frac{1}{c}(\mathrm{ch}\,y\cos\beta - 1)B_\beta$

$B_\xi(y, \beta)$, $B_\eta(y, \beta)$ nie zależeć od φ (for any vorticity N)

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Toroidal (γ,β) $\gamma+i\beta = \ln\frac{z+i(\rho+a)}{z+i(\rho-a)}$ не зависит от $\varphi \Rightarrow B_\beta(\gamma,\beta) = B_\rho(\rho,z)$

$B_\varphi(\rho,z)$ не зависит от φ for any N. $B_x + iB_y = (B_\rho + iB_\varphi)e^{i\varphi} =$

$= (B_\rho + iB_\varphi)(\cos\varphi + i\sin\varphi)$, $B_x = B_\rho\cos\varphi - B_\varphi\sin\varphi$, $B_y = B_\rho\cos\varphi + B_\rho\sin\varphi$

$B_x(\rho,\varphi,z) = B_\rho(\rho,z)\cos\varphi - B_\varphi(\rho,z)\sin\varphi$, $B_y(\rho,\varphi,z) = B_\rho(\rho z)\sin\varphi + B_\varphi(\rho z)\cos\varphi$

Очевидно, $G_x = \int dV B_x(\vec r) = 0$, $G_y = \int dV B_y(\vec r) = 0$ т.к. axially symmetric

V (cylinder, камушек) $\int_0^{2\pi} d\varphi \sin\varphi = 0$, $\int_0^{2\pi} d\varphi\cos\varphi = 0$, $G_x = 0$, $G_y = 0$.

Хотя $G_\varphi = \int dV B_\varphi(\rho,z) = 2\pi\int_0^R d\rho\,\rho\int_{-l/2}^{l/2} dz\, B_\varphi(\rho,z) \neq 0$ т.к. $G_\beta = \int dV B_\beta$ definition (2).

(1) $G_\beta(\vec r) = \int dV' \sum_\beta B_\beta(\vec r')\left(\vec e_\beta(\vec r)\cdot\vec e_\beta(\vec r')\right)$, (2) $G_\beta = \int dV B_\beta(\vec r)$

если $\vec e_\beta(\vec r)\cdot\vec e_\beta(\vec r') = \delta_{\beta\beta}$, то (1) $G_\beta(\vec r) = \int dV' \sum_\beta B_\beta(\vec r')\delta_{\beta\beta} = \int dV' B_\beta(\vec r') = \int dV B_\beta(\vec r) =$

$= const(\vec r)$

Аналогично (2) $G_\beta = \int dV B_\beta(\vec r)$

$\vec e_\varphi = -\sin\varphi\,\hat x + \cos\varphi\,\hat y = \vec e_\varphi(\vec r)$, $\vec e_\rho(\vec r) = \hat x\cos\varphi + \hat y\sin\varphi$, $\vec e_z = \hat z$

$\vec e_\rho(\vec r)\cdot\vec e_\rho(\vec r') = (\hat x\cos\varphi + \hat y\sin\varphi)\cdot(\hat x\cos\varphi' + \hat y\sin\varphi') = \cos\varphi\cos\varphi' + \sin\varphi\sin\varphi' =$

$\vec e_\varphi(\vec r)\cdot\vec e_\varphi(\vec r') = (-\sin\varphi\hat x + \cos\varphi\hat y)\cdot(-\sin\varphi'\hat x + \cos\varphi'\hat y) = \cos(\varphi-\varphi')$ $= \cos(\varphi-\varphi')$

$\vec e_\rho(\vec r)\cdot\vec e_\varphi(\vec r') = (\hat x\cos\varphi + \hat y\sin\varphi)\cdot(-\sin\varphi'\hat x + \cos\varphi'\hat y) = -\cos\varphi\sin\varphi' + \sin\varphi\cos\varphi' =$

$\vec e_\varphi(\vec r)\cdot\vec e_\rho(\vec r') = (-\sin\varphi\hat x + \cos\varphi\hat y)\cdot(\hat x\cos\varphi' + \hat y\sin\varphi') = -\sin\varphi\cos\varphi' +$ $= \sin(\varphi-\varphi')$

$+\cos\varphi\sin\varphi' = \sin(\varphi-\varphi')$

$\vec e_z(\vec r)\cdot\vec e_{\rho,\varphi}(\vec r') = \hat z\cdot\vec e_{\rho,\varphi}(\vec r') = 0$, $\vec e_z(\vec r)\vec e_z(\vec r') = \hat z\cdot\hat z = 1$.

т.е. $\vec e_\alpha(\vec r)\cdot\vec e_\beta(\vec r') \neq \delta_{\alpha\beta}$ где $\alpha,\beta = \rho,\varphi$

Definition $\vec G$ by (1), (2) eqs. are not equivalent (!), где (ρ,φ,z) cyl. coord.

Какое из них правильное вернее? Расчет Вибор Thiele et al. (1972, 1973)

$\frac{d\vec M}{dt} = \vec a$, $\frac{\partial\vec M}{\partial X_i}\cdot\left(\vec M\times\frac{d\vec M}{dt}\right) = \frac{\partial\vec M}{\partial X_i}\cdot\left(\vec M\times\frac{\partial\vec M}{\partial X_j}\frac{dX_j}{dt}\right) = v_j\frac{\partial\vec M}{\partial X_i}\cdot\left(\vec M\times\frac{\partial\vec M}{\partial X_j}\right) = -v_j\vec M\cdot\left[\frac{\partial\vec M}{\partial X_i}\times\frac{\partial\vec M}{\partial X_j}\right] =$

$= -v_j g_{ij}$, $g_{ij} = \vec M\cdot\left[\frac{\partial\vec M}{\partial X_i}\times\frac{\partial\vec M}{\partial X_j}\right]$, $\vec M(\vec r,t) = \vec M\left(\vec r,\vec X(t)\right)$, $v_j = dX_j/dt$, можно

$\vec X = (X,Y,Z)$ можно $\vec X = (X_\rho, X_\varphi, X_z)$, $X_z = Z$ — cylindrical components.

Gyrocoupling density $g_{ij} = \vec m\cdot\left[\frac{\partial\vec m}{\partial X_i}\times\frac{\partial\vec m}{\partial X_j}\right]$, rigid soliton $\vec M(\vec r,\vec X) =$

$\frac{\partial\vec M}{\partial X_j} = \frac{\partial\vec M}{\partial x_i}(-1)$, $\vec r = \rho\hat\rho + (\vec r\cdot\hat\varphi)\hat\varphi + (\vec r\cdot\hat z)\hat z = \rho\hat\rho + z\hat z$. Gyroforce Inertion

$\vec r = x\hat x + \hat y y + z\hat z$, $(\vec r\cdot\hat\varphi) = x(\hat x\cdot\hat\varphi) + y(\hat y\cdot\hat\varphi) + z(\hat z\cdot\hat\varphi) = x(-\sin\varphi) + y\cos\varphi$

$(\vec r\cdot\hat\rho) = x(\hat x\cdot\hat\rho) + \hat y(\hat y\cdot\hat\rho) = x\cos\varphi + y\sin\varphi = \rho\cos^2\varphi + \rho\sin^2\varphi = \rho$

$x = \rho\cos\varphi$, $y = \rho = \sin\varphi$ $(\vec r\cdot\hat\varphi) = -\sin\varphi\rho\cos\varphi + \rho\cos\varphi\sin\varphi = 0$. $\vec r = \rho\hat\rho + z\hat z$

$\vec r - \vec X = (\rho\hat\rho + z\hat z) - (X_\rho\hat\rho + X_\varphi\hat\varphi + Z\hat z) = (\rho - X_\rho)\hat\rho - X_\varphi\hat\varphi + (z-Z)\hat z$.

X_φ не нужно брать вообще?

Сделали $\vec m(\vec r,\vec X) \to \vec m(\vec r - \vec X)$ in cartesian coordinates, $g_{ij} = \vec m\cdot\left[\frac{\partial\vec m}{\partial x_i}\times\frac{\partial\vec m}{\partial x_j}\right]$

g_{ij} - antisymmetric tensor. $g_{ij} = F^e_{ij}$ (Gas'EPL'16), Emergent $(\vec B)_i = -\frac{1}{2}\epsilon_{ijk}F^e_{jk}$ in field

$v_j\cdot g_{ij} = v_j(-\frac{1}{2})\epsilon_{ijk}F^e_{jk} = -\frac{1}{2}(*)$, где $g_{ij}v_j = \vec g\cdot\vec v = \vec g\times\vec v$, $(\hat g)_{ij} = g_{ij}$ matrix

$\hat g = \begin{pmatrix} 0 & -g_3 & g_2 \\ g_3 & 0 & -g_1 \\ -g_2 & g_1 & 0 \end{pmatrix}$, $\vec g = (g_1,g_2,g_3)$ не обязательно (cartesian components g_i).

Dual pseudovector (Bilken) $g_i = \frac{1}{2}\epsilon_{ijk}g_{jk}$

$g_i = \frac{1}{2}\epsilon_{ijk}F^e_{jk} = -B^e_i$, $g_i \to \vec g = (g_1,g_2,g_3) = (g_{23}, g_{31}, g_{12})$, $B_\varphi = -g_\varphi = -g_{31}$.

Поэтому $\vec g = -\vec B$, $v_j g_{ij} = g_{ij}v_j = \vec g\vec v = \vec g\times\vec v$, $\vec v = d\vec X/dt$ Надо Thiele eq

$\int dV(*)$. $\int dV g_{ij}v_j = \int dV\vec g\times\vec v = \vec G\times\vec v$, $\vec G = \int dV\vec g = -\int dV\vec B$ — теорема.

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Gyro force $\vec{F}_g = \int dV (\vec{j} \times \vec{v})$, $\vec{j}_g = \vec{j} \times \vec{v}$ momo b cylindrical coordinates $\vec{j}(j_s, j_\varphi, j_z)$ $\vec{v}(v_s, v_\varphi, v_z)$

$\vec{v} = \frac{d\vec{X}}{dt} = \frac{d}{dt}\left[X_s \hat{s} + X_\varphi \hat{\varphi} + z\hat{z} \right]$, $\frac{d\hat{s}}{dt} = \frac{\partial \hat{s}}{\partial \varphi}\dot{\varphi} = \hat{\varphi}\dot{\varphi}$, $\hat{s} = \hat{x}\cos\varphi + \hat{y}\sin\varphi$

$\frac{d\hat{\varphi}}{dt} = \frac{\partial \hat{\varphi}}{d\varphi}\frac{d\varphi}{dt} + \frac{\partial \hat{\varphi}}{\partial z}\frac{dz}{dt} = \frac{\partial}{\partial \varphi}(-\hat{x}\sin\varphi + \hat{y}\cos\varphi) = (-\hat{x}\cos\varphi + \hat{y}\sin\varphi)\dot{\varphi} = \hat{s}\dot{\varphi}$. $\hat{\varphi} = -\sin\varphi\hat{x} + \cos\varphi\hat{y}$

$\frac{d\hat{s}}{dt} = \frac{\partial \hat{s}}{\partial \varphi}\frac{d\varphi}{dt} + \frac{\partial \hat{s}}{\partial z}\frac{dz}{dt} = \frac{\partial}{\partial \varphi}(\hat{x}\cos\varphi + \hat{y}\sin\varphi) = (-\hat{x}\sin\varphi + \hat{y}\cos\varphi)\dot{\varphi} = \hat{\varphi}\dot{\varphi}$, $\hat{s} = \hat{s}(\varphi)$, $\hat{\varphi} = \hat{\varphi}(\varphi)$

$\frac{d\hat{\varphi}}{dt} = \frac{\partial \hat{\varphi}}{\partial \varphi}\frac{d\varphi}{dt} + \frac{\partial \hat{\varphi}}{\partial z}\frac{dz}{dt} = \frac{\partial}{\partial \varphi}(-\sin\varphi\hat{x} + \cos\varphi\hat{y}) = (-\cos\varphi\hat{x} - \sin\varphi\hat{y}) = -\hat{s}\dot{\varphi}$, $\frac{d\vec{X}}{dt} = \left[\dot{X}_s\hat{s} + X_s\frac{d\hat{s}}{dt} + \dot{X}_\varphi\hat{\varphi} + X_\varphi\frac{d\hat{\varphi}}{dt} + \dot{z}\hat{z} \right] =$

$= \left[\dot{X}_s\hat{s} + X_s\dot{\varphi}\hat{\varphi} + \dot{X}_\varphi\hat{\varphi} + X_\varphi\dot{\varphi}(-\hat{s}) + \dot{z}\hat{z} \right] = (\dot{X}_s - X_\varphi\dot{\varphi})\hat{s} + (\dot{X}_\varphi + X_s\dot{\varphi})\hat{\varphi} + \dot{z}\hat{z}$

$\vec{j}_g = \vec{j} \times \vec{v} \to \vec{B} \times \vec{v} = \vec{B} \times \frac{d\vec{X}}{dt}$ // $\vec{X} = (X, y, z)$, to $\vec{F}_g = \int dV \vec{B} \times \frac{d\vec{X}}{dt} = \left(\int dV \vec{B}\right) \times \frac{d\vec{X}}{dt} =$ Cm. Wiki "Vector fields"

$= \vec{G} \times \frac{d\vec{X}}{dt}$, $\vec{G} = \int dV \vec{B}$ - $d\vec{X}$ no yabucar or $\vec{v}$ - coordinates.

B cyl. coordinates (s, φ, z), $\vec{v} = \frac{d\vec{X}}{dt} = (\dot{X}_s - X_\varphi\dot{\varphi})\hat{s} + (\dot{X}_\varphi + X_s\dot{\varphi})\hat{\varphi} + \dot{z}\hat{z}$ yabucar $\vec{v}$ zo/ve y $\dot{\varphi}$. Cual momento $\dot{\varphi}$?

$\vec{B} \times \vec{v} = \begin{vmatrix} \hat{s} & \hat{\varphi} & \hat{z} \\ B_s & B_\varphi & B_z \\ v_s & v_\varphi & v_z \end{vmatrix} = \hat{s}(B_\varphi v_z - B_z v_\varphi) + \hat{\varphi}(B_z v_s - B_s v_z) + \hat{z}(B_s v_\varphi - B_\varphi v_s)$

$v_s, v_\varphi = f(\varphi)$ cosine, $v_z = \frac{dz}{dt}$ o-φ no yabucar

$\vec{F}_g = \int dV \vec{B} \times \vec{v} = v_z \int dV (\hat{s}B_\varphi - \hat{\varphi}B_s) + \int dV \left[B_z(v_s\hat{\varphi} - \hat{s}v_\varphi) + \hat{z}(B_s v_\varphi - B_\varphi v_s) \right]$

$\vec{F}_g = v_z \left[\hat{s}\int dV B_\varphi - \hat{\varphi}\int dV B_s \right] + \int dV B_z(v_s\hat{\varphi} - \hat{s}v_\varphi) + \hat{z}\int dV (B_s v_\varphi - B_\varphi v_s)$

Bugno, no $G_\varphi = \int dV B_\varphi$ y $G_s = \int dV B_s$ onpegeneno: zato go gy'bpame $\vec{F}_g$

$v_s = \left(\frac{d\vec{X}}{dt}\right)_s = \dot{X}_s - X_\varphi\dot{\varphi}$, $v_\varphi = \dot{X}_\varphi + X_s\dot{\varphi}$

$\vec{F}_g = v_z\left[\hat{s}\int dV B_\varphi - \hat{\varphi}\int dV B_s\right] + \int dV B_z \cdot \left[\dot{X}_s\hat{\varphi} - \dot{X}_\varphi\hat{s}\right] + \int dV B_z\left[-X_\varphi\dot{\varphi}\hat{\varphi} - X_s\dot{\varphi}\hat{s}\right] +$

$+ \hat{z}\int dV\left[B_\varphi\dot{X}_\varphi - B_\varphi\dot{X}_s\right] + \hat{z}\int dV\left[B_s X_s\dot{\varphi} - B_\varphi(-X_\varphi\dot{\varphi})\right]$

$\vec{F}_g = v_z\left[\hat{s}\int dV B_\varphi - \hat{\varphi}\int dV B_s\right] + (\dot{X}_s\hat{\varphi} - \dot{X}_\varphi\hat{s})\int dV B_z + \hat{z}\int dV\left[B_s\dot{X}_\varphi - B_\varphi\dot{X}_s\right] -$

$- \dot{\vec{X}}_p \int dV B_z\dot{\varphi} + \hat{z}\int dV \vec{B}\cdot\vec{X}_p\dot{\varphi}$, $\vec{X}_p = X_s\hat{s} + X_\varphi\hat{\varphi}$ - in-plane part of $\vec{X}$.

Bugno, no $G_s = \int dV B_s$, $G_\varphi = \int dV B_\varphi$, $G_z = \int dV B_z$ onpegeneno: regular part of gyroforce. Toto "irregular" part of gyroforce v $\dot{\varphi}$ (nekaka zavora?)

→ Definition (2) $G_z = \int dV B_z$ umes pomy. cubica u/m $d = (s, \varphi, z)$ cyl. components.

Moment impulse $\frac{d\vec{X}}{dt} = \frac{d}{dt}\left[X_s\hat{s} + X_\varphi\hat{\varphi} + z\hat{z}\right] = \dot{X}_s\hat{s} + \dot{X}_\varphi\hat{\varphi} + \dot{z}\hat{z}$ ESy $\dot{\varphi}$-terms ?

kan (youtube) Grunfeld $\vec{X}(t) = X_s(t)\hat{s} + X_\varphi(t)\hat{\varphi} + z(t)\hat{z}$ - trajectory in covariant basis $(\hat{s}, \hat{\varphi}, \hat{z})$

n/u $B_s = 0$, $V \to \infty$, $G_z \to 0$, $G_\varphi = 0$, $\vec{F}_g = v_z\hat{s}G_\varphi - \hat{z}G_\varphi\dot{X}_s$

$\vec{F}_g = G_\varphi(\dot{z}\hat{s} - \hat{z}\dot{X}_s)$ for $(G_\varphi = 1)$ hopfion in infinite media.

$\vec{X} = X^a\hat{e}_a$, $\hat{e}_a = (\hat{s}, \hat{\varphi}, \hat{z})$ covariant basis vectors, $X^a = X^a(t)$

(3/12) Z. Hlousek, J. Shertzer, Ph. Rev. D 37, 1279 (1988)

$\vec{n} = Z^+\vec{\sigma}Z$, Hopf map. Em. vector potential $\vec{A}$; $A_i = iZ^+\partial_i Z$

Y.B. Wu, A. Zee. Nucl. Phys. B 324, 623 (1989) Gladikowski ansatz for a toroidal hopfion.

J. Gladikowski. Thesis, 1997. Three forms of maps mapping $S_3 \to S_2$.

J. Kietarinta, garnag. 2007. $\vec{m} = (Y_1^*\ Y_2)\vec{\sigma}\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$, $Y_1 = g(y)e^{i(p\varphi}$, $Y_2 = \sqrt{1-g^2}e^{iq\varphi}$

$g(y)$ is smooth monotonous function. Toroidal coo'd. (y, ξ, φ). $Q_H = pq$

$g(\infty) = 0$, $g(0) = \pm 1$. Ref. Aratyn et. al. 1999. H. Aratyn.

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• H. Aratyn, L.A. Ferreira, A.H. Zimerman. Phys. Rev. lett. 83, 1723 (1999)

$F_{\mu\nu} = \vec{m}\cdot(\partial_\mu \vec{m} \times \partial_\nu \vec{m})$, $m_x + i m_y = \frac{2w}{1+|w|^2}$, $m_z = \frac{|w|^2-1}{|w|^2+1}$

$F_{\mu\nu} = -\frac{2i}{(1+|w|^2)^2}\left[\partial_\mu w \partial_\nu w^* - \partial_\nu w \partial_\mu w^*\right]$ – кривизна, like exch. energy.

$w(y,\beta,\varphi) = f(y)\, e^{i(m\beta + n\varphi)}$, (y,β,φ) – toroidal coordinates

Коэффициенты $x = \frac{a}{c}\frac{\sin\beta}{\operatorname{ch} y}$, $z = \frac{a}{c}\frac{\operatorname{sh} y}{\operatorname{ch} y}$, $\tau = \operatorname{ch} y - \cos\beta$

B.c. $\vec{m} \to (0,0,1)$, $|w| \to \infty$, $f(y=0) \to \infty$. $\vec{m} \to (0,0,-1)$, $w \to 0$, $f(y=\infty) = 0$

$f^2(y) = \frac{\operatorname{ch} y - \sqrt{n^2/m^2 + \operatorname{sh}^2 y}}{\sqrt{1 + \frac{m^2}{n^2}\operatorname{sh}^2 y} - \operatorname{ch} y}$, при $m=n$, $f^2(y) = \frac{1}{\operatorname{sh}^2 y}$ и $w(y,\beta,\varphi) = \frac{1}{\operatorname{sh} y} e^{in(\beta+\varphi)}$

$m_z(y) = \frac{1/\operatorname{sh}^2 y - 1}{1/\operatorname{sh}^2 y + 1} = \frac{1-\operatorname{sh}^2 y}{1+\operatorname{sh}^2 y} = \frac{1-\operatorname{sh}^2 y}{\operatorname{ch}^2 y} = \frac{1}{\operatorname{ch}^2 y} - \operatorname{th}^2 y = 1 - 2\operatorname{th}^2 y$

$m_z(y) = 1 - 2\operatorname{th}^2 y$ верно не только при $m=n=1$, for any $m=n$.

$\Phi_i(y)$ явно даны, $i = 1,\dots,4$, for any m,n. $w = \frac{\Phi_1 + i\Phi_2}{\Phi_3 + i\Phi_4} = \frac{Z_1}{Z_2}$, $|Z_1|^2 + |Z_2|^2 = 1$

$\vec{m} = Z^+ \vec{\sigma} Z$, $Z^+ = (Z_1^*, Z_2^*)$, $Z = \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}$ spinors.

Vector potential $\vec{A}$: $A_\alpha = \frac{i}{2}(Z^+\partial_\alpha Z - \partial_\alpha Z^+ Z)$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$A_y = \tau \frac{\partial}{\partial y}(\Phi_3\Phi_4)$, $A_\beta = -m\tau(\Phi_1^2 + \Phi_2^2)$, $A_\varphi = -\frac{n\tau}{\operatorname{sh} y}(\Phi_3^2 + \Phi_4^2)$

$\vec{A} = (A_y, A_\beta, A_\varphi)$. $\vec{B} = \operatorname{rot}\vec{A}$, $B_i = \frac{1}{2}\varepsilon_{ijk}F_{jk}$, $Q_H = \frac{1}{(4\pi)^2}\int dV\, \vec{A}\cdot\vec{B} = -nm$.

Shnir 2018 (§8.3) $W = \frac{m_x + i m_y}{(1+m_z)} = \frac{Z_1^n}{Z_0^m}$, $Z_0 = 1 - r^2 - 2iz$, $Z_1 = 2(x - iy)$

Выбор gauge of $\vec{A}$ определяется by spinors Z_1, Z_2, т.е. $\Phi_i(y)$ будем

брать $m=n=1$. $\Phi_1^2, \Phi_2^2 \sim \frac{\operatorname{ch} y - \sqrt{\alpha^2 + \operatorname{sh}^2 y}}{(\frac{1}{\alpha} - 1)} = g(y,\alpha)$, $\alpha = \frac{m}{n}$, $\alpha = \frac{n}{m}$.

$\lim_{\alpha\to 1} g(y,\alpha) = \lim_{\alpha\to 1}\frac{\alpha}{(1-\alpha)}\left[\operatorname{ch} y - \sqrt{\alpha^2 + \operatorname{sh}^2 y}\right] = \lim_{\alpha\to 1}\frac{1}{(-1)}\frac{(-1)\cdot 2\alpha}{2\sqrt{\alpha^2+\operatorname{sh}^2 y}} =$

$= \frac{1}{\sqrt{1+\operatorname{sh}^2 y}} = \frac{1}{\operatorname{ch} y}$. $\Phi_1^2(y) \underset{\alpha=\frac{m}{n}\to 1}{=} g(y,\alpha)\frac{\cos^2 m\beta}{(\alpha^2+\operatorname{sh}^2 y)^{1/2}} = \frac{\cos^2\beta}{\operatorname{ch} y}\frac{1}{\operatorname{ch} y} = \left(\frac{1}{\operatorname{ch}^2 y}\right)\cdot\cos^2\beta$.

$A_\beta = -\tau(\Phi_1^2 + \Phi_2^2) = -\frac{\tau}{\operatorname{ch}^2 y}$. $A_\beta(\text{Glad}) = \frac{2}{a}\tau(m_z+1)$, $M=1$ $\Phi_2^2(y) = \frac{1}{\operatorname{ch}^2 y}\sin^2\beta$.

$m_z + 1 = 1 - 2\operatorname{th}^2 y + 1 = 2(1 - \operatorname{th}^2 y) = 2\frac{\operatorname{ch}^2 y - \operatorname{sh}^2 y}{\operatorname{ch}^2 y} = \frac{2}{\operatorname{ch}^2 y}$. $A_\beta(\text{Glad}) = \frac{2\tau}{a}\cdot\frac{2}{\operatorname{ch}^2 y}$

A_β (Aratyn) совпадает с A_β (Glad) с точностью до множителя (!).

$\Phi_3^2(y) = \left[\frac{\sqrt{1+\alpha^2\operatorname{sh}^2 y} - \operatorname{ch} y}{(\alpha-1)}\right]\frac{\cos^2\varphi}{\sqrt{\frac{1}{\alpha^2}+\operatorname{sh}^2 y}} = \frac{1}{\operatorname{ch}^2 y}\cos^2\varphi$, $\Phi_4(y) = \frac{1}{\operatorname{ch} y}(-1)\sin\varphi$

$\alpha = \frac{m}{n} \to 1$

$A_y = \tau\frac{\partial}{\partial y}[\Phi_3\Phi_4] = \tau\frac{\partial}{\partial y}\left[\frac{1}{\operatorname{ch}^2 y}\sin\varphi\cos\varphi\right] = \frac{\tau}{2}\sin 2\varphi\,\frac{\partial}{\partial y}\left(\frac{1}{\operatorname{ch}^2 y}\right)$ – зависит сам!

$A_y(\text{Glad}) = 0$.

$A_\varphi = -\frac{n\tau}{\operatorname{sh} y}\left[\Phi_3^2 + \Phi_4^2\right] = -\frac{\tau}{\operatorname{sh} y}\frac{1}{\operatorname{ch}^2 y}$. $A_\varphi(\text{Glad}) = \frac{2\tau}{a}\frac{1}{\operatorname{sh} y}(m_z - 1)$

$m_z - 1 = 1 - 2\operatorname{th}^2 y - 1 = (-2)\operatorname{th}^2 y$, $A_\varphi(\text{Glad}) = \frac{2\tau}{a}\frac{1}{\operatorname{sh} y}\frac{(-2)\operatorname{sh}^2 y}{\operatorname{ch}^2 y}$

A_φ (Aratyn) не совп. с A_φ (Glad). A_y (Aratyn) $\neq 0$ – странное. Проверить.

(4/IX) Работа над Hopfion $\vec{E}$ manuscript.

H. J. de Vega, 1978 дает для $|Q_H| = 1$ hopfion, $\vec{B} = \frac{1}{4\pi c^2}\frac{r^2}{\operatorname{sh} y}\left[\hat{e}_\beta + \operatorname{sh} y\,\hat{e}_\varphi\right]\sin\theta\frac{\partial\theta}{\partial y}$

$m_z = \cos\theta(y)$, $m_x + i m_y = \sin\theta(y)\, e^{i(\beta-\varphi)}$. (y,β,φ) toroidal coordinates

$B_y = 0$, $B_\beta = \frac{1}{4\pi c^2}\frac{r^2}{\operatorname{sh} y}\sin\theta\frac{\partial\theta}{\partial y}$, $B_\varphi = \frac{1}{4\pi c^2}r^2\operatorname{sh}\theta\frac{\partial\theta}{\partial y}$, $r = \operatorname{ch} y - \cos\beta$

Glad-версия: 97: $B_y = 0$. $B_\beta = -\frac{2M}{a^2}\frac{r^2}{\operatorname{sh} y}\frac{\partial m_z}{\partial y}$, $B_\varphi = \frac{2M}{a^2}r^2\frac{\partial m_z}{\partial y}$,

$\sin\theta\frac{\partial\theta}{\partial y} = -\frac{\partial\cos\theta}{\partial y} = -\frac{\partial m_z}{\partial y}$. $\vec{B}$(Vega) и $\vec{B}$(Glad) совп. с точностью до множителя, $N=1, M=-1$.

$\vec{A}(\eta,\beta,\varphi) = \frac{1}{4\pi\sqrt{2}} \frac{\varepsilon}{\sinh\eta} \left[\sinh\eta\,(1-m_0)\,\vec{e}_\beta + (1+m_0)\,\vec{e}_\varphi \right]$, $A_\eta = 0$, $A_\beta = \frac{1}{4\pi\sqrt{2}}\varepsilon(1-m_0)$

$A_\varphi = \frac{1}{4\pi\sqrt{2}}\varepsilon(1+m_0)$

Caladi-Kowalski: $A_\eta = 0$, $A_\beta = \frac{2\pi}{\alpha}\varepsilon(1+m_0)$, $A_\varphi = \frac{2\pi}{\alpha}\frac{\varepsilon}{\sinh\eta}(m_0 - 1)$

$\vec{A}'(\eta,\beta,\varphi)$ = $\vec{A}'(\eta,\beta)$ potencial vector de un monopolo con momento magnético m_0 (no tiene componente azimutal)

Hopf map $\pi: S^3 \to S^2$ con $\pi(\eta,\beta,\varphi) = f(\eta)\exp[i(m\beta + n\varphi)]$

delloque las líneas de (1), cada una cumple $m_1 = m_2 = f(\eta)\,e^{i(\beta-\varphi)}$ con $|p_n| = 1$

hopfion in toroidal (η,β,φ) coordinates y gen. Abrikosov β, $\vec{A}'$ of toroidal hopfion.

Refiere una G^m hopfion magnetic

(5/12) D. H. Nicole 1978: S^3 y S^2 map, S^3 coordinates $\vec{N}(N_1,N_2,N_3,N_4)$

S^2 coordinates x_i $(i=1,2,3)$ $N_1 = \frac{2x_1}{1+R^2}$, $N_4 = \frac{1-R^2}{1+R^2}$, $R^2 = \sum_i x_i^2$ $\vec{N}^2 = 1$

$m_z = \cos\theta$, $m_x + i m_y = \sin\theta\, e^{i\varphi}$, $w = \tan\frac{1}{2}\theta\, e^{i\varphi}$.

Hopf map $w = \frac{N_4 + iN_3}{N_1 + iN_2}$. Note 1-index coordinates, 2 → x, 3 → y.

$m_0 = N_1^2 + N_2^2 - N_3^2 - N_4^2$, $m_x = 2(N_1N_4 + N_2N_3)$, $m_y = 2(N_1N_3 - N_2N_4)$

Hopfion $(m_x, m_y, m_z) = \vec{m}$, $\vec{m}^2 = 1$ corresponde (c solución de ecuaciones)

a $\vec{\xi}(\xi_1,\xi_2,\xi_3)$ coordenadas S^2, tal que $\sum_{i=1}^{3} u_i^2 = 1$, $\xi_i = \xi_i(\vec{u})$, $i=1,2,3$.

N. Nicole usa (5) → (4) coincide con x_i (ver Shnir '18

B. (Nicole) $= \frac{1}{8\pi}\varepsilon_{ijk}\varepsilon_{abc} n_a \partial_i n_b \partial_k n_c = \frac{1}{4\pi}\varepsilon_{ijk}\sin\theta\,\partial_i\theta\,\partial_k\varphi = \frac{1}{4\pi}\sin\theta(\nabla\theta\times\nabla\varphi)_i$

B_i (Nicole) $= \frac{1}{8\pi i}\varepsilon_{ijk}(1+|w|^2)^2(\partial_j\bar{w})(\partial_k w)$ mismo coincide con Fmn (Atatyn)

Hopf map (Nicole): $w = \frac{1-R^2+2ix_3}{2(x_1+ix_2)} = \frac{1-R^2+2iz}{2(x+iy)}$, con $x_3 = z$, también $x_1 = x$, $x_2 = y$

Hopf map (Shnir) $w = \frac{z_1}{z_0} = \frac{2(x-iy)}{(1-r^2-2iz)}$. w (Shnir) gear (con $\bar{w}$ Nicole)

Gus: $m_x = \frac{4}{(1+r^2)^2}\left[2yz + x(1-r^2)\right]$, $m_y = \frac{4}{(1+r^2)^2}\left[2zx - y(1-r^2)\right]$, $m_z = 1 - \frac{8(x^2+y^2)}{(1+r^2)^2}$

m_y (Gus) difiere de m_y (Shnir) signo, m_y (Gus) $= -m_y$ (Shnir) pero con w.

Hago w (Shnir corrected) $= \frac{2(x+iy)}{(1-r^2+2iz)}$, i.e. $w \to \bar{w}$, toma $m_y \to -m_y$, a=1 gaussiano.

w (Shnir) $= \frac{2(x+iy)}{(1-r^2+2iz)} = \left|\begin{matrix} s = \frac{1}{c}\sinh\eta \\ z = \frac{1}{c}\sin\beta \end{matrix}\right| = \frac{2se^{i\varphi}}{(1-s^2-z^2+2iz)}$, (η,β,φ) - toroidal coord.

$1 - s^2 - z^2 + 2iz = 1 - \frac{1}{c^2}\sinh^2\eta - \frac{1}{c^2}\sin^2\beta + \frac{2i}{c}\sin\beta = -\frac{2}{c}e^{-i\beta}$, $c = \cosh\eta - \cos\beta$

w (Shnir) $= \frac{2se^{i\varphi}}{(1-s^2-z^2+2iz)} = \frac{(\frac{2}{c})\sinh\eta\, e^{i\varphi}}{(-\frac{2}{c}e^{-i\beta})} = -\sinh\eta\, e^{i(\varphi+\beta)} = F(\eta)e^{i(\varphi+\beta)}$

$F(\eta) = -\sinh\eta$, $m_0 = \frac{1-|w|^2}{1+|w|^2} = 1 - 2\tanh^2\eta$, Shnir prueba que vortices $n=1, m=1$.

Hopf $\forall n,m$ $w = \frac{z_1^n}{z_0^m}$. $z_1 = 2(x+iy)$, $z_0 = 1 - r^2 + 2iz$ $(z_1 \to \bar{z}_1, z_0 \to \bar{z}_0$ y Shnir)

w (Shnir corrected) $= \frac{[2(x+iy)]^n}{(1-r^2+2iz)^m} = \frac{[\frac{2}{c}\sinh\eta\, e^{i\varphi}]^n}{[-\frac{2}{c}e^{-i\beta}]^m} = (-1)^m 2^{n-m} c^{m-n} e^{i(n\varphi+m\beta)} \sinh^n\eta$

w (Shnir) $= F_{mn}(\eta,\beta)\, e^{i(n\varphi+m\beta)}$, $F_{mn}(\eta,\beta) = (-1)^m 2^{n-m} c^{m-n} \sinh^n\eta$ — Esparza,

Hago $m_z = \frac{1-|w|^2}{1+|w|^2} = \frac{1-|F_{mn}|^2}{1+|F_{mn}|^2} = f(\eta,\beta)$ porque $c(\eta,\beta)$, No $m_z(\eta)$ solamente.

Hago profesor Shnir 'corrected' = correcto también. Luego profesor Atatyn '99 con $w = f(\eta)e^{i(n\varphi+m\beta)}$, y también $w = \frac{\varphi_1 + i\varphi_2}{\varphi_3 + i\varphi_4}$ — (ver Atatyn (n,m), Zo profesor Shnir '18. Atatyn)

Bueno recordar Shnir 2018, decía que correctos, usaba (Nicole, Battye-Kamitsu, Protogeros

Shnir'2018 ch. 15/III. Поляну $\vec{A}(\text{wing}) = -2\nabla\gamma + A^e$ (17) дает вектор Q_H, где $\gamma = \gamma(\beta,\varphi) = \beta(\vartheta,\varphi) + \text{const} - \text{helicity}$, $\Phi = n\varphi + \gamma$, β is toroidal angle (γ,β,φ).

$S^3(\Phi,\vartheta,\alpha)$, $Z = \begin{pmatrix} \cos\frac{\vartheta}{2} e^{i\alpha} \\ \sin\frac{\vartheta}{2} e^{i(\varphi+\alpha)} \end{pmatrix}$, $m_x = 2(x_1x_3 + x_2x_4)$, $m_y = 2(x_1x_4 - x_2x_3)$, $m_z = (x_1^2 + x_2^2 - x_3^2 - x_4^2)$, $\vec{m} = Z^+\vec{\sigma}Z$.

По готовым Word template RRL "Hopfion" manuscript is submitted to Rapid Res. Lett. - pss as invited contribution

A., A., 10 Хаджи 4/IV to "Caixa" Bank и бесплатно получает Antoni's tarjeta 28,00 € x4 + 36,00 € (15/12/2022) = 148,00 € и Caixa service price 6,00 € (31/12/22) возврату на карточку (несмотря на Cuenta joven), т.к. обязательство выполнено не позднее 3 months after события (2) Cuenta joven Antonia надо 115 € 29 Jan. 2023 могут вернуть) переведена на Work cuenta, когда при условии salary > 600 €/months, на gratis tarjeta, на service price выполняться не будут.

(6/IV) Pawel сообщил, что я выиграл HAM-travel grant 9,800 PLN. Как оплачивать airtickets и hotel?

Shnir Spinor $Z = \begin{pmatrix} \cos\frac{\Theta}{2} e^{id} \\ \sin\frac{\Theta}{2} e^{i(\Phi+d)} \end{pmatrix}$ и $\vec{m} = Z^+\vec{\sigma}Z$ дает $m_x = \sin\Theta\cos\Phi$, $m_y = \sin\Theta\sin\Phi$, $m_z = \cos\Theta$, $\vec{m} = \vec{m}(\Theta,\Phi)$, $\vec{m}^2 = 1$

Shnir доказывает, что $m_x = 2(x_1x_3 + x_2x_4)$, $m_y = 2(x_1x_4 - x_2x_3)$, $m_z = x_1^2 + x_2^2 - x_3^2 - x_4^2$ is Hopf map of degree one, $|Q_H| = 1$, Почему? Proof, $m_\alpha(\vec{x})$ универсальны.

$Z = \begin{pmatrix} Z_1 \\ Z_0 \end{pmatrix}$ $Z_1 = \cos\frac{\Theta}{2} e^{id}$, $Z_0 = \sin\frac{\Theta}{2} e^{i(\Phi+d)}$, $w = \frac{Z_1}{Z_0} = \frac{\cos\Theta/2\, e^{id}}{\sin\frac{\Theta}{2} e^{i(\Phi+d)}} = \operatorname{ctg}\frac{\Theta}{2} e^{-i\Phi}$

Ну или $w = Z_0/Z_1 = \operatorname{tg}\frac{\Theta}{2} e^{i\Phi}$, обычное w для расчета $\vec{m}$, нигде не дает.

(7/IV) Shnir, Eq. (8.22), Gribanovski gauge $Z_1 = 2(x - iy)$, $Z_0 = 1 - r^2 - 2iz$, $w = \frac{2(x-iy)}{1-r^2-2iz}$,

Надо это $w \to \bar{w}$, т.е. corrected Shnir $w = \frac{2(x+iy)}{1-r^2+2iz}$. $Z_1 = 2(x+iy)$, $Z_0 = 1-r^2+2iz$

$|Z_1|^2 + |Z_0|^2 = 4\rho^2 + |1-r^2+2iz|^2 = 4(x^2+y^2) + (1-r^2+2iz)(1-r^2-2iz) = 4\rho^2 + (1-\rho)^2 + 4z^2 = 4r^2 + (1-r^2)^2 = (1+r^2)^2$ и где $|Z_1|^2 + |Z_0|^2 = 1$?

(для S^3 parametrization), см. далее, нормальное $(Z_1 Z_0)$.

Nicole: $\sum_{i=1}^4 N_i^2 = \sum_{i=1}^3 \frac{(2x_i)^2}{(1+R^2)^2} + \frac{(1-R^2)^2}{(1+R^2)^2} = \frac{1}{(1+R^2)^2}\left[\sum_{i=1}^3 4x_i^2 + (1-R^2)^2\right] =$

$= \frac{1}{(1+R^2)^2}\left[4R^2 + (1-R^2)^2\right] = \frac{(1+R^2)^2}{(1+R^2)^2} = 1.$

Shnir $\vec{m}(\vec{x})$, $\sum x_i^2 = 1$ совпадает с $\vec{\pi}_3(\vec{u})$ - Kamchatnov'82, $\vec{\pi}_3 \to \vec{m}$, $\vec{u} \to \vec{x}(x_1 x_2 x_3 x_4)$ (с точностью до знака my-Shnir, где совпадение него $m_y \to -m_y$ у Shnir) $\vec{m}(\vec{x})$ Jäykkä 2009 совпадает с $\vec{m}(\vec{x})$ Kamchatnov'82.

Идея: вычислить vector potential $\vec{A}$ через компоненты spinor $Z = \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}$ in toroidal coordinates. $\vec{m} = Z^+\vec{\sigma}Z$.

Aratyn'99 дает $w = \frac{\Phi_1 + i\Phi_2}{\Phi_3 + i\Phi_4}$, $Z_1 = \Phi_1 + i\Phi_2$, $Z_2 = \Phi_3 + i\Phi_4$, for any (n,m) $\Phi_i(n,m)$, $\sum_{i=1}^4 \Phi_i^2 = 1$

Shnir'18 дает $w = \frac{Z_1^n}{Z_0^m}$ for any (n,m)-axial hopfion, следуя Sutcliffe Proc.RS'2007.

Кто из них прав? Shnir (Aratyn) и w (Shnir) эквивалентны? Shnir прав

Aratyn'99: $\sum_{i=1}^4 \Phi_i^2 = |d = m/n| = \frac{\operatorname{ch}y + \left(\frac{1}{2}z^2 + \operatorname{sh}^2 y\right)^{1/2}}{(|d|-1)\left[\frac{1}{2}z^2 + \operatorname{sh}^2 y\right]^{1/2}} + \frac{(1+d^2\operatorname{sh}^2 y)^{1/2} - \operatorname{ch}y}{(|d|-1)\left(\frac{1}{2}z^2 + \operatorname{sh}^2 y\right)^{1/2}} =$

$= \frac{1}{(|d|-1)\left[\frac{1}{2}z^2 + \operatorname{sh}^2 y\right]^{1/2}}\left[\sqrt{1+d^2\operatorname{sh}^2 y} - \sqrt{\frac{1}{2}z^2 + \operatorname{sh}^2 y}\right] = \frac{1}{(|d|-1)\sqrt{\frac{1}{2}z^2 + \operatorname{sh}^2 y}}\left[\sqrt{d^2\left(\frac{1}{2}z^2 + \operatorname{sh}^2 y\right)} - \sqrt{\frac{1}{2}z^2 + \operatorname{sh}^2 y}\right] =$

$= \frac{d^2+1}{|d|-1} = 1$, $\sum_{i=1}^4 \Phi_i^2 = 1$, верно. $\Phi_1 + i\Phi_2 = g_1(\gamma,d)e^{im\beta}$ $\Phi_3 + i\Phi_4 = g_2(\gamma,d)e^{-in\varphi}$

$w(\text{Aratyn}) = (\Phi_1 + i\Phi_2)/(\Phi_3 + i\Phi_4) = [g_1(\gamma,d)/g_2(\gamma,d)]\, e^{i(m\beta + n\varphi)} = f(\gamma,d)e^{i(m\beta + n\varphi}$

Escaneado con CamScanner

$W(\text{Araújo}) = W(\gamma,\beta,\varepsilon) = f(\gamma,\alpha)\, e^{i(m\beta+n\varepsilon)}$ имеет правильную структуру,

$m_2 = m_2(\gamma) = \sqrt{1-f^2(\gamma,\alpha)}$, и согласуется с $W(\gamma,\beta,\varepsilon)$ – Gladekowski '97 и

Зависимость $W_{(\text{Araújo})}(m,n)$ через $\alpha = m/n$ близка к Nietarinta '99.

Shmez '18 for any (m,n), $w = Z_1^n / Z_0^m$, $Z_1 = \sin f \sin\theta e^{i\varphi}$, $Z_0 = \cos f + i \sin f \cos\theta$

$Z_1 = \frac{(x+iy)}{r} \sin f$, $Z_0 = \cos f + \frac{i}{r} \sin f \frac{z}{r}$, $(x+iy) = \rho e^{i\varphi}$, $\rho = r\sin\theta$, $\cos\theta = \frac{z}{r}$, $f(r)$.

$r^2 = x^2+y^2+z^2$. $|Z_1|^2 + |Z_0| = \sin^2 f \sin^2\theta + \cos^2 f + \sin^2 f \cos^2\theta = 1$.

$f \doteq f(r,m,n)$. При $m=n=1$, $f(r) = \arctan\left(\frac{2r}{1-r^2}\right)$, $\sin f = \frac{2r}{1+r^2}$, $\cos f = \frac{1-r^2}{1+r^2}$,

и $Z_1 = \frac{2r}{(1+r^2)} \sin\theta e^{i\varphi}$, $Z_0 = \frac{(1-r^2)}{(1+r^2)} + i \frac{2r}{(1+r^2)} \cos\theta$

$W(\text{Shmez}, m=n=1) = \frac{Z_1}{Z_0} = \frac{2r\sin\theta e^{i\varphi}}{(1-r^2) + i 2r\cos\theta} = \frac{2(x+iy)}{1-r^2+2iz}$, совпадает Shmez Eq. (8.24)

Чуть шире ($m,n=1$). Найдем $\vec{A}$ по eqs. Shmez, $Z_1 = \frac{2\rho}{(1+r^2)} e^{i\varphi}$, $Z_0 = \frac{(1-r^2)}{(1+r^2)} + i\frac{2z}{(1+r^2)}$

Araújo '99: $A_\alpha = \frac{i}{2}(Z^+ \partial_\alpha Z - \partial_\alpha Z^+ Z) = \frac{i}{2}(Z_1^*, Z_2^*)\begin{pmatrix}\partial_\alpha Z_1 \\ \partial_\alpha Z_2\end{pmatrix} - \frac{i}{2}(\partial_\alpha Z_1^*, \partial_\alpha Z_2^*)\begin{pmatrix}Z_1 \\ Z_2\end{pmatrix} =$

$= \frac{i}{2}\left[Z_1^* \partial_\alpha Z_1 + Z_2^* \partial_\alpha Z_2 - \partial_\alpha Z_1^* Z_1 - \partial_\alpha Z_2^* Z_2\right] = \frac{i}{2}\left[Z_i^* \partial_\alpha Z_i - \partial_\alpha Z_i^* Z_i\right]$

Это $\vec{A}$ совп. с Eq. (7.140) – Shmez, $A_\mu = \frac{i}{2}\left[Z_a^* \partial_\mu Z_a - Z_a \partial_\mu Z_a^*\right]$, $a,i = 1,2$.

Spinor $Z = \begin{pmatrix}Z_1 \\ Z_2\end{pmatrix}$, $Z_1 = \frac{2\rho}{(1+r^2)} e^{i\varphi}$, $Z_2 = Z_0 = \frac{(1-r^2)}{(1+r^2)} + \frac{2iz}{(1+r^2)}$, x,y,z в eq. 9, hopfion radius

перейдем к toroidal coordinates $\rho = \frac{\sh\gamma}{\tau}$, $z = \frac{\sin\beta}{\tau}$, $\varphi = \varphi$, $\tau = \ch\gamma - \cos\beta$,

$1+r^2 = 1+\rho^2+z^2 = 1 + \frac{\sh^2\gamma}{\tau^2} + \frac{\sin^2\beta}{\tau^2} = \frac{1}{\tau^2}\left[\ch^2\gamma + \cos^2\beta - 2\ch\gamma\cos\beta + \right.$

$\left. + \sh^2\gamma + \sin^2\beta\right] = \frac{1}{\tau^2}\left[1 + \ch^2\gamma + \ch^2\gamma - 1 - 2\ch\gamma\cos\beta\right] = \frac{2\ch\gamma}{\tau^2}(\ch\gamma - \cos\beta) = \frac{2\ch\gamma}{\tau}$

$(1+r^2) = \frac{2}{\tau}\ch\gamma$. $1-r^2+2iz = 1 - \frac{1}{\tau^2}\sh^2\gamma - \frac{1}{\tau^2}\sin^2\beta + 2i\frac{1}{\tau}\sin\beta =$

$= \frac{1}{\tau^2}\left[(\ch\gamma - \cos\beta)^2 - \sh^2\gamma - \sin^2\beta + 2i(\ch\gamma - \cos\beta)\sin\beta\right] = \frac{1}{\tau^2}\left[\ch^2\gamma + \cos^2\beta - \right.$

$\left. - 2\ch\gamma\cos\beta - \sh^2\gamma - \sin^2\beta + 2i(\ch\gamma - \cos\beta)\sin\beta\right] = \frac{1}{\tau^2}\left[1 + \cos^2\beta - 2\ch\gamma\cos\beta - \sh^2\gamma + \right.$

$\left. + 2i\sin\beta(\ch\gamma - \cos\beta)\right] = \frac{2}{\tau^2}\left[\cos^2\beta - \ch\gamma\cos\beta + i\sin\beta(\ch\gamma - \cos\beta)\right] =$

$= \frac{2}{\tau^2}\left[\cos\beta(\cos\beta - \ch\gamma) + i\sin\beta(\ch\gamma - \cos\beta)\right] = \frac{2}{\tau^2}\tau\left[-\cos\beta + i\sin\beta\right] = -\frac{2}{\tau}e^{-i\beta}$.

$1-r^2+2iz = -\frac{2}{\tau}e^{-i\beta}$ (см. 14/IX/2020).

$Z_1 = \frac{2\rho e^{i\varphi}}{(1+r^2)} = 2e^{i\varphi}\frac{\sh\gamma}{\tau}\frac{1}{\frac{2}{\tau}\ch\gamma} = \th\gamma\, e^{i\varphi}$, $Z_2 = \frac{1}{(1+r^2)}\left[1-r^2+2iz\right] =$

$= \frac{1}{\frac{2}{\tau}\ch\gamma}\left[-\frac{2}{\tau}e^{-i\beta}\right] = -\frac{1}{\ch\gamma}e^{-i\beta}$. $Z_1 = \th\gamma\, e^{i\varphi}$, $Z_2 = -\frac{1}{\ch\gamma}e^{-i\beta}$, $\det = 1$, $Z_2 = Z_0$.

$w = Z_1/Z_2 = \frac{\th\gamma\, e^{i\varphi}}{\left(-\frac{1}{\ch\gamma}e^{-i\beta}\right)} = -\sh\gamma\, e^{i(\varphi+\beta)}$, верно, $m_2 = 1 - 2\sh^2\gamma$.

Hietarinta 2000: $w = f(\gamma)e^{i(\varphi+\beta)}$

$Z_i^* \partial_\alpha Z_i - \partial_\alpha Z_i^* Z_i = F - \bar{F} = 2i\,\mathrm{Im}(F)$, $A_\alpha = \frac{i}{2}\left(Z_i^* \partial_\alpha Z_i - \partial_\alpha Z_i^* Z_i\right) = \frac{i}{2}\,2i\,\mathrm{Im}(F)$

$A_\alpha = \frac{i}{2}\cdot 2i\,\mathrm{Im}\left(Z_i^* \partial_\alpha Z_i\right) = -\mathrm{Im}\left(Z_i^* \partial_\alpha Z_i\right)$, Sum over $i = 1,2$.

$\partial_\gamma Z_1 = \frac{\partial}{\partial\gamma} Z_1 = \frac{\partial}{\partial\gamma}\th\gamma\, e^{i\varphi} = \frac{1}{\ch^2\gamma} e^{i\varphi}$, $\partial_\varphi Z_1 = \th\gamma\, i e^{i\varphi}$, $\partial_\beta Z_1 = \frac{\partial Z_1}{\partial\beta} = 0$

$\partial_\gamma Z_2 = \frac{\partial Z_2}{\partial\gamma} = \frac{\partial}{\partial\gamma}\left(-\frac{1}{\ch\gamma}\right)e^{-i\beta} = e^{-i\beta}\frac{\sh\gamma}{\ch^2\gamma}$, $\partial_\varphi Z_2 = 0$, $\partial_\beta Z_2 = \frac{i}{\ch\gamma}e^{-i\beta}$

$\alpha = \gamma$, $Z_i^* \partial_\gamma Z_i = Z_1^* \partial_\gamma Z_1 + Z_2^* \partial_\gamma Z_2 = \th\gamma\, e^{-i\varphi}\frac{1}{\ch^2\gamma}e^{i\varphi} + \left(-\frac{1}{\ch\gamma}\right)e^{i\beta}e^{-i\beta}\frac{\sh\gamma}{\ch^2\gamma} =$

$= \frac{\th\gamma}{\ch^2\gamma} - \frac{\sh\gamma}{\ch^3\gamma} = 0$ (!). $\alpha = \beta \to Z_i^* \partial_\beta Z_i = Z_1^* \partial_\beta Z_1 + Z_2^* \partial_\beta Z_2 =$

$= \th\gamma\, e^{-i\varphi}\cdot 0 + \left(-\frac{1}{\ch\gamma}\right)e^{i\beta}\frac{i}{\ch\gamma}e^{-i\beta} = -\frac{i}{\ch^2\gamma}$, $A_\beta = -\mathrm{Im}\left(Z_i^* \partial_\beta Z_i\right) = -\mathrm{Im}\left(-\frac{i}{\ch^2\gamma}\right) = \frac{1}{\ch^2\gamma}$

$A_\beta = \frac{1}{\ch^2\gamma}$. У Gladkowski '97: $A_\beta = \frac{2M}{a}\tau(m_2+1)$ отличается.

Escaneado con CamScanner

Hago $\partial_\alpha = \nabla_\alpha$ $\nabla F = \frac{1}{h_i}\frac{\partial F}{\partial q_i}\vec{e}_i = \frac{1}{h_\rho}\frac{\partial F}{\partial \rho}\hat{\rho} + \frac{1}{h_\varphi}\frac{\partial F}{\partial \varphi}\hat{\varphi} + \frac{1}{h_\eta}\frac{\partial F}{\partial \eta}\hat{\eta} =$

$= \left| h_\eta = h_\rho = \frac{a}{\tau}, h_\varphi = \frac{a}{\sh\eta \cdot \tau} \right| = \frac{\tau}{a}\left[\frac{\partial F}{\partial \eta}\hat{\eta} + \frac{\partial F}{\partial \rho}\hat{\rho} + \frac{1}{\sh\eta}\frac{\partial F}{\partial \varphi}\hat{\varphi}\right] \equiv \frac{\tau}{a}\nabla_\alpha F$

$A_\alpha = -\Im m\left(Z_i^* \nabla_\alpha Z_i\right)$, $\nabla_\alpha = \left(\frac{\tau}{a}\frac{\partial}{\partial\eta}, \frac{\tau}{a}\frac{\partial}{\partial\rho} + \frac{\tau}{a}\frac{1}{\sh\eta}\frac{\partial}{\partial\varphi}\right)$, $\nabla_\eta = \frac{\tau}{a}\frac{\partial}{\partial\eta}$

$\nabla_\rho = \frac{\tau}{a}\frac{\partial}{\partial\rho}$, $\nabla_\varphi = \frac{\tau}{a\sh\eta}\frac{\partial}{\partial\varphi}$. $Z_i^* \nabla_\eta Z_i = \frac{\tau}{a} Z_i^* \frac{\partial Z_i}{\partial \eta} = \frac{\tau}{a}\cdot 0 = 0$.

$Z_i^* \nabla_\rho Z_i = \frac{\tau}{a} Z_i^* \frac{\partial Z_i}{\partial \rho} = \frac{\tau}{a}\left(-i\frac{1}{\ch^2\eta}\right)$, $A_\eta = -\Im m\left(Z_i^* \nabla_\eta Z_i\right) = -\Im m(0) = 0$.

$A_\rho = -\Im m\left(Z_i^* \nabla_\rho Z_i\right) = -\Im m\left[\frac{\tau}{a}\left(-\frac{i}{\ch^2\eta}\right)\right] = \frac{\tau}{a}\frac{1}{\ch^2\eta}$, $m_2 + 1 = 2 - 2\th^2\eta =$

$= 2\left(1 - \th^2\eta\right) = \frac{2}{\ch^2\eta}$, $A_\rho(\text{Glad}) = \frac{2M}{a}\tau\left(m_2 + 1\right) = \frac{4M}{a}\frac{\tau}{\ch^2\eta}$, $A_\rho(\text{nuce}) = \frac{\tau}{a}\frac{1}{\ch^2\eta}$

A_φ: $Z_i^* \frac{\partial Z_i}{\partial \varphi} = Z_1^* \frac{\partial Z_1}{\partial \varphi} + Z_2^* \frac{\partial Z_2}{\partial \varphi} = \th\eta e^{-i\varphi}\th\eta\, i e^{i\varphi} + \left(-\frac{1}{\ch\eta}e^{i\beta}\right)\cdot 0 = i\th^2\eta$

$A_\varphi = -\Im m\left(Z_i^* \nabla_\varphi Z_i\right) = -\Im m\left[Z_i^* \frac{\tau}{a\sh\eta}\frac{\partial Z_i}{\partial \varphi}\right] = -\Im m\left(i\th^2\eta\right)\frac{\tau}{a\sh\eta} = -\frac{\tau}{a}\frac{\th^2\eta}{\sh\eta}$

$A_\varphi(\text{nuce}) = -\frac{\tau}{a}\frac{\th^2\eta}{\sh\eta}$. $A_\varphi(\text{Glad}) = \frac{2N}{a}\frac{\tau}{\sh\eta}\left(m_2 - 1\right) = \frac{2N}{a}\frac{\tau}{\sh\eta}\left(-2\th^2\eta\right) = -\frac{4N}{a}\frac{\tau}{\sh\eta}\th^2\eta$

$\vec{A}(\text{nuce})$: $A_\eta = 0$, $A_\rho = \frac{\tau}{a}\frac{1}{\ch^2\eta}$, $A_\varphi = -\frac{\tau}{a}\frac{\th^2\eta}{\sh\eta}$

$\vec{A}(\text{Glad})$: $A_\eta = 0$, $A_\rho = \frac{4M}{a}\tau\frac{1}{\ch^2\eta}$, $A_\varphi = -\frac{4N}{a}\frac{\tau\,\th^2\eta}{\sh\eta}$, $M = N = 1$.

$\vec{A}(\text{Glad}) \to \vec{A}(\text{Glad})/2$, no $\vec{A}(\text{nuce}) = \vec{A}(\text{Glad})/2$, b 2-paza ucnpaBue corrected $\vec{A}(\text{Glad})$.

Corrección

Protogerov '2006, $\vec{m} = \bar{\chi}\vec{\sigma}\chi$, $\chi = \begin{pmatrix}\chi_1\\ \chi_2\end{pmatrix}$ u $\vec{A} = i\left[\chi_i \nabla \chi_i^* - c.c.\right] = -2\,\Im m\left(\chi_i^* \nabla \chi_i\right)$

he совпадает с $\vec{A}(\text{nuce})$, b 2 раза больше (с учетом gg знака)

Если взять $A_\alpha = i\left[Z^+\partial_\alpha Z - \partial_\alpha Z^+ Z\right] = -2\,\Im m\left(Z_i^* \nabla_\alpha Z_i\right)$. то $\vec{A}$ совпадает с $\vec{A}(\text{Glad})$ corrected. Можно еще взять $\sqrt{2}$ в definition of Z_1, Z_2.

и $\vec{A}$(Proposen.)

→ Note [Shnir, Ch. 7] that the skyrme field $\mathbf{U}$, $\mathbf{U}$ is $Su(2)$-matrix, is a map of S_3 to $Su(2)$ group space labeled by topol. charge $Q = \pi_3(S_3)$

Если introduce $Z_1 = \Phi_1 + i\Phi_2$, $Z_2 = \Phi_3 + i\Phi_4$, то topol. charge Q

$Q = -\frac{1}{12\pi^2}\int d^3x\, \varepsilon_{abcd}\,\varepsilon^{ijk}\,\Phi^a \partial_i\Phi^b\,\partial_j\Phi^c\,\partial_k\Phi^d = -\frac{1}{4\pi^2}\int d^3x\, \varepsilon_{ijk} A_i F_{jk}$

$F_{jk} = \partial_j A_k - \partial_k A_j$ - field tensor, $A_\mu = \frac{i}{2}\left[\partial_\mu Z_a \cdot Z_a^* - Z_a \partial_\mu Z_a^*\right]$, $Z_a^* Z_a = 1$

$B_i = \frac{1}{2}\varepsilon_{ijk}F_{jk}$, $Q = -\frac{1}{2\pi^2}\int d^3x\, B_i A_i = -\frac{1}{2\pi^2}\int d^3\vec{A}\cdot\vec{B} \sim Q_H = -\frac{1}{(4\pi)^2}\oint d\vec{A}\vec{B}$

Q is topol. charge of S_3 to $Su(2)$ group space; Q_H - S_3 to S_2 map.

mapping

Наиден vector potential $\vec{A}$. $A_\mu = -\Im m\left(Z_i^* \nabla_\mu Z_i\right)$ в cylindrical coordinates (ρ, φ, z) и (θ, Φ, χ) coordinates.

Cylindr. • $Z_1 = \frac{2\rho e^{i\varphi}}{(1+r^2)}$, $Z_2 = \frac{1}{(1+r^2)}\left[1 + r^2 + 2iz\right]$, $Z_i(\rho, \varphi, z)$, w - function в cyl. coordinates, $w(\eta, \beta, \varphi) = f(\eta)\, e^{i(\varphi+\beta)}$ не нужен, так как $Z_i(\rho, \varphi, z)$ - уже пр.

$\nabla = \hat{\rho}\frac{\partial}{\partial\rho} + \hat{\varphi}\frac{1}{\rho}\frac{\partial}{\partial\varphi} + \hat{z}\frac{\partial}{\partial z}$, $\nabla_\rho = \frac{\partial}{\partial\rho}$, $\nabla_\varphi = \frac{1}{\rho}\frac{\partial}{\partial\varphi}$, $\frac{\partial}{\partial r^2} = \nabla_z$. $r^2 = \rho^2 + z^2$

A_φ: $Z_i^* \frac{1}{\rho}\frac{\partial}{\partial\varphi} Z_i = \frac{1}{\rho} Z_i^* \frac{\partial Z_i}{\partial\varphi}$, $\frac{\partial Z_1}{\partial\varphi} = \frac{i2\rho e^{i\varphi}}{(1+r^2)}$, $\frac{\partial Z_2}{\partial\varphi} = 0$.

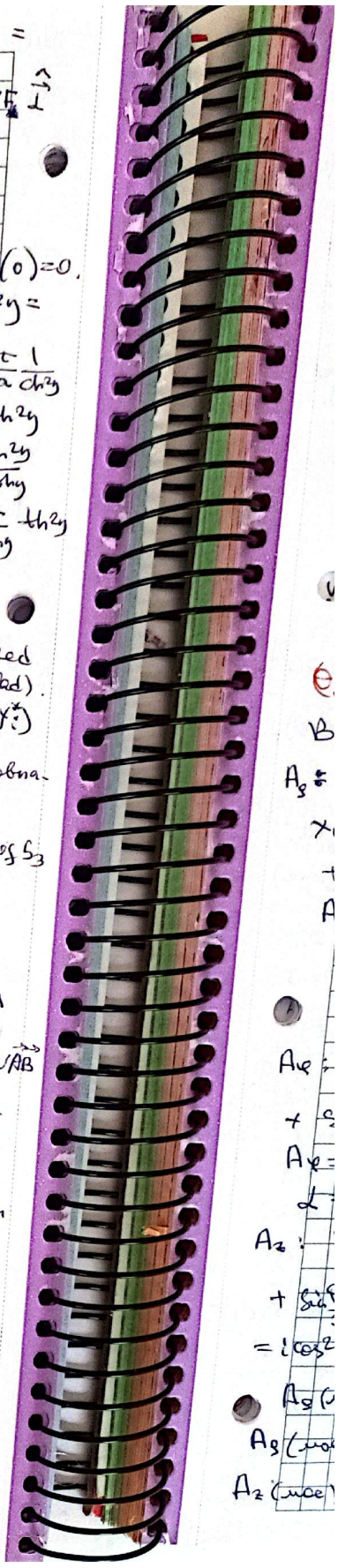
$Z_i^* \frac{\partial Z_i}{\partial\varphi} = Z_1^* \frac{\partial Z_1}{\partial\varphi} + Z_2^* \frac{\partial Z_2}{\partial\varphi} = \frac{2\rho e^{-i\varphi}}{(1+r^2)} \cdot \frac{i2\rho e^{i\varphi}}{(1+r^2)} = i\frac{4\rho^2}{(1+r^2)^2}$

$A_\varphi = -\Im m\left(Z_i^* \nabla_\varphi Z_i\right) = -\Im m\left[\frac{1}{\rho} Z_i^* \frac{\partial Z_i}{\partial\varphi}\right] = -\Im m\left[\frac{1}{\rho}\, i\frac{4\rho^2}{(1+r^2)^2}\right] = -\frac{4\rho}{(1+r^2)^2}$

$A_\varphi(\text{wing'og}) = \frac{n}{\rho}\left(1 - \cos\Theta\right)$, $m_2 = 1 - \frac{8\rho^2}{(1+r^2)^2}$, $1 - m_2 = \frac{8\rho^2}{(1+r^2)^2}$

$A_\varphi(\text{wing}) = n\frac{8\rho}{(1+r^2)^2}$, $A_\varphi(\text{nuce}) = -\frac{4\rho}{(1+r^2)^2}$.

$A_\rho = -\Im m\left(Z_i^* \nabla_\rho Z_i\right)$, $\nabla_\rho = \frac{\partial}{\partial\rho}$, $\frac{\partial Z_1}{\partial\rho} = 2e^{i\varphi}\frac{\partial}{\partial\rho}\left[\frac{\rho}{(1+\rho^2+z^2)}\right] = 2e^{i\varphi}\frac{\partial}{\partial\rho}[\ldots]$



Escaneado con CamScanner

$Z_1^* \frac{\partial Z_1}{\partial s} = \frac{2s e^{-i\varphi}}{(1+r^2)} e^{i\varphi} \frac{1}{2} \frac{s}{(1+r^2)}$ is real, we get Bmag $\vec{B} \parallel \vec{A}_s$.

$Z_2^* \frac{\partial Z_2}{\partial s} = \frac{(1-r^2-2ir)}{(1+r^2)} \frac{\partial}{\partial s}\left[\frac{1-r^2+2ir}{(1+r^2)}\right] = \frac{(\dots)}{(1+r^2)} \cdot \frac{-2s[1+r^2]-(1-r^2+2ir)2s}{(1+r^2)^2} =$

$= \frac{(\dots)}{(1+r^2)^3}\left[-4s - 2is\right] = \frac{1}{(1+r^2)^3}(1-r^2-2ir)(-4s)(1+i) = -\frac{4s}{(1+r^2)^3}\left[-2r^2 + ir(1-r^2)\right] =$

$= \frac{-4s}{(1+r^2)^3} ir\left[-2+1-r^2\right] = ir\frac{4s}{(r^2+1)^2}$, $A_s = -\mathrm{Im}\left(Z_i^* \frac{\partial Z_i}{\partial s}\right) = -\frac{4rs}{(1+r^2)^2}$

$A_s(\text{Wang}) = -(1+\cos\theta)\frac{\partial \gamma}{\partial s}$, $\frac{\partial \gamma}{\partial s} = \frac{\partial \varphi}{\partial s} = \frac{\partial}{\partial s}\operatorname{arctg}\left(\frac{2r}{r^2-1}\right) = \frac{1}{1+\frac{4r^2}{(r^2-1)^2}} \times$

$\times 2r \frac{\partial}{\partial s}\frac{1}{(r^2-1)} = \frac{(r^2-1)^2}{(r^2-1)^2+4r^2} 2r \frac{(-1)}{(r^2-1)^2} \cdot 2s = -\frac{4rs}{(r^2-1)^2+4r^2}$

$A_s(\text{Wang}) = -\left[2 - \frac{8s^2}{(1+r^2)^2}\right]\frac{(-1)4rs}{(r^2-1)^2+4r^2}$, $1 - \frac{4s^2}{(1+r^2)^2} = \frac{1}{(1+r^2)^2}\left[(1+r^2)^2 - 4s^2\right] =$

$= \frac{1}{(1+r^2)^2}\left[(1+s^2+r^2)^2 - 4s^2\right] = \frac{1}{(1+r^2)^2}\left[(1+s^2)^2 - 4s^2 + 2r^2(s^2+1) + r^4\right] =$

$= \frac{1}{(1+r^2)^2}\left[(1-s^2)^2 + 2r^2(1+s^2) + r^4\right]$, $(r^2-1)^2 + 4r^2 = (s^2+r^2-1)^2 + 4r^2 =$

$= (s^2-1)^2 + 2r^2(s^2-1) + r^4 + 4r^2 = (s^2-1)^2 + 2r^2(s^2+1) + r^4$

$A_s(\text{Wang}) = 2\left[1 - \frac{4s^2}{(1+r^2)^2}\right]\frac{4rs}{(r^2-1)^2+4r^2} = \frac{8}{(1+r^2)^2}\frac{4rs}{\left[(1-s^2)^2+2r^2(1+s^2)+r^4\right]}$

$A_s(\text{Wang}) = \frac{8rs}{(1+r^2)^2}$, $A_s(\text{моё}) = -\frac{4rs}{(1+r^2)^2}$

Видно, $\vec{A}(\text{моё}) = -\frac{1}{2}\vec{A}(\text{Wang})$, хотя A_z не проверен. Ищем $\vec{B}(s,\varphi,z)$ для $(s,\varphi,z) \to$ Описаны $Z_1 = \cos\frac{\theta}{2}e^{id}$, $Z_2 = \sin\frac{\theta}{2}e^{i(\varphi+d)}$, $\vec{n}(\theta,\varphi)$, $\theta(\vec{r}), \varphi(\vec{r})$, $d(\vec{r})$

В силу аксиальной симметрии $\theta = \theta(s,z)$, $\varphi = n\varphi + \gamma(s,z)$, $d = d(s,z)$.

$A_s = Z_i^* \frac{\partial Z_i}{\partial s}$. $\frac{\partial Z_1}{\partial s} = -\sin\frac{\theta}{2}\cdot\frac{1}{2}\frac{\partial\theta}{\partial s}e^{id} + i\cos\frac{\theta}{2}e^{id}\frac{\partial d}{\partial s}$, $\frac{\partial Z_2}{\partial s} = \cos\frac{\theta}{2}\frac{1}{2}\frac{\partial\theta}{\partial s} \times$

$\times e^{i(\varphi+d)} + i\sin\frac{\theta}{2}e^{i(\varphi+d)}\left[\frac{\partial\gamma}{\partial s} + \frac{\partial d}{\partial s}\right]$, $Z_1^* \frac{\partial Z_1}{\partial s} = \frac{1}{2}\cos\frac{\theta}{2}\left[-\sin\frac{\theta}{2}\frac{\partial\theta}{\partial s}e^{id} + \right.$

$\left. + i\cos\frac{\theta}{2}e^{id}\frac{\partial d}{\partial s}\right]e^{id} = \cos\frac{\theta}{2}\left[-\sin\frac{\theta}{2}\frac{\partial\theta}{\partial s}\frac{1}{2} + i\cos\frac{\theta}{2}\frac{\partial d}{\partial s}\right]$.

$A_s = -\mathrm{Im}\left(Z_i^* \frac{\partial Z_i}{\partial s}\right) = -\mathrm{Im}\left[i\cos^2\frac{\theta}{2}\frac{\partial d}{\partial s} + \sin\frac{\theta}{2}e^{-i(\varphi+d)}\left\{\frac{1}{2}\cos\frac{\theta}{2}\frac{\partial\theta}{\partial s}e^{i(\varphi+d)} + \right.\right.$

$\left.\left. + i\sin\frac{\theta}{2}e^{i(\varphi+d)}\left(\frac{\partial\gamma}{\partial s} + \frac{\partial d}{\partial s}\right)\right\}\right] = -\mathrm{Im}\left[i\cos^2\frac{\theta}{2}\frac{\partial d}{\partial s} + i\sin^2\frac{\theta}{2}\left(\frac{\partial\gamma}{\partial s} + \frac{\partial d}{\partial s}\right)\right]$

$A_s = -\frac{\partial d}{\partial s} - \sin^2\frac{\theta}{2}\frac{\partial\gamma}{\partial s} = -\frac{\partial d}{\partial s} - \frac{1}{2}(1-\cos\theta)\frac{\partial\gamma}{\partial s}$, $A_s(\text{Wang}) = -(1+\cos\theta)\frac{\partial\gamma}{\partial s}$

A_φ: $Z_i^* \frac{1}{s}\frac{\partial Z_i}{\partial\varphi} = \frac{1}{s}Z_1^*\frac{\partial Z_1}{\partial\varphi} + \frac{1}{s}Z_2^*\frac{\partial Z_2}{\partial\varphi} = \frac{1}{s}\cos\frac{\theta}{2}e^{-id}\left(\cos\frac{\theta}{2}e^{id}\right)i\frac{\partial d}{\partial\varphi} + \frac{1}{s}\sin\frac{\theta}{2}e^{-i(\varphi+d)} \times$

$\times \sin\frac{\theta}{2}e^{i(\varphi+d)}i\left[n + \frac{\partial d}{\partial\varphi}\right] = \frac{i}{s}\cos^2\frac{\theta}{2}\frac{\partial d}{\partial\varphi} + \frac{i}{s}\sin^2\frac{\theta}{2}\left(n + \frac{\partial d}{\partial\varphi}\right) = \frac{i}{s}\frac{\partial d}{\partial\varphi} + \frac{in}{s}\sin^2\frac{\theta}{2}$

$A_\varphi = -\mathrm{Im}\left(Z_i^*\frac{1}{s}\frac{\partial Z_i}{\partial\varphi}\right) = -\frac{1}{s}\frac{\partial d}{\partial\varphi} - \frac{n}{s}\frac{1}{2}(1-\cos\theta)$, $A_\varphi(\text{Wang}) = \frac{n}{s}(1-\cos\theta)$

$d = d(s,\varphi,z)$ тоже не нулевая $\vec{A} \to \vec{A} - \nabla d$, и не даёт вклада $\theta \parallel \vec{n}$.

A_z: $Z_i^*\frac{\partial Z_i}{\partial z} = Z_1^*\frac{\partial Z_1}{\partial z} + Z_2^*\frac{\partial Z_2}{\partial z} = \left|\text{можно } \frac{\partial}{\partial\varphi} = 0\right| = \cos\frac{\theta}{2}e^{-id}\left[-\sin\frac{\theta}{2}\frac{1}{2}\frac{\partial\theta}{\partial z}e^{id} + i\cos\frac{\theta}{2}e^{id}\frac{\partial d}{\partial z}\right] +$

$+ \sin\frac{\theta}{2}e^{-i\varphi-id}\left[\frac{1}{2}\cos\frac{\theta}{2}\frac{\partial\theta}{\partial z}e^{i(\varphi+d)} + \sin\frac{\theta}{2}ie^{i(\varphi+d)}\left(\frac{\partial\gamma}{\partial z} + \frac{\partial d}{\partial z}\right)\right] =$

$= i\cos^2\frac{\theta}{2}\frac{\partial d}{\partial z} + i\sin^2\frac{\theta}{2}\left(\frac{\partial\gamma}{\partial z} + \frac{\partial d}{\partial z}\right)$, $A_z = -\mathrm{Im}\left(Z_i^*\frac{\partial Z_i}{\partial z}\right) = -\frac{\partial d}{\partial z} - \frac{1}{2}(1-\cos\theta)\frac{\partial\gamma}{\partial z}$

$A_s(\text{моё}) = -\frac{\partial d}{\partial s} + \frac{1}{2}(1+\cos\theta)\frac{\partial\gamma}{\partial\varphi}\frac{\partial\varphi}{\partial s} = -\frac{\partial d}{\partial s} - \frac{\partial\gamma}{\partial s} + \left(-\frac{1}{2}\right)(-1)(1+\cos\theta)\frac{\partial\gamma}{\partial s}$, $A_z(\text{Wang}) = -(1+\cos\theta)\frac{\partial\gamma}{\partial z}$

$A_s(\text{моё}) = -\frac{\partial d}{\partial s} - \frac{\partial\gamma}{\partial s} - \frac{1}{2}A_s(\text{Wang})$, $A_\varphi(\text{моё}) = -\frac{1}{s}\frac{\partial d}{\partial\varphi} - \frac{1}{2}\frac{n}{s}(1-\cos\theta)$

$A_z(\text{моё}) = -\frac{\partial d}{\partial z} + \left(-\frac{1}{2}\right)(-1)(1+\cos\theta)\frac{\partial\varphi}{\partial z} - \frac{\partial\gamma}{\partial z} = -\frac{\partial d}{\partial z} - \frac{\partial\gamma}{\partial z} - \frac{1}{2}A_z(\text{Wang})$ $\quad A_\varphi(\text{Wang})$

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В (θ,φ,α) представлении $\vec{A}(\text{mee}) = -\nabla\alpha - \nabla\gamma \sim \frac{1}{2}\vec{A}(\text{Wang})$ $\partial\beta/\partial\varphi = 0$.

Цил (ρ,φ,z) цилиндр. расчет $\vec{A} = -\operatorname{Im}(Z_i^* \nabla Z_i)$, $\vec{A}(\text{mee}) = -\frac{1}{2}\vec{A}(\text{Wang})$

Цил (γ,β,φ) тороид. расчет $\vec{A}(\text{mee}) = \frac{1}{2}\vec{A}(\text{Glad corrected})$.

Выводы: 1) Надо $\vec{A} = -2\operatorname{Im}(Z_i^* \nabla Z_i)$, тогда $\vec{A}(\text{mee}) = \vec{A}(\text{Glad}) = -\vec{A}(\text{Wang})$

2) Wang-Berardo неправильно взял $\vec{A}$, надо $\vec{A}(\text{Wang}) \to -\vec{A}(\text{Wang})$ и знак $Q_H = \frac{1}{(4\pi)^2}\int dV\, \vec{B}\cdot\vec{A}$ неправильно, $\vec{A} \to -\vec{A}$ и $Q_H \to -Q_H$ дает верный знак Q_H.

3) $\vec{A}(\text{mee}) = \vec{A}(\text{Glad}) - 2\nabla\alpha - 2\nabla\gamma$, представление $\vec{A}(\theta,\varphi,\alpha) = \vec{A}(\text{Glad})$, даст gauge $\nabla(\alpha+\gamma) = 0$, $\alpha(\vartheta,\varphi,z) = -\gamma(\vartheta,z) + \text{const} = -\beta(\vartheta,z) + \text{const}$

β is the poloidal angle $\beta(\vartheta,z) = \operatorname{arctg}\left(\frac{2az}{\vartheta^2+z^2-a^2}\right)$ of toroidal coord. (γ,β,φ).

Gauge of Gladikowski $Z_1 = \cos\frac{\Theta}{2}e^{i\alpha}$, $Z_2 = \sin\frac{\Theta}{2}e^{i(\varphi+\alpha)}$ есть особая, $\alpha(\vec{r}) = -\gamma(\vartheta,z) + \text{const}$, $\alpha(\vec{r})$ не влияет на $w = \operatorname{tg}\frac{\Theta}{2}e^{i\varphi} = \left(\frac{Z_1}{Z_2}\right)$, $\vec{m}$ и $\vec{B}$, но влияет на вектор потенциал $\vec{A}$.

Найдем A_z (цил.) $Z_1 = \frac{z\vartheta e^{i\varphi}}{(1+r^2)}$, $Z_2 = \frac{1}{(1+r^2)}[1-r^2+2iz]$

A_z: $Z_i^* \frac{\partial Z_i}{\partial z} = Z_1^* \frac{\partial Z_1}{\partial z} + Z_2^* \frac{\partial Z_2}{\partial z} = \frac{z\vartheta e^{i\varphi}}{(1+r^2)}\frac{\partial}{\partial z}\frac{z\vartheta e^{i\varphi}}{(1+r^2)} + \frac{(1-r^2-2iz)}{(1+r^2)} \times$

$\times \frac{\partial}{\partial z}\frac{(1-r^2+2iz)}{(1+r^2)} = \frac{(1-r^2-2iz)}{(1+r^2)}\frac{(-2z+2i\vartheta^2)(1+r^2)-(1-r^2+2iz)\cdot 2z}{(1+r^2)^2} =$

$= \frac{1}{(1+r^2)^3}\left[(1-r^2+2iz)\cdot\left\{-4z+2i+2ir^2-4iz^2\right\}\right] = \frac{1}{(r^2+1)^3}\left[2i(1+r^2+2z^2)(1-r^2)+8iz^2\right]$

$(1+r^2+2z^2)(1-r^2)+4z^2 = (1+r^2)(1-r^2)-2z^2(1-r^2)+4z^2 = (1+r^2)(1-r^2)+2z^2(1-\vartheta^2-z^2)+$

$+4z^2 = (1+r^2)(1-r^2)+2z^2(\vartheta^2+z^2)-2z^2+4z^2 = (1+r^2)(1-r^2)+2z^2(\vartheta^2+z^2+1)$

$Z_i^* \frac{\partial Z_i}{\partial z} = \frac{1\cdot 2i}{(1+r^2)^3}\left[(1+r^2)(1-r^2)+2z^2(r^2+1)\right] = \frac{2i}{(1+r^2)^2}\left[1-r^2+2z^2\right] = \frac{2i}{(1+r^2)^2}\left[1-\vartheta^2+z^2\right]$

$A_z = -2\operatorname{Im}\left(Z_i^* \frac{\partial Z_i}{\partial z}\right) = -2\operatorname{Im}\left[\frac{2i}{(1+r^2)^2}(1-\vartheta^2+z^2)\right] = -\frac{4(1-\vartheta^2+z^2)}{(1+r^2)^3} = A_z(\text{mee})$

$A_z(\text{Wang}) = -(1+m_z)\frac{\partial\beta}{\partial z} = -(1+m_z)\frac{\partial}{\partial z}\operatorname{arctg}\left(\frac{2z}{r^2-1}\right) = -(1+m_z)\frac{1}{1+\frac{4z^2}{(r^2-1)^2}} \times$

$\times \frac{2(r^2-1)-2z\cdot 2z}{(r^2-1)^2} = -(1+m_z)\frac{(r^2-1)^2}{(r^2-1)^2+4z^2}\frac{2(r^2-2z^2-1)}{(r^2-1)^2} =$

$= -2\left[1-\frac{4\vartheta^2}{(1+r^2)^2}\right]\frac{2(r^2-1-2z^2)}{\left[(r^2-1)^2+4z^2\right]} = -\frac{2}{(1+r^2)^2}\frac{\left[(1-\vartheta^2)^2+2z^2(\vartheta^2+1)+z^4\right]\cdot 2(\vartheta^2-1-z^2)}{\left[(r^2-1)^2+4z^2\right]} =$

$= -4\frac{(\vartheta^2-1-z^2)}{(1+r^2)^2} = 4\frac{(1-\vartheta^2+z^2)}{(1+r^2)^2}$, $\vec{A}_z(\text{mee}) = -A_z(\text{Wang})$.

$\vec{A}$ вектор потенциал

$\vec{A}$ (mee corrected): $A_\vartheta = -\frac{8z\vartheta}{(1+r^2)^2}$, $A_\varphi = -\frac{8\vartheta}{(1+r^2)^2}$, $A_z = -4\frac{(1-\vartheta^2+z^2)}{(1+r^2)^2}$ Hopf fibration $Q_H = 1$

$\vec{B} = \operatorname{rot}\vec{A}$, $B_\varphi = \frac{\partial A_\vartheta}{\partial z} - \frac{\partial A_z}{\partial\vartheta} = \frac{\partial}{\partial z}(-8)\frac{z\vartheta}{(1+r^2)^2} - \frac{\partial}{\partial\vartheta}(-4)\frac{(1-\vartheta^2+z^2)}{(1+r^2)^2} =$

$= -8\vartheta\frac{\partial}{\partial z}\frac{z}{(1+r^2)^2} + 4\frac{\partial}{\partial\vartheta}\frac{(1-\vartheta^2+z^2)}{(1+r^2)^2} = -\frac{8\vartheta}{(1+r^2)^4}\left[(1+r^2)^2 - z\cdot 2(1+r^2)\cdot 2z\right] + \frac{4}{(1+r^2)^4} \times$

$\times\left[-2\vartheta(1+r^2)^2 - (1-\vartheta^2+z^2)\cdot 2(1+r^2)2\vartheta\right] = -\frac{8\vartheta}{(1+r^2)^3}\left[1+r^2-4z^2\right] + \frac{4}{(1+r^2)^3}\left[-2\vartheta(1+r^2) - 4\vartheta(1-\vartheta^2+z^2)\right] =$

$= \frac{8\vartheta}{(1+r^2)^3}\left[-1-r^2+4z^2-1-r^2-2(1-\vartheta^2+z^2)\right] = \frac{8\vartheta}{(1+r^2)^3}\left[-2-2r^2+4z^2-2(1-r^2+2z^2)\right] =$

$= -\frac{32\vartheta}{(1+r^2)^3}$, $B_\varphi = -\frac{32a^3\vartheta}{(a^2+\vartheta^2+z^2)^3} = B_\varphi(\text{mee})$, $B_\varphi(\text{Wang}) = +\frac{32a^3\vartheta}{(a^2+r^2)^3}$

Верное $B_\varphi(\text{mee})$. т.к. $\vec{A}(\text{mee}) = -\vec{A}(\text{Wang})$ то и $\vec{B}(\text{mee}) = -\vec{B}(\text{Wang})$

$B_\varphi(\text{Glad}) = \frac{M}{a^2}r^2\frac{\partial w}{\partial\vartheta} = -\frac{4M}{a^2}r^2\frac{1}{1+r^2}$. $B_\varphi(\text{Glad}) < 0$ при $M > 0$, $M = 1$, то $B_\varphi(\text{mee})$.

В Popad' PRL'13 а здесь $M = -1$ $\Rightarrow$ $B_\varphi = B_\varphi(\text{Wang}) > 0$, $A_\varphi(\text{Wang}) > 0$, $A_\varphi(\text{mee}) = -A_\varphi(\text{Glad}) < 0$

Вывод: все получили hopf fibr $Q_H = \text{integer}$, надо Gladikowski-gauge $\alpha(\vec{r}) = -\beta(\vartheta,z)$ - poloidal angle

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(8/IV) Уеорма дома. Q-дом - Бълг. Hopfion vector potential $\vec{A}$

(9/IV) Получих 7/IV компоненты vector potential $\vec{A}$ toroidal hopfion $Q_H=1$ в toroidal (η,β,φ), cylindrical (ρ,φ,z) - coordinates и (Θ,Φ,α) предварителни в cylindr. coord.

Основа $\vec{m}=Z^+\vec{\sigma}Z$, $Z=\begin{pmatrix}z_1\\z_2\end{pmatrix}$ spinor, $w=\frac{z_1}{z_2}$, z_1,z_2 give $Q_H=1$

abno gauge Gutsche, Rs. 2007 $Z_1=\frac{2\rho}{1+r^2}e^{i\varphi}$, $Z_2=\frac{1}{(1+r^2)}\left[1-r^2+2iz\right]$. Eq. (8.4) Shnir '2018. $m_x+im_y=\frac{2w}{1+|w|^2}$. Dof. $\vec{A}=-2\,\mathrm{Im}\left(Z_i^*\nabla Z_i\right)$

Gladikowski gauge give общо (Θ,Φ,α) предварителни с $\vec{m}=\vec{m}(\Theta,\Phi)$

$\alpha(\vec{r})=-\beta(\vec{r})+\text{const}$, β - poloidal angle. $Z=\begin{pmatrix}z_1\\z_2\end{pmatrix}=\begin{pmatrix}\cos\frac{\Theta}{2}e^{i\alpha}\\ \sin\frac{\Theta}{2}e^{i(\Phi+\alpha)}\end{pmatrix}$

$w(\Theta,\Phi,\alpha)=\frac{z_2}{z_1}=\mathrm{tg}\frac{\Theta}{2}e^{i\Phi}$ не зависи от $\alpha(\vec{r})$

$\Phi=n\varphi+\gamma$, $\Phi+\alpha=n\varphi+(\gamma+\alpha)=n\varphi$, $z_1=\cos\frac{\Theta}{2}e^{i\alpha}=\cos\frac{\Theta}{2}e^{-i\gamma}$

$z_2=\sin\frac{\Theta}{2}e^{in\varphi}$, $w=\frac{z_2}{z_1}=\frac{\sin\frac{\Theta}{2}e^{in\varphi}}{\cos\frac{\Theta}{2}e^{-i\gamma}}=\mathrm{tg}\frac{\Theta}{2}e^{in\varphi}e^{i\gamma}$,

$\gamma=m\beta\to w(\eta,\beta,\varphi)=\mathrm{tg}\frac{\Theta}{2}e^{i(n\varphi+m\beta)}=F(\eta)e^{i(n\varphi+m\varphi)}$ - Glad. 97, Hietarinta 2000

Toroidal coordinates (η,β,φ) особени, т.к. $\Theta=\Theta(\eta)$ и $m_z=m_z(\eta)$. $w(\eta,\beta,\varphi)$ предава.

Дава $Q_H=1$, $m_z(\eta)=1-2\,\mathrm{th}^2\eta$. Въпр $m_z(\eta),\Theta(\eta)$ дава $Q_H>1$ невъзможен.

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A. Yershova et al. '2010 (wiki)

Hopf coordinates (Θ,Φ,Ψ), (Θ,Φ) are spherical angles on S^2. S_3 sphere by $\vec{X}$

$\vec{X}(x_1,x_2,x_3,x_4)$, $\sum_{i=1}^{4}x_i^2=1$, $x_1=\cos\frac{\Theta}{2}\cos\frac{\Psi}{2}$, $x_2=\cos\frac{\Theta}{2}\sin\frac{\Psi}{2}$, $x_3=\sin\frac{\Theta}{2}\cos(\Phi+\frac{\Psi}{2})$

$x_1+ix_2=\cos\frac{\Theta}{2}e^{i\Psi/2}$, $\frac{x_3}{+ix_4}=\sin\frac{\Theta}{2}e^{i(\Phi+\Psi/2)}$ $x_4=\sin\frac{\Theta}{2}\sin(\Phi+\frac{\Psi}{2})$

$\vec{m}=m_x+im_y=\sin\Theta e^{i\Phi}$, $m_z=\cos\Theta$, $w=\frac{x_3+ix_4}{x_1+ix_2}=\mathrm{tg}\frac{\Theta}{2}e^{i\Phi}$

С тождество за gauge $e^{i\Psi/2}$, $x_1+ix_2=\cos\frac{\Theta}{2}e^{i\Phi}$, $x_3+ix_4=\sin\frac{\Theta}{2}e^{i(\Phi+\Psi)}$ и т.н.

Заместване в (Θ,Φ,α) предварителни Shnir'18. $Z_1=\cos\frac{\Theta}{2}e^{i\alpha}$, $Z_2=\sin\frac{\Theta}{2}e^{i(\Phi+\alpha)}$

Ест. Jacobian $(\Theta,\Phi,\Psi)\to(x_1,x_2,x_3,x_4)$ transformation $J=\left(\frac{\partial x_i}{\partial\Theta},\frac{\partial x_i}{\partial\Phi},\frac{\partial x_i}{\partial\Psi}\right)$

M. Zamboj '2021 (wiki): Spinor components Z_1,Z_2, $Z=\begin{pmatrix}z_1\\z_2\end{pmatrix}$.

The Hopf fibration (mapping) $(m_x+im_y,m_z)=f(Z_1,Z_2)=\left(2Z_1Z_2^*,|Z_1|^2-|Z_2|^2\right)$

$Z_1=\cos\gamma e^{i\alpha}$, $Z_2=\sin\gamma e^{i\beta}$ дава $m_x+im_y=\sin2\gamma e^{i(\alpha-\beta)}$, $m_z=\cos2\gamma$.

Вадим $2\gamma=\Theta$, $\alpha-\beta=\Phi$, $m_x+im_y=\sin\Theta e^{i\Phi}$, $m_z=\cos\Theta$, $\vec{m}(\Theta,\Phi)$. (Θ,Φ,α) Shnir

$m_x+im_y=\frac{2w}{1+|w|^2}=\left|w=Z_1/Z_2\right|=\frac{2Z_1/Z_2}{1+|Z_1|^2/|Z_2|^2}=\frac{2Z_1|Z_2|^2}{Z_2}\cdot\frac{1}{|Z_1|^2+|Z_2|^2}$

$=\frac{2Z_1}{Z_2}Z_2Z_2^*=2Z_1Z_2^*$, т.е $m_x+im_y=2Z_1Z_2^*\cos\theta$. $w=\frac{Z_1}{Z_2}$.

$m_z=|Z_1|^2-|Z_2|^2=1-2|Z_2|^2$, $|Z_1|^2+|Z_2|^2=1$. $m_z=2|Z_1|^2-1$,

$Z_1=\mathrm{th}\eta e^{i\alpha}\to m_z=2\,\mathrm{th}^2\eta-1$ дава $Q_H=1$. $|Z_1|$ и $|Z_2|=f(\eta)$ только

in toroidal coordinates (η,β,φ). $Z_1=|Z_1|e^{in\varphi}$, $Z_2=|Z_2|e^{im\beta}$

дава correct $\vec{m}$ с $w=\frac{|Z_1|}{|Z_2|}e^{i(n\varphi+m\beta)}=f(\eta)e^{i(n\varphi+m\beta)}$ Glad. '97, Hietarinta 2000

$m_x+im_y=2Z_1Z_2^*=2(x_1+ix_2)(x_3-ix_4)=2\left[x_1x_3+x_2x_4+i(x_2x_3-x_1x_4)\right]$ $f(\eta)=|Z_1|/|Z_2|$.

$m_z=|Z_1|^2-|Z_2|=x_1^2+x_2^2-x_3^2-x_4^2$. $m_x+im_y=2Z_1Z_2^*$ са определени.

$f(\eta)=|Z_1|/|Z_2|$ невъзможна дава $Q_H>1$.

Escaneado con CamScanner

Y/He. H. Guo, Phys. Lett. B '2014: $\vec{m} = Z^+ \vec{\sigma} Z$, $Z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$ spinor

Sg (x_1, x_2, x_3, x_4). $z_1 = x_1 + i x_2$, $z_2 = x_3 + i x_4$.

Field tensor $F_{\mu\nu} = \varepsilon_{ijk} m_i \partial_\mu m_j \partial_\nu m_k = \vec{m} \cdot (\partial_\mu \vec{m} \times \partial_\nu \vec{m})$

$F_{\mu\nu}(z) = -2i \left[\partial_\mu z_i^* \partial_\nu z_i - \partial_\nu z_i^* \partial_\mu z_i \right] = -2i \left[\partial_\mu Z^+ \partial_\nu Z - \partial_\nu Z^+ \partial_\mu Z \right]$

Также $F_{\mu\nu}$ можно выразить через $F_{\mu\nu}(\vec{m})$ вводя $\vec{m}(x)$ и используя $\varepsilon^{abc}(\dots)$

$\varepsilon^{abc} \sigma^b_{ij} \sigma^c_{kl} = i \left[\sigma^a_{il} \delta_{jk} - \sigma^a_{kj} \delta_{il} \right]$ где Паули матрицы $\vec{\sigma} = (\sigma^x, \sigma^y, \sigma^z)$

Не: $A_\mu = -i \left[z_i^* \partial_\mu z_i - \partial_\mu z_i^* z_i \right] = -i \left[Z^+ \partial_\mu Z - \partial_\mu Z^+ Z \right] = 2 \operatorname{Im} \left(Z^+ \partial_\mu Z \right)$

A_μ (Аратьян) $= \frac{i}{2} \left[Z^+ \partial_\mu Z - \partial_\mu Z^+ Z \right] = -\operatorname{Im} \left(Z^+ \partial_\mu Z \right)$

A_μ (Гус) $= -2 \operatorname{Im} \left(Z^+ \partial_\mu Z \right)$, $\vec{A}$ (Гус) $= -2 \operatorname{Im} \left(Z^+ \nabla Z \right)$, отличается знаком от A_μ (Не). Подставим $A_\mu = c \operatorname{Im} \left(Z^+ \partial_\mu Z \right)$ в $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$F_{\mu\nu} = \partial_\mu c \operatorname{Im} Z^+ \partial_\nu Z - \partial_\nu c \operatorname{Im} \left(Z^+ \partial_\mu Z \right) = c \operatorname{Im} \left[\partial_\mu \left(Z^+ \partial_\nu Z \right) - \partial_\nu \left(Z^+ \partial_\mu Z \right) \right]$

$F_{\mu\nu} = c \operatorname{Im} \left[\partial_\mu Z^+ \partial_\nu Z + Z^+ \partial_\mu \partial_\nu Z - \partial_\nu Z^+ \partial_\mu Z - Z^+ \partial_\nu \partial_\mu Z \right] =$

$= c \operatorname{Im} \left[\partial_\mu Z^+ \partial_\nu Z - \partial_\nu Z^+ \partial_\mu Z \right] = \frac{c}{2i} \left[\partial_\mu Z^+ \partial_\nu Z - \partial_\nu Z^+ \partial_\mu Z - \left(\partial_\mu Z \partial_\nu Z^+ - \partial_\nu Z \partial_\mu Z^+ \right) \right] =$

$= \frac{c}{i} \left[\partial_\mu Z^+ \partial_\nu Z - \partial_\nu Z^+ \partial_\mu Z \right]$, $\frac{c}{i} = -2i$, $c = 2$ – как Не-знак.

Но $\frac{c}{i} = 2i$, $c = -2$ как Гус-знак, $F_{\mu\nu}$ (Гус) $= 2i \left[\partial_\mu Z^+ \partial_\nu Z - \partial_\nu Z^+ \partial_\mu Z \right]$

Берем A_μ (Гус) $= i \left[Z^+ \partial_\mu Z - \partial_\mu Z^+ Z \right] = -2 \operatorname{Im} \left[Z^+ \partial_\mu Z \right]$.

Если $A_\mu \to A_\mu + \frac{\partial g}{\partial \mu} = A_\mu + \partial_\mu g$, $F_{\mu\nu}(A'_\mu = A_\mu + \partial_\mu g) = \partial_\mu \left[A_\nu + \partial_\nu g \right] -$

$- \partial_\nu \left[A_\mu + \partial_\mu g \right] = F_{\mu\nu}(A_\mu) + \partial_\mu \partial_\nu g - \partial_\nu \partial_\mu g = F(A_\mu)$

$A_\mu \to A'_\mu + \partial_\mu g$, $\vec{A} \to \vec{A}' = \vec{A} + \nabla g$ сохраняет $F_{\mu\nu}$ и $\vec{B}$ defined via $F_{\mu\nu}$.

т.е. $\vec{A} = -2 \operatorname{Im} \left(Z^+ \nabla Z \right)$ определено с точностью до ∇g. У Гладковского и Wang-Bratkas'99 брали $\nabla g = 0$, используя $Z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$ прямо из Hopf map.

$F_{\mu\nu}$ (Аратьян) $= -\frac{2i}{(1+|w|^2)^2} \left[\partial_\mu w \partial_\nu w^* - \partial_\nu w \partial_\mu w^* \right]$, $\left| w = \frac{z_1}{z_2} \right|$

$\partial_\mu \left(z_1 / z_2 \right) \partial_\nu \left(z_1^* / z_2^* \right) - \partial_\nu \left(\frac{z_1}{z_2} \right) \left(z_1^* / z_2^* \right) = \frac{1}{z_2^2} \left[\partial_\mu z_1 z_2 + z_1 \partial_\mu z_2 \right]^* \times$

$\times \left[\partial_\mu z_1^* z_2^* - z_1^* \partial_\mu z_2^* \right] \frac{1}{(z_2^*)^2} - (\mu \to \nu)$, $1 + |w|^2 = 1 + \frac{|z_1|^2}{|z_2|^2} =$

$= \frac{1}{|z_2|^2} \left[|z_1|^2 + |z_2|^2 \right] = \frac{1}{|z_2|^2} \cdot \left[\partial_\mu z_1 z_2 - z_1 \partial_\mu z_2 \right] \left[\partial_\nu z_1^* z_2^* - z_1^* \partial_\nu z_2^* \right] =$

$= \partial_\mu z_1 \partial_\nu z_1^* |z_2|^2 - \partial_\mu z_1 \partial_\nu z_2^* z_2 z_1^* - \partial_\mu z_2 \partial_\nu z_1^* z_1 z_2^* + |z_1|^2 \partial_\mu z_2 \partial_\nu z_2^*$

$z_1 z_1^* + z_2 z_2^* = 1$, $\partial z_1 z_1^* + z_1 \partial z_1^* + \partial z_2 z_2^* + z_2 \partial z_2^* = 0$.

Если $\partial_\mu z_1 \partial_\nu z_2^*$ – вместо сокращающихся, то $[\mu][\nu] = 0$? Наверное.

Тогда $\left[\partial_\mu z_1 z_2 - z_1 \partial_\mu z_2 \right] \left[\partial_\nu z_1^* z_2^* - z_1^* \partial_\nu z_2^* \right] = \partial_\mu z_1 \partial_\nu z_1^* |z_2|^2 -$

$- \partial_\mu z_2 \partial_\nu z_1^* z_1 z_2^* - \partial_\mu z_1 \partial_\nu z_2^* z_1^* z_2 - |z_1|^2 \partial_\mu z_2 \partial_\nu z_2$ – такая путь больше $F_{\mu\nu}$ из $F_{\mu\nu}(w)$ и $w = z_1 / z_2$ прямоугольник. Вместо $\vec{m}(Z)$ подставим в $F_{\mu\nu}(\vec{m})$.

(10/IV) Гус: $\vec{A}_\mu = 2 \left(\partial_\mu \vec{n} \times \vec{n} \right)$, $n_x + i n_y = \sin\frac{\theta}{2} e^{i\varphi}$, $n_z = \cos\frac{\theta}{2}$. $A^e_\mu = \left(\vec{A}_\mu \cdot \vec{m} \right)$

Также A^e_μ дает $A^e_\mu = (1 - \cos\theta) \partial_\mu \varphi$ natural gauge, т.к. $\vec{n}(\theta, \varphi)$, $\vec{m}(\theta, \varphi)$

Emergent mag. field $B_\lambda = \frac{1}{2} \varepsilon_{\lambda\mu\nu} F_{\mu\nu}$, $A_\lambda = -2 \operatorname{Im} \left[Z^+ \partial_\lambda Z \right]$, $F_{\mu\nu}(Z)$.

Можно ввести инвариант в spinor representation $Q_H(Z) = -\frac{1}{(4\pi)^2} \int dV A_\alpha B_\alpha$

$Q_H(Z) = -\frac{1}{(4\pi)^2} \int dV \, i \left[Z^+ \partial_\lambda Z - \partial_\lambda Z^+ Z \right] \frac{1}{2} \varepsilon_{\lambda\mu\nu} (2i) \left[\partial_\mu Z^+ \partial_\nu Z - \partial_\nu Z^+ \partial_\mu Z \right]$

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$$Q_H(Z) = \frac{1}{(4\pi)^2} \varepsilon_{\mu\nu\lambda} \int dV \left[Z^+ \partial_\mu Z - \partial_\mu Z^+ Z \right] \left[\partial_\nu Z^+ \partial_\lambda Z - \partial_\lambda Z^+ \partial_\nu Z \right], \quad \partial_\mu = \frac{\partial}{\partial x_\mu} \quad (6)_\mu$$

6/IX 2020 : Hopf index in toroidal coordinates [Faddeev-Niemi '97]

$$Q_H = -\frac{1}{(4\pi)^2} \int dV \vec{A}\cdot\vec{B} = -\frac{1}{(4\pi)^2} \int dV (-2) \frac{NM\, \tau^3}{a^3 \sinh\gamma} \frac{\partial w}{\partial \gamma} = \frac{2NM}{(4\pi)^2} \int dV \frac{\tau^3}{a^3 \sinh\gamma} \frac{\partial w}{\partial \gamma}$$

$$dV = \frac{a^3}{\tau^3} \sinh\gamma \, d\gamma d\theta d\varphi, \quad Q_H = \frac{2NM}{(4\pi)^2} \int_0^\infty d\gamma \int_0^{2\pi} d\theta \int_{-\pi}^{\pi} d\varphi \, \frac{a^3}{\tau^3} \sinh\gamma \cdot \frac{\tau^3}{a^3 \sinh\gamma} \frac{\partial w}{\partial \gamma} = \frac{2NM}{(4\pi)^2} (2\pi)(2\pi) \int_0^\infty d\gamma \frac{\partial w}{\partial \gamma}$$

$$Q_H = \frac{1}{2} NM \int_0^\infty d\gamma \frac{\partial w}{\partial \gamma} = \frac{1}{2} NM \left[w(\infty) - w(0) \right] = \frac{1}{2} NM \left[m_z(\infty) - m_z(0) \right]$$

$m_z(\gamma \to \infty) = -1$ (т.к. $\rho = a$), $m_z(\gamma \to 0) = 1$ By Hopf fibration definition.

$Q_H = \frac{1}{2} NM [(-1) - 1] = -NM$, integer. $Q_H = \frac{2NM}{(4\pi)^2} \int d\gamma \int d\theta \int d\varphi \frac{\partial w}{\partial \gamma}$

Сомасно 17/III $\vec{A} = -2\nabla\chi + \vec{A}^e$, $\vec{A}^e = (1-\cos\theta)\nabla\varphi$, helicity χ из $\Phi(\vec{r}) = n\varphi + \chi(\rho,z)$

$\vec{A}\cdot\vec{B} = (-2\nabla\chi + \vec{A}^e) \sin\theta [\nabla\theta \times \nabla\varphi] = -2\sin\theta \nabla\chi \cdot [\nabla\theta \times \nabla\varphi]$ — Hopf charge density

$\chi(\rho,z) = N\beta(\rho,z)$, χ в Φ можно интерпретировать — не дает вклада в $\vec{A}\cdot\vec{B}$.

$\beta(\rho,z) = \operatorname{arctg}\left(\frac{2az}{\rho^2+z^2-a^2}\right)$, $\operatorname{th}\gamma = \frac{2\rho a}{(z^2+\rho^2+a^2)}$ for any N,M, это нормально ли

Т.к. тангенс $\beta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, а надо $\beta \in [-\pi, \pi]$ [coord. $(\gamma,\beta,\varphi) \to$ cyl. (ρ,φ,z)]

$$2\int d\gamma \int d(\cos\theta) \int d\Phi = 2 \int_{-\pi}^{\pi} d(N\beta) \int d(\cos\theta) \int d(N\varphi) = 2MN \int_{-\pi}^{\pi} d\beta \int d(\cos\theta) \int_0^{2\pi} d\varphi =$$

$$= 2MN \cdot (2\pi)(2\pi) \int d(\cos\theta) \Big|_{-\pi}^{\pi} = 8\pi^2 MN \left[\cos\theta(\infty) - \cos\theta(0) \right] = 8\pi^2 MN (-2) = -16\pi^2 MN$$

$$\frac{2}{(4\pi)^2} \int d\gamma \int d(\cos\theta) \int d\Phi = \frac{1}{(4\pi)^2} (-16)\pi^2 MN = -MN = Q_H \qquad (\gamma=\infty \to \gamma=0)$$

$$\vec{A}\cdot\vec{B} = -2\sin\theta \nabla\chi \cdot [\nabla\theta \times \nabla\varphi] = -2\frac{N}{\rho} \sin\theta \left[\frac{\partial\chi}{\partial z}\frac{\partial\theta}{\partial\rho} - \frac{\partial\chi}{\partial\rho}\frac{\partial\theta}{\partial z} \right]$$ — Hopf density cyl. coord.

$$Q_H = -\frac{1}{(4\pi)^2} \int dV \vec{A}\cdot\vec{B} = \frac{2NM}{(4\pi)^2} \int d\rho\rho \int d\varphi \int dz \frac{1}{\rho} \sin\theta \left[\frac{\partial\beta}{\partial z}\frac{\partial\theta}{\partial\rho} - \frac{\partial\beta}{\partial\rho}\frac{\partial\theta}{\partial z} \right]$$

RRL: Kyg-Sys-23

$$\int d\rho\rho \int d\varphi \int dz \, J(\chi,\theta,\Phi)_{\rho,\varphi,z} = \int d\chi \int d\theta \sin\theta \int d\Phi$$

$$J(\chi,\theta,\Phi;\rho,\varphi,z) = \begin{pmatrix} \frac{\partial\rho}{\partial\chi} & \frac{\partial\rho}{\partial\theta} & \frac{\partial\rho}{\partial\Phi} \\ \frac{\partial\varphi}{\partial\chi} & \frac{\partial\varphi}{\partial\theta} & \frac{\partial\varphi}{\partial\Phi} \\ \frac{\partial z}{\partial\chi} & \frac{\partial z}{\partial\theta} & \frac{\partial z}{\partial\Phi} \end{pmatrix} = \begin{pmatrix} \frac{\partial\rho}{\partial\chi} & 0 & \frac{\partial\rho}{\partial\Phi} \\ \frac{\partial\varphi}{\partial\chi} & 0 & \frac{\partial\varphi}{\partial\Phi} \\ \frac{\partial z}{\partial\chi} & \frac{1}{N}\frac{\partial z}{\partial\theta} & \frac{\partial z}{\partial\Phi} \end{pmatrix}$$

$$\det J = \frac{\partial\rho}{\partial\chi}\frac{1}{N}\frac{\partial z}{\partial\theta} - \frac{\partial\rho}{\partial\theta}\frac{1}{N}\frac{\partial z}{\partial\chi} = \left| \frac{\partial\rho}{\partial\theta} = N, \operatorname{tg}\chi \right| = \frac{N}{\rho}\left[\frac{\partial\rho}{\partial z}\frac{\partial\theta}{\partial\rho} - \frac{\partial\rho}{\partial\rho}\frac{\partial\theta}{\partial z} \right]$$

$$Q_H = -\frac{2}{(4\pi)^2} \int d\chi \int d\theta \sin\theta \int d\Phi = -\frac{2}{(4\pi)^2} \int dV_\rho \, J(\chi,\theta,\Phi)_{z,\varphi,\rho} = -\frac{2}{(4\pi)^2} \int d\rho\rho \int d\varphi \int dz \times$$

$$\times \frac{N}{\rho}\left[\frac{\partial\chi}{\partial z}\frac{\partial\theta}{\partial\rho} - \frac{\partial\chi}{\partial\rho}\frac{\partial\theta}{\partial z} \right] = -\frac{e\cdot 2\pi N}{(4\pi)^2} \int d\rho\rho \int dz \frac{1}{\rho}\left[\frac{\partial\chi}{\partial z}\frac{\partial\theta}{\partial\rho} - \frac{\partial\chi}{\partial\rho}\frac{\partial\theta}{\partial z} \right] =$$

$$= -\frac{N}{4\pi} \int d\rho\, dz \left[\frac{\partial\chi}{\partial z}\frac{\partial\theta}{\partial\rho} - \frac{\partial\chi}{\partial\rho}\frac{\partial\theta}{\partial z} \right] = Q_H(\text{wang})$$ — можно ещё $\chi = N\beta + \text{const}$

Вывод: $\int d\chi d\theta \sin\theta d\Phi = \int d\rho\rho \int d\varphi \int dz \det J(\chi,\theta,\Phi;\rho,\varphi,z)$, т.е. Hopf charge density $\vec{A}\cdot\vec{B} = -\frac{2N}{\rho}(\cdot)$ в Jacobian перехода от (ρ,φ,z) cyl. к (χ,θ,Φ) coordinates.

Точнее надо ввести $m_z = \cos\theta$ и считать $\det J(\chi, m_z, \Phi; \rho,\varphi,z)$ Jacobian.

$$\det J(\chi, m_z, \Phi; \rho,\varphi,z) = -\sin\theta \frac{N}{\rho}\left[\frac{\partial\chi}{\partial z}\frac{\partial\theta}{\partial\rho} - \frac{\partial\chi}{\partial\rho}\frac{\partial\theta}{\partial z} \right]$$

$$\int d\chi \int dm_z \int d\Phi = \int d\rho\rho \int d\varphi \int dz \det J(\chi, m_z, \Phi; \rho,\varphi,z) = \int d\rho\rho \int d\varphi \int dz \det J(\chi, m_z, \Phi; \rho,\varphi,z)$$

→ Hopf charge density $\vec{A}\cdot\vec{B} = -2\sin\theta \nabla\chi [\nabla\theta \times \nabla\Phi] = -2\nabla\chi [\nabla m_z \times \nabla\Phi] = 2\det J(\chi, m_z, \Phi; \rho,z)$

Т.е. helicity $\chi(\rho,z) = N\beta(\rho,z)$ имеет еще одну интерпретацию.

Escaneado con CamScanner

Якобиан (m₂, β, φ) тороид. переход к (ρ, φ, z) цилиндр. координаты

$$J\begin{pmatrix} m_2,\beta,\varphi \\ \rho,\varphi,z \end{pmatrix} = \begin{pmatrix} \frac{\partial m_2}{\partial \rho} & \frac{\partial m_2}{\partial \varphi}\frac{1}{\rho} & \frac{\partial m_2}{\partial z} \\ \frac{\partial \beta}{\partial \rho} & \frac{\partial \beta}{\partial \varphi}\frac{1}{\rho} & \frac{\partial \beta}{\partial z} \\ \frac{\partial \varphi}{\partial \rho} & \frac{1}{\rho}\frac{\partial \varphi}{\partial \varphi} & \frac{\partial \varphi}{\partial z} \end{pmatrix} = \begin{pmatrix} \frac{\partial m_2}{\partial \rho} & 0 & \frac{\partial m_2}{\partial z} \\ \frac{\partial \beta}{\partial \rho} & 0 & \frac{\partial \beta}{\partial z} \\ 0 & \frac{1}{\rho} & 0 \end{pmatrix}$$

$\rho = \frac{\sin\eta}{\cos\xi}$, $z = \frac{\sin\beta}{\cos\xi}$, $\varphi = \varphi$

$\eta(\rho,z)$, $\beta(\rho,z)$

$$\det J(m_2,\beta,\varphi;\rho\varphi z) = -\frac{1}{\rho}\frac{\partial\beta}{\partial\rho}\frac{\partial m_2}{\partial z} - \frac{1}{\rho}\frac{\partial\beta}{\partial z}\frac{\partial m_2}{\partial\rho} = \frac{1}{\rho}\left[\frac{\partial\beta}{\partial\rho}\frac{\partial m_2}{\partial z} - \frac{\partial\beta}{\partial z}\frac{\partial m_2}{\partial\rho}\right]$$

$Q_H \sim \int dx\,dy\,dz\,\frac{\partial m_2}{\partial y} = \int d(m_2)\int d\beta\int d\varphi = \int d\rho\rho\int d\varphi\int dz\,\frac{1}{\rho}\left[\frac{\partial\beta}{\partial\rho}\frac{\partial m_2}{\partial z} - \frac{\partial\beta}{\partial z}\frac{\partial m_2}{\partial\rho}\right] =$

$= \int d\rho\rho\int d\varphi\int dz\,\det J(m_2,\beta,\varphi;\rho\varphi z)$, т.е. Q_H density $\vec{A}\cdot\vec{B} \sim \det J(m_2,\beta,\varphi;\rho\varphi z)$

$\sim$ Якобиан $(\rho,\varphi,z) \to (\eta,\beta,\varphi)$ тороидальное (цил.) преобразование координат.

Неудивительно. Всегда (β, m_2, Φ) и (β, m_2, φ) совпадают $\Phi = M\beta + c$ $\Leftrightarrow = N\varphi$.

Оплачено CA. Portugal toll card (20 €, saldo 19/IV 23 = 24,39 €)
Начислено оплатить Hyundai-i30 и Toyota RAV4 car taxes by „MiPago"
(deadline was Apr. 3, 2023). Забыл в среду.
(14/IV) 10. Оплачена Car Taxes в Ayuntamiento Izurza со штрафом (9 €)

Named "Toroidal Hopfion gauge" monograph.
Ссылка S.S. Pershoguba, D. Andreoli, J. Zang. PRB 104, 075102 (2021)
Electrons scattering off a magnetic hopfion. $\langle\vec{B}(\vec{r})\rangle_m = 0$, поэтому
начинаем toroidal moment $\vec{T} = \frac{1}{2}\int dV\,\vec{r}\times\vec{B}$, есть (skyrmion) Hall effect.
$\langle\vec{B}\rangle_m = 0$, но topol. Hall effect есть. Это проявление topological
суперпозиции что the Hall effect из-за $\langle\vec{B}\rangle \sim \vec{G} \neq 0$.
Итак $\vec{B} = (B_r, B_\theta, B_\varphi)$ in $\vec{r}(r,\theta,\varphi)$ spherical coordinates. $\langle B_r\rangle \neq 0$, $\langle B_\theta\rangle \neq 0$
Согласно Lin-Nagaosa PRL'22 toroidal moment $\frac{1}{2}\int d^3\vec{r}\,\vec{r}\times\vec{B} = \int dV\vec{A} = \frac{\langle B_\varphi\rangle}{1} \neq 0$
Yao Nagaosa определяет $\vec{A} = \frac{1}{2}(1-\cos\theta)\nabla\Phi$.
У меня для hopfion $|Q_H| = 1$ $\vec{A}(A_\rho, A_\varphi, A_z)$, $A_\rho = -\frac{8z\rho}{(1+r^2)^2}$, $A_\varphi = -\frac{8\rho}{(1+r^2)^2}$, A_z
$A_z = (-4)\frac{(1-\rho^2+z^2)}{(1+r^2)^2}$, ρ, z in units of „a".
$T_\rho = 0$, $T_\varphi \neq 0$, T_z надо считать, $T_\varphi = f(R)$ где cyl.-dist.
Sugic 2021 Вернулся к „Sugic et al. Nat. Com. 2021 (5/III, 12/III).
El. field transverse components $\vec{E} = E_R\hat{e}_R + E_L e_L$, $\hat{e}_{R,L} = \frac{1}{\sqrt{2}}(\hat{x} \pm i\hat{y})$.
$E_{R,L} = E'_{R,L} + iE''_{R,L}$, $|E_R|^2 + |E_L|^2 = 1$, $E_R = \cos\frac{\beta}{2}e^{\frac{i}{2}(\gamma-\delta)}$, $E_L = \sin\frac{\beta}{2}e^{\frac{i}{2}(\gamma+\delta)}$
Stokes vector $\vec{S}$ (analogue to $\vec{m}$) $\vec{S} = \vec{E}^+\vec{\sigma}\vec{E}$, $\vec{m} = Z^+\vec{\sigma}Z$
$Z = \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}$ spinor, $\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \sin\beta/2\, e^{\frac{i}{2}(\alpha+\gamma)} \\ \cos\beta/2\, e^{i/2(\gamma-\alpha)} \end{pmatrix}$ сравн. с Shnirz $\begin{pmatrix} \sin\frac{\Theta}{2}e^{i(\Phi+\alpha)} \\ \cos\frac{\Theta}{2}e^{i\gamma} \end{pmatrix}$
$w = \frac{Z_1}{Z_2} = \operatorname{tg}\beta/2\, e^{i\alpha} \sim w(\text{Shnirz}) = \operatorname{tg}\frac{\Theta}{2}e^{i\Phi}$. $\Theta = \beta$, $\alpha = \Phi$, $\vec{m}(\Theta,\Phi)$.
$\begin{pmatrix} \sin\Theta/2\, e^{i(\Phi+\delta)/2} \\ \cos\Theta/2\, e^{-i(\Phi-\delta)/2} \end{pmatrix}$ отличие от Shnirz $\times e^{i(\Phi+\delta)/2} \to \begin{pmatrix} \sin\frac{\Theta}{2}e^{i(\Phi+\delta)} \\ \cos\frac{\Theta}{2}e^{i\delta} \end{pmatrix}$
$\delta(\text{Sug}) = \alpha(\text{Shnirz})$. $Z_1 = E_L$, $Z_2 = E_R e^{i\delta}$ сходно.
S_3 to R^3 projection $n_i = \frac{2x_i}{1+r^2}$, $n_w = \frac{1+r^2}{1+r^2}$, $r^2 = \sum_i x_i^2$, $i = 1,2,3$ $R^3(x_1,x_2,x_3)$
$U = n_w + in_z = \frac{1-r^2+2iz}{1+r^2}$, $V = n_x + in_y = \frac{2(x+iy)}{(1+r^2)}$ $S_3(n_i, n_w)$
Shnirz для $Q_H = 1$ $Z_1 = \frac{2(x+iy)}{(1+r^2)}$, $Z_2 = Z_0 = \frac{(1+r^2+2iz)}{(1+r^2)}$
(1, IV)
следовательно $Z_2 = U$, $Z_1 = V$, $w = Z_1/Z_2$. $X_i \to x_i$ вместо (X,Y,Z)

Escaneado con CamScanner

S_3 $n_w^2+n_x^2+n_y^2+n_z^2=1$. $Z_1=n_y+in_y=\sin\omega\sin\ell\, e^{i\tilde\varphi}$, $Z_2=n_w+in_z=\cos\omega+i\sin\omega\cos\ell$

Hypersheare S_3 angles $(\omega,\ell,\tilde\varphi)$. Toroid. coordinates $(\tau,\beta,\tilde\varphi)\to(\gamma,\beta,\varphi)$

$\alpha(S_{ug})=\Phi$, $\beta(S_{ug})=\theta$. $\tilde\varphi=\operatorname{arctg}\frac{n_y}{n_x}=\arg Z_1$, $\beta=\operatorname{arctg}\left(\frac{2i\tau}{\tau^2-1}\right)=\arg Z_2$

$\arg Z_2=\arg F_R=\frac12(\gamma-\varphi)$, $\arg Z_1=\frac12(\gamma+\varphi)$. $z\to Z$.

У меня полоидальный угол $\beta=\operatorname{arctg}\left(\frac{2z}{r^2-1}\right)$ в γ^4 единицах. Это не тороидальный.

$(\operatorname{th}\tau)\to\operatorname{ch}\gamma=\frac{r^2+1}{\sqrt{(r^2+1)^2\dots}}=\frac{r^2+1}{\sqrt{r^4+1-2(s^2-z^2)}}=|u|^{-1}=\frac{1}{\cos\frac\theta2}=\sqrt{\frac{2}{1+\cos\theta}}$

$u=Z_2=\cos\omega+i\sin\omega\cos\ell$, $|u|=\sqrt{\cos^2\omega+\sin^2\omega\cos^2\ell}$

У меня $\operatorname{th}\gamma=\frac{2s}{(r^2+1)}$, $\operatorname{th}^2\gamma+\frac{1}{\operatorname{ch}^2\gamma}=1$, $\frac{1}{\operatorname{ch}^2\gamma}=1-\operatorname{th}^2\gamma=1-\frac{4s^2}{(r^2+1)^2}=$

$=\frac{1}{(r^2+1)^2}\left[(r^2+1)^2-4s^2\right]=\frac{1}{(s^2+z^2)^2}\left[(s^2+z^2)^2-4s^2\right]=\frac{1}{(r^2+1)^2}\times$

$\times\left[(s^2+1)^2+2z^2(s^2+1)+z^4-4s^2\right]=\frac{1}{(r^2+1)^2}\left[(s^2-1)^2+2z^2(s^2+1)+z^4\right]$

$(r^4)+1)^2=2s^2+2r^2=r^4+1-2s^2+2z^2=(s^2+z^2)^2+1-2s^2+2z^2=$

$=s^4+2z^2s^2+z^4+1-2s^2+2z^2$

$[\dots]=s^4+1-2s^2+2z^2s^2+2s^2+z^4$ — не сокращается.

Eq. Sugic (s,z) неверное? Ch. 21(III), $\operatorname{ch}\gamma=\frac12\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$, $x=\frac{z^2+(s+a)^2}{z^2+(s-a)^2}$

$\operatorname{ch}\gamma=\frac{1}{2\sqrt{x}}(x+1)$, $\operatorname{th}\gamma=\frac{x-1}{x+1}$, $\operatorname{th}^2\gamma+\frac{1}{\operatorname{ch}^2\gamma}=\frac{(x-1)^2}{(x+1)^2}+\frac{4x}{(x+1)^2}=\frac{(x+1)^2}{(x+1)^2}=1$

$\operatorname{th}\gamma=\frac{z^2+(s+a)^2-[z^2+(s-a)^2]}{z^2+(s+a)^2+z^2+(s-a)^2}=\frac{4sa}{2z^2+2s^2+2a^2}=\frac{2sa}{(z^2+s^2+a^2)}$ — верно.

$\operatorname{ch}\gamma=\sqrt{\frac{2}{1+\cos\theta}}$, $\cos\theta=w_z=1-2\operatorname{th}^2\gamma$ — верно для $Q_H=1$.

Eqs. Suppl. Sugic замечает, что W,X,Y,Z — cartesian coordinates in 4-dimensional space to describe S_3 sphere. $\operatorname{ch}(X,Y,Z)$ — Sugic номер более верный, т.к. $x\neq X$, $y\neq Y$, $z\neq Z$. Сверх $x_i\subset X_i$ не ясно.

Real 3D space coordinates (x,y,z) given $\bar u=\frac{r^2-1+2iz}{r^2+1}$, $v=\frac{2(x+iy)}{r^2+1}$

Sugic: $\operatorname{Re}\bar u=-\operatorname{Im}u$, $\operatorname{Re}u=\frac{r^2-1}{r^2+1}$, $\operatorname{Im}u=\frac{2y}{1+R^2}$, $R^2=X^2+Y^2+Z^2$

$z^2=x^2+y^2+z^2$

В каком сверх $(x,y,z)\in(X,Y,Z)$? (у Sugic'2011). Suppl. 2.

$u(s,\varphi,z)=\frac{s^2+z^2-1+2iz}{s^2+z^2+1}$, $v(s,\varphi,z)=\frac{2se^{i\varphi}}{s^2+z^2+1}$

$F_R=\frac{s^2+z^2-1+2iz}{s^2+z^2+1}$, $F_L=\frac{2se^{i\varphi}}{s^2+z^2+1}$ in real space cyl. coord. (s,φ,z).

Toroidal coordinates Sugic $(\tau,\beta,\tilde\varphi)\to(\gamma,\beta,\tilde\varphi)$ are in (X,Y,Z) space.

$\operatorname{ch}\gamma$ (Sugic) $=f(X,Y,Z)$ и сам параметр γ is in (X,Y,Z) space (?).

Poloid. β (Sugic) $=\operatorname{arctg}\left(\frac{2iZ}{R^2-1}\right)$. Real space poloid. $\beta=\operatorname{arctg}\left(\frac{2z}{r^2-1}\right)$

$\operatorname{ch}\gamma$ (Sugic) $=\frac{R^2+1}{\sqrt{R^4+1-2(X^2+Y^2-Z^2)}}$, Real space $\operatorname{ch}\gamma$ пр. ур. $\operatorname{th}\gamma=\frac{2s}{(s^2+1)}$

$R^2=X^2+Y^2+Z^2$ $r^2=x^2+y^2+z^2$.

$\tilde\varphi$ (Sugic) $=\operatorname{arctg}\frac{Y}{X}$, real space $\varphi=\operatorname{arctg}\frac{y}{x}$

$m_2=1-2\operatorname{th}^2\gamma$ — инвариантно и дано γ (Sugic) и γ (real space) тороид. параметры

Два вихрика (n,m) Sugic дает $F_R=Z_2=\bar u^n$, $F_L=Z_1=v^m$

$Z_2=\frac{1}{\operatorname{ch}^n\gamma}e^{in\beta}$, $Z_1=\operatorname{th}^m\gamma\, e^{im\varphi}$, (s,φ,z) real space cyl. $\leftrightarrow$ тороид. координаты $(\gamma,\beta,\tilde\varphi)$.

γ (Sugic) $=m\varphi+n\beta$, $\alpha(S_{ug})=\Phi\cdot m-n\beta$. $\cos\beta(S_{ug})=\cos\theta=\frac{\operatorname{sch}^{2n}\gamma-\operatorname{th}^{2n}\gamma}{\operatorname{sch}^{2n}\gamma+\operatorname{th}^{2n}\gamma}$

$m_2(\gamma)=\frac{1-\operatorname{th}^{2n}\gamma}{1+\operatorname{th}^{2n}\gamma}$ задает число полоидальных вихрей? (?)

Escaneado con CamScanner

Z_1, Z_2 (Sugic) бинуси к моим $Z_1 = th\gamma e^{i\varepsilon}$, $Z_2 = -\frac{1}{ch\gamma} e^{-i\beta}$ для $Q_H = 1$.

$Z_1 = v^m$, $Z_2 = u^n$. $v = \frac{2\xi e^{i\varphi}}{\xi^2+\tau^2+1} = th\gamma e^{i\varepsilon}$, $th\gamma = \frac{2\xi}{\tau^2+1}$, v – верно.

$u = \frac{\tau^2-1+2i\tau}{\tau^2+1} = \frac{\tau^2-1}{\tau^2+1}\left[1+\frac{2i\tau}{\tau^2-1}\right] = \frac{\tau^2-1}{\tau^2+1}\left[1+i\frac{1}{\xi}\beta\right] = \frac{1}{\cos\beta}\left(\frac{\tau^2-1}{\tau^2+1}\right)e^{i\beta}$

$\frac{\tau^2-1}{\tau^2+1} = \left|\xi = \frac{1}{\tau}sh\gamma,\ \tau = \frac{1}{\tau}\sin\beta\right| = \frac{\frac{1}{\tau^2}(sh^2\gamma+\sin^2\beta)-1}{\frac{1}{\tau^2}(sh^2\gamma+\sin^2\beta)+1} =$

$\tau^2 = ch\gamma - \cos\beta$

$= \frac{sh^2\gamma+\sin^2\beta-(ch\gamma-\cos\beta)^2}{sh^2\gamma+\sin^2\beta+(ch\gamma-\cos\beta)^2} = \frac{sh^2\gamma+\sin^2\beta-ch^2\gamma-\cos^2\beta+2ch\gamma\cos\beta}{sh^2\gamma+\sin^2\beta+ch^2\gamma+\cos^2\beta-2ch\gamma\cos\beta} =$

$= \frac{-1+\cos^2\beta+2ch\gamma\cos\beta}{2ch^2\gamma-1+1-2ch\gamma\cos\beta} = \frac{1}{2ch\gamma}\frac{-2\cos^2\beta+2ch\gamma\cos\beta}{(ch\gamma-\cos\beta)} = \frac{\cos\beta}{ch\gamma}\frac{(ch\gamma-\cos\beta)}{(ch\gamma-\cos\beta)} = \frac{\cos\beta}{ch\gamma}$

$u = \frac{1}{\cos\beta}e^{i\beta}\frac{\cos\beta}{ch\gamma} = \frac{1}{ch\gamma}e^{i\beta}$, $u^n = \frac{1}{ch^n\gamma}e^{in\beta}$ – верно. Для m,n разные

$w = \frac{Z_1}{Z_2} = \frac{th^m\gamma e^{im\varepsilon}}{\left(\frac{1}{ch\gamma}\right)^n e^{in\beta}} = ch^n\gamma \cdot th^m\gamma\, e^{(im\varepsilon - in\beta)}$

$m_z = \frac{1-|w|^2}{1+|w|^2} = \frac{1-ch^{2n}\gamma th^{2m}\gamma}{1+ch^{2n}\gamma th^{2m}\gamma}$, Sugic bzen $m_z = \frac{1-sh^{2n}\gamma}{1+sh^{2n}\gamma}$ u/m $m=n$,

Сделал баг, $\neq 0$ m_z for any (m,n).

Хотя $w = \frac{Z_1^n}{Z_2^m}$ возможно для axially symmetric hopfions

согласно Sutcliffe, PRS, 2007 и Shnir 2018, Z_1, Z_2 зависят от (m,n).

$Z_1 = \frac{\xi}{\tau}\sin f(\tau)\cdot e^{i\varepsilon}$, $Z_2 = \cos f + i\frac{z}{\tau}\sin f$, $f(\tau)$, $\tau^2 = x^2+y^2+z^2$, $f(0)=\pi$, $f(\infty)=0$.

Баг $f(\infty) = f_{mn}(\tau)$. Для $m=1, n=1$, $f_{11}(\tau) = \frac{2\tau}{(1-\tau^2)}$, $\sin f = \frac{2\tau}{1+\tau^2}$, $\cos f = \frac{1-\tau^2}{1+\tau^2}$

Тут верно для $m=1, n=1$ и получаю $Z_1 = th\gamma e^{i\varepsilon}$, $Z_2 = -\frac{1}{ch\gamma}e^{-i\beta}$

Взять Z_1, Z_2 при $m=1, n=1$ и определить для (m,n) any

$Z_1 = \left[Z_1(1,1)\right]^m$, $Z_2 = \left[Z_2(1,1)\right]^n$ как Sugic 2011 Needs. $Z_{1,2}(m,n)$

и баг $Z_1(m,n)$, $Z_2(m,n)$ неизвестен. Eqs. (5.4.18) – Sugic нет f_{mn} для Z_1, Z_2.

$Z_1 = \frac{\xi}{\tau}\sin f\cdot e^{i\varepsilon}$, $Z_2 = \cos f + i\frac{z}{\tau}\sin f$, $f = f_{mn}(\tau)$, согласно Sutcliffe, 2007.

Sugic 2011: $Z_1 = F_L = \sin\omega\cdot\sin\theta\, e^{i\varepsilon}$, $Z_2 = F_R = \cos\omega + i\sin\omega\cos\theta$ совсем иначе

баг Z_1 (Sutcliffe) $= \sin f\cdot\sin\theta\, e^{i\varphi}$, $Z_2 = \cos f + i\cos\theta\sin f$ (Sutcliffe, 2007).

Но у Sutcliffe (τ,θ,φ) real space spherical coordinates, а Sugic $(\omega,\varepsilon,\psi)$ are angles of S_3 hypersphere. Но $\omega = f(\tau)$ интересно.

→ Найти $\vec{A}$ for any (n,m) и сравнить с $\vec{A}$ (Glad.) и $\vec{A}$ (Wang) – тороид. сферич. coordinates.

Sugic: $\chi(Sug) = m\varphi + n\beta$, $\Phi = m\varphi - n\beta$ в (γ,β,φ) тороид. coordinates

Параметрически $\Phi(\tau) = m\varphi - n\beta(\xi,\tau)$ и я попытаюсь найти решение $\vec{B}, \vec{A}$ и Q_H.

Для $(m=n=1)$ $Z_1(Sug) = \sin\frac{\theta}{2}e^{\frac{i}{2}(\Phi+\chi)} = \sin\frac{\theta}{2}e^{im\varphi} = \sin\frac{\theta}{2}e^{i\varphi}$, сом.

$\Rightarrow Z_1 = v = th\gamma e^{i\varepsilon}$ (Sugic), $\cos\theta = 1-2\sin^2\frac{\theta}{2} = 1-2th^2\gamma$, $th\gamma = \sin\frac{\theta}{2}$.

$Z_2 = \cos\frac{\theta}{2}e^{i/2(\chi-\Phi)} = \cos\frac{\theta}{2}e^{i\beta}$, совпадение с $Z_2 = u = \frac{1}{ch\gamma}e^{i\beta}$ (Sugic),

т.е. $Z_1(Sug) = \sin\frac{\theta}{2}e^{i\varphi}$, $Z_2(Sug) = \frac{1}{ch\gamma}e^{i\beta} = \cos\frac{\theta}{2}e^{i\beta}$ иначе баг (θ,Φ,χ)

For any (m,n) $Z_1 = F_{mn}(\gamma)e^{im\varphi}$, $Z_2 = G_{mn}(\gamma)e^{in\beta}$, $|Z_1|^2+|Z_2|^2=1$ Shnir '18.

$m_z(\gamma) = |Z_1|^2 - |Z_2|^2 = 1-2|Z_2|^2 = 2|Z_1|^2 - 1$, все определяет баг $m_z(\gamma) = m_z(n,m)$

Escaneado con CamScanner

Hopfion $m_2(y,m,n)$ намел Arafyn'93 for particular Lagrangeans и намел Sugic (2021), но невярно. Обещи недостатъци в Sutcliffe'2007.

(12/IV) Начисляване Hopfion gauge MS. < Л... "Електрическо" Oliveri.

(13/IV) Аванс заплати април по Grupos Consolidados project за 2019-2022 (33.960€) ако и tax office May 2, 12ое (проучване Л. Сарафова) project # GIU 18/192

Изчисляване бюджета за Form 10-T за 2022: 1) Jan - Apr. 2022, 2) May - Dec 2022

1) Jan.-Apr. 22, B/нa. 27.044,32, retención 7.580,80, deducibles 3.154,20 €
2) May - Dec. 22, B/нa. 54.148,64, retención 15.161,80, deducibles 3.154,20

Всичко Form 10-T за 2022: B/нa 81.222,96 €, retención 22.742,40, deducibles 3.154,20 Общо 6.308,40 €

Том 2022

Bruto Sebang - (Soc. Sec.) - (Bonificación) - (declar. conjunta) - (minoración de cuota) =

= заето B/нa, което на годишна основа. Дом. бонификации: 1) dependents (Axer.) 2) hipoteca

Годишна брутна B/нa 2022: 7.107,44 (6 месеца) + 1.777,50 + 5.351,19 = 14.236,13 €

(14/IV) Начисляване Hopf gauge paper.

$F_{\mu\nu} = \vec{m}\cdot(\partial_\mu\vec{m}\times\partial_\nu\vec{m})$. Но ясно, че $\vec{m} = Z^+\vec{\sigma}Z$ според?

$\partial_\mu m^i = \frac{\partial}{\partial x_\mu}(\dots) = \partial_\mu(Z^+\vec{\sigma}Z) = \partial_\mu Z^+\vec{\sigma}Z + Z^+\vec{\sigma}\partial_\mu Z$

$\partial_\mu\vec{m}\times\partial_\nu\vec{m} = [\partial_\mu Z^+\vec{\sigma}Z + Z^+\vec{\sigma}\partial_\mu Z]\times[\partial_\nu Z^+\vec{\sigma}Z + Z^+\vec{\sigma}\partial_\nu Z] =$

$= \partial_\mu Z^+\vec{\sigma}Z + \partial_\nu Z^+\vec{\sigma}Z + \partial_\mu Z^+\vec{\sigma}Z\times Z^+\vec{\sigma}\partial_\nu Z + Z^+\vec{\sigma}\partial_\mu Z\times\partial_\nu Z^+\vec{\sigma}Z +$

$+ Z^+\vec{\sigma}\partial_\mu Z\times Z^+\vec{\sigma}\partial_\nu Z$, (1) (2) (3) (4)

Ако $\vec{\sigma}\times\vec{\sigma} = \frac{i}{2}\vec{\sigma}$ и каквото ползне.

$\partial_\mu Z^+\vec{\sigma}Z\times\partial_\nu Z^+\vec{\sigma}Z = \partial_\mu Z^+(\vec{\sigma}Z)\dots$ $(\vec{a}\cdot\vec{\sigma})\times(\vec{b}\cdot\vec{\sigma}) = 2i(\vec{a}\times\vec{b})\vec{\sigma}$ и т.н.?..

(1)+(2) $\partial_\mu Z^+[\vec{\sigma}Z\times\partial_\nu Z^+\vec{\sigma}Z + \vec{\sigma}Z\times Z^+\vec{\sigma}\partial_\nu Z]$

$\partial_\mu Z^+\vec{\sigma}Z\times\partial_\nu Z^+\vec{\sigma}Z = \partial_\mu Z^+\partial_\nu Z^+(\vec{\sigma}\times\vec{\sigma})Z = \frac{i}{2}\partial_\mu Z^+\partial_\nu Z^+\vec{\sigma}Z$, нещо.

$\vec{m}\cdot\frac{i}{2}\partial_\mu Z^+\partial_\nu Z^+\vec{\sigma}Z = Z^+\vec{\sigma}Z\cdot\frac{i}{2}\partial_\mu Z^+\partial_\nu Z^+\vec{\sigma}Z$, $\vec{\sigma}\cdot\vec{\sigma} = Z\sigma_i^2 = 3\begin{pmatrix}1&0\\0&1\end{pmatrix}$

$F_{\mu\nu} = \varepsilon_{ijk}m_i\partial_\mu m_j\partial_\nu m_k = \varepsilon_{ijk}Z^+\sigma_i Z\cdot\partial_\mu(Z^+\sigma_j Z)\partial_\nu(Z^+\sigma_k Z) =$

$= \varepsilon_{ijk}Z^+\sigma_i Z\left[(\partial_\mu Z^+\sigma_j Z + Z^+\sigma_j\partial_\mu Z)(\partial_\nu Z^+\sigma_k Z + Z^+\sigma_k\partial_\nu Z)\right] =$

$= \varepsilon_{ijk}Z^+\sigma_i Z\left[\partial_\mu Z^+\sigma_j Z\,\partial_\nu Z^+\sigma_k Z + \dots\right]$ $m_i = Z^+\sigma^i Z = \sigma^i_{k\ell}Z^*_k Z_\ell$

$F_{\mu\nu} = \varepsilon^{ijk}m_i\partial_\mu(\sigma^i_{k\ell}Z^*_k Z_\ell)\partial_\nu(\sigma^k_{pq}Z^*_p Z_q) = \varepsilon^{ijk}m_i\partial_\mu(\sigma^i_{n\ell}Z^*_n Z_\ell)\partial_\nu(\sigma^k_{pq}Z^*_p Z_q) =$

$= \varepsilon^{ijk}\sigma^j_{n\ell}\sigma^k_{pq}m_i\partial_\mu(Z^*_n Z_\ell)\partial_\nu(Z^*_p Z_q) = m_i\,i\left[\sigma^i_{nq}\delta_{\ell p} - \sigma^i_{p\ell}\delta_{nq}\right]\partial_\mu(Z^*_n Z_\ell)\partial_\nu(Z^*_p Z_q) =$

$= im_i\left[\sigma^i_{nq}\partial_\mu(Z^*_n Z_\ell)\partial_\nu(Z^*_\ell Z_q) - \sigma^i_{p\ell}\partial_\mu(Z^*_n Z_\ell)\partial_\nu(Z^*_p Z_n)\right] =$

$= i\varepsilon^i_{jk}Z^*_j Z_k\left[\sigma^i_{nq}\partial_\mu(Z^*_n Z_\ell)\partial_\nu(Z^*_\ell Z_q) - \sigma^i_{p\ell}\partial_\mu(Z^*_n Z_\ell)\partial_\nu(Z^*_p Z_n)\right]$

$\varepsilon^i_{jk}\sigma^i_{nq} \Rightarrow$ нещо нещо как умножение.

Нямам с $F_{\mu\nu} = -\frac{2i}{(1+|w|^2)^2}[\partial_\mu w\,\partial_\nu w^* - \partial_\nu w\,\partial_\mu w^*]$ и $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

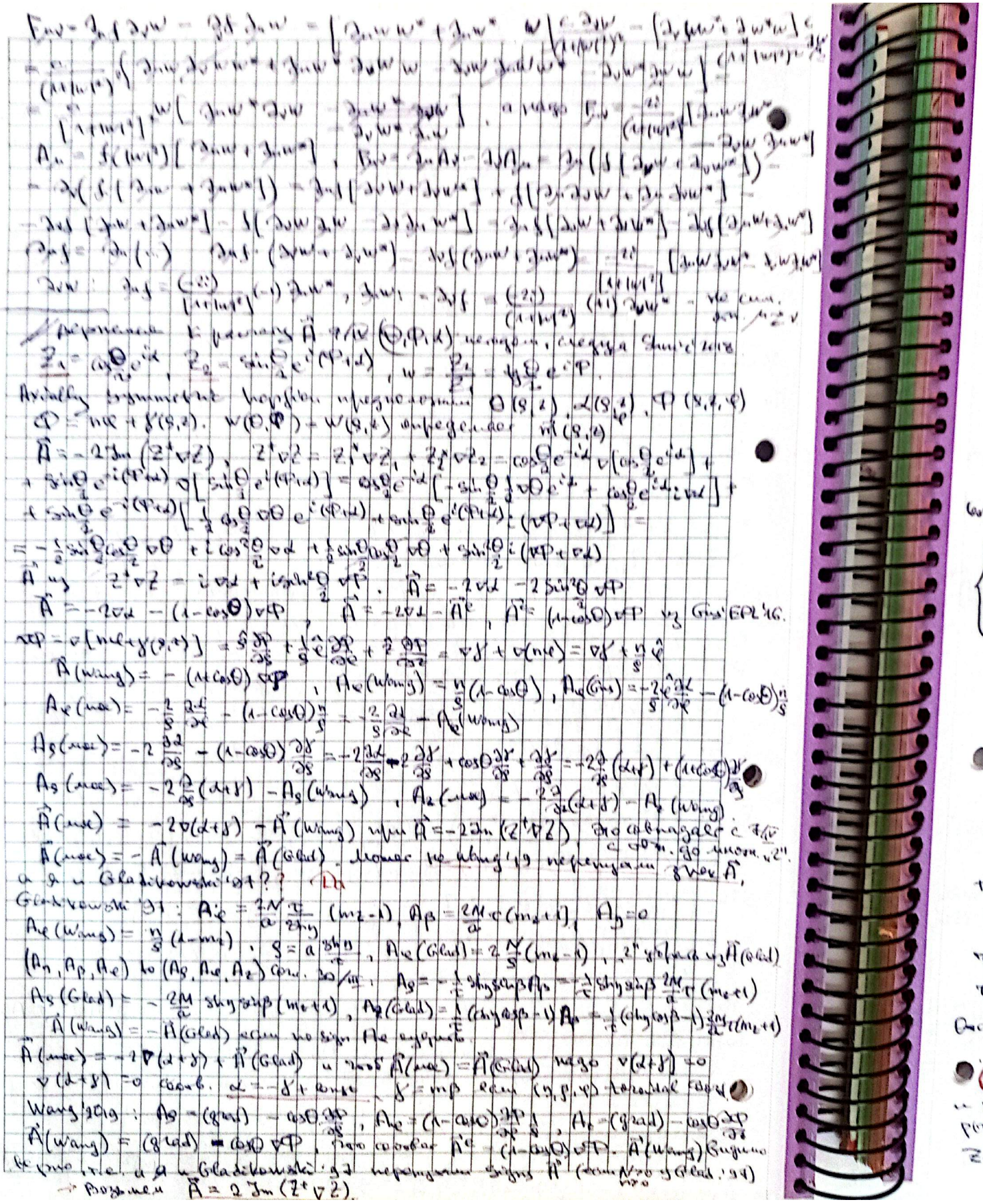
Намирам $\vec{A}$: $A_\mu = f(w)\partial_\mu w$. $A_\mu = -\mathrm{Im}(Z^+\partial_\mu Z)$.

$F_{\mu\nu} = \partial_\mu[f(w)\partial_\nu w] - \partial_\nu[f(w)\partial_\mu w] = \partial_\mu f\,\partial w + f\partial_\mu\partial_\nu w - \frac{\partial f}{\partial x_\nu}\partial_\mu w - f\frac{\partial^2 w}{\partial x_\mu\partial x_\nu} =$

$= \partial_\mu f\,\partial_\nu w - \frac{\partial f}{\partial x_\nu}\partial_\mu w = -\frac{2i}{(1+|w|^2)^2}[\partial_\mu w\,\partial_\nu w^* - \partial_\nu w\,\partial_\mu w^*]$

$-\partial_\nu f = \frac{f(x^\nu)}{(1+|w|^2)}\partial_\nu w^*$ и $\partial_\mu f = +\frac{(2i)}{(1+|w|^2)^2}\partial_\mu w^*$, $f(w) = \frac{c}{1+|w|^2}$; $\frac{\partial f}{\partial x} = \frac{c}{(1+|w|^2)^2}$

$+\partial_\mu(w\bar{w}) = \frac{c}{(1+|w|^2)^2}[\partial_\mu w w^* + \partial_\mu w^* w]$



Escaneado con CamScanner

Then $\vec{A}(\text{noe}) = 2\vec{v}(\alpha+\gamma) + \vec{A}(\text{Wang})$. $\vec{A}(\text{noe})$ sign совпадает с Eq.(12) $\vec{A}$ (Ke-Gno).

$\vec{v}(\alpha+\gamma)=0$ дает $\vec{A}'(\text{noe}) = \vec{A}'(\text{Wang}) = -2\vec{v}\gamma + \vec{A}^e$ (conv. 17/III).

(15/IV) Описание 3D rotations of coordinate system - Vorobyev-Lovitch-1928

(1) Euler angles (α, β, γ); (2) Rotation axis $\vec{n}(\theta, \varphi)$ + rotation angle $\omega \in [0, \pi]$ (3) Cayley-Klein parameters (§ 1.4)

(3) Cayley-Klein: $\hat{x} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$, $X = \begin{pmatrix} z & x+iy \\ x-iy & -z \end{pmatrix} = \sum_i x_i \sigma_i^T$

rotation of coord. system, $S(x,y,z) \to S'(x',y',z')$

X' (coordinates in S'), $X' = UXU^{-1}$, $U' = U^{-1}$, $U = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}$ - $SU(2)$ element

$|a|^2 + |b|^2 = 1$.

Cayley parameters $a = \cos\frac{\beta}{2} e^{-\frac{i}{2}(\alpha+\gamma)}$, $b = \sin\frac{\beta}{2} e^{\frac{i}{2}(\alpha-\gamma)}$ через Euler (α, β, γ)

- Klein

$\cos\beta = |a|^2 - |b|^2$ cos-b, $m_z = \cos\Theta$

Cayley-Klein parameters через $\vec{n}(\theta, \varphi)$ и ω: $a = \cos\frac{\omega}{2} - i\sin\frac{\omega}{2}\cos\theta$, $b = -i\sin\frac{\omega}{2}\sin\theta e^{i\varphi}$

$w = \frac{b}{a} = \frac{\sin\beta/2 \cdot e^{\frac{i}{2}(\alpha-\gamma)}}{\cos\beta/2 \cdot e^{-\frac{i}{2}(\alpha+\gamma)}} = \operatorname{tg}\frac{\beta}{2} e^{i\alpha}$, $\beta = \Theta$, $\alpha = \Phi$ $\in m'(\Theta, \Phi)$ и не забыть α, γ - Euler angle.

Можно $Z = \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$, $a = \cos\frac{\omega}{2} - i\sin\frac{\omega}{2}\cos\theta$, $b = -i\sin\frac{\omega}{2}\sin\theta e^{i\varphi}$

$Z_1 = a$, $Z_2 = b$ и $w = Z_2/Z_1$. $Z_1 = \cos\frac{\omega}{2} - i\sin\frac{\omega}{2}\cos\theta$, $Z_2 = -i\sin\frac{\omega}{2}\sin\theta e^{i\varphi}$

переключается с Sutcliffe $\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \frac{x+iy}{r}\sin f \\ \cos f + i\sin f \frac{z}{r} \end{pmatrix}$ mapping of spatial

coord. $(x_1, x_2, x_3) = (x, y, z) \in R^3$ to the unit three-sphere. $S^3_{1,3} \subset C^2$, $f(0) = \pi$

$$\begin{cases} Z_2(\text{Sut}) = \sin\Theta \sin f e^{i\varphi}, & Z_2(\text{Sut}) = \cos f + i\sin f\cos\Theta \\ Z_2 = -i\sin\frac{\omega}{2}\sin\theta e^{i\varphi}, & Z_1 = \cos\frac{\omega}{2} - i\sin\frac{\omega}{2}\cos\theta \\ Z_2(\text{Sugic}) = \sin\omega\sin\theta e^{i\varphi}, & Z_1(\text{Sugic}) = \cos\omega + i\sin\omega\cos\theta \end{cases}$$

$r^2 = x^2 + y^2 + z^2$, $f = f(r)$, $f(\infty) = 0$

(r, θ, φ) - spherical coordinates

$\rho = r\sin\Theta$, $z = r\cos\Theta$.

$\omega(\text{Sug}) = f(r)$ - Sutcliffe.

Ротация матрица вокруг $\vec{n}$ на угол ω: $\hat{R}(\vec{n}, \omega) = e^{i\omega\vec{n}\cdot\vec{J}}$, $\vec{J} = \frac{1}{2}\vec{\sigma}$ момент. бозон

Sutcliffe $w = \frac{m_x + im_y}{1+m_z}$, $w = \frac{Z_1^n}{Z_0^m}$ или $w = \frac{Z_2^n}{Z_1^m}$ в наших обозн. - $Z = \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}$

$Z_1(\text{Sut}) = \cos f + i\sin f \frac{z}{r}$ дополнено $\sim e^{-i\varphi}$, тогда $w \sim e^{i(n\varphi + m\beta)}$ in toroidal coordinates (γ, β, φ). $\rho = a\frac{\sinh\gamma}{\tau}$, $z = a\frac{\sin\beta}{\tau}$, $\tau = \cosh\gamma - \cos\beta$. $r^2 = \rho^2 + z^2 = \frac{a^2}{\tau^2}[\sinh^2\gamma + \sin^2\beta]$

$\frac{z}{r} = \frac{a\sin\beta}{\tau\sqrt{\sinh^2\gamma + \sin^2\beta}} \frac{a}{\tau} = \frac{\sin\beta}{\sqrt{\sinh^2\gamma + \sin^2\beta}}$, $Z_1(\text{Sut}) = \cos f + i\sin f \frac{\sin\beta}{\sqrt{\sinh^2\gamma + \sin^2\beta}} =$

$= -\frac{\sin f}{\sqrt{\sinh^2\gamma + \sin^2\beta}}\left[-\operatorname{ctg} f\sqrt{\sinh^2\gamma + \sin^2\beta} - i\sin\beta\right]$, надо $-\operatorname{ctg} f\sqrt{\sinh^2\gamma + \sin^2\beta} = \cos\beta$, тогда $Z_1 \sim e^{-i\beta}$

т.е. $-\operatorname{tg} f(r) = \frac{1}{\cos\beta}\sqrt{\sinh^2\gamma + \sin^2\beta}$, $r = \frac{a}{\tau}\sqrt{\sinh^2\gamma + \sin^2\beta}$, $-\operatorname{tg} f(r) = \frac{r\tau}{a\cos\beta}$ - возможно и?

$\frac{r}{\cos\beta}$ должна быть $g(r)$, $\frac{(\cosh\gamma - \cos\beta)}{\cos\beta} = g(r)$? $\sinh^2\gamma + \sin^2\beta = \cosh^2\gamma - \cos^2\beta$

$r = \frac{a}{\tau}\sqrt{\cosh^2\gamma - \cos^2\beta} = \frac{a}{\tau}\sqrt{(\cosh\gamma - \cos\beta)(\cosh\gamma + \cos\beta)} = a\sqrt{\frac{\cosh\gamma + \cos\beta}{\cosh\gamma - \cos\beta}}$, $a = 1$

$r^2 - 1 = \frac{\cosh\gamma + \cos\beta}{\cosh\gamma - \cos\beta} - 1 = \frac{\cosh\gamma + \cos\beta - (\cosh\gamma - \cos\beta)}{(\cosh\gamma - \cos\beta)} = \frac{2\cos\beta}{(\cosh\gamma - \cos\beta)}$

Однако $\frac{\cosh\gamma - \cos\beta}{\cos\beta} = \left(\frac{1}{2}\right)\frac{2}{(r^2-1)} = g(r)$, $-\operatorname{tg} f(r) = r\frac{\tau}{\cos\beta} = r\cdot g(r) = \frac{2r}{(r^2-1)}$

(!) $\operatorname{tg} f(r) = \frac{2r}{(1-r^2)}$, $\sin f(r) = \frac{2r}{1+r^2}$, $\cos f(r) = \frac{1-r^2}{1+r^2}$. Именно $\operatorname{tg} f(r) = \frac{2r}{(1-r^2)}$

и публикация из Eq. (12,26) Shnir '18. $\sin f(r) = 2r/P(r)$, $\cos f(r) = \frac{1-r^2}{P(r)}$

$P(r) = 1 + r^2$ надо еще сделать. $Z_1(\text{Sut}) = -\sin f \left[\cos\beta + i\sin\beta\right] = -\frac{\sin f \cdot e^{-i\beta}}{\sqrt{\sinh^2\gamma + \sin^2\beta}}$

$Z_2(\text{Sut}) = \frac{\rho}{r}\sin f e^{i\varphi}$. $\sqrt{\sinh^2\gamma + \sin^2\beta} = r\tau$, $a = 1$!

$Z_1(\text{out}) = -\sin f\, e^{i\beta}\frac{1}{r\tau}$, $Z_2(\text{out}) = \frac{s}{\tau}\sin f\, e^{i\varphi}$, $\frac{s}{\tau} = \frac{\sh y}{r\tau}$, $\tau = \ch y - \cos\beta$

$|Z_1|^2 + |Z_2|^2 = \sin^2 f\frac{1}{r^2\tau^2} + \sin^2 f\frac{\sh^2 y}{r^2\tau^2} = \frac{\sin^2 f}{r^2\tau^2}(1+\sh^2 y) = \ch^2 y\frac{\sin^2 f}{r^2\tau^2} = 1$

$\sin f = \frac{2r}{\rho(r)}$, $\frac{\ch^2 y}{r^2\tau^2}\frac{4r^2}{\rho^2(r)} = 1$, $\frac{4\ch^2 y}{r^2\rho^2(r)} = 1$, $\rho(r) = \frac{2}{\tau}\ch y = 2\frac{\ch y}{(\ch y - \cos\beta)}$, $r^2+1 = \frac{\ch y + \cos\beta}{\ch y - \cos\beta} + 1 = \frac{2\ch y}{(\ch y - \cos\beta)} = \frac{2}{\tau}\ch y$

Όπως για $\rho(r) = (1+r^2)$ και β είναι $\sin^2 f = \frac{4}{\rho^2}$

$Z_2(\text{out}) = \frac{s}{r\tau}\sin f(r)\, e^{i\varphi}$, $Z_1(\text{out}) = -\frac{1}{r\tau}\sin f\, e^{-i\beta}$.

$w = \frac{Z_2}{Z_1} = \frac{s}{r}\frac{\sin f\, e^{i\varphi}}{(-1)\frac{1}{r\tau}\sin f\, e^{-i\beta}} = -\tau s\, e^{i(\varphi+\beta)} = -\sh y\, e^{i(\varphi+\beta)} = w(y,\beta,\varphi)$

Corrado Sutcliffe PRS (2007) και Shmir 2018, για $m,n \neq 1$ $w = \frac{Z_2^n}{Z_1^m}$

$-\sh y = \sqrt{1-m_2^2}$, $m_2^2 = 1-\sh^2 y = 2-\ch^2 y$, πολ/σε $\to$ θέτω

Άρα $m_2(y) = 1-2\th^2 y$ για $m=1, n=1$. $\sqrt{1-m_2^2} = \pm\frac{1}{\sh y}$, $1-m_2^2 = \frac{1}{\sh^2 y}$

$m_2^2 = 1-\frac{1}{\sh^2 y}$?. $m_2 = \frac{1-|w|^2}{1+|w|^2} = \frac{1-\sh^2 y}{1+\sh^2 y} = \frac{1-\sh^2 y}{\ch^2 y}\frac{1}{\ch^2 y} = -\th^2 y$

$m_2(y) = 1-2\th^2 y$, $\frac{1}{\ch^2 y} + \th^2 y = 1$.

$W(y,\beta,\varphi) = -\sh y\, e^{i(\varphi+\beta)}$ όπου $m=n=1$. $W(y,\beta,\varphi) = \frac{Z_2^n}{Z_1^m}$ for any integer (m,n)

$W(y,m,n) = \frac{\left[\frac{s}{\tau}\sin f\, e^{i\varphi}\right]^n}{\left[(-1)\frac{1}{r\tau}\sin f\, e^{-i\beta}\right]^m} = \left|(-1)\text{ έβεγαν }\beta\right| = \frac{s^n}{\tau^n}\frac{\sin^n f}{\sin^m f}\tau^m r^{m-n} e^{i(n\varphi+\beta m)}$

$W(y,m,n) = |W(y,m,n)|\, e^{i(n\varphi+m\beta)}$, $|w| = s^n\tau^m r^{m-n}\sin^{n-m} f$

$s = \frac{\sh y}{\tau}$, $|w| = \frac{\sh^n y}{\tau^n}\tau^m r^{m-n}\sin^{n-m} f = \sh^n y\,(\tau r)^{m-n}\sin^{n-m} f = \sh^n y\left(\frac{\sin f}{r\tau}\right)^{n-m}$

$\frac{\sin f}{r\tau}$ χαιρόμουν όλους $f(y)$. $\frac{\sin f}{r\tau} = \frac{2r}{(1+r^2)\,r\tau} = \frac{2}{(1+r^2)\tau}$.

$r^2+1 = \frac{2}{\tau}\ch y$, $\frac{\sin f}{r\tau} = \frac{2}{2\ch y} = \frac{1}{\ch y}$.

$|w| = \sh^n y\left(\frac{\sin f}{r\tau}\right)^{n-m} = \frac{\sh^n y}{\ch^{n-m} y}\cdot\frac{1}{\ch^n y} = \ch^m y\,\th^n y$ (!)

$m_2(y) = \frac{1-|w|^2}{1+|w|^2} = \frac{1-\ch^{2m} y\,\th^{2n} y}{1+\ch^{2m} y\,\th^{2n} y}$. Για $n=m=1$, $m_2 = \frac{1-\sh^2 y}{\sh^2 y+1} = 1-2\th^2 y$.

$m_{\text{τελικό}}(y,m,n) = \sqrt{1-m_2^2(y)}\, e^{i(n\varphi+m\beta)}$ coord. Gladkowski 97.

$U(y,\beta,\varphi) = f(y)\, e^{i(m\beta+n\varphi)}$ - Hierarchical 2000 coord. $f^2(y) = (1-m_2^2(y)) = |w|^2_{(y)}$.

τότε $f(y)$ Hierarchical συνάρτηση $f(y) = f(y,m,n) = |w|_{(y,m,n)} = \ch^m y\,\th^n y$.

MathCad "Hopf-quaternion.mcd". $m_2(y,m,n)$ Γκαουσιανό. Βρες $n^{\text{ο}}$ μαγνητικό hopfion, $m^{\text{ο}}$ ομοιόμορφο hopfion, με $m=1, n=1$ hopfion $\to m_2 = 1-2\th^2 y$.

μόνιμο βλέπω hopfion Radius $y_c(m,n)$, όπου $m_2(y_c) = 0$, $y_c = 0.6 \div 1.5$

$\th y = \frac{2as}{s^2+r^2+a^2}$. Παίρνω $m_2(s, r=0)$ ως $\th y = \frac{2s}{s^2+1}$, s βεβ. $a^{\text{ο}}$.

→ // Nicole '78: $X_4 = \frac{1-r^2}{1+r^2}$, $X_i = \frac{2x_i}{(1+r^2)}$, $i=1,2,3$ $S_3(X_i, i=1,4)$ το R^3 (x_1,x_2,x_3) S_2 mapping.

Sutcliffe 2021, $n_i = \frac{2x_i}{1+r^2}$, $n_w = \frac{1-r^2}{1+r^2}$, $S_3(n_1,n_2,n_3,n_w)$ $n_1^2+n_2^2+n_3^2+n_w^2 = 1$

$S_2(\vec{n})$ το R^3 (x,y,z) projection.

γ μονάδα $(X_i, i=1,4)$ $Z_1 = X_1+iX_2$, $Z_2 = X_3+iX_4$, $w = \frac{Z_2}{Z_1}$, $|w| = \frac{|Z_2|}{|Z_1|}$.

$m_2 = \frac{1-|w|^2}{1+|w|^2} = \frac{1-|Z_2|^2/|Z_1|^2}{1+|Z_2|^2/|Z_1|^2} = |Z_1|^2-|Z_2|^2$, $m_2+\ch y = \frac{2}{(1+|w|^2)}$ $w = \frac{Z_2}{Z_1} = \frac{X_3+iX_4}{X_1+iX_2} = (\cdot)\frac{(X_3+iX_4)(X_1-iX_2)}{|Z_1|^2} = 2\left[X_3X_4+X_2X_4+i(X_4X_1-X_2X_3)\right]$

Escaneado con CamScanner

$\frac{1}{2}(m_x+im_y) = X_1X_3 + X_2X_4 + i(X_1X_4 - X_2X_3)$, $m_x+im_y = 2Z_1Z_2^* = 2(X_1+iX_2)(X_3-iX_4) =$
$= 2[X_1X_3 + X_2X_4 + i(X_2X_3 - X_1X_4)]$, т.е. $w = \frac{Z_1}{Z_2}$. Надо $w = \frac{Z_2}{Z_1}$, $m_y \to -m_y$

$m_x + im_y = 2Z_1^*Z_2 = 2(X_1-iX_2)(X_3+iX_4)$. Надо $w \sim (x+iy) \sim (X_1+iX_2)$

$w = \frac{Z_2}{Z_1}$ хочем. Тогда $Z_2 = X_1+iX_2 = \frac{2}{(1+r^2)}(x+iy)$ и $Z_1 = X_3+iX_4$

$Z_1 = \frac{1}{(1+r^2)}[2z + i(1-r^2)]$. Другие $Z_1 = X_4+iX_3$, $X_4 = \frac{1-r^2}{(1+r^2)}$, $X_i = \frac{2x_i}{(1+r^2)}$, $i=1,3$

$w = \frac{Z_2}{Z_1} = \frac{X_1+iX_2}{X_4+iX_3} = \frac{2(x+iy)}{(1-r^2+2iz)}$, поэтому w нельзя выбрать $1/w$.

$Z_2 = \frac{1}{(1+r^2)}2(x+iy) = \frac{2s}{(1+r^2)}e^{i\varphi}$, $Z_1 = \frac{1}{(1+r^2)}[1-r^2+2iz]$, $Z_1 \rightleftarrows Z_2$.

$Z_2 = th\gamma\, e^{i\varphi}$, $Z_1 = -\frac{1}{ch\gamma}e^{-i\beta}$, $w = \frac{Z_2}{Z_1} = -sh\gamma\, e^{i(\varphi+\beta)}$

$m_x+im_y = \frac{2}{(1+|w|^2)|Z_1|^2}\left((X_1+iX_2)(X_4-iX_3)\right) = 2\left[X_1X_4 + X_2X_3 + i(X_2X_4 - X_1X_3)\right]$

$m_x+im_y = 2Z_2Z_1^*$, $m_z = |Z_1|^2 - |Z_2|^2$.

(16/IV) Расчет IVA «Constanta» от Ikb. 2023. Abs. на SNM'23, Prague – Hopfion $\vec{G}$.

(17/IV) С.Л. в «Lidl» Нетания. Л. поехали в Киев: Л.Т. 9 kg + taxa 8 kg – 50 €.

(18/IV) trip to Porto. Reflexão em Boavondadesillas.

(19/IV) Ужинали в Matosinhos, «Valentim» hotel. Ужин в ресторане.

(20/IV) С Пасхой. Ужин в номере и обед в ресторане. Ainda Doce.

(21/IV) Написание hopfion MS. Вектор потенциал $\vec{A} = 2\,Im(Z^+\nabla Z)$, $Z = \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}$

сон. 1/IV ($\lambda = 2$): $Z_1 = -\frac{1}{ch\gamma}e^{-i\beta}$, $Z_2 = th\gamma\, e^{i\varphi}$, $w = \frac{Z_2}{Z_1} = -sh\gamma\, e^{i(\varphi+\beta)}$.

Для m,n arbitrary integer надо $w = \frac{Z_2^n}{Z_1^m}$, т.е. $Z_2 \to Z_2^n$, $Z_1 \to Z_1^m$, $m,n = 1,-$

$Z^+\nabla Z = (Z_1^*, Z_2^*)\begin{pmatrix}\nabla Z_1 \\ \nabla Z_2\end{pmatrix} = Z_1^*\nabla Z_1 + Z_2^*\nabla Z_2$, $\nabla = \frac{\tau}{a}\left(\frac{\partial}{\partial\gamma}, \frac{\partial}{\partial\beta}, \frac{1}{sh\gamma}\frac{\partial}{\partial\varphi}\right)$

$A_\gamma = 2Im\left[Z_1^*\frac{\tau}{a}\frac{\partial}{\partial\gamma}Z_1 + Z_2^*\frac{\tau}{a}\frac{\partial}{\partial\gamma}Z_2\right]$, $Z_1 = (-1)^m\frac{1}{ch^m\gamma}e^{-im\beta}$, $Z_2 = th^n\gamma\, e^{in\varphi}$

$[\dots]$ is real, т.к. $e^{im\beta}e^{-im\beta}$, $e^{-in\varphi}e^{in\varphi}$ are real. $A_\gamma = 0$.

$A_\beta = 2Im\frac{\tau}{a}\left[Z_1^*\frac{\partial Z_1}{\partial\beta} + Z_2^*\frac{\partial Z_2}{\partial\beta}\right] = \frac{2\tau}{a}Im\left[(-1)^m\frac{1}{ch^m\gamma}e^{im\beta}\frac{(-1)^m}{ch^m\gamma}(-im)e^{-im\beta} + \right.$

$\left. + th^n\gamma\, e^{-in\varphi}th^n\gamma\frac{\partial}{\partial\beta}e^{in\varphi}\right] = \frac{2\tau}{a}Im\left[\frac{(-1)^{2m}}{ch^{2m}\gamma}(-im)\right] = \frac{2\tau}{a}\frac{1}{ch^{2m}\gamma}(-m)$, $A_\beta = -\frac{2\tau m}{a\,ch^{2m}\gamma}$

$A_\beta(Glad) = \frac{2M}{a}\tau(m_z+1)$, $m_z+1 = \frac{1-ch^{2m}\gamma\, th^{2n}\gamma}{1+ch^{2m}\gamma - th^{2n}\gamma} + 1 = \frac{2}{1+ch^{2m}\gamma\, th^{2n}\gamma}$ отсюда $A_\beta(нов)$

При $m=1, n=1$, $A_\beta(нов) = -\frac{2\tau m}{a\,ch^2\gamma}$, $A_\beta(Glad) = \frac{2M}{a}\tau\frac{2}{1+th^2\gamma+th^2\gamma} = \frac{2M\tau}{a}\frac{2}{ch^2\gamma}$

– совпадение с точн. выражением $\times(-2)$.

$A_\varphi = 2Im\frac{\tau}{a\,sh\gamma}\left[Z_1^*\frac{\partial Z_1}{\partial\varphi} + Z_2^*\frac{\partial Z_2}{\partial\varphi}\right] = 2\frac{\tau}{a\,sh\gamma}Im\left[-th^n\gamma\, e^{\frac{in\varphi}{\partial\varphi}}th^n\gamma\, e^{in\varphi}\right] = \frac{2\tau}{a\,sh\gamma}\times$

$\times th^{2n}\gamma\, Im\left[e^{-in\varphi}(in)e^{in\varphi}\right] = \frac{2\tau n}{a\,sh\gamma}th^{2n}\gamma$, $A_\varphi(нов) = \frac{2\tau n}{a\,sh\gamma}th^{2n}\gamma$.

$A_\varphi(Glad) = \frac{2\tau N}{a\,sh\gamma}(m_z-1)$, $m_z - 1 = -\frac{2\,ch^{2m}\gamma\, th^{2n}\gamma}{1+ch^{2m}\gamma\, th^{2n}\gamma}$. При $m=1, n=1$

$A_\varphi(Glad) = \frac{2\tau N}{a\,sh\gamma}(-2)\frac{sh^2\gamma}{1+sh^2\gamma} = \frac{2\tau N}{a\,sh\gamma}(-2)th^2\gamma$, $A_\varphi(нов) = \frac{2\tau}{a\,sh\gamma}th^2\gamma$ – отлич.

от Glad-ова $\times(-2)$ множителем (-2).

Почему $\vec{A}(нов)$ и $\vec{A}(Glad)$ отличаются лишь при $m=1, n=1$? $m_z(\gamma, m, n)$ – нов. коэффициент?

$w = Z_2^n/Z_1^m$ – неверно?

нов $\vec{A}$: $A_\gamma = 0$, $A_\beta = -\frac{2\tau m}{a}\frac{1}{ch^{2m}\gamma}$, $A_\varphi = \frac{2\tau n}{a\,sh\gamma}th^{2n}\gamma$.

$\vec{B} = \nabla\times\vec{A} = \begin{vmatrix} \frac{\tau}{a}\frac{\partial}{\partial\gamma} & \frac{\tau}{a}\frac{\partial}{\partial\beta} & \frac{\tau}{a\,sh\gamma}\frac{\partial}{\partial\varphi} \\ A_\gamma & A_\beta & A_\varphi \end{vmatrix} = \hat{\gamma}\left[\frac{\tau}{a}\frac{\partial A_\varphi}{\partial\beta} + \frac{\tau}{a\,sh\gamma}\frac{\partial A_\beta}{\partial\varphi}\right] + \hat{\beta}\left[\frac{\tau}{a\,sh\gamma}\frac{\partial A_\gamma}{\partial\varphi} - \frac{\tau}{a}\frac{\partial A_\varphi}{\partial\gamma}\right] +$

Escaneado con CamScanner

$$+\hat{q}\left[\frac{\tau}{a}\frac{\partial}{\partial\beta}A_\beta - \frac{\tau}{a}\frac{\partial}{\partial\beta}A_y\right] = \hat{y}\left[\frac{\partial A_\varphi}{\partial\beta} - \frac{1}{\operatorname{sh}y}\frac{\partial A_\beta}{\partial\varphi}\right]\frac{\tau}{a} + \hat{\beta}\left(-\frac{\tau}{a}\right)\frac{\partial A_\varphi}{\partial y} + \hat{\varphi}\frac{\tau}{a}\frac{\partial A_\beta}{\partial y}$$

Нет $2\pi\vec{A}$ см 25/IV

$A_y = 0$

$$B_\beta = \left[\frac{\partial A_\varphi}{\partial\beta} - \frac{1}{\operatorname{sh}y}\frac{\partial A_\beta}{\partial\varphi}\right]\frac{\tau}{a}, \quad B_\varphi = -\frac{\tau}{a}\frac{\partial A_\beta}{\partial y}, \quad B_y = \frac{\tau}{a}\frac{\partial A_\varphi}{\partial y}$$

$$B_y = \frac{\tau}{a}\left[\frac{\partial}{\partial\beta}\frac{2\varepsilon n}{a\operatorname{sh}y}\operatorname{th}^{2n}y - \frac{1}{\operatorname{sh}y}\frac{\partial}{\partial\varphi}\left(-\frac{2\varepsilon m}{a}\right)\frac{1}{\operatorname{ch}^{2m}y}\right] = \frac{\tau}{a}\frac{2n\operatorname{th}^{2n}y}{a\operatorname{sh}y}\frac{\partial\tau}{\partial\beta}$$

$\tau = \operatorname{ch}y - \cos\beta$, $B_y = \frac{2n\tau}{a^2}\frac{\operatorname{th}^{2n}y}{\operatorname{sh}y}\sin\beta$. $\tau = \operatorname{ch}y - \cos\beta$.

$$B_\beta = -\frac{\tau}{a}\frac{\partial}{\partial y}A_\varphi = -\frac{\tau}{a}\frac{\partial}{\partial y}\left(\frac{2\varepsilon n}{a\operatorname{sh}y}\operatorname{th}^{2n}y\right) = -\frac{2n\varepsilon}{a^2}\frac{\partial}{\partial y}\left(\frac{\tau}{\operatorname{sh}y}\operatorname{th}^{2n}y\right)$$

$$B_\varphi = \frac{\tau}{a}\frac{\partial A_\beta}{\partial y} = \frac{\tau}{a}\frac{\partial}{\partial y}\left(-\frac{2\varepsilon m}{a}\frac{1}{\operatorname{ch}^{2m}y}\right) = -\frac{2\varepsilon m\tau}{a^2}\frac{\partial}{\partial y}\left(\frac{\tau}{\operatorname{ch}^{2m}y}\right)$$

$$\vec{A}\cdot\vec{B} = A_\beta B_\beta + A_\varphi B_\varphi = -\frac{2\varepsilon m}{a}\frac{1}{\operatorname{ch}^{2m}y}\frac{(-n2\varepsilon)}{a^2}\frac{\partial}{\partial y}\left(\frac{\tau}{\operatorname{sh}y}\operatorname{th}^{2n}y\right) +$$

$$+\frac{2\varepsilon n\tau}{a^2\operatorname{sh}y}\operatorname{th}^{2n}y\left(-\frac{2\varepsilon m\tau}{a^2}\frac{\partial}{\partial y}\left(\frac{\tau}{\operatorname{ch}^{2m}y}\right)\right) = \frac{4\varepsilon^2 mn\tau^2}{a^4}\frac{1}{\operatorname{ch}^{2m}y}\frac{\partial}{\partial y}\left(\frac{\tau}{\operatorname{sh}y}\operatorname{th}^{2n}y\right) -$$

$$-\frac{4\varepsilon^2 mn\tau^2\operatorname{th}^{2n}y}{a^4\operatorname{sh}y}\frac{\partial}{\partial y}\left(\frac{\tau}{\operatorname{ch}^{2m}y}\right), \quad dV = \frac{a^3}{\tau^3}\operatorname{sh}y\,dy\,d\beta\,d\varphi$$

$$\int dV\vec{A}\cdot\vec{B} = \int dy\int d\beta\int d\varphi\,\frac{a^3}{\tau^3}\operatorname{sh}y\,\frac{4\varepsilon^2 mn\tau^2}{a^4}\left[\frac{1}{\operatorname{ch}^{2m}y}\frac{\partial}{\partial y}\left(\frac{\tau}{\operatorname{sh}y}\operatorname{th}^{2n}y\right) - \frac{\operatorname{th}^{2n}y}{\operatorname{sh}y}\frac{\partial}{\partial y}\left(\frac{\tau}{\operatorname{ch}^{2m}y}\right)\right] =$$

$$= 4\varepsilon^2 mn\int_0^\infty dy\int_0^\pi d\beta\int_0^{2\pi} d\varphi\,\frac{\operatorname{sh}y}{\tau}\left[\frac{\tau}{\operatorname{ch}^{2m}y}\frac{\partial}{\partial y}\left(\frac{\tau}{\operatorname{sh}y}\operatorname{th}^{2n}y\right) - \frac{\operatorname{th}^{2n}y}{\operatorname{sh}y}\frac{\partial}{\partial y}\left(\frac{\tau}{\operatorname{ch}^{2m}y}\right)\right]$$

$$\int dV\vec{A}\cdot\vec{B} = 8\pi^2 mn\int_0^\infty dy\int_0^\pi d\beta\,\frac{\operatorname{sh}y}{\tau}\left[\frac{\tau}{\operatorname{ch}^{2m}y}\frac{\partial}{\partial y}\left(\frac{\tau}{\operatorname{sh}y}\operatorname{th}^{2n}y\right) - \frac{\operatorname{th}^{2n}y}{\operatorname{sh}y}\frac{\partial}{\partial y}\left(\frac{\tau}{\operatorname{ch}^{2m}y}\right)\right]$$

$$[\ldots] = \frac{1}{\operatorname{ch}^{2m}y}\left[\frac{1}{\operatorname{sh}y}\operatorname{th}^{2n}y\frac{\partial\tau}{\partial y} + \tau\operatorname{th}^{2n}y\left(-\frac{\operatorname{ch}y}{\operatorname{sh}^2y}\right) + \frac{\tau}{\operatorname{sh}y}\frac{\partial}{\partial y}\left(\operatorname{th}^{2n}y\right)\right] -$$

$$-\frac{\operatorname{th}^{2n}y}{\operatorname{sh}y}\left[\frac{\operatorname{sh}y}{\operatorname{ch}^{2m}y} + \tau\frac{\partial}{\partial y}\left(\frac{1}{\operatorname{ch}^{2m}y}\right)\right] = \frac{1}{\operatorname{ch}^{2m}y}\left[\operatorname{th}^{2n}y - \frac{\tau\operatorname{ch}y\operatorname{th}^{2n}y}{\operatorname{sh}^2y} + \frac{\tau}{\operatorname{sh}y}\frac{\partial}{\partial y}\left(\operatorname{th}^{2n}y\right)\right] -$$

$$-\frac{\operatorname{th}^{2n}y}{\operatorname{sh}y}\left[\frac{\operatorname{sh}y}{\operatorname{ch}^{2m}y} + \tau\frac{\partial}{\partial y}\left(\frac{1}{\operatorname{ch}^{2m}y}\right)\right] = \frac{1}{\operatorname{ch}^{2m}y}\frac{\tau}{\operatorname{sh}y}\left[\frac{\partial}{\partial y}\left(\operatorname{th}^{2n}y\right) - \operatorname{cth}y\cdot\operatorname{th}^{2n}y\right] +$$

$$+\frac{\tau}{\operatorname{sh}y}\operatorname{th}^{2n}y\,(-1)\frac{\partial}{\partial y}\left(\frac{1}{\operatorname{ch}^{2m}y}\right), \quad \frac{\operatorname{sh}y}{\tau}[\ldots] = \frac{1}{\operatorname{ch}^{2m}y}\left[\frac{\partial}{\partial y}\left(\operatorname{th}^{2n}y\right) - \operatorname{cth}y\cdot\operatorname{th}^{2n}y\right] -$$

$$-\operatorname{th}^{2n}y\frac{\partial}{\partial y}\left(\frac{1}{\operatorname{ch}^{2m}y}\right), \quad \int dV\vec{A}\cdot\vec{B} = 8\pi^2 mn\int_0^\infty dy\int_0^\pi d\beta\,\frac{\operatorname{sh}y}{\tau}[\ldots] = 8\pi^2 mn\cdot 2\pi\times$$

$$\times\int_0^\infty dy\left\{\frac{1}{\operatorname{ch}^{2m}y}\left[\frac{\partial}{\partial y}\left(\operatorname{th}^{2n}y\right) - \operatorname{cth}y\operatorname{th}^{2n}y\right] - \operatorname{th}^{2n}y\frac{\partial}{\partial y}\left(\frac{1}{\operatorname{ch}^{2m}y}\right)\right\}$$

$$\int dy\left[u\frac{\partial v}{\partial y} - v\frac{\partial u}{\partial y}\right] = \int dy[\ldots] = \int dv\,u - \int du\,v, \quad \int uv' = uv - \int vu'$$

$$J_y = \int_0^\infty dy\,\operatorname{cth}y\frac{\operatorname{th}^{2n}y}{\operatorname{ch}^{2m}y} = \int_0^\infty dy\,\frac{\operatorname{sh}^{2n-1}y}{\operatorname{ch}^{2n}y\cdot\operatorname{ch}^{2m-1}y} = \left|\begin{matrix}\operatorname{ch}y = x\\ \operatorname{sh}y\,dy = dx\end{matrix}\right| = \int_1^\infty dx\,\frac{\operatorname{sh}^{2n-2}y}{\operatorname{ch}^{2n+2m-1}}$$

$\operatorname{sh}^2y = \operatorname{ch}^2y - 1$, $J_y = \int_1^\infty dx\cdot\frac{(x^2-1)^{n-1}}{x^{2n+2m-1}}$ — сходится.

$$\int dV\vec{A}\cdot\vec{B} = 16\pi^2 mn\left[\int_0^\infty dy\left\{\frac{1}{\operatorname{ch}^{2m}y}\frac{\partial}{\partial y}\left(\operatorname{th}^{2n}y\right) - \operatorname{th}^{2n}y\frac{\partial}{\partial y}\left(\frac{1}{\operatorname{ch}^{2m}y}\right)\right\} - J_y\right]$$

Вывод: если $w = z_2^n/z_1^m$, $z_1 = -\frac{1}{\operatorname{ch}y}e^{-i\beta}$, $z_2 = \operatorname{th}y\,e^{i\varphi}$, то получим $\vec{A}(\ldots)$, которое отличается от $\vec{A}$ (Gkd) при $m\neq 1$, $n\neq 1$.

(22/IV) В расслоении с Λ, Гиббон и его напарник.

(23/IV) В расслоении $\vec{A}$ — корабли (m,n) отмотка. $z_1 = -\frac{1}{\operatorname{ch}y}e^{-i\beta}$, $z_2 = \operatorname{th}y\,e^{i\varphi}$

$z_1 \to z_1^m = (-1)^m\frac{1}{\operatorname{ch}^m y}e^{-im\beta}$, $z_2 \to z_2^n = \operatorname{th}^n y\,e^{in\varphi}$. $\tilde{z}_1 = z_1^m$ и $\tilde{z}_2 = z_2^n$

Надо нормировать: $|\tilde{z}_1|^2 + |\tilde{z}_2|^2 = 1$, $\left(\frac{1}{\operatorname{ch}^{2m}y} + \operatorname{th}^{2n}y\right)N^{-1} = 1$, $N = \left(\frac{1}{\operatorname{ch}^{2m}y} + \operatorname{th}^{2n}y\right)^{+1}$

$\tilde{z}_1 = \frac{1}{\sqrt{N}}\frac{(-1)^m}{\operatorname{ch}^m y}e^{-im\beta}$, $\tilde{z}_2 = \frac{1}{\sqrt{N}}\operatorname{th}^n y\,e^{in\varphi}$. Нормировка не важна для

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$\vec{m}$ - magnetization, $w = \tilde{Z}_2/\tilde{Z}_1$, но важна идея расчета $\vec{A}(Z)$.

$N = N(y) = \frac{1}{ch^{2m}y} + th^{2n}y$. $m_2 = |\tilde{Z}_1|^2 - |\tilde{Z}_2|^2 = \frac{1}{N}\left[\frac{1}{ch^{2m}y} + th^{2n}y\right]$

$m_2(y) = \frac{1 - ch^{2m}y\, th^{2n}y}{1 + th^{2m}y\, th^{2n}y}$ как 15/IV. $\vec{A} = 2\,Im\,(Z^* \nabla Z)$.

В тороид. координатах (γ, β, φ) $\nabla = \frac{\tau}{a}\left(\frac{\partial}{\partial\gamma}, \frac{\partial}{\partial\beta}, \frac{1}{sh\gamma}\frac{\partial}{\partial\varphi}\right)$, $A_\gamma = 0$

$A_\beta = -\frac{2\tau m}{a}\frac{1}{N(y)}\frac{1}{ch^{2m}y} = -\frac{2m\tau}{a}\frac{1}{1 + ch^{2m}y\, th^{2n}y}$, $A_\varphi = \frac{2n\tau}{a\,sh\gamma}\frac{-th^{2n}y}{N(y)}$

$A_\varphi = \frac{2n\tau}{a\,sh\gamma}\frac{ch^{2m}y\, th^{2n}y}{1 + ch^{2m}y\, th^{2n}y}$. $m_2 + 1 = \frac{2}{1 + ch^{2m}y\, th^{2n}y}$, $m_2 - 1 = -\frac{2ch^{2m}y\, th^{2n}y}{1 + ch^{2m}y\, th^{2n}y}$

Мое $\vec{A}$: $A_\gamma = 0$, $A_\beta = -\frac{m\tau}{a}(m_2 + 1)$, $A_\varphi = -\frac{n\tau}{a\,sh\gamma}(m_2 - 1)$

$\vec{A}$ - Gladikowski'ya: $A_\gamma = 0$, $A_\beta = \frac{2M\tau}{a}(m_2 + 1)$, $A_\varphi = \frac{2N\tau}{a\,sh\gamma}(m_2 - 1)$

у Gladikowski имеется „2" из-за определения $\vec{B}$ и $\vec{A}$.

Т.е. мое $\vec{A}$ и $\vec{A}$ (Glad) совпадают с точностью до знака, $\vec{A}(мое) = -\vec{A}(Glad)$.

Мое $\vec{A} = 2\,Im\,(Z^* \nabla Z)$ соответствует $\vec{A}$ (wang'ю), но $\vec{A}(мое) = -\vec{A}$ (Glad corrected) см. 14/IV.

// Интересно, как Gladikowski'ят, которые не знали вида $m_2(\gamma)$, смог записать A_β, A_φ через $m_2(\gamma)$? $\tau = ch\gamma - \cos\beta$

(24/IV) С А.-Ю., Гловин в ресторане Бали.

(25/IV) Memory day, в hotel с П., Ю., Гловин. поездка в тор. Center. Mor". Отчет „пункт" экзамена.

(26/IV) Закончил и послал Якубову отчет GIU18 project (2019-2022). Составил и submitted Abstract „Спорт для всех" на SPIM'23, Prague.

(27/IV) Сделать фото: (1) ~~отчет~~ Шотландии Embassy (паспорт Анжела);

(2) Glas Insome тем К., П. (переделать 2 V) и Ю., А. (сделать позже);

3) подготовить Т-10 forms Ю. и А.; 4) COST-project отчет Оксане.

5) подать appl. Distinguished Scientist; 6) update SolSkyMgg program.

(7) Билеты в Poznan June 25 - July 8(9); 8) Сделать BP-paper перевести Окс.

$\vec{B} = rot\vec{A} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{\gamma} & \hat{\beta} & \hat{\varphi} \\ \frac{\tau}{a}\frac{\partial}{\partial\gamma} & \frac{\tau}{a}\frac{\partial}{\partial\beta} & \frac{\tau}{a\,sh\gamma}\frac{\partial}{\partial\varphi} \\ 0 & A_\beta & A_\varphi \end{vmatrix} = \hat{\gamma}\left[\frac{\tau}{a}\frac{\partial A_\varphi}{\partial\beta} - \frac{\tau}{a\,sh\gamma}\frac{\partial A_\beta}{\partial\varphi}\right] +$

$+\hat{\beta}\left[-\frac{\tau}{a}\frac{\partial}{\partial\gamma}A_\varphi\right] + \hat{\varphi}\left[\frac{\tau}{a}\frac{\partial}{\partial\gamma}A_\beta\right]$. $\vec{A} = \vec{A}(\gamma, \beta)$, $A_\gamma = 0$

$B_\gamma = \frac{\tau}{a}\frac{\partial A_\varphi}{\partial\beta}$, $B_\beta = -\frac{\tau}{a}\frac{\partial A_\varphi}{\partial\gamma}$, $B_\varphi = \frac{\tau}{a}\frac{\partial A_\beta}{\partial\gamma}$. $A_\beta = -\frac{m\tau}{a}(m_2 + 1)$, $A_\varphi = \frac{-n\tau}{a\,sh\gamma}(m_2 - 1)$

$B_\gamma = \frac{\tau}{a}\frac{\partial}{\partial\beta}\left[-\frac{n\tau}{a\,sh\gamma}(m_2 - 1)\right] = \frac{\tau}{a}\frac{(n)}{a\,sh\gamma}(m_2 - 1)\frac{\partial\tau}{\partial\beta} = -\frac{n\tau}{a^2 sh\gamma}(m_2 - 1)\sin\beta$

$B_\beta = -\frac{\tau}{a}\frac{\partial}{\partial\gamma}\left[-\frac{n\tau}{a\,sh\gamma}(m_2 - 1)\right] = \frac{n\tau}{a^2}\frac{\partial}{\partial\gamma}\left[\frac{\tau}{sh\gamma}(m_2 - 1)\right]$, $B_\beta(Glad) = -\frac{2N\tau^2}{a^2}\frac{\partial m_2}{\partial\gamma}$

$\frac{\partial}{\partial\gamma}\left[\frac{\tau}{sh\gamma}(m_2 + 1)\right] = \frac{(m_2 - 1)}{sh\gamma} + \frac{\tau(-1)}{sh^2\gamma}ch\gamma\,(m_2 - 1) + \frac{\tau}{sh\gamma}\frac{\partial m_2}{\partial\gamma}$.

$(m_2 - 1)\left[1 - \frac{ch\gamma}{sh^2\gamma}(ch\gamma - \cos\beta)\right] = \frac{(m_2 - 1)}{sh^2\gamma}\left[sh^2\gamma - ch^2\gamma + ch\gamma\cos\beta\right] = \frac{(m_2 + 1)}{sh^2\gamma}\left[ch\gamma\cos\beta - 1\right]$

B_β (мое) из $\vec{B} = rot\vec{A}$ и B_β (Glad) не совпадают.

$B_\varphi = \frac{\tau}{a}\frac{\partial}{\partial\gamma}\left[-\frac{m\tau}{a}(m_2 + 1)\right] = -\frac{m\tau}{a^2}\frac{\partial}{\partial\gamma}\left[\tau(m_2 + 1)\right] = -\frac{m\tau}{a^2}\left[sh\gamma\,(m_2 + 1) + \tau\frac{\partial m_2}{\partial\gamma}\right]$, $B_\varphi = \frac{2n\tau^2}{a^2}\frac{\partial m_2}{\partial\gamma}$ (Glad)

$B_\varphi(Glad) = \frac{2M\tau^2}{a^2}\frac{\partial m_2}{\partial\gamma}$. $B_\varphi(мое) = -\frac{m\tau}{a^2}sh\gamma\,(m_2 + 1) - \frac{m\tau^2}{a^2}\frac{\partial m_2}{\partial\gamma}$.

$B_\beta(Glad) = -\frac{2N}{a^2}\frac{\tau^2}{sh\gamma}\frac{\partial m_2}{\partial\gamma}$. $B_\beta(мое) = \frac{n\tau}{a^2}\frac{(m_2 - 1)}{sh^2\gamma}(ch\gamma\cos\beta - 1) + \frac{n\tau^2}{a^2 sh\gamma}\frac{\partial m_2}{\partial\gamma}$.

$B_\gamma(Glad) = 0$. $B_\gamma(мое) = -\frac{n\tau}{a^2 sh\gamma}(m_2 - 1)\sin\beta$. Вот чем да (!).

// $B_\gamma, B_\beta, B_\varphi$ у меня и Gladikowski не совпадают, даже учитывая множитель (-1).

Поиск в toroidal coordinates (η, β, φ)? Да, получим

$\text{rot}\vec{A} = \frac{\hat{\eta}}{h_\beta h_\varphi}\left[\frac{\partial}{\partial\beta}(h_\varphi A_\varphi) - \frac{\partial}{\partial\varphi}(h_\beta A_\beta)\right] + \frac{\hat{\beta}}{h_\eta h_\varphi}\left[\frac{\partial}{\partial\varphi}(h_\eta A_\eta) - \frac{\partial}{\partial\eta}(h_\varphi A_\varphi)\right] +$

$+ \frac{\hat{\varphi}}{h_\eta h_\beta}\left[\frac{\partial}{\partial\eta}(h_\beta A_\beta) - \frac{\partial}{\partial\beta}(h_\eta A_\eta)\right]$. $h_\eta = h_\beta = \frac{a}{\tau}$, $h_\varphi = \frac{a\,\text{sh}\eta}{\tau}$, $\tau = \text{ch}\eta - \cos\beta$.

$\text{rot}\vec{A} = \hat{\eta}\frac{\tau^2}{a^2\text{sh}\eta}\left[\frac{\partial}{\partial\beta}\left(a\frac{\text{sh}\eta}{\tau}A_\varphi\right) - \frac{\partial}{\partial\varphi}\left(\frac{a}{\tau}A_\beta\right)\right] + \hat{\beta}\frac{\tau^2}{a^2\text{sh}\eta}\left[\frac{\partial}{\partial\varphi}\left(\frac{a}{\tau}A_\eta\right) - \frac{\partial}{\partial\eta}\left(\frac{a\,\text{sh}\eta}{\tau}A_\varphi\right)\right] +$

$+ \hat{\varphi}\frac{\tau^2}{a^2}\left[\frac{\partial}{\partial\eta}\left(\frac{a}{\tau}A_\beta\right) - \frac{\partial}{\partial\beta}\left(\frac{a}{\tau}A_\eta\right)\right]$, $A_\eta = 0$, $\vec{A}(\eta,\beta)$ только, $m_z = m_z(\eta)$.

$B_\eta = \frac{\tau^2}{a^2\text{sh}\eta}\left[\frac{\partial}{\partial\beta}\left(\frac{\text{sh}\eta}{\tau}A_\varphi\right)\right] = \frac{\tau^2}{a\,\text{sh}\eta}\frac{\partial}{\partial\beta}\left[\frac{\text{sh}\eta}{\tau}\frac{(-n)\tau}{a\,\text{sh}\eta}(m_z - 1)\right] = \frac{-n\tau^2}{a^2\text{sh}\eta}\frac{\partial}{\partial\beta}(m_z - 1) = 0$

$B_\beta = \frac{\tau^2}{a^2\text{sh}\eta}(+a)\frac{\partial}{\partial\eta}\left[\frac{\text{sh}\eta}{\tau}\frac{(-n)\tau}{a\,\text{sh}\eta}(m_z - 1)\right] = \frac{\tau^2 n}{a^2\text{sh}\eta}\frac{\partial}{\partial\eta}(m_z - 1) = \frac{n\tau^2}{a^2\text{sh}\eta}\frac{\partial m_z}{\partial\eta}$

$B_\varphi = \frac{\tau^2}{a^2}a\frac{\partial}{\partial\eta}\left[\frac{1}{\tau}(+m)\frac{\tau}{a}(m_z + 1)\right] = -\frac{m\tau^2}{a^2}\frac{\partial}{\partial\eta}(m_z + 1) = -\frac{m\tau^2}{a^2}\frac{\partial m_z}{\partial\eta}$

т.е., с точностью до (-2) - Glebikowski to (1), $\vec{B}$(мое) совпадает с $\vec{B}$(Glebi.).

$\vec{A}\cdot\vec{B} = A_\beta B_\beta + A_\varphi B_\varphi = -\frac{m\tau}{a}(m_z + 1)\frac{n\tau^2}{a^2\text{sh}\eta}\frac{\partial m_z}{\partial\eta} + \frac{(-n)\tau}{a\,\text{sh}\eta}(m_z - 1)\frac{(-m)\tau^2}{a^2}\frac{\partial m_z}{\partial\eta}$

$\vec{A}\cdot\vec{B} = mn\frac{\tau^3}{a^3\text{sh}\eta}\left[-(m_z + 1) + (m_z - 1)\right]\frac{\partial m_z}{\partial\eta} = (-2)mn\frac{\tau^3}{a^3\text{sh}\eta}\frac{\partial m_z}{\partial\eta}$

$Q_H = -\frac{1}{(4\pi)^2}\int dV\,\vec{A}\cdot\vec{B} = -\frac{1}{(4\pi)^2}(-2)mn\int_0^\infty d\eta\int_0^{2\pi}d\beta\int_0^{2\pi}d\varphi\,\frac{a^3\text{sh}\eta}{\tau^3}\frac{\tau^3}{a^3\text{sh}\eta}\frac{\partial m_z}{\partial\eta} =$

$= \frac{2\cdot mn}{16\pi^2}(2\pi)(2\pi)\int_0^\infty d\eta\frac{\partial m_z}{\partial\eta} = mn\left(\frac{1}{2}\right)\left[m_z(\infty) - m_z(0)\right]$. $m_z(\infty) = -1$, $\eta = a$, $z = 0$

$Q_H = -mn$, где $m_z(0) = +1$, $m_z(\eta\to\infty) = -1$. $Q_H = mn\left[m_z(\infty) - m_z(0)\right]$ не зависит от вида $m_z(\eta)$ - профиля, но $\vec{A}$ и $\vec{B}$ зависят от вида $m_z(\eta)$, см. 10/IV

28/IV $A_\eta = 0$, $A_\beta = +\frac{m\tau}{a}(m_z + 1)$, $A_\varphi = -\frac{n\tau}{a\,\text{sh}\eta}(m_z - 1)$, $\eta = a\frac{\text{sh}\eta}{\tau}$, $z = a\frac{\sin\beta}{\tau}$, $\varphi = \varphi$.

$\vec{A}(A_\rho, A_\varphi, A_z)$ в цил. координатах, можно найти $\text{tg}\beta = \left(\frac{2za}{z^2 - a^2}\right)$, $\text{th}\eta = \frac{2\rho a}{(z^2 + a^2)}$

transforming (η,β,φ) toroidal to (ρ,φ,z) cylindrical system.

Сам. 30/III ($m_z = m_z(\eta)$, $A_\eta = 0$): $A_\rho = -\frac{1}{\tau}\sin\beta\,\text{sh}\eta\,A_\beta = -\frac{1}{\tau}\sin\beta\,\text{sh}\eta\frac{(-m)\tau}{a}(m_z + 1) = \frac{\sin\beta\,\text{sh}\eta}{a}m\times(m_z + 1)$

$A_\varphi = A_\varphi$, $A_z = \frac{1}{\tau}(\text{ch}\eta\cos\beta - 1)\cdot A_\beta = -\frac{m}{a}(m_z + 1)(\text{ch}\eta\cos\beta - 1)$ надо было $(\eta,\beta)\to(\rho,z)$.

Имеем (A_ρ, A_φ, A_z) через Z_1, Z_2 см. 7/III, (1,2) $Z_1 = \frac{1}{(1+\tau^2)}\left[1 - \tau^2 + 2i\tau\right]$, $Z_2 = \frac{2\rho e^{i\varphi}}{(1+\tau^2)}$.

$w = \frac{Z_2}{Z_1}$. $Z_1\to Z_1^m$, $Z_2\to Z_2^n$ и $w = Z_2^n/Z_1^m$.

$\tilde{Z}_1 = \frac{1}{\sqrt{N}}\frac{(1-\tau^2+2i\tau)^m}{(1+\tau^2)^m}$, $\tilde{Z}_2 = \frac{1}{\sqrt{N}}\frac{2^n\rho^n e^{in\varphi}}{(1+\tau^2)^n}$, $|\tilde{Z}_1|^2 + |\tilde{Z}_2|^2 = 1$ определяет N.

$N = N(\rho,z)$ и при расчете $\vec{A} = 2\text{Im}\left(\vec{Z}^+\nabla Z\right)$, $Z = \begin{pmatrix}\tilde{Z}_1\\ \tilde{Z}_2\end{pmatrix}$ надо учитывать $\frac{\partial N}{\partial\rho}$, $\frac{\partial N}{\partial z}$, получается

В тороид. (η,β,φ) работает 23/IV. $\tilde{Z}_1 = \frac{1}{\sqrt{N}}\frac{(-1)^m}{\text{ch}^m\eta}e^{-im\beta}$, $\tilde{Z}_2 = \frac{1}{\sqrt{N}}\text{th}^n\eta\,e^{in\varphi}$, $N = N(\eta)$,

$dN/d\eta \neq 0$ то не берем, т.к. A_η (зависимость от $dN/d\eta$) $= 0$, A_β, A_φ остаются

3/IV Без $N(\eta)$ имеем $A_\beta = -\frac{2mn\tau}{a\,\text{ch}^2\eta}$, $A_\varphi = \frac{2n\tau}{a\,\text{sh}\eta}\text{th}^{2n}\eta$ и нужно $\times\frac{1}{N(\eta)}$. т.к. $A_{\beta,\varphi}\sim(\tilde{Z}_{1,2})^2$

$N(\eta) = \frac{1}{\text{ch}^{2m}\eta} + \text{th}^{2n}\eta$. В цил. координатах $N(\rho,z) = \frac{|1-\tau^2+2i\tau|^{2m}}{(1+\tau^2)^{2m}} + \frac{2^{2n}\rho^{2n}}{(1+\tau^2)^{2n}}$ и

$dN/d\rho$, dN/dz будут присутствовать в A_β, A_φ, A_z будут присутствовать.

Имеем $A_\beta = \frac{m}{a}\text{sh}\eta\sin\beta(m_z + 1)$, $A_\varphi = -\frac{n}{a}(m_z - 1)$, $A_z = -\frac{m}{a}(\text{ch}\eta\cos\beta - 1)(m_z + 1)$ и

$(\eta,\beta)\to(\rho,z)$. Знаем (Корякин, см. и Wang '2019) что $A_\rho = -\frac{(1+m_z)}{\rho}\frac{\partial\chi}{\partial z}$, $A_z = (1+m_z)\frac{\partial\chi}{\partial\rho}$,

где $\varphi = n\varphi + \gamma(\rho,z)$, $\gamma(\rho,z) = m\beta$ находят $-\frac{\partial\chi}{\partial\rho} = \frac{m}{a}\text{sh}\eta\sin\beta$, и $-\frac{\partial\chi}{\partial z} = -\frac{m}{a}(\text{ch}\eta\cos\beta - 1)$.

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$\frac{\partial\beta}{\partial s} = -\frac{1}{a}\,\mathrm{sh}\gamma\sin\beta$ и $\frac{\partial\beta}{\partial z} = \frac{1}{a}(\mathrm{ch}\gamma\cos\beta - 1)$, $\beta(s,z) = \mathrm{arctg}\left(\frac{2za}{r^2 - a^2}\right)$.

$\frac{\partial\beta}{\partial s}(z/\bar{w}) = -\frac{1}{a}\frac{4rsa^2}{(r^2-a^2)^2 + 4z^2a^2}$, нижадато $\mathrm{sh}\gamma\sin\beta = \frac{4rsa^2}{[(s^2+z^2-a^2)^2 + 4z^2a^2]}$

$\mathrm{sh}\gamma\sin\beta = \frac{r^2}{a^2}sz$. $r^2 = (\mathrm{ch}\gamma - \cos\beta)^2 = \mathrm{ch}^2\gamma - 2\mathrm{ch}\gamma\cos\beta + \cos^2\beta = 1 + \mathrm{sh}^2\gamma - 2\mathrm{ch}\gamma\cos\beta + \cos^2\beta$, $r^2 = \frac{4a^2}{[(s^2+z^2-a^2)^2 + 4z^2a^2]}$ - gowno bomo. emerce.

$r = \mathrm{ch}\gamma - \cos\beta$, $\mathrm{tg}\beta = \left(\frac{2za}{r^2-a^2}\right)$, $\mathrm{th}\gamma = \frac{2sa}{(r^2+a^2)}$, $\frac{1}{\mathrm{ch}^2\gamma} + \mathrm{th}^2\gamma = 1$.

$\mathrm{ch}\gamma = \frac{1}{\sqrt{1-\mathrm{th}^2\gamma}}$, $\mathrm{tg}\beta = \frac{\sin\beta}{\cos\beta}$, $\mathrm{tg}^2\beta = \frac{\sin^2\beta}{\cos^2\beta} = \frac{1-\cos^2\beta}{\cos^2\beta} = \frac{1}{\cos^2\beta} - 1$

$\cos\beta = (\pm 1)\frac{1}{\sqrt{\mathrm{tg}^2\beta + 1}}$, $r = \frac{1}{\sqrt{1-\mathrm{th}^2\gamma}} - \frac{1}{\sqrt{\mathrm{tg}^2\beta+1}}$.

(29/IV) Написание Hopfion gauge manuscript. Final lunch с П.и Ю. Matagiukos.

(30/IV) Trip Porto → Tordesellas. Рест. с П.и Ю.

(1/V) Hopfion gauge MS. Current e-mails. Bmw. Сделать visas Ukr, Emb.

(2/VI) Meeting с преподавателями Carlos S. Maria? A. passport - May 30, 13^{20}, June 5, 12^{00}.

Library, с Ю., Julian. Julian в INTERMAG 2026 in Donostia, to be decided.

Skyrme с Nenad (positive ref. responses on Hopfion $\vec{G}$ - RRL); $\vec{G}$ in different coord. Systems.

Вт. 15, 17/IV 2022. Eq. Thiele $\dot{\vec{M}} = -\gamma\vec{M}\times\vec{H}$ (LLG), $f^G_i = (\vec{g}\times\vec{V})_i = \frac{\partial w}{\partial x_i}$, $w(\vec{M})$ - energy density in LLG, $\vec{H} = -\frac{\partial w}{\partial \vec{M}}$ effective field. $\frac{\partial}{\partial m_i} \to \frac{\partial}{\partial \vec{M}}$, $w(\vec{m}, \frac{\partial\vec{m}}{\partial x_i})$.

$g_{ij} = \frac{M_s}{\gamma}\vec{m}\cdot\left[\frac{\partial\vec{m}}{\partial x_i}\times\frac{\partial\vec{m}}{\partial x_j}\right]$ - gyrocoupling density, $g_i = -\frac{1}{2}\varepsilon_{ijk}g_{jk}$ - dual vector, gyrovector density $\vec{g}$. $\vec{G} = \int dV\vec{g}$ - gyrovector, $\vec{V} = d\vec{X}/dt$, $\vec{X}(t)$ - soliton center position.

$\vec{f}_G = \vec{g}\times\vec{V}$, gyroforce $\vec{F}_g = \vec{F}_G = \int dV\vec{g}\times\vec{V} = \vec{G}\times\vec{V}$. $\vec{g} = \vec{B}$ - emergent mag. field (с конт. до инвариантов). $\vec{G} = \int dV\vec{B}$, на в Thiele eq. go $\int dV(\ldots) = \int d^3r(\ldots)$

Вторая $\vec{f}_G = \vec{B}\times\vec{V}$. $\vec{F}_G = \int dV\vec{f}_G = \int dV[\vec{B}\times\vec{V}]$ такого и вывели. $\vec{V} = \frac{d\vec{X}}{dt}$.

$|\vec{f}_G| = |\vec{B}\times\vec{V}| = |\vec{B}||\vec{V}|\sin(\vec{V}\vec{B})$. In cyl. coordinates (s,φ,z), $\vec{B} = (B_s, B_\varphi, B_z)$, $\vec{B} = \vec{B}(r)$.

Очевидно $|\vec{F}_G| = |\vec{G}\times\vec{V}| \neq |\int dV\vec{f}_G| \neq \int dV|\vec{f}_G|$. Gyrovection $\vec{G}$

$$\vec{B}\times\vec{V} = \begin{vmatrix} \hat{s} & \hat{\varphi} & \hat{z} \\ B_s & B_\varphi & B_z \\ V_s & V_\varphi & V_z \end{vmatrix} = \hat{s}[B_\varphi V_z - B_z V_\varphi] + \hat{\varphi}[B_z V_s - V_z B_s] + \hat{z}[B_s V_\varphi - B_\varphi V_s]$$

Axially sym. hopfion $B_s, B_\varphi, B_z = f(s,z)$.

$f^G_s = B_\varphi V_z - B_z V_\varphi$, $f^G_\varphi = B_z V_s + V_z B_s$, $f^G_z = B_s V_\varphi - B_\varphi V_s$, см. 2/IV.

$B_\varphi, V_\varphi = -V_x\sin\varphi + V_y\cos\varphi$, $V_\rho = V_x\cos\varphi + V_y\sin\varphi$

$F^G_s \to F^G_s = \int dV[B_\varphi V_z - B_z V_\varphi] = \int ds\,s\int d\varphi\,dz\left[B_\varphi(s,z)V_z - B_z(s,z)(-V_x\sin\varphi + V_y\cos\varphi)\right] =$

$= \int dV B_\varphi\cdot V_z + 0$ (после $\int d\varphi(\ldots)$) $= G_\varphi V_z$ т/ж $G_\varphi = \int dV B_\varphi$, $dV = s\,ds\,d\varphi\,dz$.

$F^G_\varphi = \int dV[B_z V_s - V_z B_s] = \int ds\,s\,d\varphi\,dz\left[B_z(s,z)(V_x\cos\varphi + V_y\sin\varphi) - V_z B_s\right] =$

$= -V_z\int dV B_s = -G_s V_z$

$F^G_z = \int dV[B_s V_\varphi - B_\varphi V_s] = \int ds\,s\,d\varphi\,dz\left[B_s(s,z)(-V_x\sin\varphi + V_y\cos\varphi) - B_\varphi(V_x\cos\varphi + V_y\sin\varphi)\right] =$

$= 0$, т.к. V_x, V_y, V_z не зависят от φ. $\vec{F}_G = (G_\varphi V_z, -G_s V_z, 0)$

Но $G_s = 0$ в силу симметрии, $\vec{F}_G = (G_\varphi V_z, 0, 0)_{\hat{s},\hat{\varphi},\hat{z}} = G_\varphi V_z\hat{s}$ т/ж $\int ds\,s\,d\varphi\int dz(\ldots)$

$V_s\hat{\varphi} - \hat{s}V_\varphi = (V_x\cos\varphi + V_y\sin\varphi)(\hat{x}\cos\varphi + \hat{y}\sin\varphi) - (\hat{x}\cos\varphi + \hat{y}\sin\varphi)(-V_x\sin\varphi + V_y\cos\varphi) =$

$= \hat{x}[-V_x\cos\varphi\sin\varphi - V_y\sin^2\varphi + V_x\cos\varphi\sin\varphi - V_y\cos^2\varphi] +$

$+ \hat{y}[V_x\cos^2\varphi + V_y\cos\varphi\sin\varphi + V_x\sin^2\varphi - V_y\sin\varphi\cos\varphi] = -\hat{x}V_y + \hat{y}V_x$.

$\vec{F}_G = G_\varphi V_z\hat{s}$ направлена 2/IV, где $(V_y\hat{\varphi} - \hat{s}V_\varphi)$ равен исходно т/ж $\int ds\,s\,d\varphi\int dz(\ldots)$

Gyrodoxa $G_s \equiv 0$, а G_z входит в $\vec{F}_G$ не зависит - т.к. $\int d\varphi V_\varphi = 0$, $\int d\varphi V_s = 0$.

2/IV нужен, что $\vec{G} = \int dV\vec{B}(r)$ и $G_z = \int dV B_z$ - разные, сейчас имею в виду конкр. конфиг.

Escaneado con CamScanner

(3/X) Noemie 11 offers to be section Chairs SolStorMag - 2023. Все согласились, кроме Tsenkovych просит перенести это на Friday, June 23.

(4/X) Dr Bocchi Referees Koppion G manuscript - PRL и переделка MS. Arc. e-mails

(5/X) Tax office Tolosa visit. Моя и Л. Tax Return Forms. Вернул S. 4199,90 €

Andrea уходит из dependants. т.к. он получает salary в 2020 as autonoma.

Noemie to coauthors Revised Koppion G manuscript и отвечает Referees.

(6/X) Зачитал Report task #6 POLS project и послал to Olena и Maciej

Вместе с Resolution.

Vector potential of toroidal Koppion $\vec{A}$, $\vec{A}(r,\theta,z) = \vec{A}$ (Wang PRL 2019)

$A_\beta = -(1+m_z)\frac{\partial \gamma}{\partial \beta}$, $A_\varphi = \frac{n}{\beta}(1-m_z)$, $A_z = -(1+m_z)\frac{\partial \gamma}{\partial z}$.

Helicity $\gamma(\beta,z) = m\beta(\beta,z)$, β - poloidal angle. $\beta(\beta,z) = \operatorname{arctg}\left(\frac{2za}{r^2-a^2}\right)$, $r^2 = \beta^2+z^2$

$\frac{\partial \beta}{\partial \beta} = -4\frac{\beta z a}{[z^2+(\beta+a)^2][z^2+(\beta-a)^2]}$, $\frac{\partial \beta}{\partial z} = 2a\frac{(\beta^2-z^2-a^2)}{[z^2+(\beta+a)^2][z^2+(\beta-a)^2]}$

$[-][-] = [z^2+\beta^2+a^2+2a\beta][z^2+\beta^2+a^2-2a\beta] = [r^2+a^2+2a\beta][r^2+a^2-2a\beta]$

$(r^2-a^2)^2 + 4z^2a^2 = r^4+a^4-2r^2a^2+4r^2a^2 = r^4+a^4+2r^2a^2-2\beta^2a^2$

$[-] = (r^2+a^2)^2 - 2\cdot 2a^2\beta^2 = r^4+a^4+2r^2a^2-4a^2\beta^2 = r^4+a^4+2r^2a^2-2\beta^2a^2$

$[z^2+(\beta+a)^2][z^2+(\beta-a)^2] = (r^2-a^2)^2+4z^2a^2$ или $(r^2-1)^2+4z^2$ in units a^4.

$\frac{\partial \beta}{\partial \beta} = -\frac{4}{a}\frac{\beta z}{[(r^2-1)^2+4z^2]}$, $\frac{\partial \beta}{\partial z} = \frac{2}{a}\frac{(\beta^2-z^2-1)}{[(r^2-1)^2+4z^2]} = \frac{2}{a}\frac{(r^2-1-2z^2)}{[(r^2-1)^2+4z^2]}$

$A_\beta = -m(1+m_z)\frac{\partial \beta}{\partial \beta}$, $A_\varphi = \frac{n}{\beta}(1-m_z)$, $A_z = -m(1+m_z)\frac{\partial \beta}{\partial z}$

$m_z(y) = \frac{1-ch^{2m}y\ th^{2n}y}{1+ch^{2m}y\ th^{2n}y}$ for any m,n. $th\,y = \frac{2\beta a}{(r^2+a^2)}$, $r^2 = \beta^2+z^2$.

$\vec{A}(A_\beta, A_\varphi, A_z)$ изображено для any (m,n) в цил. coordinates (β, z, φ).

Для n/m $m=1$, $n=1$ (7/X), $m_z = 1-2th^2y$, $A_\beta = \frac{2z\beta}{(1+r^2)^2}\frac{1}{a}$, $A_\varphi = \frac{4\beta}{(1+r^2)^2}\frac{1}{a}$, $A_z = 4\frac{(1-\beta^2+z^2)}{(1+r^2)^2}\frac{1}{a}$

$|\vec{A}|^2 = A_\beta^2+A_\varphi^2+A_z^2 = \left(\frac{4}{a}\right)^2\frac{1}{(1+r^2)^4}\left[(2z\beta)^2+(2\beta)^2+(1-\beta^2+z^2)^2\right] =$

$= (\cdot)^2[4z^2\beta^2+4\beta^2+(z^2+1)^2+\beta^4-2\beta^2(1+z^2)] = (\cdot)^2[(z^2+1)^2+\beta^4+2\beta^2(1+z^2)] =$

$= (\cdot)^2(z^2+1+\beta^2)^2 = \left(\frac{4}{a}\right)^2\frac{1}{(1+r^2)^4}(1+r^2)^2 = \left(\frac{4}{a}\right)^2\frac{1}{(1+r^2)^2}$.

Corrected Glad. 97 [т.е. $\vec{A}, \vec{B}$ - Glad. $\times(-\frac{1}{2})$] : $A_\varphi = -\frac{1}{a}\frac{r}{sh\gamma}(m_z-1)$, $A_\beta = -\frac{1}{a}\tau(m_z+1)$, $A_\gamma = 0$ ($M=N=1$)

$|\vec{A}|^2(Glad) = \frac{1}{a^2}\frac{r^2}{sh^2y}(m_z-1)^2 + \frac{r^2}{a^2}(m_z+1)^2$, $m_z(M=1,N=1) = 1-2th^2y$

$|\vec{A}|^2 = \frac{r^2}{a^2}\left[\frac{1}{sh^2y}(m_z-1)^2+(m_z+1)^2\right] = \frac{r^2}{a^2}\left[\frac{1}{sh^2y}4th^4y + 2^2(1-th^2y)^2\right] = 4\frac{r^2}{a^2}\times$

$\times\left[\frac{1}{sh^2y}\frac{sh^4y}{ch^4y} + \frac{1}{ch^4y}\right] = 4\frac{r^2}{a^2}\frac{1}{ch^4y}[sh^2y+1] = 4\frac{r^2}{a^2}\frac{1}{ch^2y}$. $r = ch\gamma - \cos\beta$

Цил. coordinates $|\vec{A}|^2 = \left(\frac{4}{a}\right)^2\frac{1}{(1+r^2)^2}$. тороидал coord. $|\vec{A}|^2 = 4\frac{r^2}{a^2}\frac{1}{ch^2y}$.

$r^2 = \beta^2+z^2 = \frac{sh^2y}{r^2}+\frac{\sin^2\beta}{r^2}$, $1+r^2 = 1+\frac{1}{r^2}(sh^2y+\sin^2\beta) = \frac{1}{r^2}[r^2+sh^2y+\sin^2\beta] = \frac{1}{r^2}[ch^2y-2\cos\beta ch\gamma+\cos^2\beta+sh^2y+\sin^2\beta] = \frac{1}{r^2}[ch^2y-2\cos\beta ch\gamma+1+sh^2y] =$

$= \frac{1}{r^2}2ch\gamma[ch\gamma-\cos\beta] = \frac{2ch\gamma}{r^2}$. $|\vec{A}|^2 = \left(\frac{4}{a}\right)^2\frac{1}{(1+r^2)^2} = \left(\frac{4}{a}\right)^2\frac{1}{\left[\frac{2}{r}ch\gamma\right]^2} = \frac{4}{a^2}\frac{r^2}{ch^2y}$

$1+r^2 = \frac{2}{r}ch\gamma$

Т.е. преобразование $\vec{A}(A_\gamma, A_\beta, A_\varphi) \to \vec{A}(A_\beta, A_\varphi, A_\varphi)$ сохраняет $|\vec{A}|^2$.

$|\vec{A}|^2 = \frac{16}{a^2}\frac{1}{(1+r^2)^2} = \frac{4}{a^2}\frac{r^2}{ch^2y}$

Em. field: $B_\beta = \frac{32}{a^2}\frac{\beta z}{(1+r^2)^3}$, $B_\varphi = \frac{16}{a^2}\frac{(1+z^2-\beta^2)}{(1+r^2)^3}$, $B_\varphi = \frac{32}{a^2}\frac{\beta}{(1+r^2)^3}$

$|\vec{B}|^2 = \left(\frac{16}{a^2}\right)^2\frac{1}{(1+r^2)^{3\cdot 2}}[(2\beta z)^2+(1+z^2-\beta^2)^2+(2\beta)^2] = \left(\frac{16}{a^2}\right)^2\frac{1}{(1+r^2)^4}$

Escaneado con CamScanner

Corrected $\vec{B}$ (Glad.) [t.e. $\times(-\frac{1}{2})$]: $B_\varphi = -\frac{1}{a^2}\tau^2\frac{\partial w}{\partial \eta}$, $B_\eta = \frac{1}{a^2}\frac{\tau^2}{\sin\eta}\frac{\partial w}{\partial \varphi}$, $B_\tau = 0$, $w = m_2$. $(M=1, N=1)$

$|\vec{B}|^2 = B_\varphi^2 + B_\eta^2 + B_\tau^2 = \frac{\tau^4}{a^4}\left(\frac{\partial m_2}{\partial \eta}\right)^2\left[1+\frac{1}{\sinh^2\eta}\right] = \frac{\tau^4}{a^4}\left(\frac{\partial m_2}{\partial \eta}\right)^2\frac{1}{\sinh^2\eta}\cosh^2\eta$.

$\frac{\partial m_2}{\partial \eta} = \frac{\partial}{\partial \eta}(1-2\tanh^2\eta) = (-2)\frac{\partial}{\partial \eta}\tanh^2\eta = (-2)\,2\tanh\eta\cdot\frac{1}{\cosh^2\eta} = -4\frac{\tanh\eta}{\cosh^2\eta}$.

$|\vec{B}|^2 = \frac{\tau^4}{a^4}16\frac{\tanh^2\eta}{\cosh^4\eta}\frac{\cosh^2\eta}{\sinh^2\eta} = \frac{16}{a^4}\tau^4\frac{1}{\cosh^4\eta}$ in toroidal coord.

$|\vec{B}|^2_{(\text{Cyl.})} = \frac{(16)^2}{a^4}\frac{1}{(1+\tau^2)^4} = \frac{(16)^2}{a^4}\frac{1}{\left(\frac{2}{\tau}\cosh\eta\right)^4} = \frac{16}{a^4}\frac{\tau^4}{\cosh^4\eta}$. $(1+\tau^2) = \frac{2}{\tau}\cosh\eta$.

t.e. $\vec{B}(B_\eta, B_\varphi, B_\tau) \to \vec{B}(B_\varphi, B_\tau, B_z)$ compared $|\vec{B}|^2 = \left(\frac{4}{a}\right)^4\frac{1}{(1+\tau^2)^4} = \frac{16}{a^4}\frac{\tau^4}{\cosh^4\eta}$.

$|\vec{G}|^2$ не сохраняется чем $(\eta, \varphi, \tau) \to (\varphi, \tau, z) \to (x, y, z)$ coord. transformations.

$|\vec{B}(\vec{r})|^2 = \frac{4^4}{a^4(1+\tau^2)^4} = (4a)^4\frac{1}{(a^2+\tau^2)^4}$ в разных единицах.

$G_\alpha = \int dV B_\alpha$, $\sum_\alpha G_\alpha^2 = |\vec{G}|^2 = \sum_\alpha\left(\int dV B_\alpha\right)^2 = \sum_\alpha\left(\overline{B}_\alpha\right)^2$, $|\vec{B}|^2 = \sum_\alpha B_\alpha^2$.

$|\vec{B}|^2 - |\vec{G}|^2 = \sum_\alpha B_\alpha^2 - \sum_\alpha\left(\overline{B}_\alpha\right)^2 = \sum_\alpha\left[B_\alpha^2 - \left(\overline{B}_\alpha\right)^2\right]$, $G_\alpha = \overline{B}_\alpha$.

$\mathcal{D}(\vec{B}) = \sum_\alpha(B_\alpha - \overline{B}_\alpha)^2 = \sum_\alpha\left[B_\alpha^2 + \left(\overline{B}_\alpha\right)^2 - 2B_\alpha\overline{B}_\alpha\right] = |\vec{B}|^2 + |\vec{G}|^2 - 2\vec{B}\cdot\vec{G}$

(7/IX) Вернемся к анализу $\vec{G}$ - фурье-вектор в произвольной координатной системе с единичными векторами $\vec{e}_\alpha(\vec{r})$ см 2/IX.

Цилиндр. координаты $\vec{e}_\varphi(\vec{r}) = -\sin\varphi\,\hat{x} + \cos\varphi\,\hat{y}$, $\vec{e}_\eta(\vec{r}) = \hat{x}\cos\varphi + \sin\varphi\,\hat{y}$, $\hat{z} = \hat{z}$

$\vec{e}_\eta(\vec{r})\cdot\vec{e}_\eta(\vec{r}') = \cos(\varphi-\varphi')$, $\vec{e}_\varphi(\vec{r})\cdot\vec{e}_\varphi(\vec{r}') = \cos(\varphi-\varphi')$, $G_\alpha = \int dV B_\alpha$

$\vec{e}_\eta(\vec{r})\cdot\vec{e}_\varphi(\vec{r}') = \sin(\varphi-\varphi')$, $\vec{e}_\varphi(\vec{r})\cdot\vec{e}_\eta(\vec{r}') = -\sin(\varphi-\varphi')$.

$\vec{G} = \int dV\vec{B}$, $G_\alpha(\vec{r}) = \vec{G}\cdot\vec{e}_\alpha(\vec{r}) = \int dV'\sum_\beta B_\beta(\vec{r}')\left(\vec{e}_\alpha(\vec{r})\cdot\vec{e}_\beta(\vec{r}')\right) =$

$G_\eta = \int dV'\sum_\beta B_\beta(\vec{r}')\left(e_\eta(\vec{r})\cdot e_\beta(\vec{r}')\right) = \int dV'\left[B_\eta\left(\vec{e}_\eta(\vec{r})\cdot\vec{e}_\eta(\vec{r}')\right) + B_\varphi\left(e_\eta(\vec{r})\cdot e_\varphi(\vec{r}')\right) +\right.$

$\left. + B_z\underbrace{\left(e_\eta(\vec{r})\cdot\hat{z}\right)}_{=0}\right] = \int dV'\left[B_\eta(\vec{r}')\cos(\varphi-\varphi') + B_\varphi(\vec{r}')\sin(\varphi-\varphi')\right]$, $\vec{B}(\vec{r}) = \vec{B}(\eta, z)$ аксиальная симметрия.

$\int d\varphi' B_\eta(\eta', z')\cos(\varphi-\varphi') = 0$ и $\int d\varphi' B_\varphi(\eta' z')\sin(\varphi-\varphi') = 0$, $\vec{G}_\eta = 0$ (!), хотя $G_\eta \neq 0$.

$G_\varphi = \int dV'\sum_\beta B_\beta(\vec{r}')\left(\vec{e}_\varphi(\vec{r})\cdot\vec{e}_\beta(\vec{r})\right) = \int dV'\left[B_\varphi\left(e_\varphi(\vec{r})\cdot e_\varphi(\vec{r}')\right) + B_\eta\left(e_\varphi(\vec{r})\cdot e_\eta(\vec{r}')\right)\right] =$

$= \int dV'\left[B_\varphi(\vec{r}')\cos(\varphi-\varphi') + B_\eta(\vec{r}')(-1)\sin(\varphi-\varphi')\right] = 0$, $\vec{G}_\varphi = 0$, т.к. $B(\vec{r}') = \vec{B}(\eta' z')$.

$G_z = G_z \neq 0$ в бинте sample. $G_\eta = 0, G_\varphi = 0$ и $\vec{G} = (0, 0, G_z)$.

$G_x = \vec{G}\cdot\hat{x} = \int dV'\sum_\beta B_\beta(\vec{r}')\left(\hat{x}\cdot\vec{e}_\beta(\vec{r}')\right) = \int dV'\left[B_\eta\cos\varphi + B_\varphi(-1)\sin\varphi\right] = 0$ чем $\int d\varphi(\ldots)$

т.к. $\vec{B} = \vec{B}(\eta, z)$.

в цилиндрических (η, φ, z) и Декартовых (x, y, z) координатах $\vec{G} = (0, 0, G_z)$ чем

дословно $\vec{G} = \int dV\vec{B}$.

(8-9/V) Референса Hopfion-$\vec{G}$ manuscript и replies referees.

(10/V) Submitted Hopfion $\vec{G}$ ms в RRL ещё сразу принимать к печати.

(11/V) Paperpaper (scanned), travel report Азата и отдел Eli. Рец. и Kaskantzis с N.N., Julian и его ученик. Emails. Extra figs Hopfion-ms новые улучшенные Figs. to RRL.

(12/V) Референса SciSkyMag'23. Вопрос к поездке в Madrid

(14-16/V) Trip to Madrid, Hotel "Ozuna", Флоренция Ант. пассажир на 1 под, до June 6, 2024. Принимать Proofs Hopfion RRL paper.

(17/V) В Decision on a приглашение, Lourdes уехала на неделю. Сдали Boe на обед и 13 € на подарок. Встреча с Grzegorz - numer report по SW $w(\vec{r})$ in square dot arrays.

(18/V) Proofs RRL - Hopfion paper и PRA - BP paper с Термодин.

Oral talk in Laguardia June 1, Quantum Center., Hope D. не сохр. Hood E. (West. Un.)

Escaneado con CamScanner

(19/V) Перемисла с Ruben (travel reports и GPU) и EQ: (PM'2023 conf. see).

Hopf index Q_H как Jacobian перехода от тороидал (γ,β,φ) к (ρ,φ,z) цилиндрические координаты (Ch. 10/IV и Burke '2021).

В переменных (γ,θ,Φ) можно и $Q_H = -\frac{2}{(4\pi)^2}\int d\gamma \int d\theta \sin\theta \int d\Phi =$

$= \left|\gamma = m\beta,\ \beta\in[-\pi,\pi],\ \Phi = n\varphi + \gamma(\beta,z) \to \Phi = n\varphi,\ \text{т.к. } \gamma \text{ уже } 2\pi\text{-пер.},\ \varphi=[0,2\pi]\right| =$

$= -\frac{2}{(4\pi)^2} m \int_{-\pi}^{\pi} d\beta \int_0^{2\pi} d\theta \sin\theta \cdot n \int_0^{2\pi} d\varphi = -\frac{2mn}{(4\pi)^2}\cdot 2\pi\cdot 2\pi\,(+\cos\theta)\Big|_0^{\pi} = -\frac{8mn\pi^2}{16\pi^2}\left[1-(-1)\right] =$

$= -\frac{16mn\pi^2}{16\pi^2} = mn(-1)$, $Q_H = -mn$ (!) как у Pelekovski '1997 ? Да.

Поправки $\vec{A} = -2\nabla\gamma + \vec{A}^e$, $\vec{A} = (1-\cos\theta)\nabla\Phi$ из Gu 'EPL'2016.

$\int d\gamma \int d\theta \sin\theta \int d\Phi = \int dV_s \sin\theta \det J\begin{pmatrix}\gamma,\theta,\Phi\\ s,\varphi,z\end{pmatrix} = \int dV_s \sin\theta \det J\begin{pmatrix}\gamma,\theta,\varphi\\ s,\varphi,z\end{pmatrix}$

$\det J = \begin{pmatrix}\gamma_s & 0 & \gamma_z\\ \theta_s & 0 & \theta_z\\ \Phi_s & \frac{1}{s}\Phi_\varphi & \Phi_z\end{pmatrix} = \gamma_z\theta_s\frac{1}{s}\Phi_\varphi - \gamma_s\theta_z\frac{1}{s}\Phi_\varphi = \frac{1}{s}\frac{\partial\Phi}{\partial\varphi}\left[\frac{\partial\gamma}{\partial z}\frac{\partial\theta}{\partial s} - \frac{\partial\gamma}{\partial s}\frac{\partial\theta}{\partial z}\right]$

$\int d\gamma d\theta \sin\theta d\Phi = \int dV_s \sin\theta \frac{1}{s}\frac{\partial\Phi}{\partial\varphi}\left[\frac{\partial\gamma}{\partial z}\frac{\partial\theta}{\partial s} - \frac{\partial\gamma}{\partial s}\frac{\partial\theta}{\partial z}\right] = \left|\Phi = n\varphi + \underbrace{\text{const}}_{\text{fix } z} \right| =$

$= n\int ds\, s\int d\varphi \int dz \frac{1}{s}\sin\theta\left[\frac{\partial\gamma}{\partial z}\frac{\partial\theta}{\partial s} - \frac{\partial\gamma}{\partial s}\frac{\partial\theta}{\partial z}\right] = 2\pi n\int ds\int dz \sin\theta\left[\frac{\partial\gamma}{\partial z}\frac{\partial\theta}{\partial s} - \frac{\partial\gamma}{\partial s}\frac{\partial\theta}{\partial z}\right]$

$Q_H = -\frac{2}{(4\pi)^2}\int d\gamma \int d\theta \sin\theta \int d\Phi = -\frac{2}{(4\pi)^2} 2\pi n \int ds \int dz\left[\frac{\partial\gamma}{\partial z}\frac{\partial\theta}{\partial s} - \frac{\partial\gamma}{\partial s}\frac{\partial\theta}{\partial z}\right] =$

$= \frac{n}{4\pi}\int ds\int dz\left[\frac{\partial\gamma}{\partial s}\frac{\partial\theta}{\partial z} - \frac{\partial\gamma}{\partial z}\frac{\partial\theta}{\partial s}\right]$ — это и помогло Q_H (Wang '2021).

Q_H (Wang) переверна: $Q_H(W) = \frac{1}{(4\pi)^2}\int dV \vec{A}\cdot\vec{B} = \frac{1}{(4\pi)^2}\int dV\left[+(1+\cos\theta)\frac{\partial\gamma}{\partial s}\frac{n}{s}\sin\theta\frac{\partial\theta}{\partial z} + \right.$

$+\frac{n}{s}(1-\cos\theta)\sin\theta\left[\theta_s\gamma_s - \theta_s\gamma_z\right] - (1+\cos\theta)\frac{\partial\gamma}{\partial z}\frac{n}{s}\sin\theta\frac{\partial\theta}{\partial s}\Big] = \frac{1}{(4\pi)^2}\int dV \frac{n}{s}\sin\theta\times$

$\times\left[(1+\cos\theta)\gamma_s\theta_z + (1-\cos\theta)(\theta_z\gamma_s - \theta_s\gamma_z) - (1+\cos\theta)\gamma_z\theta_s\right] = \frac{n}{(4\pi)^2}\int ds\int_0^{2\pi} d\varphi\int dz\,\frac{\sin\theta}{s}\times$

$\times\left[2(\gamma_s\theta_z - \gamma_z\theta_s)\right] = \frac{n}{4\pi}\int ds\int dz\sin\theta\left[\frac{\partial\gamma}{\partial s}\frac{\partial\theta}{\partial z} - \frac{\partial\gamma}{\partial z}\frac{\partial\theta}{\partial s}\right]$ — верно.

В тороидальных координатах $\vec{A}\cdot\vec{B} = -2mn\frac{c^3}{a^3 s h\eta}\frac{\partial m_z}{\partial\gamma}$, $Q_H(\text{top}) = +\frac{1}{(4\pi)^2}\int dV \vec{A}\cdot\vec{B} =$

$= \frac{1}{(4\pi)^2}\int d\gamma\int d\alpha\int d\beta\,\frac{a^3 sh\eta}{c^3}\left(-2mn\right)\frac{c^3}{a^3 sh\eta} = -\frac{2mn}{(4\pi)^2}\int_0^{\infty} d\eta\int_0^{2\pi} d\beta\int_0^{2\pi} d\varphi\,\frac{\partial m_z}{\partial\eta} = \left|\begin{matrix}m_z(\infty) = -1\\ m_z(0) = +1\end{matrix}\right| =$

$= -\frac{2mn}{(4\pi)^2}\int_1^{-1} dm_z\int d\beta\int d\varphi = \int dm_z\, d\beta\int d\alpha\cdot\gamma$, $\gamma = -\frac{2mn}{(4\pi)^2} = \text{const}$. $m_z = m_z(\eta)$.

$\int d\gamma\int d\theta\sin\theta\int d\Phi = -mn\int d\beta\int dm_z\int d\alpha = -mn\int dV_s\, J\begin{pmatrix}m_z,\beta,\varphi\\ s,\varphi,z\end{pmatrix} =$

$= -mn\int ds\, s\int d\alpha\int dz\,\frac{1}{s}\left[\frac{\partial\beta}{\partial s}\frac{\partial m_z}{\partial z} - \frac{\partial\beta}{\partial z}\frac{\partial m_z}{\partial s}\right] = -mn\cdot 2\pi\int ds\, s\int dz\left[\frac{\partial\beta}{\partial s}\frac{\partial m_z}{\partial z} - \frac{\partial\beta}{\partial z}\frac{\partial m_z}{\partial s}\right]$

$Q_H = -\frac{2}{(4\pi)^2}\int d\gamma\int d\theta\sin\theta\int d\Phi = \frac{2mn}{(4\pi)^2}\int dV_s\, J\begin{pmatrix}m_z,\beta,\varphi\\ s,\varphi,z\end{pmatrix}$, $J = \frac{1}{s}\left[\frac{\partial\beta}{\partial s}\frac{\partial m_z}{\partial z} - \frac{\partial\beta}{\partial z}\frac{\partial m_z}{\partial s}\right]$

$Q_H = \frac{2mn}{(4\pi)^2}\int ds\, s\int d\alpha\int dz\,\frac{1}{s}\left[\frac{\partial\beta}{\partial s}\frac{\partial m_z}{\partial z} - \frac{\partial\beta}{\partial z}\frac{\partial m_z}{\partial s}\right] = \frac{mn}{4\pi}\int ds\int dz\left[\frac{\partial\beta}{\partial s}\frac{\partial m_z}{\partial z} - \frac{\partial\beta}{\partial z}\frac{\partial m_z}{\partial s}\right]$

верно с учетом $\frac{\partial\gamma}{\partial s} = m\frac{\partial\beta}{\partial s}$, $\frac{\partial\gamma}{\partial z} = m\frac{\partial\beta}{\partial z}$, $\gamma = m\beta + \text{const}$.

$\nabla\gamma\cdot\left[\nabla\theta\times\nabla\Phi\right] = \nabla\gamma\cdot\left[\nabla\theta\times\left(\frac{\partial\Phi}{\partial s}\hat{s} + \frac{n}{s}\hat{\varphi}\right)\right] = \frac{n}{s}\nabla\gamma\left[\nabla\times\theta\right] = \frac{n}{s}\hat{\varphi}\cdot\left[\nabla\theta\times\nabla\gamma\right] =$

$= \frac{n}{s}\hat{\varphi}\cdot\left(\frac{\partial\gamma}{\partial s}\hat{s} + \frac{\partial\gamma}{\partial z}\hat{z}\right)\times\left(\frac{\partial\theta}{\partial s}\hat{s} + \frac{\partial\theta}{\partial z}\hat{z}\right) = \frac{n}{s}\hat{\varphi}\cdot\left[\frac{\partial\theta}{\partial s}\frac{\partial\gamma}{\partial z}\hat{\varphi} + \frac{\partial\gamma}{\partial z}\frac{\partial\theta}{\partial s}\hat{\varphi}\right] = \left[\frac{\partial\theta}{\partial s}\frac{\partial\gamma}{\partial z} - \frac{\partial\gamma}{\partial s}\frac{\partial\theta}{\partial z}\right]\frac{n}{s}$

$Q_H = +\frac{2}{(4\pi)^2}\int dV\vec{A}\cdot\vec{B} = \frac{1}{(4\pi)^2}(-2)\int dV\sin\theta\frac{n}{s}\left[\frac{\partial\gamma}{\partial z}\frac{\partial\theta}{\partial s} - \frac{\partial\gamma}{\partial s}\frac{\partial\theta}{\partial z}\right] = \frac{(-2)}{(4\pi)^2}\int ds\, s\int d\varphi\int dz\times$

$\times\frac{n}{s}\left[\gamma_z\theta_s - \theta_z\gamma_s\right] = \frac{n}{4\pi}\int ds\, dz\left[\frac{\partial\theta}{\partial s}\frac{\partial\gamma}{\partial z} - \frac{\partial\theta}{\partial z}\frac{\partial\gamma}{\partial s}\right] = Q_H$ (Wang) — верно.

D: $S = \frac{1}{4\pi}\int d\theta\sin\theta\int d\Phi = -\frac{1}{4\pi}\int dm_z\int d\Phi = \left|\Phi = n\varphi + \Phi_0\right| = -\frac{2\pi n}{4\pi}\left[m_z(\infty) - m_z(0)\right] =$

$= -\frac{2\pi n}{4\pi}\left[-p - p\right] = pn$, $p = m_z(0)$ is sky-rain polarity. $S = pn$.

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$\int d\theta \sin\theta \int d\varphi = \int ds\, \sin\theta\, J(\theta,\varphi;s,\varphi)$, $J(\theta,\varphi;s,\varphi) = \det\begin{pmatrix} \frac{\partial\theta}{\partial s} & \frac{\partial\theta}{\partial\varphi}\frac{1}{s} \\ \frac{\partial\varphi}{\partial s} & \frac{1}{s}\frac{\partial\varphi}{\partial\varphi} \end{pmatrix} =$

$= \frac{1}{s}\left[\frac{\partial\theta}{\partial s}\frac{\partial\varphi}{\partial\varphi} - \frac{\partial\varphi}{\partial s}\frac{\partial\theta}{\partial\varphi}\right] = \frac{n}{s}\frac{\partial\theta}{\partial s}$.

$S = \frac{1}{4\pi}\int d\theta \int d\varphi \sin\theta = \frac{1}{4\pi}\int dS_s \sin\theta \frac{n}{s}\frac{\partial\theta}{\partial s} = \frac{n}{4\pi}\int ds\, s \int d\varphi \sin\theta \frac{1}{s}\frac{\partial\theta}{\partial s} =$

$= \frac{n}{4\pi}\cdot 2\pi(-1)\int ds \frac{\partial(\cos\theta)}{\partial s} = -\frac{2\pi n}{4\pi}\left[m_z(\infty) - m_z(0)\right] = -\frac{2\pi n}{4\pi}[-p-p] = pn$.

Kamchatnov '22 abre formula $\vec{A}$ y $\vec{B}$ que $Q_H = 1$ y $\vec{B} = \mathrm{rot}\vec{A} = \frac{4}{(1+r^2)^2}\vec{A}$

$B^2 = 1/(1+r^2)^4$. Hubbard.

(20/X) Kamchatnov Hopfion $\vec{A}, \vec{B}$ paper. Thanks.

(21/X) Kamchatnov Hopfion paper. Norman at Rice. Ochanomizu Osawa Madrid. May 28. received "Sullen" La Guardia May 31. A. at meeting "Sullen" on 31/X.

Con. 6/X que $Q_H = 1$: $|\vec{A}|^2 = \left(\frac{4}{a}\right)^2 \frac{1}{(1+r^2)^2}$, $|\vec{B}|^2 = \left(\frac{4}{a}\right)^4 \frac{a^8}{(a^2+r^2)^4}$, $r^2 = s^2+z^2$ in a units

$\frac{|\vec{B}|^2}{|\vec{A}|^2} = \left(\frac{4}{a}\right)^4 \frac{1}{(1+r^2)^4}\frac{a^2}{4^2}(1+r^2)^2 = \frac{16}{a^2}\frac{1}{(1+r^2)^2}$ obs. C Kamchatnov '22.

$A_s = -\frac{8zs}{(1+r^2)^2}\frac{1}{a}$, $A_\varphi = -\frac{8s}{(1+r^2)^2 a}$, $A_z = -4\frac{(1-s^2+z^2)}{(1+r^2)^2 a}$ (7/IX)

$B_s = \frac{32}{a^2}\frac{sz}{(1+r^2)^3}$, $B_\varphi = \frac{32}{a^2}\frac{s}{(1+r^2)^3}$, $B_z = \frac{16}{a^2}\frac{(1+z^2-s^2)}{(1+r^2)^3}$.

$\vec{A} \to \vec{A}(-1)$. Luego $\vec{B} = (B_s, B_\varphi, B_z) = \frac{4}{a}\frac{1}{(1+r^2)}\vec{A}$ (!) Bor rau ge.

Emergent field que $Q_H = 1$ $\vec{B} = \frac{4}{a}\frac{1}{(1+r^2)}\vec{A}$ hopfion Toroid vrome

Con. Liu-Nagaosa Toroid moment $\vec{T} = \frac{1}{2}\int dV\, \vec{r}\times\vec{B} = \int dV \vec{A}$ (11/IX).

$T_s = \langle A_s\rangle = 0$, $T_\varphi \neq 0$, $T_z \neq 0$. $T_x = 0$, $T_y = 0$, no $T_\varphi \neq 0$. $T_z \neq 0$, no bueno $T_z \to 0$, $R\to\infty$.

Nave $\frac{1}{2}\int dV\, \vec{r}\times\vec{B} = \vec{A}$ y Liu-Nagaosa '22 que $\vec{A}^e = \frac{1}{2}(1-\cos\theta)\vec{\nabla\Phi}$ y $\vec{B}^e = \vec{B}$.

Liu-Nagaosa reinterpretan $T_z \neq 0$ teorema b y (1) mag. hopfion, $\vec{T} = T_z\hat{z}$.

no T_z raro $\to \langle B_z\rangle$, $T_z = \langle A_z\rangle$ gaussiano $\to 0$ que infinite sample, $R\to\infty$.

$A_z = \frac{4}{a}\frac{(1-s^2+z^2)}{(1+r^2)^2} = 4a\frac{(a^2-s^2+z^2)}{(a^2+r^2)^2}$, $T_z = 2\pi\int_0^R ds\, s \int_{-L/2}^{L/2} dz\, 4a\frac{(a^2-s^2+z^2)}{(a^2+r^2)^2}$

$T_z(R) \sim R$, paralogues $\forall\, R\to\infty$, $\beta = \frac{L}{R} = \mathrm{const}$. T_z $T_z(\text{infinite sample}) = \infty$

y no explique que description of hopfion dynamics. Liu-Nagaosa habrá en.

Gnuacad Doroshogula PRB 104, 075418 (2021) $T_z = \frac{2}{3}\int d^3r \sin^2\gamma(r)\frac{\partial\gamma}{\partial r}$, $\gamma(r)$ is arbitrary monotonic function of $r = \sqrt{x^2+y^2+z^2}$

$T_z = \frac{2}{3}\int dr\, r^2 \int d\theta \sin\theta \int d\varphi = \frac{2}{3}\cdot 4\pi\int_0^R dr\, r^2 \sin^2\gamma(r)\frac{\partial\gamma}{\partial r}$, $\gamma(0) = 0$, $\gamma(\infty) = \pi$.

paralogues $\forall\, R\to\infty$. Ahora bueno $\gamma(r)$ mono nacen y nacen $T_z(R) \sim R$ $\forall\, R\to\infty$.

Mover b Liu-Nagaosa '22 hago $T_z = \int dV A_z^e$, que $A_z^e = (1-\cos\theta)\frac{\partial\Phi}{\partial z}$?

a interpolación $A_z = A_z(\text{wang}) = -(1+\cos\theta)\frac{\partial\beta}{\partial z}$. $\Phi = n\varphi + \gamma(s,z)$, $\gamma = m\beta$.

$\vec{A}(\text{nue}) = \vec{A}(\text{wang}) = -2\nabla\gamma + \vec{A}^e$ b cyl (s,φ,z) coordinates.

$m_z(Q_H = \pm 1) = 1 - \frac{8s^2a^2}{(a^2+s^2+z^2)^2}$, $A_z^e = (1-m_z)\frac{\partial\Phi}{\partial z} = \frac{8s^2a^2}{(a^2+r^2)^2}\frac{\partial\beta}{\partial z}$, $\frac{\partial\beta}{\partial z} = 2a\frac{(s^2-z^2-a^2)}{[z^2+(s+a)^2][z^2+(s-a)^2]}$

$T_z(A^e) = 2\pi\int_0^R ds\, s\int_{-L/2}^{L/2} dz \frac{16s^2a^3}{(a^2+r^2)^2}\frac{(s^2-z^2-a^2)}{[z^2+(s+a)^2][z^2+(s-a)^2]} \sim \int_0^R ds\int_{-L/2}^{L/2} dz\, s\frac{1}{s^2}\frac{1}{s^2}$ converge $\forall\, R\to\infty$

Mathcad "Hopf-Skyrmion.mcd" que $T_z(A_z^e)$ gau finite limit $R\to\infty$ (Saturation)

+-e. $T_z(\vec{A}^e)$ under curves, a $T_z(\vec{A}) \to \infty$ $\forall\, R\to\infty$.

Bueno, toroidal moment Liu-Nagaosa $T_z(\vec{A}^e)$ es finite, no $\vec{A}^e$ no gau integer Q_H.

$\vec{A}$ (nue) gau integer Q_H, no $T_z(\vec{A}) \to \infty$ $\forall\, R\to\infty$. Bueno $\vec{A} = -2\nabla\gamma + \vec{A}^e$.

Definition $\vec{T}$ resp $\vec{A}^e$ Liu-Nagaosa '22 bozmeno, no no Gocbes. integer Q_H.

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Условие на hopfion vector potential A, $A_z = 0$ странное (!?).

Согласно Eq. (59) Wang-Brataas PRL'2019 и модели Eq. (18)

$A_z = -(1+m_z)\frac{\partial \gamma}{\partial z}$, $\Phi(\vec{r}) = n\varphi + \gamma(\rho,z)$, $\gamma(\rho,z) = \Delta\Phi(\rho,z)$ у Wang'2019.

$A_z = 0$ значит либо $m_z = -1$ (плохо) или $\frac{\partial \gamma}{\partial z} = 0$ (?)

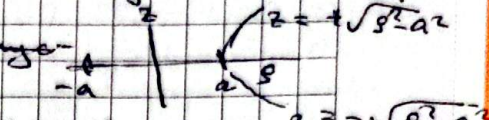
$\gamma(\rho,z) = m\beta(\rho,z)$, β – poloidal angle, $\beta(\rho,z) = \operatorname{arctg}\left(\frac{2az}{\rho^2+z^2-a^2}\right)$.

$\frac{\partial \gamma}{\partial z} = m\frac{\partial \beta}{\partial z}$, $m \neq 0$ (?), когда $Q_H = 0$. $\frac{\partial \beta}{\partial z} = 0$ (?). $\frac{\partial \beta}{\partial z} = \frac{2}{a}\frac{(r^2-1-2z^2)}{[(r^2-1)^2+4z^2]}$

$\frac{\partial \beta}{\partial z} = \frac{2}{a}\frac{(r^2-a^2-2z^2)}{[(r^2-a^2)^2+4z^2]}$ в безразм. единицах. $r^2 = \rho^2+z^2$, $\rho^2 = x^2+y^2$ – cyl radius

$\frac{\partial \beta}{\partial z} = 0$ при $r^2 = a^2+2z^2$, $\rho^2 = a^2+z^2$ – это конус-

невернулся в (ρ,z) coordinates.



$\partial\gamma/\partial z$ не может выполняться для всех $\vec{r}(\rho,\varphi,z)$.

Q_H (при $A_z=0$) $= \frac{1}{(4\pi)^2}\int dV[A_\rho B_\rho + A_\varphi B_\varphi] = \frac{1}{(4\pi)^2}\int dV\left[-(1+m_z)\frac{\partial\gamma}{\partial\rho}\times\right.$

$\times(-n)\frac{\sin\theta}{\rho}\frac{\partial\theta}{\partial z} + \frac{n}{\rho}(1-m_z)\sin\theta\left(\frac{\partial\theta}{\partial z}\frac{\partial\gamma}{\partial\rho} - \frac{\partial\theta}{\partial\rho}\frac{\partial\gamma}{\partial z}\right)\Big] = \frac{1}{(4\pi)^2}\times$

$\left.+\int dV\left[-\frac{n}{\rho}(1-m_z)\sin\theta\frac{\partial\theta}{\partial\rho}\frac{\partial\gamma}{\partial z} + \frac{2n}{\rho}\sin\theta\frac{\partial\gamma}{\partial\rho}\frac{\partial\theta}{\partial z}\right]\right.$. $A_z = 0$ означает $\frac{\partial\gamma}{\partial z} = 0$. $\gamma(\rho)$ только

$Q_H(A_z=0) = \frac{1}{(4\pi)^2}\int dV\frac{2n}{\rho}\sin\theta\frac{\partial\gamma}{\partial\rho}\frac{\partial\theta}{\partial z} - \int dV\frac{1}{\rho}\sin\theta\frac{\partial\gamma}{\partial\rho}\frac{\partial\theta}{\partial z} = \int d\rho\,\rho\int d\varphi\int dz\times$

$\times\frac{1}{\rho}\sin\theta\frac{\partial\gamma}{\partial\rho}\frac{\partial\theta}{\partial z} = 2\pi\int d\rho\int dz\sin\theta\frac{\partial\gamma}{\partial\rho}\frac{\partial\theta}{\partial z}$, $\gamma = \gamma(\rho)$, $\theta = \theta(\rho,z)$.

$\int dz\sin\theta\frac{\partial\theta}{\partial z} = -\int dz\frac{\partial(\cos\theta)}{\partial z} = (-1)\cos\theta(\rho,z)\Big|_{z=-L/2}^{z=L/2}$

$m_z(\gamma) = \frac{1-\operatorname{ch}^{2m}\gamma\,\operatorname{th}^{2n}\gamma}{1+\operatorname{ch}^{2m}\gamma\,\operatorname{th}^{2n}\gamma}$, $\gamma(\rho,z) = \operatorname{arcth}\left(\frac{2\rho a}{r^2+a^2}\right)$. $\cos\theta(\rho,z) = m_z(\rho,z)$.

$\gamma\left(\rho, z=\pm\frac{L}{2}\right) = \operatorname{arcth}\left(\frac{2\rho a}{\rho^2+\left(\frac{L}{2}\right)^2+a^2}\right)$; $\gamma(\rho,z)$ is even function of z.

поэтому $\cos\theta(\rho,z=\frac{L}{2}) - \cos\theta(\rho,z=-\frac{L}{2}) = m_z\left(\gamma,z=\frac{L}{2}\right) - m_z\left(\gamma,z=-\frac{L}{2}\right) = 0$

$\int dz\sin\theta(\rho,z)\frac{\partial\theta}{\partial z} = 0$ при сим. интеграле по z, $z = \pm L/2$.

Тогда $Q_H(A_z=0) = 0$ из-за симметрии.

Вывод: 1) $A_z = 0$ невозможно из-за $\frac{\partial\gamma}{\partial z} \neq 0$; 2) $Q_H(A_z=0) = 0$, при m_z realistic

$A_z = -(1+m_z)\frac{\partial\gamma}{\partial z} = 0$ из-за $m_z = -1$. Плохо, но если непрерывность?

Тогда $A_\rho = 0$, $A_z = 0$, $A_\varphi = \frac{2n}{\rho}$. $Q_H = \frac{1}{(4\pi)^2}\int dV A_\varphi B_\varphi = \frac{1}{(4\pi)^2}\int dV\times$

$\times\frac{2n}{\rho}\sin\theta\left(\frac{\partial\theta}{\partial z}\frac{\partial\gamma}{\partial\rho} - \frac{\partial\theta}{\partial\rho}\frac{\partial\gamma}{\partial z}\right) = \frac{1}{(4\pi)^2}\cdot 2n\int d\rho\,\rho\int d\varphi\int dz\frac{1}{\rho}\sin\theta\left(\frac{\partial\theta}{\partial z}\frac{\partial\gamma}{\partial\rho} - \frac{\partial\theta}{\partial\rho}\frac{\partial\gamma}{\partial z}\right)$

$Q_H(A_z=0, m_z=-1) = \frac{n}{4\pi}\int d\rho\,dz\sin\theta\left(\frac{\partial\theta}{\partial z}\frac{\partial\gamma}{\partial\rho} - \frac{\partial\theta}{\partial\rho}\frac{\partial\gamma}{\partial z}\right)$ (!) формально верное

Q_H как Eq. (S10) Wang-Brataas'2019.

Вывод: $A_z = 0$ возможно при непрерывном $m_z = -1$, $Q_H(A_z=0, m_z=-1)$ – верный.

$m_z = 1$ дает $\vec{A} = (A_\rho, A_\varphi, A_z) = (0, A_\varphi, 0)$, $A_\varphi = \frac{2n}{\rho}$.

$A_\varphi^e = (1-\cos\theta)\frac{\partial\Phi}{\partial\rho}$, $\Phi = n\varphi + \gamma(\rho,z)$. $A_\varphi^e = (1-\cos\theta)\frac{n}{\rho} = \frac{2n}{\rho}$ при $\cos\theta = -1$

Resume. Hopfion MS to chaos

(12/VIII) Hopfion MS was accepted for publ. Начислено 2,000 PLN.

(13/VII) Report on internship G. Cenzala in Donostia. Proofs Hopfion MS to Julian Ortiz.

(14/VII) Meeting c Julian (gave eng 1,500 €). Сл. на Mercadona.

Sent to "Chaos" corrected proofs.

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(15/VII) Trip to SMM'23 y NAP'23: BIO/EAS (Sept.3) → Prague → train Prague → Bratislava (Sept.9) → Brat. (9-16/IX) → Brat. → BIO (16/IX) (3-9/IX)

Вдъл билети BIO → Praga, 17:10 Sept. 3, Wien → Santander 12:30 Sept. 17,
(8) и Hotel Vienna Sept. 16/17.

(16-17/VII) travel report Poznan, June 24 – July 8, 2023.
Nocar Hopfion gauge, Chaos, paper группа M. Krawczyk.

(18/VII) Переписка с I. Smalyukh, обяснихме, че gauge $A_z = 0$ возможно, дори вероятно Hopf index Q_H, но нерешителна, т.к. $m_z = -1$.
Оксана съобщила, че new Ministerio project „Curvalinear geometry" получил висока оценка и бъдещо подкрепа, в 200 k€ на ICMM, где я member of Res. team. Расчет IVA „Constanto".

(19/VII) С п.и Ю. на плаж Орео. Хубаво време.

(20-21/VII) Расчет IVA „Constanto" II кв. 2023 и приходи за 2022.
Установихме, че во II кв. 2022 в расчета и declaration указан IVA base 31,342,71 €, а в расчета IVA 2022 и declaration за 2022 указан IVA base 32,300,00 €, т.е. завишена IVA received за 2022.
Проверих calc. IVA base за II кв. 2022, и уредена и тази сумма 3.000 € tourbureau (получ. в Мае 2022). Да.

(22/VII) С п. в „Carrefour Market". Расчет IVA II кв. 2023.

(23/VII) С п.и Ю. на плаж Орео. Хубаво време. Бъмъ.

(24/VII) Запознах IVA II кв. 2023 и подадох Tax Form 300 – IVA II кв. 2023.

(25/VII) Расчет приходи „Constanto" 2022. 3,000 € tourismo вписаны в расчет IVA II кв. 22. Conf. fee received II кв. 22 постъпила вярно, но тази в разход вписана 1690 € (ИК). Не знам, защо това? Тази сумма в Bank разход I, III кв. 2022. Имаме 1690 € transfer, dated June 30, 2022

(26/VII) Confirm. letter to Rosa. Реш. SW Central JMMM'2023. Euskaltel (Santander) Спасявам double payment за Ап. cell phone 15 € x2, направено в June 2023.

(27/VII – 28/VII) Edited the MS draft „BP-DW motion in cyl. nanowire" и послах на Оксана.

(29/VII – 31/VII) Работа в София. Бъмъ. Хубаво време. Реш. MS Gonzalez-Gutierrez et al. ACS Nano „Soaring spin probe" 31/III – printed papers in University.

(1/VIII) С п. в Carrefour Обаборена, използ. тел. 25 €.

(2/VIII) Реш. MS Gonzalez-Gutierrez. Хубаво време.

(3/VIII) Skype с Оксана: обсъждане на Nanowire project, обсъждане задн. „BP Dynamics" Comm. Phys., новия new MS BP-DW dynamics / Gyromodes" и Vortex string oscillations с Тейо. Оксана обещава сделать Simulations для new MS „BP-DW gyrotropic dynamics" до конца Oct. 2023 и сама соавтором.

(4/VIII) С п. в „Froski" Urbil. Сделан Bratislava 'NAP'23 registration и получен invoice conf. fee 400 €. Topol. Hall effect

(5-6/VIII) С п. в „Carrefour Market". Хубаво време. E-mail of I. Smalyukh.

(7/VIII) Добре to I. Smalyukh & Consultancy с Japan Center WPI-SKCM. Обсъд. инт. Top. Hall effect.

(8/VIII) Tatara, Phys. Rep. 2008: the Hall force due to $\vec{m}(\vec{r},t)$ texture Eq. (245) $\vec{F}_{Hall} = -\frac{Pa^3}{2eC}(\vec{j} \times \vec{G})$, gyrovector $G_i = \frac{2\pi S}{a^3}\varepsilon_{ijk}\Phi_{jk}$, где

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the emergent field tensor $\Phi_{ij} = \frac{1}{4\pi S^3}\int d^3x\, \vec{S}\cdot(\partial_i\vec{S}\times\partial_j\vec{S})$, $\vec{m}=\vec{S}/S$

The Hall force $\sim \vec{G}$ - gyrovector global - volume average em. magnetic field $\vec{B}_{em}$.

$\vec{F}^{Hall}$ - force exerted by the electron (due to the electron) - emergent electromagnetic field of localized spins $\vec{S}(\vec{r},t) = S\vec{m}(\vec{r},t)$ on conduction electrons. $\vec{F} = -\left\langle \frac{\partial \mathcal{H}_{sd}}{\partial \vec{X}} \right\rangle$.

$\vec{F}^{Hall}$ - emergent gyromagnetic force of Thiele eq. of motion of magnetization texture, $\vec{X}(t)$. $\vec{F}^{Hall} \sim (\vec{j}\times\vec{G})$ is a back reaction of the Hall effect due to spin chirality.

- N. Kanazawa et. al. PRL 106, 156603 (2011). Large topological Hall eff. in MnGe. Lorentz force in emergent mag field $\vec{F} = e\vec{v}\times\vec{B}$ leads to the Hall resistivity $\rho_{yx} = R_0 B_z$ (R_0 is normal Hall coef.). T.k. B_z равно, то это дано 2D case с $F_{\mu\nu} \Rightarrow F_{xy} = \vec{m}\cdot\left(\frac{\partial\vec{m}}{\partial x}\times\frac{\partial\vec{m}}{\partial y}\right)$ field tensor.

- A. Neubauer, C. Pfleiderer et al. PRL 2009: MnSi

$\vec{B}_{eff} = \Phi_0 \vec{b}$, $\Phi_0 = h/e$, $b_\mu = \frac{1}{8\pi}\varepsilon_{\mu\nu\lambda}\vec{m}\cdot(\partial_\nu\vec{m}\times\partial_\lambda\vec{m})$ - emergent mag field

Topol. Hall effect $\Delta\rho_{xy} = PR_0 B_{eff}^2$ (again 2D)

Ref. to THE theory P. Bruno PRL 2004 и G. Tatara J. Ph. Soc. Jap. 2007

- P. Bruno et al. PRL 2004. Emergent mag. field $B_z = -\frac{\Phi_0}{4\pi}\vec{m}\cdot\left(\frac{\partial\vec{m}}{\partial x}\times\frac{\partial\vec{m}}{\partial y}\right)$

$\Phi_0 = \frac{hc}{e}$ (flux quantum), $B_z = (\nabla\times\vec{A})_z$, $\vec{A}_i = -\frac{\pi\Phi_0}{(1+m_z)}[m\times\partial_i m_y - m_y\partial_i m_x]$

Hall conductivity $\sigma_{xy} \sim B_z$ (no dot array) и $\rho_{xy} \simeq \frac{\sigma_{xy}}{\sigma_{xx}^2} = \left(\frac{1}{ecn_z}\right)B_z$ (2D)

- G. Tatara et. al. J. Ph. Soc. Jap. 2007 - то же, что и в Tatara Ph. Rep. 2008

$\vec{F}^{Hall} = -\frac{Pa^3}{e\sigma_S}(\vec{j}\times\vec{G})$ вклад в eq. Thiele для $\vec{X}(t)$.

- B. J. Chapman et. al. PRB 2013, MnSi: $b_i = \frac{\Phi_0}{8\pi}\varepsilon_{ijk}\vec{m}(\partial_j\vec{m}\times\partial_k\vec{m})$

$\Phi_0 = h/|e|$. Hexagonal Sk-lattice in MnSi uniform $\|0z \Rightarrow \vec{b} = (0,0,b_z)$. 2D

$\rho_{yx}^T = R_H B_{eff}$, $B_{eff} = f\cdot b_z$ - the topol. Hall resistance, $f = 0.05 \div 0.45$.

- P. Ritz, Halder M. et al. Nat. 2013, Ph. Rev. B 2013, MnSi

$\rho_{xy}^{top} = PR_0 B_z^{eff}$, $\vec{B}^{eff}$ - emergent mag. field.

- C. Franz, F. Freimuth et al. PRL 2014, MnFeSi:

$\rho_{xy}^{top} \sim B_z^{eff}$ (2D)

→ Выводы Topol Hall effect: 1) Наблюдался экспериментально на Skyrmion lattices B20 (MnSi, ...); 2) $\rho_{xy}^{top} \sim B_z$ emergent field 2D; 3) теория - расчет Bruno PRL 2004 $\rho_{xy} \sim B_z$ на 2D array of cylindrical dots; 4) есть Berry effect of $\vec{m}(\vec{r})$-texture. 5) Нужны расчеты по 3-компонентному $3D$ $\vec{B}(\vec{r})$ emergent mag. field в MnSi по p-компонентам не нашел, все 2D и B_z.

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(9/VII) Some Experiment and Theory of Topol. Hall effect is in

T. Schulz et al. Nat. Phys. 8, 301 (2011) Emergent electrodynamics of skyrmions in a chiral magnet.

N. Nagaosa et al. Nat. Nanotechnology 8, 899 (2013); Emergent fields

Emergent mag. field $B_\alpha = \frac{1}{2}\varepsilon_{\alpha\beta\gamma}\vec{m}\cdot(\partial_\beta\vec{m}\times\partial_\gamma\vec{m})$

Emergent electric field $E_\alpha = \vec{m}\cdot(\partial_\alpha\vec{m}\times\dot{\vec{m}})$

$\frac{d}{dt}(\vec{m}(\vec{r},t))_\beta = \dot{m}_\beta(\vec{r},\vec{X}) \to$ rigid model $\frac{dm_\beta}{dt} = \frac{d}{dt}m_\beta(\vec{r}-\vec{X}) = -\frac{\partial m_\beta}{\partial x_\gamma}\frac{dX_\gamma}{dt}$

$E_\alpha = \vec{m}\cdot(\partial_\alpha\vec{m}\times\dot{\vec{m}}) = \vec{m}\cdot\left(\partial_\alpha\vec{m}\times\frac{\partial\vec{m}}{\partial x_\gamma}(-)\frac{dX_\gamma}{dt}\right) = -\vec{m}(\partial_\alpha\vec{m}\times\partial_\gamma\vec{m})v_\gamma$

$v_\gamma = \frac{dX_\gamma}{dt} = \dot{X}_\gamma$. $E_\alpha = -F_{\alpha\gamma}v_\gamma$, $F_{\mu\nu} = \vec{m}\cdot(\partial_\mu\vec{m}\times\partial_\nu\vec{m})$ em. field tensor

$(\vec{v}\times\vec{B})_\alpha = \varepsilon_{\alpha\beta\gamma}v_\beta B_\gamma = \varepsilon_{\alpha\beta\gamma}v_\beta\frac{1}{2}\varepsilon_{\gamma\mu\nu}\vec{m}\cdot(\partial_\mu\vec{m}\times\partial_\nu\vec{m})$

$\sum_\gamma \varepsilon_{\alpha\beta\gamma}\varepsilon_{\gamma\mu\nu} = \sum_\gamma \varepsilon_{\gamma\alpha\beta}\varepsilon_{\gamma\mu\nu} = \delta_{\alpha\mu}\delta_{\beta\nu} - \delta_{\alpha\nu}\delta_{\beta\mu}$

$(\vec{v}\times\vec{B})_\alpha = \frac{1}{2}v_\beta(\delta_{\alpha\mu}\delta_{\beta\nu} - \delta_{\alpha\nu}\delta_{\beta\mu})\vec{m}\cdot(\partial_\mu\vec{m}\times\partial_\nu\vec{m}) = \frac{1}{2}v_\beta(\delta_{\alpha\mu}\delta_{\beta\nu} - \delta_{\alpha\nu}\delta_{\beta\mu})F_{\mu\nu}$.

$\delta_{\alpha\mu}\delta_{\beta\nu}F_{\mu\nu} = F_{\alpha\beta}$, $\delta_{\alpha\nu}\delta_{\beta\mu}F_{\mu\nu} = F_{\beta\alpha} = -F_{\alpha\beta}$

$(\vec{v}\times\vec{B})_\alpha = \frac{1}{2}v_\beta(F_{\alpha\beta} - (-F_{\alpha\beta})) = F_{\alpha\beta}v_\beta$. $E_\alpha = -F_{\alpha\gamma}v_\gamma$.

$(\vec{v}\times\vec{B})_\alpha = -E_\alpha$. $\vec{E} = -\vec{v}\times\vec{B}$ gue $\vec{v} = \frac{d\vec{X}}{dt}$ u rigid model $\vec{m} = \vec{m}(\vec{r}-\vec{X}(t))$.

$\vec{E} \sim \frac{d\vec{m}}{dt}$. $\vec{m}$ moveno bounce. Solved within rigid model $\vec{m}(\vec{r}-\vec{X}(t))$. Emergent fields

// Ременник Хоменкова gue uniaxial ferromagnet. ФММ 1982.

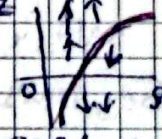
$E(\vec{m}) = A(\nabla\vec{m})^2 - K\cos^2\theta$. Cylindrical coordinates (ρ,φ)

$\cos\theta(\rho,z) = ch^{-1}\left[\frac{z}{\Delta} - |S|\ln\frac{\rho}{\Delta}\right]$, $\Phi(\varphi) = S\varphi + \varphi_0$, $S = 0,\pm1,\pm2,\dots$

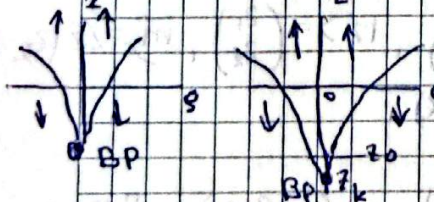
$\cos\theta(\rho,z) = \pm th\left[\frac{z}{\Delta} - S\ln\frac{\rho}{\Delta}\right] = m_z(\rho,z)$ — 3D domain wall

$m_z = 0$ на повърхността $\approx$ конуса $\rho = \Delta\exp\left(\frac{z}{\Delta S}\right)$

Bulk, no magnetostatics, but 3D.



Motion in $\vec{H}\parallel Oz$: $\ddot{Z}_0(t) = \frac{d}{1+d^2}\gamma\Delta H_z(t)$, $\Delta = \sqrt{A/K}$. d – Gilbert damp.



BP в точка $z_k = \frac{\Delta S}{\ln\frac{a}{\Delta}} = \Delta S\ln\frac{a}{\Delta}$. a – lattice constant

$z_k \gg \Delta, r_{nuc}, \Delta \gg a$.

DW – BP velocity $V(t) = \dot{Z}_0(t) = \frac{d}{1+d^2}\gamma\Delta H_z(t)$

$v_{DW} \simeq 0.01\cdot 3\frac{MHz}{Oe}\cdot 2\pi\cdot 10\,nm\cdot 100\,Oe = 3\cdot10^{-2}\cdot6\cdot10^3\,nm\cdot Oe\frac{MHz}{Oe} =$

$= 18\cdot10^{21}\,nm\cdot\frac{10^6}{s} \simeq 20\cdot10^{-3}m/s\cdot10^7 = 0.2\frac{m}{s}$ – стана нала $\Delta = 10\,nm$, $H_z = 100\,Oe$.

Walker DW solution $v_{DW} = \frac{\gamma\Delta}{1}H_z$. $d \to \frac{1}{d}$, $10^{-2}\to10^2$ дава $v_{DW} \approx 200\frac{m}{s}$.

$m_z(\rho,z) = th\left(\frac{z}{\Delta} - S\ln\frac{\rho}{\Delta}\right)$. $r_{nuc} = 0$, $S = 1$, $m_z = th\ln\left(\frac{\Delta}{\rho}\right) = \frac{e^{\ln x} - e^{-\ln x}}{e^{\ln x} + e^{-\ln x}} =$

$= \frac{x - \frac{1}{x}}{x + \frac{1}{x}} = \frac{x^2-1}{x^2+1} = \frac{(\Delta/\rho)^2-1}{(\Delta/\rho)^2+1} = \frac{\Delta^2-\rho^2}{\Delta^2+\rho^2}$ + Доп. точки с $z_0 = \Delta$, точно

$r_{nuc}/\Delta \gg 1$, $m_z(\rho,z) = \pm1$ – domains с $\vec{m}\parallel Oz$ nanowire axis Skyrm

$\Phi = S\varphi + \Phi_0$, берем $S=1$, $\Phi_0 = \pm\pi/2$, так чтоб $m_\rho(\rho,z)=0$.

Trial function $m_z(\rho,z) = th\left[\frac{z}{\Delta} - ln\frac{\rho}{\Delta}\right]$, где Δ – BP size

$m_z(\rho, z=0) = \frac{\Delta^2-\rho^2}{\Delta^2+\rho^2}$, это не vortex, а Bloch skyrmion.

При $v_{tw} = v \neq 0$ $m_z(\rho,z) = th\left[\frac{(z-vt)}{\Delta} - f ln\frac{\rho}{\Delta}\right]$

$f = f(v)$ расширяет функцию v, $f(0)=1$.



Ansatz de Bonte [Aranda '2018] $\cos\Theta(r) = th\, f(r)$, $f(r) = \frac{r-r_s}{\delta} + ln\left(\frac{r}{r_s}\right)$

Особенность $m_z(\rho,z) = th\left[\frac{z}{\Delta} - ln\frac{\rho}{\Delta}\right]$ не дает плоской DW.

Может при $v \neq 0$ отличие в ширине ansatz $\Delta(v)$ и $\Delta(0)$ надает $v\uparrow$.

(10/VIII) // New $m_z(\rho,z)$ ansatz подставить в $E[\vec m]$ и найти $E(\Delta)$, есть ли Δ_{min}? Не стоит. // Решение Хоменков '1982 $\cos\Theta(\rho,z) = th\left[\frac{z}{\Delta} - f ln\frac{\rho}{\Delta}\right]$ – хотя и описывает „bobber" (поплавок) с Bloch point, но оно skyrmion-like $m_z(\rho) = \frac{\Delta^2-\rho^2}{\Delta^2+\rho^2}$ при $z=0$ и не подходит для описания vortex-like Bloch point DW в cyl. nanowires.

(11/VIII) С Л. и Ю. начали в Орио. Сомнения, удар. Забыли принадл.

(12/VIII–13/VIII) Более-менее разобрались.

(14/VIII) Обзор работ по 3D magnetism (A.B. Borisov '2011, 2020).

(15/VIII) Saji–Tronoso ArXiv '2023 „Magnonic Hall effect and magnonic holography of Hopfions"

$Su(2)$ as representation of rotation group $SO(3)$. Hopfions: there

$Su(2)$ $\hat R = R_0 I_2 + i\sum_\mu R^\mu \hat\sigma_\mu$, $\vec m = \hat R^{-1}\partial_z \hat R = \vec m(\vec r)$ soliton magnetization

Due Hopfion $\hat R = \exp\left[i f(r)\frac{\vec r}{r}\right]$, $\vec R = (R_1, R_2, R_3) = (R^\mu)$, $f(0)=0$, $f(\infty)=\pi$ – toroidal hopfion in ∞-space. Откуда $\vec R$ взяли?

General (p,q) hopfion described by toroidal p and poloidal q cycles in spherical coordinates $\vec r = (r, \theta, \varphi)$, $\vec m(\vec r)$:

$m_x + i m_y = \sin\chi \cdot e^{i(q\varphi - p\Theta + \gamma)}$, $m_z = \cos\chi$, where

$tg\Theta(r,\theta) = tg f(r)\cdot\cos\theta$, $\sin\chi(r,\theta) = \sin f(r)\cdot\sin\theta$.

Без DMI $(\varkappa=0)$ $\vec B^{eff} = \vec B^{em} = rot\,\vec A^z$, т.е. $\vec A^{em} = \vec A^z$,

$A^k_\mu \sigma_k = (\vec A_\mu \cdot \vec\sigma) = i\hat R^{-1}\partial_\mu \hat R$, $\hat R = \begin{pmatrix} z_1 & z_2 \\ -z_2^* & z_1^* \end{pmatrix}$, $|\vec z\rangle = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$, $\vec m_\mu = \langle \vec z | \sigma_\mu | \vec z \rangle$

$|\vec z\rangle = \begin{pmatrix} \cos\chi\, e^{i\varphi\Theta} \\ i\sin\chi\, e^{iq\varphi} \end{pmatrix}$, $tg\Theta = tg f(r)\cos\theta$, $\vec r = (r,\theta,\varphi)$

Note: $\vec A^z = \vec A^{em}$ (Saji) не совпадает с $\vec A^{em}$ (Pershoguba '2021). В частности в ref. Pershoguba нет $\vec e_\theta$-component of $\vec A$ и $\vec r$-component другая. Где ошибка?

Согласно Gusynin '2023 $\vec A(z) = 2\,Im(z^+\nabla z)$, $z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$

Можно явно найти $\vec A$-components $\vec A(A_r, A_\theta, A_\varphi)$ в сферических координатах $\vec r(r,\theta,\varphi)$

→ Идея о сферич. координатах: взять $\vec m(\vec r)$ hopfion при $|\vec r| > R$, R – sphere radius, и сделать inversion $z \to 1/z$, т.е. найти $\vec m(\vec r)$ при $r < R$ (внутри сферы) аналитически. Это и/или близкие решения при $R < a$ hopf.

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(16-17/VIII) Отдых дома. Блин. Inversion. R, O, P, P'' — точка P внутри сферы, $r_p < R$, инверсируется

(18/VIII) Hopfion in spherical particle.

в точку P' вне сферы, $r_{p'} > R$. $|OP| \cdot |OP'| = r_p \cdot r_{p'} = R^2$. The same polar and azimuthal angles for the vectors $\vec{r}_p(r_p, \theta, \varphi)$ и $\vec{r}_{p'} = (r_{p'}, \theta, \varphi)$.

Wiki: Dupin cyclides. Inversion at the sphere $x^2+y^2+z^2=R^2$ is

$\vec{r}(x,y,z) \to \vec{r}'(x',y',z') = R^2 \frac{(x,y,z)}{x^2+y^2+z^2}$. $\vec{r}' = R^2 \frac{\vec{r}}{|\vec{r}|^2}$, $\vec{r} \to \vec{r}'$ и наоборот $\vec{r}' \to \vec{r}$.

$\vec{m}(\vec{r})$ описывается complex function $w = \frac{m_x + i m_y}{1+m_z}$.

Для Hopfion с $|Q_H| = 1$ $w(\vec{r}) = \frac{2(x+iy)}{1-r^2+2iz}$, (x,y,z) in units of "a" - Hopfion radius.

r вне сферы радиуса R, $r > R$. При $R \to 0$ это ∞ infinite space.

$\vec{r}$ $\vec{r}' = R^2 \frac{\vec{r}}{|\vec{r}|^2}$ внутри сферы, $|\vec{r}'| < R$.

R В $\vec{m}(\vec{r})$ field заменим $\vec{r}$ на $\vec{r}'$, $|\vec{r}| < R$, $rr' = R^2$

$w(\vec{r}) = \frac{2a(x+iy)}{a^2 - r^2 + 2iza}$. $\vec{r} = \frac{|\vec{r}|^2}{R^2}\vec{r}' = \frac{r^2}{R^2}\vec{r}'$, $(x,y,z) = \frac{r^2}{R^2}(x',y',z')$

$w(\vec{r}') = \frac{2a \frac{r^2}{R^2}(x'+iy')}{a^2 - R^4 + 2ia\frac{r^2}{R^2}z'}$, $\frac{r}{R} = \frac{R}{r'}$ Hopfion in sphere

$w(\vec{r}') = \frac{2a\frac{R^2}{r'^2}(x'+iy')}{a^2 - \frac{R^4}{r'^2} + 2ia z' \frac{R^2}{r'^2}} = \frac{2aR^2(x'+iy')}{a^2 r'^2 - R^4 + 2iaR^2 z'}$

$w(\vec{r}') = \frac{2a(x'+iy')}{\left(\frac{a^2}{R^2}\right)r'^2 - R^2 + 2iaz'} = \frac{R^2}{a^2}\cdot\frac{2a(x'+iy')}{\left(r'^2 - \frac{R^4}{a^2} + 2iz'\frac{R^2}{a}\right)}$, $w(\vec{r}) = \frac{2a(x+iy)}{(a^2 - r^2 + 2iza)}$

$\tilde{w}(\vec{r}') = -\frac{R^2}{a^2}\cdot\frac{2a(x'+iy')}{\left[\frac{R^4}{a^2} - r'^2 - 2iz'\frac{R^2}{a}\right]} = -\frac{2a'(x'+iy')}{(a'^2 - r'^2 - 2ia'z')}$, $a' = \frac{R^2}{a}$.

T.e. $\tilde{w}(\vec{r}') = -w(\vec{r} \to \vec{r}', a \to a') = -w(x',y',-z',a')$, $w(x,y,z,a) = \frac{2a(x+iy)}{(a^2 - r^2 + 2iaz)}$ вне сферы

$\vec{m}(\vec{r}')$ внутри сферы R описывается через $w(\vec{r}')$.

$w(\vec{r},a) = w(x,y,z,a)$ хорошая функция с $r > R$, $w(\vec{r}') = -w(x',y',-z',a')$

$w(\vec{r}')$ с точностью до знака это $w(x',y',-z',a')$, $a' = R^2/a$.

$m_z(\vec{r}) = 1 - \frac{8s^2a^2}{(a^2+s^2+z^2)^2}$ вне сферы R. Внутри сферы $m_z(\vec{r}') = 1 - \frac{8s'^2a'^2}{(a'^2+s'^2+z'^2)^2}$ $(a \to a')$

По смыслу $R < a$, $R < r$, чтобы вокруг нового солитона был вакуум. Hopfion $\vec{m}(\vec{r})$ вне сферы. $m_z(z'=0, s'=a') = -1$ при $s' = a' = R^2/a < R$

→ Вывод: magnetization of hopfion внутри spherical particle of radius R описывается by function $\tilde{w}(\vec{r})_{r<R} = -w(x,y,-z,a')$, где function $w(x,y,z,a) = w(\vec{r},a) = \frac{2a(x+iy)}{[a^2 - r^2 + 2iaz]}$ описывает hopfion in ∞-space, $a' = R^2/a < R$ — eff. hopfion radius. Inversion $\vec{r}' = \frac{R}{r^2}\vec{r}$ of outer sphere space, $r > R$, to inner space $r' < R$. Q_H ... $Q_H(R \to 0) \to 1$.

// Можно найти hopfion energy $E(a')$ и посмотреть E_{min} $E(a')$, metastable hopfion.

$E(a') = E_{ex}(a') + E_m^{vol}(a') + E_m^{surf}(a')$. т.к. $R < a$ - мало, то $E_{ex}(a')$ should dominate.

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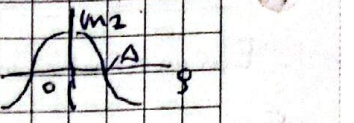
$\Phi = S\varphi + \Phi_0$, берем $S=1$, $\Phi_0 = \pm\pi/2$, так чтоб $m_\varphi(\rho, z) = 0$.

Trial function $m_z(\rho, z) = th\left[\frac{z}{\Delta} - ln\frac{\rho}{\Delta}\right]$, где Δ – BP size

$m_z(\rho, z=0) = \frac{\Delta^2 - \rho^2}{\Delta^2 + \rho^2}$, это не vortex, а Bloch skyrmion.

При $v_{tw} = v \neq 0$ $m_z(\rho, t) = th\left[\frac{(z - vt)}{\Delta} - S ln\frac{\rho}{\Delta}\right]$

$S = S(v)$ расщепление функционала v, $S(0) = 1$.



Ansatz de Bonte [Aranda '10, 18] $\cos\Theta(r) = th\, f(r)$, $f(r) = \frac{r - r_s}{\delta} + ln\left(\frac{r}{r_s}\right)$

Особенность $m_z(\rho, z) = th\left[\frac{z}{\Delta} - ln\frac{\rho}{\Delta}\right]$ не дает плоской фазы.

Может при $v \neq 0$ учесть в явном ansatz $\Delta(v)$ и $\delta(v)$ надо, $v \uparrow$.

(6/VIII) // New $m_z(\rho, z)$ ansatz подставить в $E[\vec{m}]$ и найти $E(\Delta)$, есть ли Δ_{min}?

Не спит. // Денежное Хорошков '1982 $\cos\Theta(\rho, z) = th\left[\frac{z}{\Delta} - S ln\frac{\rho}{\Delta}\right]$ – хорошо и описывает „bobber" (поплавок) с Bloch point,

но оно Skyrmion like $m_z(\rho) = \frac{\Delta^2 - \rho^2}{\Delta^2 + \rho^2}$ при $z=0$ и не подходит для описания vortex like Bloch point QW – in cyl. nanowires.

(11/VIII) С п. и Ю. полным в Орио. Совместный ужин. Забавен приезд.

(12/VIII – 13/VIII) Болен, мне paracetamol.

(14/VIII) Обзор работ по 3D magnetism (A.B. Borisov '2011, 2020).

(15/VIII) Seji-Troncoso ArXiv '2023 „Magnonic Hall effect and magnonic holography of Hopfions"

$SU(2)$ as representation of rotation group $SO(3)$. Hopfions theme

$SU(2)$ $\hat{R} = R_0 I_2 + i\sum_\mu R^\mu \hat{\sigma}_\mu$, $\vec{m} = \hat{R}^{-1}\partial_z\hat{R} = \vec{m}(\vec{r})$ soliton magnetization

Due Hopfion $\hat{R} = exp\left[i f(r)\frac{\vec{r}}{r}\right]$, $\vec{R} = (R_1, R_2, R_3) = (R^\mu)$, $f(0) = 0$, $f(\infty) = \pi$

– toroidal Hopfion in ∞-space. Q: куда $\vec{R}$ будет?

General (p, q) Hopfion described by toroidal p and poloidal q cycles in spherical coordinates $\vec{r} = (r, \theta, \varphi)$, $\vec{m}(\vec{r})$:

$m_x + i m_y = \sin\chi \cdot e^{i(q\varphi - p\Theta + \gamma)}$, $m_z = \cos\chi$, where

$tg\Theta(r, \theta) = tg f(r) \cdot \cos\theta$, $\sin\chi(r, \theta) = \sin f(r) \cdot \sin\theta$.

Без DMI ($\varkappa = 0$) $\vec{B}^{eff} = \vec{B}^{em} = rot\,\vec{A}^z$, т.е. $\vec{A}^{em} = \vec{A}^z$,

$A^k_\mu \sigma_k = (\vec{A}_\mu \cdot \vec{\sigma}) = i\hat{R}^{-1}\partial_\mu\hat{R}$, $\hat{R} = \begin{pmatrix} z_1 & z_2 \\ -z_2^* & z_1^* \end{pmatrix}$, $|\vec{z}\rangle = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$, $\vec{m}_\mu = \langle\vec{z}|\sigma_\mu|\vec{z}\rangle$

$|\vec{z}\rangle = \begin{pmatrix} \cos\frac{\chi}{2} e^{ip\Theta} \\ i\sin\frac{\chi}{2} e^{iq\varphi} \end{pmatrix}$, $tg\Theta = tg f(r)\cos\theta$, $\vec{r} = (r, \theta, \varphi)$

Note: $\vec{A}^z = \vec{A}^{em}$ (Seji) не совпадает с $\vec{A}^{em}$ (Pershoguba '2021). В каком-то реф. Першогубы нет $\vec{e}_\theta$ – component of $\vec{A}$ и $\vec{e}_r$ – component другая. Что-то Ошибка?

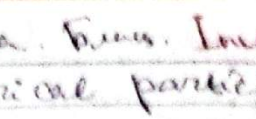
Согласно Gus'kova '2023 $\vec{A}(\vec{r}) = 2Im\left(\vec{z}^+ \nabla \vec{z}\right)$, $\vec{z} = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$

можно явно найти $\vec{A}$ – components $\vec{A}(A_r, A_\theta, A_\varphi)$ в сферических координатах $\vec{r}(r, \theta, \varphi)$

→ идея о сферич. координатах: будет $\vec{m}(\vec{r})$ – hopfion при $|\vec{r}| > R$, R – sphere radius, и сделать inversion $r \to 1/r$, т.е. найти $\vec{m}(\vec{r})$ при $r < R$ внутри сферы аналитически. Это и/или численное решение при $R < a$ (hopf.

Escaneado con CamScanner

(16-17/VIII) Отражение заряда. Метод. Inversia.



точка P внутри сферы $r_p < R$, инверсная

(18/VIII) Hopfion in spherical particle.

В точку P' вне сферы, $r_{P'} > R$. $|OP| \cdot |OP'| = r_P r_{P'} = R^2$. The same polar and azimuthal angles for the vectors $\vec{r}_P(r_P, \theta, \varphi)$ и $\vec{r}_{P'}(r_{P'}, \theta, \varphi)$

Wiki: Digital cyclides. Inversia: also sphere $x^2+y^2+z^2=R^2$:

$\vec{r}(x,y,z) \to \vec{r}'(x',y',z') = R^2 \frac{(x,y,z)}{x^2+y^2+z^2}$. $\vec{r}' = R^2 \frac{\vec{r}}{|\vec{r}|^2}$, $\vec{r} \to \vec{r}'$ и $\vec{r}' \to \vec{r}$.

$\vec{m}(\vec{r})$ отображается complex function $w = \frac{m_x + i m_y}{1+m_z}$.

Для Hopfion с $|Q_H| = 1$ $w(\vec{r}) = \frac{2(x+iy)}{1-r^2+2iz}$, (x,y,z) в units of a''.

- Hopfion Jackiw.

$\vec{r}$ вне сферы радиуса R, $r > R$. При $R \to 0$ это во infinite space.

$\vec{r}' = R^2 \frac{\vec{r}}{|\vec{r}|^2}$ внутри сферы, $|\vec{r}'| < R$.

В $\vec{m}(\vec{r})$ field Заменяем $\vec{r}$ на $\vec{r}'$, $|\vec{r}| < R$, $r r' = R^2$

$w(\vec{r}) = \frac{2a(x+iy)}{a^2-r^2+2iza}$. $\vec{r} = \frac{|\vec{r}|^2}{R^2}\vec{r}' = \frac{r^2}{R^2}\vec{r}'$, $(x,y,z) = \frac{r^2}{R^2}(x',y',z')$

$w(\vec{r}') = \frac{2a \cdot \frac{r^2}{R^2}(x'+iy')}{a^2 - \frac{R^4}{r'^2} + 2ia\frac{r^2}{R^2}z'}$, $\frac{r}{R} = \frac{R}{r'}$ Hopfion in sphere

$w(\vec{r}') = \frac{2a\frac{R^2}{r'^2}(x'+iy')}{a^2 - \frac{R^4}{r'^2} + 2ia\frac{z'R^2}{r'^2}} = \frac{2aR^2(x'+iy')}{a^2r'^2 - R^4 + 2iaR^2z'}$

$w(\vec{r}') = \frac{2a(x'+y'i)}{\left(\frac{a^2}{R^2}\right)r'^2 - R^2 + 2iaz'} = \frac{R^2}{a^2}\cdot\frac{2a(x'+iy')}{\left(r'^2 - \frac{R^4}{a^2} + 2iz'\frac{R^2}{a}\right)}$, $w(\vec{r}) = \frac{2a(x+iy)}{(a^2-r^2+2iza)}$

$\tilde{w}(\vec{r}') = -\frac{R^2}{a^2}\cdot\frac{2a(x'+iy')}{\left[\frac{R^4}{a^2} - r'^2 - 2iz'\frac{R^2}{a}\right]} = -\frac{2a'(x'+iy')}{(a'^2 - r'^2 - 2ia'z')}$, $a' = \frac{R^2}{a}$.

T.e. $\tilde{w}(\vec{r}') = -w(\vec{r}\to\vec{r}', z\to -z, a\to a') = -w(x',y',-z',a')$, $w(x,y,z,R) = \frac{2a(x+iy)}{(a^2-r^2+2iaz)}$ вне сферы

$\vec{m}(\vec{r}')$ внутри сферы R описывается через $w(\vec{r}')$.

$w(\vec{r},a) = w(x,y,z,a)$ исходная функция с $r > R$, $w(\vec{r}') = -w(x',y',-z',a')$

$w(\vec{r})$ с точностью до знака это $w(x',y',-z',a')$, $a' = R^2/a$.

$m_z(\vec{r}) = 1 - \frac{8a^2r^2}{(a^2+x^2+z^2)^2}$ вне сферы R. Внутри сферы $m_z(\vec{r}') = 1 - \frac{8a'^2r'^2}{(a'^2+x'^2+z'^2)^2}$ $(a\to a')$

По смыслу $R < a$, $R < r$, чтобы воспользоваться подобием функций Hopfion $\vec{m}(\vec{r})$ вне сферы. $m_z(z'=0, \rho'=a') = -1$ при $\rho' = a' = R^2/a <$

→ Вывод: magnetization of hopfion внутри spherical particle of radius R описывается by function $\tilde{w}(\vec{r}')_{r'<R} = -w(x,y,-z,a')$, где function $w(x,y,z,a) = w(\vec{r},a) = \frac{2a(x+iy)}{[a^2-r^2+2iaz]}$ описывает hopfion в ∞-space

$a' = R^2/a < R$ — эфф. hopfion в radius. Inversion $\vec{r}' = \frac{R}{r}\vec{r}$, of outer sphere space, $r > R$, to inner space $r' < R$. При $R \to 0$ →

// Можно найти hopfion energy $E(a')$ и минимизировать, т.е. $\min E(a')$, подходящее hopfion.

$E(a') = E_{ex}(a') + E_m^{vol}(a') + E_m^{surf}(a')$ т.к. $R < a$ — мало, то $E_{ex}(a')$ should dominate.

Escaneado con CamScanner

Small sphere radius $R < a$ hexagonal hopfion $\vec{m}(\vec{r})$, $z \le R$, c helicity inversion (18/VIII). Large sphere radius $R > a$, $\vec{m}(\vec{r})$ given как $\vec{m}(\vec{r})$ of hopfion in infinite media.

Cylindrical dot (w/ re), radius R, length L. BP hopfion & interaction.

El. current – $\vec{m}(\vec{r})$ texture interaction $\delta E_{int} = -\int dV \vec{j} \cdot \vec{A}_{em}$ Helicity or spin-torque $\vec{j}$ via $\vec{m}$, see b of shield eddy $\vec{E}$ field $\sim (\vec{j} \times \vec{r})$ Computing Tatara, Ange. Rep. Nano, Lec. 8/VIII.

Cylindrical nonaxisymmetric BP (BP Dw) vs toroidal hopfion. hyperbolic tan $\vec{j} = (0, 0, j_z)$ || ox nanowire/nanotube. $\vec{j} \cdot \vec{A} = j_z A_z$, where A_z – component of energy int. vector potential Comsol.

Dzno hopfion [Gus'Chaos'Cas]: $A_z = -(1+m_z)\frac{\partial \gamma}{\partial z}$, $\gamma(\rho,\phi) = m\beta(\rho,\phi)$ $\beta(\rho,\phi) = \arctg(2az/(\rho^2+z^2-a^2))$ poloidal toroidal angle.

15-16/II 2022 & cross-check Aggravector gives BP DW – $m_z = \frac{z}{r}\frac{B^2}{(B^2+\rho^2)}$, $m_\rho = 0$ – Thiaville ansatz, no $\vec{A}$ we cross. Cm. 3/IX 2022 gives $G_\rho \neq 0$ 20/X $\vec{G}$ (Zöller-Feldhellen).

$\vec{G} = \nabla m_z \times \nabla\varphi = \sin\theta \nabla\varphi \times \nabla\theta$, hence $\vec{G} \to \vec{g}$ aggravector density.

$\tg\varphi = \frac{m_y}{m_x}$, $m_x = \sin\theta\cos\varphi$, $m_y = \sin\theta\sin\varphi$, $m_z = \cos\theta$.

Thiaville BP ansatz: $m_\rho = 0$, $m_\varphi = \sqrt{1-m_z^2}$, $m_z(\rho,z) = \frac{z}{r}\frac{B^2}{B^2+\rho^2}$, $r^2 = \rho^2 + z^2$.

$m_\rho = m_x\cos\varphi + m_y\sin\varphi = 0$, hence $m_x = -\sin\varphi\sqrt{1-m_z^2}$, $m_y = \cos\varphi\sqrt{1-m_z^2}$ →

$m_\varphi = -(\underbrace{-}_{m_x})\sin\varphi + (\underbrace{}_{m_y})\cos\varphi = \sin^2\varphi\sqrt{1-m_z^2} + \cos^2\varphi\sqrt{(\cdot)} = \sqrt{1-m_z^2}$ $m_\rho \equiv 0$ (9/IX 22).

$\tg\Phi = \frac{(x/\rho)\sqrt{1-m_z^2}}{(-y/\rho)\sqrt{1-m_z^2}} = -\frac{x}{y}$, $\Phi = \arctg\left(\frac{x}{y}\right)(+1) = (-1)\left[-\arctg\left(\frac{x}{y}\right)^{-1} + \frac{\pi}{2}\sign\left(\frac{x}{y}\right)\right] =$

$= \arctg\left(\frac{y}{x}\right) - \frac{\pi}{2}\sign\left(\frac{x}{y}\right)$, $\Phi(\rho,\varphi,z) = \varphi + \text{const}$, где $\Phi = q\varphi + \gamma$ для BP sphere.

$\frac{\partial\Phi}{\partial\rho} = 0$, $\frac{\partial\Phi}{\partial z} = 0$, $\frac{\partial\Phi}{\partial\varphi} = 1$. $\nabla\Phi = \left[\hat\rho\frac{\partial}{\partial\rho} + \hat\varphi\frac{1}{\rho}\frac{\partial}{\partial\varphi} + \hat z\frac{\partial}{\partial z}\right]\Phi = \hat\varphi\frac{1}{\rho}$.

$\vec g = \nabla m_z \times \nabla\Phi = \vec a \times \nabla\Phi = \begin{vmatrix} \hat\rho & \hat\varphi & \hat z \\ a_\rho & a_\varphi & a_z \\ 0 & 1/\rho & 0 \end{vmatrix} = \frac{1}{\rho}a_\rho\hat z - \frac{1}{\rho}a_z\hat\rho$.

$\vec a = \nabla m_z(\rho,z) = a_\rho\hat\rho + a_z\hat z$, $a_\rho = \frac{\partial m_z}{\partial\rho}$, $a_z = \frac{\partial m_z}{\partial z}$.

$\vec g = \frac{1}{\rho}\frac{\partial m_z}{\partial\rho}\hat z - \frac{1}{\rho}\frac{\partial m_z}{\partial z}\hat\rho$. $m_z(z)$ even or odd, $\frac{\partial m_z}{\partial z} = $ even or odd $f(z)$ $G_\rho \neq 0$, $G_z = 0$.

$G_\rho \neq 0$ для BP – новый эффект 15/II 2022.

Hb emerglnt mag. field $\vec B = \vec g = \frac{1}{\rho}\frac{\partial m_z}{\partial\rho}\hat z - \frac{1}{\rho}\frac{\partial m_z}{\partial z}\hat\rho$ для BP Thiaville ansatz.

Вернемся к ур-ю Q_{II} (BP-Thiaville) $= 0$ из-за $m_\rho = 0$, $\gamma = \text{const}$ (9/IX 22)??

Вводя Q_H, $\vec A$, $\vec B$ & cyl. coordinates in Wang-Braastad'19 и Gus'Chaos'23 Сравнивая для toroidal hopfions, found и для BP.

$\vec B$ (BP) $= (B_\rho, 0, B_z)$, $B_\rho = -\frac{1}{\rho}\frac{\partial m_z}{\partial z}$, $B_z = \frac{1}{\rho}\frac{\partial m_z}{\partial\rho}$.

$\vec B = \rot\vec A = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right)\hat\varphi + \frac{1}{\rho}\left(\frac{\partial}{\partial\rho}(\rho A_\varphi) - \frac{\partial A_\rho}{\partial\varphi}\right)\hat z$

$B_\varphi = \frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho} = 0$, $B_\rho = \frac{1}{\rho}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z} = -\frac{1}{\rho}\frac{\partial m_z}{\partial z}$, $B_z = \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho A_\varphi) - \frac{\partial A_\rho}{\partial\varphi}\right] =$

Видно что $\vec A = \vec A(\rho,\varphi)$, $\frac{\partial A_z}{\partial\varphi} = 0$, $\frac{\partial A_\rho}{\partial\varphi} = 0$. $B_\varphi = \frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho} = 0$ $= +\frac{1}{\rho}\frac{\partial m_z}{\partial\rho}$.

$B_\rho = -\frac{1}{\rho}\frac{\partial A_\varphi}{\partial z} = +\frac{1}{\rho}\frac{\partial m_z}{\partial z}$, $\frac{\partial A_\rho}{\partial z} = \frac{1}{\rho}\frac{\partial m_z}{\partial z}$, $B_z = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\varphi) = \frac{1}{\rho}\frac{\partial m_z}{\partial\rho}\cdot\frac{\partial}{\partial\rho}(\rho A_\varphi) = \frac{\partial m_z}{\partial\rho}$.

Escaneado con CamScanner

$\frac{\partial A_\rho}{\partial z} = \frac{\partial A_z}{\partial \rho}$, $\frac{\partial A_\varphi}{\partial z} = \frac{1}{\rho}\frac{\partial m_z}{\partial z}$, $\frac{\partial}{\partial \rho}(\rho A_\varphi) = \frac{\partial m_z}{\partial \rho}$ eqs на A_ρ, A_φ, A_z для BP (Thiaville)

$\rho A_\varphi(\rho,z) = m_z(\rho,z) + f_1(z)$, $A_\varphi = \frac{1}{\rho}m_z + \frac{1}{\rho}f_1(z)$, $\frac{\partial}{\partial z}A_\varphi = \frac{1}{\rho}\frac{\partial m_z}{\partial z} + \frac{1}{\rho}\frac{\partial f_1}{\partial z} =$

$= \frac{1}{\rho}\frac{\partial m_z}{\partial z}$, что дает $f_1(z) = 0$. $A_\rho = \frac{1}{\rho}m_z$. Т.к. $B_\varphi = 0$, то A_φ не

Важная для Hopf index $Q_H = \frac{1}{(4\pi)^2}\int dV \vec{A}\cdot\vec{B}$.

Note: $A_\varphi(\text{Hopfion}) = (1-m_z)\frac{n}{\rho}$, $A_\varphi(BP) = \frac{1}{\rho}m_z$. $\vec{A} = (A_\rho, A_\varphi, A_z)$.

$\frac{\partial A_\rho}{\partial z} = \frac{\partial A_z}{\partial \rho}$ остается. Есть trivial solution $A_\rho = A_z = 0$ для BP.

$Q_H(\text{trivial BP}) = 0$ как посчитал 9/IX на основе интегральных eqs.

for $\vec{B}, \vec{A}$ of toroidal hopfion. Hence $Q_H(BP) = Q_H(\text{Hopfion})$.

$Q_H(\text{Hopfion}) = \frac{n}{4\pi}\int d\rho \int dz\, \vec{m}\cdot\left(\frac{\partial \vec{m}}{\partial z}\times\frac{\partial \vec{m}}{\partial \rho}\right) = \frac{n}{4\pi}\int d\rho \int dz \sin\theta\left[\frac{\partial\theta}{\partial z}\frac{\partial\varphi}{\partial\rho} - \frac{\partial\theta}{\partial\rho}\frac{\partial\varphi}{\partial z}\right]$.

$\Phi(\text{Hopfion}) = n\varphi + \chi(\rho,z)$

→ Вычисл Hopf index of BP in Thiaville-like ansatz $\vec{m}(\rho,z)$ открыт (!)

$Q_H(BP) = \frac{1}{(4\pi)^2}\int dV [A_\rho B_\rho + A_z B_z]$, B_ρ, B_z - известны, A_ρ, A_z - неизвестны $\frac{\partial A_\rho}{\partial z} = \frac{\partial A_z}{\partial \rho}$

$\operatorname{div}\vec{B}(BP) = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho B_\rho) + \frac{1}{\rho}\frac{\partial B_\varphi}{\partial\varphi} + \frac{\partial B_z}{\partial z} = |B_\varphi = 0| = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\rho\left(-\frac{1}{\rho}\right)\frac{\partial m_z}{\partial z}\right] +$

$+\frac{\partial}{\partial z}\frac{1}{\rho}\frac{\partial m_z}{\partial\rho} = -\frac{1}{\rho}\frac{\partial^2 m_z}{\partial\rho\partial z} + \frac{1}{\rho}\frac{\partial^2 m_z}{\partial z\partial\rho} \equiv 0$, т.к. $m_z(\rho,z)$ аналитич. функция.

$\operatorname{div}\vec{B}(BP\ \text{cylinder}) = 0$ и нам надо вводить Dirac string чтобы $\operatorname{div}\vec{B} \neq 0$

как в Tejo'Sa: Rep. 2021. Этот string: Tejo'21 - это дельта сферич. координат

Вводя BP-Thiaville при $b \to \infty$ сводится к BP-Feld used в Tejo'2011

Скорее $\operatorname{div}\vec{B}_0 \neq 0$ в Tejo'2011 если разумно ввести Singular $\vec{A}$.

Здесь $\vec{B}_0$ is connected by Singular $\vec{B}_S$, что $\operatorname{div}\vec{B} = 0$, $\vec{B} = \vec{B}_0 + \vec{B}_S$.

$\operatorname{div}\vec{B}_0(\text{sphere}) \neq 0$.

//Gauge $A_z(\text{cyl BP}) = 0$ дает $\vec{j}\cdot\vec{A} = j_z A_z = 0$, и electric SP current не нужен возбуждая BP в cylindrical wire. Но simulations показали, что из зависимости BP ориентация $\parallel$ оси wire Oz, будем $A_z(\text{cyl}) \neq 0$.

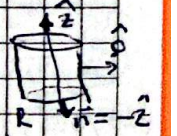
Как найти $A_\rho(BP\ \text{cyl})$, $A_z(BP\ \text{cyl})$?

$\int_S dS_z (\operatorname{rot}\vec{A})_z = \oint_C \vec{A}\,d\vec{l} = \oint A_\varphi R\,d\varphi = \int dS_z B_z$, условие на A_φ.

Lm. 17/III: $\int dV(\nabla\times\vec{A}) = \int dV\vec{B} = \oint_S dS\,\vec{n}\times\vec{A}$ - Gauss theorem, $\vec{n}$ - outward normal to surface S.

21/VIII Domag - Hopfion C" на SMM, Prague 2023.

22/VIII $\oint dS\,\vec{n}\times\vec{A} = \int_{z=L/2} dS_1\,\hat{z}\times\vec{A} + \int_{z=-L/2} dS_2\,(-\hat{z})\times\vec{A} + \int_{S=R} dS_3\,\hat{\rho}\times\vec{A}$



$\int dV\vec{B} = \int dS_{z=L/2}(A_\rho\hat{e} - A_\varphi\hat{\rho}) - \int dS_{z=-L/2}(A_\rho\hat{e} - A_\varphi\hat{\rho}) + \int dS_{S=R}(A_\varphi\hat{z} - A_z\hat{e})$

$G_\rho = \int dV B_\rho = -\int dS_{z=L/2} A_\varphi + \int dS_{z=-L/2} A_\varphi = |A_\varphi = \frac{1}{\rho}m_z| = -\int_{z=L/2} d\rho\,d\varphi\,\frac{1}{\rho}m_z + \int_{z=-L/2} d\rho\,d\varphi\,\frac{1}{\rho}m_z +$

$\int dV B_\rho = 2\pi\int_0^R d\rho\,[m_z(\rho,-\frac{L}{2}) - m_z(\rho,\frac{L}{2})] = |m_z(\rho,z) - \text{odd function in } z| = -4\pi\int_0^R d\rho\, m_z(\rho,\frac{L}{2})$.

$\int dV B_\rho = \int dz\int d\rho\,\rho\,d\varphi\left(-\frac{1}{\rho}\frac{\partial m_z}{\partial z}\right) = -2\pi\int dz\int d\rho\frac{\partial m_z}{\partial z}$

$\int dV B_\rho = -4\pi\int d\rho\, m_z(\rho,L/2) = -2\pi\int dz\int d\rho\frac{\partial m_z}{\partial z} = -2\pi\int d\rho\,[m_z(\rho,\frac{L}{2}) - m_z(\rho,-\frac{L}{2})] =$

вычисляется, что $m_z(z)$ odd f. "z"

$\int dV B_\varphi = \int_{z=L/2} dS_1 A_\rho - \int_{z=-L/2} dS_2 A_\rho - \int_{S=R} dS_3 A_z = 0$, $\int d\rho\,\rho\,2\pi[A_\rho(\rho,\frac{L}{2}) - A_\rho(\rho,-\frac{L}{2})] = \int_{S=R} dS_3 A_z$

Escaneado con CamScanner

$2\pi \int_0^R d\rho\, \rho \left[A_\varphi(\rho, \tfrac{L}{2}) - A_\varphi(\rho, -\tfrac{L}{2}) \right] = \int dz \int d\varphi R A_z(R,z)$.

$\int d\rho\, \rho \left[A_\varphi(\rho, \tfrac{L}{2}) - A_\varphi(\rho, -\tfrac{L}{2}) \right] = R \int dz\, A_z(R,z)$, $\frac{\partial A_\varphi}{\partial z} = \frac{\partial A_z}{\partial \rho}$

$\int dz \frac{\partial A_\varphi}{\partial z}(\rho,z) = A_\varphi(\rho, \tfrac{L}{2}) - A_\varphi(\rho, -\tfrac{L}{2})$, $\int d\rho\, \rho \int dz \frac{\partial A_\varphi}{\partial z} = \int d\rho\, \rho \int dz \frac{\partial A_z}{\partial \rho} = R \int dz\, A_z(R,z)$

$\int d\rho\, \rho \frac{\partial A_z}{\partial \rho}(\rho,z) = R A_z(R,z)$, уголовие на A_z.

$\rho A_z(\rho,z)\Big|_0^R - \int d\rho\, A_z(\rho,z) = R A_z(R,z)$. $R A_z(R,z) - 0 - \int d\rho\, A_z(\rho,z) = R A_z(R,z)$,

если $\rho A_z(\rho,z) \to 0$ при $\rho \to 0$. $\int d\rho\, A_z(\rho,z) = 0$, тогда допустимо $A_z \equiv 0$.

$\lim_{\rho\to 0} \rho A_z(\rho,z) = C(z)$, $\int d\rho\, A_z(\rho,z) = -C(z)$, тогда допустимо $A_z \neq 0$.

Пусть $A_z(\rho,z) = \frac{C(z)}{\rho} + F(\rho,z)$, $\rho F \xrightarrow[\rho\to 0]{} 0$. $\int d\rho\, A_z = \int d\rho \left[\frac{C(z)}{\rho} + F(\rho,z) \right] = -C(z)$

$C(z) \left(\int \frac{d\rho}{\rho} + \int d\rho\, F(\rho,z) \right) = -C(z)$, $\int d\rho\, F(\rho,z) = -C(z) \left[1 + \ln \rho \Big|_0^R \right]$, тоже возможно

если $F(\rho,z) = \frac{1}{\rho} C_1(z) + C_2(z)$, но это противоречит $\rho F \xrightarrow[\rho\to 0]{} 0$ assumed.

Отсюда A_z (BP cylinder) $= 0$, A_φ (BP cyl) $= 0$. BP cylinder

→ Вывод: для Ньюэлловского ansatz BP-цилиндр, $\vec{B}(B_\rho, 0, B_z)$, $\vec{A}(0, A_\varphi, 0)$ → Hopf index ($\sim \vec{A}\cdot\vec{B}$) $= 0$. Таковы current-BP interaction ($\sim j_z A_z$) $= 0$.

23/VIII Review papers Charilaou-Braun PRL '2020 и PR '2018.

Переписка с Оксаной о four moving BP reversal mechanism in (L=300 nm / R=30 nm) PY cylindrical nanowires. BP velocities up to 3 km/s. Strange middle plane z=0 domain с $m_z \approx \pm 1$ при $H_z < 0$ и 50 точках, $|H_z| > H_c$.

24/VIII Оформление. Ещё.



Central domain ($m_z \approx +1$) переписка со Оксаной negative H_z.

Charilaou 2018 берет $|H_z^*| > |H_n|$, $H_n < 0$ nucleation field, $H_z^* < 0$ и смотрит $\vec{m}(\vec{r},t)$ evolution. $|H_z^*| > H_c$ Брауна.

Брауна $H_c \sim \frac{1}{L}$ и для long nanowires Оксаны hysteresis loop very narrow, так что in small field $|H_z| \sim 10$–20 mT уже $\vec{m}$-reversal через motion of BP-DW along wire axis.

25/VIII Переписка с Оксаной на Charilaou PRL '2018 и ее simulations BP-DW dynamics nanowires. Запросил new invoice 4806 NAP 2023.

26–27/VIII Замена аккумулятора. Оформ. переревел Visa 200 € Santander. Собрал files билетов и hotels trip to Poznan June-July 2023.

Nikita Skoblov прислал file по SW in cylindrical thick dot с H_z || plane, где есть SW mode localized at dot faces $z = \pm L/2$. Кликнул в вопросах ding-

28/VIII Дал Visa 1800 € cash из Замка на зубы.

29/VIII Оформ Eli order Trip Poznan June-July '2023. Meet. Julian. Вернул Замок в office. Прислал file N. Strelkov SW in 1-magn. Cyl. dot.

30/VIII Skype session с Германом: localized SW mode in thick dots

Skype со Славой. Прочел по марку с Mag. Frontiers 2023

31/VIII – 2/IX hotels+place trips to Prague-Sochi и Bratislava/NAP.

3–18/IX Trip Prague → Bratislava-Vienna. Приехал поезд в Santander 17/18/IX т.н. не было bus tickets to Donostia. M. Mrusticuch – non-reciprocal SW in cylindrical nanowires: sign of H_z^3 dipolar term? Maria Salvador, Oviedo – bio-magnetism. Meet-с Jose, Boldan – BP-DW: BP-DW change sign velocity V_t as function of current, intensity.

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(19/IX) План работы (см. 29/V)

1) Vortex string GM nanowires, Оксана Simul., нереспинзав ω_n
2) BP-DW gyrotropic dynamics nanowires, Окс. Simulation Bmag
3) ~~Cites AXA, Toyota - coll. Cremo~~
4) re-submit „Mag. Frontiers" conf. proposal 2025 не submit again
5) distinguished scientist UPV/EHU appl.
6) appl. new passport Ariana, cita Uber. Embassy пода на 13/III 23 на среща.
7) EL. Signature application Cita Police - 21/III 23г.
8) review Sci Adv. Smalyukh MS subm. 14/I.
9) travel reports SMM-NAP. Glob, Ruben, Porto, Madrid
10) MS с Стрелков - SW in thick dots, Lc, mode
11) MS с Zelent - FM ring over SK-nanodot, $\vec{H}_m(\vec{r})$
12) Hopfion stability paper in ∞-sample, wc soft-hard-Gleb
13) Santander docs, Orange 15€, Bog. Стрелк, Крайтер Ann
14) Cites for new: tarjetas NIE К., П., А., new law Hipoteca
15) IVA Constanto III кв. STCM projects evaluation, Antrag Patricia, Mag. Frontiers
16) % возврат Santander, new Hipoteca 2.12% fixed. Dorog Constanto" 2023

(20/IX) И Zelent о профилиране $\vec{H}_m(\vec{r})$ ring MS. Лене 1506 West Union.
(21/IX) Travel report SMM-NAP. С П. в „Eroski" „Lidl", „Mercadona".
(22) Стрелков SW in thick cyl. dot, H_z - saturated. BP sphere
(23/IX) Bloch point sphere. $\vec{m}(\theta,\varphi)$, $\theta(\vec{r}) = p\varrho + \frac{\pi}{2}(1-p)$, $\varphi(\vec{r}) = q\varrho + \gamma$
$\vec{r}(r,\varrho,\varphi)$ spherical coordinates, $\gamma = \gamma(r,\varrho)$ - vorticity.
Уравнение на $\gamma(\vec{r})$: $\ell_e^2\left[\nabla^2\gamma + \frac{2}{r^2}\operatorname{ctg}\varrho\frac{\partial\gamma}{\partial\varrho}\right] = \sin\gamma\,(h_z + h_\varrho\operatorname{ctg}\varrho)$
В exchange approximation, бимерзъ dipolar field, hz, hϱ = 0:

$\nabla^2\gamma + \frac{2}{r^2}\operatorname{ctg}\varrho\frac{\partial\gamma}{\partial\varrho} = 0$, $\nabla^2\gamma = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\gamma}{\partial r}\right) + \frac{1}{r^2\sin\varrho}\frac{\partial}{\partial\varrho}\left(\sin\varrho\frac{\partial\gamma}{\partial\varrho}\right) + \frac{2}{r^2}\operatorname{ctg}\varrho\frac{\partial\gamma}{\partial\varrho} = 0$

$\frac{\partial}{\partial r}\left(r^2\frac{\partial\gamma}{\partial r}\right) + \frac{1}{\sin\varrho}\frac{\partial}{\partial\varrho}\left(\sin\varrho\frac{\partial\gamma}{\partial\varrho}\right) + 2\operatorname{ctg}\varrho\frac{\partial\gamma}{\partial\varrho} = 0$. $\gamma(r,\varrho) = R(r)F(\varrho)$

$F\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{R}{\sin\varrho}\frac{\partial}{\partial\varrho}\left(\sin\varrho\frac{\partial F}{\partial\varrho}\right) + 2\operatorname{ctg}\varrho\,R\frac{\partial F}{\partial\varrho} = 0$. divide by RF (Arsken)

$\frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{1}{F\sin\varrho}\frac{\partial}{\partial\varrho}\left(\sin\varrho\frac{\partial F}{\partial\varrho}\right) + \frac{2}{F}\operatorname{ctg}\varrho\frac{\partial F}{\partial\varrho} = 0$. обе страни $= Q$ (const)

$\frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) = Q$ и $\frac{1}{F\sin\varrho}\frac{\partial}{\partial\varrho}\left(\sin\varrho\frac{\partial F}{\partial\varrho}\right) + \frac{2}{F}\operatorname{ctg}\varrho\frac{\partial F}{\partial\varrho} = -Q$.

$2rR' + r^2R'' - QR = 0$, не сводится к spherical Bessel functions $x^2y'' + 2xy' + [x^2 - Q]y = 0$

$\frac{1}{\sin\varrho}\frac{\partial}{\partial\varrho}\left(\sin\varrho\frac{\partial F}{\partial\varrho}\right) + 2\operatorname{ctg}\varrho\frac{\partial F}{\partial\varrho} + QF = 0$, не сводится к legendre dif. eq. $y(x) = j_n(x)$, $Q = n(n+1)$.

$R'' + \frac{2}{r}R' - \frac{Q}{r^2}R(r) = 0$;

→ Бобоч: $\vec{m}(\vec{r})$ of Bloch point in spherical particle не сводится к известным решениям даже в exchange approximation.

$\frac{1}{\sin\varrho}\left[\cos\varrho\frac{\partial F}{\partial\varrho} + \sin\varrho\frac{\partial^2F}{\partial\varrho^2}\right] + 2\operatorname{ctg}\varrho\frac{\partial F}{\partial\varrho} + QF = 0$, $F'' + 3\operatorname{ctg}\varrho\,F' + QF = 0$.

Пусть $F(\varrho) \to F(\cos\varrho)$, $\frac{\partial F}{\partial\varrho} = -\sin\varrho\frac{1}{(-\sin\varrho\,\partial\varrho)} = -\sin\varrho\frac{\partial}{\partial(\cos\varrho)} = -\sin\varrho\frac{\partial}{\partial x}F(x)$ $x = \cos\varrho$

$F''_{\varrho\varrho} = \frac{\partial^2F}{\partial\varrho^2} = \frac{\partial}{\partial\varrho}\frac{\partial F}{\partial\varrho} = \frac{\partial}{\partial\varrho}(-1)\sin\varrho\frac{\partial}{\partial x}F(x) = -\cos\varrho\frac{\partial F}{\partial x} - \sin\varrho\frac{\partial}{\partial\varrho}\left(\frac{\partial F}{\partial x}\right) =$

$= -\cos\varrho\frac{\partial F}{\partial x} - \sin\varrho\frac{\partial}{\partial x}\frac{\partial F}{\partial\varrho} = -x\frac{\partial F}{\partial x} - \sin\varrho\frac{\partial}{\partial x}\left[-\sin\varrho\frac{\partial F(x)}{\partial x}\right] = -x\frac{\partial F}{\partial x} + \sin\varrho\frac{\partial\sin\varrho}{\partial x}\frac{\partial F}{\partial x} +$

$+ \sin^2\varrho\frac{\partial^2F(x)}{\partial x^2} = \left|x = \cos\varrho,\ \partial x = -\sin\varrho\,\partial\varrho\right| = -x\frac{\partial F}{\partial x} + \sin\varrho\frac{\partial\sin\varrho}{-\sin\varrho\,\partial\varrho}\frac{\partial F}{\partial x} + \sin^2\varrho\,F''(x) =$

$= -xF'(x) - \cos\varrho\,F'(x) + \sin^2\varrho\,F''(x) = -2xF'(x) + (1-x^2)F''(x)$

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$-2xF'(x) + (1-x^2)F''(x) + 3\operatorname{ctg}\theta\left(-\sin\theta\frac{\partial F}{\partial x}\right) + QF = 0$, $F = F(x)$, $x = \cos\theta$

$(1-x^2)F''(x) - 5xF'(x) + QF(x) = 0$. $(1-x^2)y'' - 2xy' + Qy = 0$ – Legendre eq

$y = P_n(x)$, $Q = n(n+1)$

y_p – a general $R(x)$ moreno pomuc. Ho eq. for $F(\theta)$, $F'' + 3\operatorname{ctg}\theta F' + QF = 0$

нумбодит = new special functions, т.е. ho решено в известных спец. функциях нельзя. $\Psi(r,\theta) = R(r)F(\theta)$ – helicity of BP

Hopfion Toroidal hopfion in ∞-sample. Есть ли min E as function of a – hopf. radius?

$E[\vec m] = E_{ex}[\vec m] + E_m[\vec m]$. $m_z(y) = \frac{1-\operatorname{ch}^{2n}y\cdot\operatorname{th}^{2m}y}{1+\operatorname{ch}^{2n}y\cdot\operatorname{th}^{2m}y}$, $Q_H = mn$,

$m_x + im_y = \sqrt{1-m_z^2(y)}\, e^{i(n\varphi+m\beta)}$

$\vec r(y,\varphi,\beta)$ – toroidal coordinates

E_{ex} есть в Galatkovski '1997. $E_{ex}[\vec m] = \frac{1}{2}\Lambda\int dV(\nabla\vec m)^2$ нет $g_1=0, g_2=0$.

$E_{ex}[\vec m] = \frac{2\pi}{2}\Lambda\int_0^\infty dy\int_{-\pi}^{\pi} d\beta\,\frac{a^3\operatorname{sh}y}{\tau^3}\left[\frac{(\nabla m_z)^2}{1-m_z^2} + (1-m_z^2)\left\{(\nabla v)^2 + \frac{N^2\tau^2}{a^2\operatorname{sh}^2 y}\right\}\right]$,

$\tau = \operatorname{ch}y - \cos\beta$, $v(y,\beta) = M\beta + v_0$, $v_0 = \text{const}$. $E_{ex}(a) = C_1a^3 + C_2\underset{=0}{} + C_3a$, $C_i > 0$ (?).

C_1, C_2 имеют один знак. Видимо $C_1>0$, $C_2>0$ и $E_{ex}(a)$ растет с расширением (!?) нарушается. Нет, a^* есть в ∇.

Дно провала $E_m[\vec m]$ надо $\operatorname{div}\vec m$ in toroidal coordinates.

См. 18/VIII: analytic $\vec m$ for tor. hopfion in spherical particle.

24/IX Toroidal coordinates $\vec r(y,\beta,\varphi)$. $\tau = \operatorname{ch}y - \cos\beta$, $h_\beta = h_y = \frac{a}{\tau}$, $h_\varphi = \frac{a}{\tau}\operatorname{sh}y$.

$dV = h_y h_\beta h_\varphi\, dy\,d\beta\,d\varphi = \frac{a^3}{\tau^3}\operatorname{sh}y\,dy\,d\beta\,d\varphi$

$\nabla f = \frac{\tau}{a}\left[\frac{\partial f}{\partial y}\hat e_y + \frac{\partial f}{\partial\beta}\hat e_\beta + \frac{1}{\operatorname{sh}y}\frac{\partial f}{\partial\varphi}\hat e_\varphi\right]$, $(\nabla f)^2 = \frac{\tau^2}{a^2}\left[\left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial\beta}\right)^2 + \frac{1}{\operatorname{sh}^2 y}\left(\frac{\partial f}{\partial\varphi}\right)^2\right]$

Galatkovski '97: $E_{ex}[\vec m] = \pi\Lambda a^3\int dy\,d\beta\,\frac{\operatorname{sh}y}{\tau^3}\left[\frac{(\nabla m_z)^2}{(1-m_z^2)} + (1-m_z^2)\left\{M^2(\nabla\beta)^2 + \frac{N^2\tau^2}{a^2\operatorname{sh}^2 y}\right\}\right]$

$m_z = m_z(y)$ только, $\nabla\beta$ (нет $v_0 = \text{const}$) $= \frac{\tau}{a}\frac{\partial}{\partial\beta}(\beta)\hat e_\beta = \frac{\tau}{a}\hat e_\beta$, $(\nabla\beta)^2 = \frac{\tau^2}{a^2}$

$\nabla m_z = \frac{\tau}{a}\frac{\partial m_z}{\partial y}\hat e_y$, $(\nabla m_z)^2 = \frac{\tau^2}{a^2}\left(\frac{\partial m_z}{\partial y}\right)^2$.

$E_{ex}[\vec m] = \pi\Lambda a^3\int dy\int d\beta\,\frac{\operatorname{sh}y}{\tau^3}\left[\frac{\tau^2}{a^2}\left(\frac{\partial m_z}{\partial y}\right)^2\frac{1}{(1-m_z^2)} + (1-m_z^2)\frac{\tau^2}{a^2}\left\{M^2 + \frac{N^2}{\operatorname{sh}^2 y}\right\}\right]$

$E_{ex}[\vec m] = \pi\Lambda a\int dy\int d\beta\,\frac{\operatorname{sh}y}{\tau}\left[\frac{1}{(1-m_z^2)}\left(\frac{\partial m_z}{\partial y}\right)^2 + (1-m_z^2)\left\{M^2 + \frac{N^2}{\operatorname{sh}^2 y}\right\}\right]$

$E_{ex} \sim a$ (!?). можно брать $\int d\beta\,\frac{1}{\tau} = \int_{-\pi}^{\pi}\frac{d\beta}{(\operatorname{ch}y - \cos\beta)}$.

Note that $\vec m(\vec r)$ of toroidal hopfion слабо (Gus'Chaos'23 не зависит от hopfion radius a^*, если брать y в eg. a^*: $y/a = \operatorname{th}\left(\frac{2a\rho}{\rho^2+z^2+a^2}\right)$

$\rho = \frac{a}{\tau}\operatorname{sh}y$, $z = \frac{a}{\tau}\sin\beta$, φ – общая с цил. координатами $\vec r(\rho,\varphi,z)$.

$E_{ex}[\vec m] = A\int dV(\nabla\vec m)^2$, $m_z = m_z(y/a)$, $m_x + im_y = \sqrt{1-m_z^2(y/a)}\,e^{i(n\varphi+m\beta)}$

$(\nabla m_z)^2 = \frac{\tau^2}{a^2}\left(\frac{\partial m_z}{\partial y}\right)^2$. $d=3$, y – dimensionless. $m_{x,y} = m_{x,y}(y/a)$

$E_{ex}[\vec m] = A\int dV\sum_d(\nabla m_d)^2$ нет $d = x,y,z$. $(\nabla m_d)^2 = \frac{\tau^2}{a^2}\left[\left(\frac{\partial m_d}{\partial y}\right)^2 + \left(\frac{\partial m_d}{\partial\beta}\right)^2 + \frac{1}{\operatorname{sh}^2 y}\left(\frac{\partial m_d}{\partial\varphi}\right)^2\right]$

$(\nabla m_d)^2 \sim \frac{1}{a^2}$, $E_{ex}[\vec m] = A\int dy\,d\beta\,d\varphi\,\frac{a^3}{\tau^3}\operatorname{sh}y\sum_d(\nabla m_d)^2 \sim a^3\cdot\frac{1}{a^2} = a$.

В ∞-sample есть только volume charges $\operatorname{div}\vec m$.

$\operatorname{div}\vec m = \frac{\tau^3}{a^3\operatorname{sh}y}\left[\frac{\partial}{\partial y}\left(\frac{a^2}{\tau^2}\operatorname{sh}y\, m_y\right) + \frac{\partial}{\partial\beta}\left(\frac{a^2}{\tau^2}\operatorname{sh}y\, m_\beta\right) + \frac{\partial}{\partial\varphi}\left(\frac{a^2}{\tau^2}m_\varphi\right)\right] =$

$= \frac{1}{a}\frac{\tau^3}{\operatorname{sh}y}\left[\frac{\partial}{\partial y}\left(\frac{\operatorname{sh}y}{\tau^2}m_y\right) + \frac{\partial}{\partial\beta}\left(\frac{\operatorname{sh}y}{\tau^2}m_\beta\right) + \frac{1}{\tau^2}\frac{\partial m_\varphi}{\partial\varphi}\right]$. перейти m_x, m_y, m_z.

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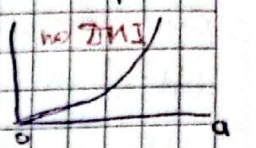
Если y in units of "a", то $m_y = m_y(y, \beta, \varphi)$, где y – dimensionless

$\operatorname{div}\vec{m} \sim \frac{1}{a}$, $E_m[\vec{m}] = \frac{M_s^2}{2}\int dV \int dV' \frac{\operatorname{div}\vec{m}\cdot \operatorname{div}\vec{m}(\vec{r}')}{|\vec{r}-\vec{r}'|} \sim a^3 a^3 \frac{1}{a}\frac{1}{a}\frac{1}{a} \sim a$

$E_m[\vec{m}] \sim a^3$. $E[\vec{m}] = E_{ex}[\vec{m}] + E_m[\vec{m}] = C_1 a + C_2 a^3$. $C_1, C_2 > 0$

no chiral sym. и magnetostatic energy. Тогда $E(a)$ растет с ростом "a", $\min E(a)$ при finite "a" нет.

$\min E(a)$ достигается при $a \to 0$, $\min E(a=0) = 0$ – $E(\text{uniform } \vec{m})$.



Вывод: toroidal hopfion. В ∞-sample при уходе E_{ex}, E_m неограничены, но сохраняются в $a \to 0$ uniform $\vec{m}(\vec{r})$ state.

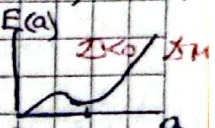
Hopfion ∞ stability

Если добавить DMI? $E_D[\vec{m}] = D\int dV(\vec{m}\cdot \operatorname{rot}\vec{m})$, $\operatorname{rot}\vec{m} \sim \frac{1}{a}$

$E_D[\vec{m}] = D\int dV(\vec{m}\cdot\operatorname{rot}\vec{m}) = D\int dy\,d\varphi\,d\alpha \frac{a^3}{L^3} \sin\gamma\,(\vec{m}\cdot\operatorname{rot}\vec{m}) \sim a^3 \frac{1}{a} \sim \operatorname{sign}(D)\cdot a^2$.

$E[\vec{m}] = E_{ex}[\vec{m}] + E_D[\vec{m}] + E_m[\vec{m}] = C_1 a - C_2 a^2 + C_3 a^3$, $C_i > 0$, $D < 0$.

В зависимости от соотн. a_1, C_1, C_3 может быть $\min E(a)$, E_D и E_m обе важны для $\min E(a)$ of hopfion.



Чтобы $\min E(a)$ был только при $D<0$, chiral hopfions.

Можно ли достичь $\min E(a)$ и $\min E(a) < E_{\text{uniform}}$ (stable hopfion) при реальных A, D, M_s? Это зависит от hopfion $\vec{m}(\vec{r})$.

Удобно $E(a) = C_1 a - \frac{1}{2}C_2 a^2 + \frac{1}{3}C_3 a^3$, $C_i > 0$. $\frac{\partial E}{\partial a} = C_1 - C_2 a + C_3 a^2 = 0$

Equil. a из $C_3 a^2 - C_2 a + C_1 = 0$, $a_{1,2} = \frac{1}{2C_3}\left[C_2 \pm \sqrt{C_2^2 - 4C_1C_3}\right]$

Очевидно "+" корень. $\min E(a)$, $a_0 = \frac{1}{2C_3}\left[C_2 + \sqrt{C_2^2 - 4C_1C_3}\right]$.

Для $\min E(a)$, надо $C_2 > 0$ и $C_2^2 \geq 4C_1C_3$, $|C_2| = C_2 \geq 2\sqrt{C_1C_3}$, т.е. Big DMI at real a_0.

Условие $C_2 \geq 2\sqrt{C_1C_3}$ впервые показано что toroidal hopfion.

(25/IX) Рассм. направлений на application Distinguished Scientist UPV/EHU E-mails to Patricia y Julian. dot localized SW (Strelkov).

(26/IX) Printed receipts, scanned report и поехал/отдал to Eli:

SW С Л. и А. в Mercadona Andorra. Новая old calculations SW-cylinder

1998-99: Calculations SW in in-plane magnetized dot, Gus' JAP'2000.

2003 Jan.: SW in out-of-plane magnetized dot, Kakazei' APL'2004

2020 SW in thick cyl. dot, Nanoscale'2020, спроэу $\int dz(\dots)$ $G_{zz}(\vec{r}-\vec{r}')$

Jan., Apr.-May-2019. Если зависимость от z' даст MSW (Damon-Eshbach).

(27/IX) Расчет SW in Cylinder. dot (Стрелков). Основа Jan. 2003.

SW

$-\frac{1}{\gamma}\dot{\vec{M}} = \vec{M}\times\vec{H}$, $\vec{M} = \vec{M}_0 + \vec{m}(\vec{r},t)$, $\vec{M}_0 \parallel Oz$, $\vec{H} = \vec{H}_0 + \vec{h}$

$\vec{M}\times\vec{H} = (\vec{M}_0+\vec{m})\times(\vec{H}_0+\vec{h}) = \vec{M}_0\times\vec{H}_0 + \vec{M}_0\times\vec{h} + \vec{m}\times\vec{H}_0$, $\vec{M}_0 = M_s\hat{z}$

$\vec{M}_0\times\vec{h} = M_s\hat{z}\times\vec{h} = M_s\begin{vmatrix}\hat{x}&\hat{y}&\hat{z}\\0&0&1\\h_x&h_y&h_z\end{vmatrix} = M_s(\hat{y}h_x - \hat{x}h_y)$, $\vec{m} = (m_x, m_y, 0)$

$\vec{m}\times\vec{H}_0 = \begin{vmatrix}\hat{x}&\hat{y}&\hat{z}\\m_x&m_y&0\\H_0^x&H_0^y&H_0^z\end{vmatrix} = \hat{x}m_y H_0^z + \hat{z}[m_x H_0^y - H_0^x m_y] - \hat{y}m_x H_0^z$

$-\frac{1}{\gamma}\dot{\vec{m}} = M_s(\hat{y}h_x - \hat{x}h_y) + \hat{x}m_y H_0^z - \hat{y}m_x H_0^z + \hat{z}[m_x H_0^y - m_y H_0^x]$, где $\vec{m}/M_s \to \vec{m}$

$-\frac{1}{\gamma}\dot{\vec{m}} = \hat{y}h_x - \hat{x}h_y + \hat{x}m_y H_0^z - \hat{y}m_x H_0^z + \hat{z}[m_x H_0^y - m_y H_0^x]$, $m_\pm(\rho,\varphi) = u(\rho)e^{\pm i\varphi_0}$

Azim. Symmetric modes: $m_x = m_x(\rho)$, $m_y = m_y(\rho)$, $m_\pm = m_x \pm i m_y$

Static field $\vec{H}_0 = \vec{H}_0^{e_x} + \vec{H}_0^{m}$, $\vec{H}_0^{e_x} = 0$, т.к. $\vec{M}_0 = M_s \hat{z} = \text{const}$, $\vec{H}_0 = 0$

$\vec{H}_0 = (H_0^x, H_0^y, H_0^z) = (H_0^\rho, 0, H_0^z)$, $H_0^{\rho,z} = H_0^{\rho,z}(\rho, z)$ в силу осевой симметрии $\vec{M}_0$, $H_0^\varphi = 0$.

$$H_0^\rho(\rho,z) = \int d^3\vec{r}' \left[G_{\rho\rho}(\vec{r}\vec{r}') M_0^\rho + G_{\rho\varphi}(\vec{r}\vec{r}') \underbrace{M_0^\varphi}_{=0} + G_{\rho z}(\vec{r}\vec{r}') \underbrace{M_0^z}_{=M_s} \right]$$

$$H_0^\rho(\rho,z) = \int d^3\vec{r}' \, G_{\rho z}(\vec{r}\vec{r}') M_s \qquad G_{\alpha\beta}(\vec{r}\vec{r}') = -\frac{\partial}{\partial x_\alpha \partial x_\beta'} \frac{1}{|\vec{r}-\vec{r}'|}$$

$$\frac{1}{|\vec{r}-\vec{r}'|} = \int dk \, e^{-k|z-z'|} J_0(k|\vec{\rho}-\vec{\rho}'|) = \int dk \, e^{-k|z-z'|} \sum_\mu J_\mu(k\rho) J_\mu(k\rho') e^{i\mu(\varphi-\varphi')}$$

$$G_{\rho z}(\vec{r}\vec{r}') = -\frac{\partial}{\partial \rho}\frac{\partial}{\partial z'} \int dk e^{-k\Delta z} \sum_\mu J_\mu(k\rho) J_\mu(k\rho') e^{i\mu(\varphi-\varphi')}$$

$$\int d\varphi' \, G_{\rho z}(\vec{r}\vec{r}') = \int d\varphi' \left[-\frac{\partial^2}{\partial\rho\partial z'} \right] \int dk e^{-k\Delta z} \sum_\mu J_\mu J_\mu(\cdot) e^{i\mu(\varphi-\varphi')} =$$

$$= -\frac{\partial^2}{\partial\rho\partial z'} \int dk e^{-k\Delta z} 2\pi J_0(k\rho) J_0(k\rho') \quad \text{— в общем } \mu/\mu \beta \neq \varphi$$

$$\int d\varphi' \, G_{\alpha\beta}(\vec{r}\vec{r}') = -\frac{\partial^2}{\partial x_\alpha \partial x_\beta'} \int dk e^{-k\Delta z} 2\pi J_0(k\rho) J_0(k\rho')$$

$$H_0^\rho(\rho,z) = M_s \int d\varphi' \int d\rho' \rho' \int dz' \, G_{\rho z}(\vec{r}\vec{r}') = -M_s \int d^2\vec{\rho}' \, 2\pi \frac{\partial^2}{\partial\rho\partial z'} \int dk e^{-k\Delta z} J_0(k\rho) J_0(k\rho') \quad (\vec{\rho}' d\rho' dz')$$

$$H_0^\rho(\rho,z) = -2\pi M_s \int d\rho' \rho' \int dz' \frac{\partial}{\partial\rho}\frac{\partial}{\partial z'} \int dk e^{-k|z-z'|} J_0(k\rho) J_0(k\rho')$$

$$\int_0^R d\rho' \rho' J_0(k\rho') = |k\rho' = x| = \frac{1}{k^2} \int_0^{kR} dx \, x J_0(x) = \frac{1}{k^2} x J_1(x)\Big|_0^{kR} = \frac{R}{k} J_1(kR)$$

$$\rightarrow H_0^\rho(\rho,z) = -2\pi M_s \frac{\partial}{\partial\rho}\left(\frac{\partial}{\partial z'}\right)^0 \int dz' \frac{\partial}{\partial z'} \int_0^\infty dk e^{-k|z-z'|} J_0(k\rho) \frac{R}{k} J_1(kR)$$

$$H_0^z(\rho,z) = \int d^3\vec{r}' \left[G_{z\rho} \underbrace{M_0^\rho}_{=0} + G_{z\varphi} \underbrace{M_0^\varphi}_{=0} + G_{zz} M_z^\varphi \right] = M_s \int d^3\vec{r}' \, G_{zz}(\vec{r}\vec{r}')$$

$$H_0^z(\rho,z) = M_s \int d\rho' \rho' \int dz' \underbrace{\int d\varphi' G_{zz}(\vec{r}\vec{r}')}_{}, \quad \int d\varphi' G_{zz}(\vec{r}\vec{r}') = -\left(\frac{\partial^2}{\partial z \partial z'}\right) 2\pi \int dk e^{-k\Delta z} J_0(k\rho) J_0(k\rho')$$

$$H_0^z(\rho,z) = M_s \int d\rho' \rho' \int dz' \left(-2\pi \frac{\partial^2}{\partial z \partial z'}\right) \int dk e^{-k\Delta z} J_0(k\rho) J_0(k\rho') = -2\pi M_s \int dz' \frac{\partial^2}{\partial z \partial z'} \int dk e^{-k\Delta z} J_0(\cdot)$$

$$\rightarrow H_0^z(\rho,z) = -2\pi M_s \frac{\partial}{\partial z} \int dz' \frac{\partial}{\partial z'} \int dk e^{-k|z-z'|} J_0(k\rho) \frac{R}{k} J_1(kR) \cdot \text{можно } \int dz'(\ldots).$$

В LLG есть член $\hat{z}[m_x H_0^y - m_y H_0^x]$, $\langle LLG \rangle_\varphi = \frac{1}{2\pi} \int_0^{2\pi} d\varphi (LLG)$.

$m_x = m_x(\rho)$, $m_y = m_y(\rho)$, $\frac{1}{2\pi}\int_0^{2\pi} d\varphi \, H_0^y = \int_0^{2\pi} \frac{d\varphi}{2\pi}\left[H_0^\rho \sin\varphi - H_0^\varphi \cos\varphi \right] = \int \frac{d\varphi}{2\pi} H_0^\rho(\rho,z)$ & ho

$\sim \frac{1}{2\pi}\int_0^{2\pi} d\varphi \, H_0^x = \int_0^{2\pi} \frac{d\varphi}{2\pi} H_0^\rho(\rho,z) \cos\varphi = 0$.

УД: $-\frac{1}{\gamma}\dot{\vec{m}} = \hat{y} h_x - \hat{x} h_y + \hat{x} m_y H_0^z - \hat{y} m_x H_0^z$, предполагая $\frac{1}{2\pi}\int_0^{2\pi} d\varphi(\ldots)$.

$$\begin{cases} -\frac{1}{\gamma}\dot{m}_x = m_y H_0^z - h_y \\ -\frac{1}{\gamma}\dot{m}_y = -m_x H_0^z + h_x \end{cases}$$ — совпадает с УД Jan. 2003, кроме $\int dz(\ldots)$

$H_0^z \to H_0^z + H_z = H_t$

$$\begin{cases} -\frac{1}{\gamma}\dot{m}_+ = (m_y H_0^z - h_y) + i(-m_x H_0^z + h_x) = -i H_0^z m_+ + i(h_x + i h_y) \\ -\frac{1}{\gamma}\dot{m}_- = i H_0^z m_- - i(h_x - i h_y) \end{cases}$$ в согласии с Jan. 2003, H_0 $h_{x,y}$ сглаженные иначе.

$h_d = h_d^{e_x} + h_d^m$. $h_d^m = h_d$ поля. $h_x = \int d^3\vec{r}' \left[G_{xx} m_x + G_{xy} m_y \right] M_s$,

$h_y = \int d^3\vec{r}' \left[G_{yx} m_x + G_{yy} m_y \right] M_s$, $h_x + i h_y = \int d^3\vec{r}' \left[G_{xx} m_x + G_{xy} m_y + i G_{yx} m_x + i G_{yy} m_y \right] M_s$, $m_x = \frac{1}{2}(m_+ + m_-)$, $m_y = \frac{1}{2i}(m_+ - m_-)$

$$h_x + i h_y = \frac{1}{2}\int d^3\vec{r}' \left[G_{xx}(m_+ + m_-) + G_{xy}\frac{1}{i}(m_+ - m_-) + i G_{yx}(m_+ + m_-) + i G_{yy}\frac{1}{i}(m_+ - m_-) \right] M_s$$

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$$h_x + i h_y = \frac{1}{2} M_s \int d^3\vec{r}' \Big[\big(G_{xx} - iG_{xy} + iG_{yx} + G_{yy}\big) m_+ + \big(G_{xx} + iG_{xy} + iG_{yx} - G_{yy}\big) m_- \Big]$$

$$G_{\alpha\beta} = -\frac{\partial^2}{\partial x_\alpha \partial x_\beta} \frac{1}{|\vec{r} - \vec{r}'|} = G_{\alpha\beta}(\vec{r}, \vec{r}'), \quad \text{ман.} \quad \frac{1}{|\vec{r} - \vec{r}'|} = \frac{1}{2\pi} \int d^2\vec{k} \frac{1}{k} e^{-k|z-z'|} e^{i\vec{k}(\vec{\rho} - \vec{\rho}')}$$

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$$G_{\alpha\beta}(\vec{r}, \vec{r}') = -\frac{\partial^2}{\partial x_\alpha \partial x_\beta} \frac{1}{2\pi} \int d^2\vec{k} \frac{1}{k} e^{-k|z-z'|} e^{i\vec{k}(\vec{\rho} - \vec{\rho}')}, \text{ надо } \alpha, \beta = x, y.$$

$$\frac{\partial^2}{\partial x_\alpha \partial x_\beta} e^{i\vec{k}(\vec{\rho} - \vec{\rho}')} = \frac{\partial}{\partial x_\alpha} e^{i\vec{k}\vec{\rho}} \frac{\partial}{\partial x_\beta} e^{-i\vec{k}\vec{\rho}'} = (+ik_\alpha)(-ik_\beta) e^{i\vec{k}(\vec{\rho} - \vec{\rho}')} =$$

$$= k_\alpha k_\beta e^{i\vec{k}(\vec{\rho} - \vec{\rho}')}$$

$$G_{\alpha\beta}(\vec{r}, \vec{r}') = -\frac{1}{2\pi} \int d^2\vec{k} \frac{1}{k} e^{-k|z-z'|} k_\alpha k_\beta e^{i\vec{k}(\vec{\rho} - \vec{\rho}')}, \quad \alpha, \beta = x, y.$$

$$G_{\alpha\beta}(\vec{r}, \vec{r}') = -\frac{1}{2\pi} \int d^2\vec{k} \frac{k_\alpha k_\beta}{k} e^{-k|z-z'|} e^{i\vec{k}(\vec{\rho} - \vec{\rho}')} \quad \int_0^L dz dz' e^{-k|z-z'|} = \frac{2L}{k} f(kL)$$

$$g_{\alpha\beta}(\vec{\rho}, \vec{\rho}') = \frac{1}{L} \int dz dz' G_{\alpha\beta}(\vec{r}, \vec{r}') = -\frac{1}{2\pi} \int d^2\vec{k} \frac{k_\alpha k_\beta}{k} \frac{2L}{k} f(kL) e^{i\vec{k}(\vec{\rho} - \vec{\rho}')}$$

$$g_{\alpha\beta}(\vec{\rho}, \vec{\rho}') = \frac{1}{(2\pi)^2} \int d^2\vec{k}\, g_{\alpha\beta}(\vec{k}) e^{i\vec{k}(\vec{\rho} - \vec{\rho}')}, \quad g_{\alpha\beta}(\vec{k}) = -4\pi \frac{k_\alpha k_\beta}{k^2} f(kL),$$

совпадает с Eq. (20) Gus. Jmm '2011. Поэтому (*) верно для $\alpha, \beta = x, y$.

$G_{\alpha\beta}(\vec{r}, \vec{r}')$ теперь можно $\int d\varphi e^{(\dots)}$ и $\int d\varphi' e^{(\dots)}$.

$$\int \frac{d\varphi}{2\pi} \int \frac{d\varphi'}{2\pi} G_{xx} = \int_0^{2\pi} \frac{d\varphi}{2\pi} \int_0^{2\pi} d\varphi' G_{xx}(\vec{r}, \vec{r}') = \frac{1}{2\pi} \int d\varphi \int d\varphi' \left(-\frac{\partial}{\partial x}\right) \int d^2\vec{k} \frac{k_x^2}{k} e^{-k|z-z'|} e^{i\vec{k}(\vec{\rho} - \vec{\rho}')}$$

$$\int d\varphi \int d\varphi' e^{i\vec{k}(\vec{\rho} - \vec{\rho}')} = \int d\varphi e^{i\vec{k}\vec{\rho}} \cdot \int d\varphi' e^{-i\vec{k}\vec{\rho}'} = \left| \int d\varphi e^{i\vec{k}\vec{\rho}} \right|^2 - \text{нет}.$$

$$\int_0^{2\pi} d\varphi e^{i\vec{k}\vec{\rho}} = \int d\varphi e^{ik\rho\cos(\varphi - \varphi_k)} \quad \text{Jacobi-Anger expansion} \quad e^{ix\cos\varphi} = \sum_{n=-\infty}^{\infty} i^n J_n(x) e^{in\varphi}$$

$$\int_0^{2\pi} d\varphi e^{ix\cos\varphi} = \int_0^{2\pi} d\varphi \sum_n i^n J_n(x) e^{in\varphi} = \sum_n i^n J_n(x) 2\pi\delta_{n0} = 2\pi J_0(x)$$

$$\int_0^{2\pi} d\varphi e^{ix\cos(\varphi - \varphi_k)} = \left| \varphi - \varphi_k = \psi \right| = \int_{-\varphi_k}^{2\pi - \varphi_k} d\psi e^{ix\cos\psi} = \int_0^{2\pi} d\psi e^{ix\cos\psi} = 2\pi J_0(x)$$

$$\int_0^{2\pi} d\varphi' e^{-i\vec{k}\vec{\rho}'} = \int d\varphi' e^{-ik\rho'\cos(\varphi' - \varphi_k)} \quad \text{---} \quad \int_0^{2\pi} d\psi e^{-ik\rho'\cos\psi} = 2\pi J_0(k\rho')$$

$$\int_0^{2\pi} \frac{d\varphi}{2\pi} \int_0^{2\pi} d\varphi' \cdot e^{i\vec{k}(\vec{\rho} - \vec{\rho}')} = \frac{1}{2\pi} \cdot 2\pi J_0(k\rho) \cdot 2\pi J_0(k\rho') = 2\pi J_0(k\rho) J_0(k\rho')$$

$$\frac{1}{2\pi} \int d\varphi \int d\varphi' G_{xx}(\vec{r}, \vec{r}') = -\frac{1}{2\pi} \int d^2\vec{k} \frac{k_x^2}{k} e^{-k|z-z'|} \cdot 2\pi J_0(k\rho) J_0(k\rho') =$$

$$= -\int d^2\vec{k} \frac{k_x^2}{k} e^{-k|z-z'|} J_0(k\rho) J_0(k\rho')$$

$$\int \frac{d\varphi}{2\pi} \int d\varphi' G_{xx}(\vec{r}, \vec{r}') = (-1) \int dk\, k\, d\varphi_k \frac{1}{k} k^2 \cos^2\varphi_k e^{-k|z-z'|} J_0(k\rho) J_0(k\rho')$$

$$\int_0^{2\pi} d\varphi_k \cos^2\varphi_k = \pi, \quad \int \frac{d\varphi}{2\pi} \int d\varphi' G_{xx}(\vec{r}, \vec{r}') = -\pi \int dk\, k^2 e^{-k|z-z'|} J_0(k\rho) J_0(k\rho')$$

Очевидно $\int \frac{d\varphi}{2\pi} \int d\varphi' G_{yy}(\vec{r}, \vec{r}') = \int \frac{d\varphi}{2\pi} \int d\varphi' G_{xx}(\vec{r}, \vec{r}')$ т.к. $\int_0^{2\pi} d\varphi_k \sin^2\varphi_k = \pi$.

$\int \frac{d\varphi}{2\pi} \int d\varphi' G_{xy}(\vec{r}, \vec{r}') = \int \frac{d\varphi}{2\pi} \int d\varphi' G_{yx}(\vec{r}, \vec{r}') \equiv 0$, т.к. $\int_0^{2\pi} d\varphi_k \cos\varphi_k \sin\varphi_k = 0$.

Поэтому в 3D мы $\int_0^{2\pi} \frac{d\varphi}{2\pi}$ учредняем $h_x + i h_y = \frac{1}{2} M_s \int d^3\vec{r}' \big[2 G_{xx} m_+ + 0 \cdot m_-\big] =$

$= M_s \int d^3\vec{r}' G_{xx}(\vec{r}, \vec{r}') m_+(\rho', z')$ и 3D даёт m_+ и m_- независимые.

Def. $\int \frac{d\varphi}{2\pi} \int d\varphi' G_{xx}(\vec{r}, \vec{r}') = -\pi \int dk\, k^2 e^{-k|z-z'|} J_0(k\rho) J_0(k\rho') = g(\rho\rho', zz')$

$$-\frac{1}{\gamma} \dot{m}_+ = -i H_0^z m_+ + i(h_x^{ex} + i h_y^{ex}) + i M_s \int d^3\vec{r}' G_{xx}(\vec{r}, \vec{r}') m_+(\rho', z'), \quad \vec{h}^{ex} = q v^2 \vec{m}$$

[Mdkcn]

$$h_x^{ex} + i h_y^{ex} = q v^2 m_x + i q v^2 m_y = q v^2 m_+.$$

$$-\frac{i}{\gamma} \dot{m}_+ = H_z^0 m_+ - q v^2 m_+ - M_s \int d\rho' \rho' \int dz' g(\rho\rho', zz') m_+(\rho', z'), \quad q = \frac{2A}{M_s}$$

$$m_+(\rho, z, t) = m_+(\rho, z) e^{i\omega t} \Rightarrow \frac{\omega}{\gamma} m_+ = (H_z + H_z^0) m_+ - q v^2 m_+ - M_s \int d\rho' \rho' \int dz' g(\rho\rho', zz') m_+(\rho', z')$$

Попробуем 3D обобщающее Jan. 2003, $m_+ = m_+(\rho, z)$, $\vec{r} = (\rho, \varphi, z)$, $m_+ = m_x + i m_y$.

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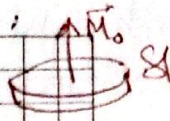
28/IV New magnetostatic kernel given SW ↑ и $m_+(\rho,z)$:



$$g(\rho,\rho',zz') = -\pi\int_0^\infty dk\, k^2 e^{-k|z-z'|} J_0(k\rho) J_0(k\rho')$$

$$\frac{1}{L}\int dz \int dz'\, g(\rho\rho', zz') = -\pi\int dk\, k^2 J_0(k\rho)J_0(k\rho')\frac{1}{L}\int_0^L dz\, e^{-k|z-z'|} =$$

$$= -2\pi\int dk \underbrace{\frac{k}{f(kL)}} J_0(k\rho) J_0(k\rho') = 2\pi \langle G_{xx}\rangle_{\rho\rho'} \quad -4\pi \text{ used in Jan. 2003}$$

given Kakazu-Aoe'ou

$$\frac{\omega}{\gamma} = (\dots) - M_s \int d\rho'\rho' \int dz' g(\rho\rho', zz')\, m_+(\rho' z') \quad \text{и т.ч.} \quad m_+(\rho,z) = m_+(\rho),\ \int_L \frac{dz}{L}(\dots)$$

$$\frac{\omega}{\gamma} = (\dots) - M_s \int d\rho'\rho' \int \frac{dz}{L}\int dz' g(\rho\rho' zz')\, m_+(\rho) = (\dots) + 2\pi M_s \int dk\, k f(kL) J_0(k\rho) \underbrace{\int_0^{\rho'} J_0(k\rho')\rho' d\rho'}_{m_+(\rho')\times}$$

$$\frac{1}{L}\int_L dz\, g(\rho\rho', zz') = -2\pi\int dk\, k f(kL) J_0(k\rho) J_0(k\rho') = 2\pi \bar{g}(\rho\rho') \quad \text{как в Jan. 2003.}$$

Тогда $\frac{\omega}{\gamma} = (\dots) + 2\pi M_s \int d\rho'\rho' \int dk\, k f(kL) J_0(k\rho) J_0(k\rho')\, m_+(\rho')$

Нерегулярность в этом доказательстве $\frac{1}{L}\int dz(\dots)$ дает $g(\rho\rho', zz')$ - вершина.

Доказать: $\langle G_{xx}\rangle_\rho + \langle G_{yy}\rangle_\rho + \langle G_{zz}\rangle_\rho = -2\pi\int dk\, k^2 e^{-k|z-z'|} J_0(k\rho)J_0(k\rho') \pm$

$$\pm 2\pi\int_0^\infty dk\, e^{-k|z-z'|} J_0(k\rho)J_0(k\rho')\left[k^2 - 2k\,\delta(z-z')\right] = -4\pi\int dk\, k e^{-k|z-z'|} J_0(k\rho)J_0(k\rho') \times$$

$$= -4\pi\int dk\, k\, J_0(k\rho)J_0(k\rho') = -4\pi\frac{1}{\rho}\delta(\rho-\rho') \quad - ? \text{ А где } \delta(z-z')? \quad \delta(z-z') =$$

$$\delta(\vec{r}-\vec{r}') = \delta(\rho-\rho')\frac{1}{\rho}\delta(\varphi-\varphi')\delta(z-z'), \quad \int_0^{2\pi} d\varphi\,\delta(\varphi-\varphi') = 1 \text{ если } \varphi'\in[0,2\pi].$$

Мы $G_{zz}(\vec{r}\vec{r}')$ дает $\int d\varphi' G_{zz}(\vec{r}\vec{r}') = -2\pi\frac{\partial^2}{\partial z\partial z'}\int dk\, e^{-k|z-z'|} J_0(k\rho)J_0(k\rho')$

$$\int\frac{d\varphi}{2\pi}\int d\varphi' G_{zz}(\vec{r},\vec{r}') = -2\pi\frac{\partial^2}{\partial z\partial z'}\int dk\, e^{-k|z-z'|} J_0(k\rho)J_0(k\rho')$$

$$\langle G_{xx}\rangle_\rho + \langle G_{yy}\rangle_\rho + \langle G_{zz}\rangle_\rho = -2\pi\int dk\, k^2 e^{-k|z-z'|} J_0(k\rho)J_0(k\rho') -$$

$$-2\pi\frac{\partial^2}{\partial z\partial z'}\int dk\, e^{-k|z-z'|} J_0(k\rho)J_0(k\rho'), \quad \frac{\partial^2 e^{-k|z-z'|}}{\partial z\partial z'} = -k^2 e^{-k|z-z'|} + 2k\delta(z-z') \ ?$$

$$\sum_\alpha \langle G_{\alpha\alpha}\rangle_\rho = -2\pi\int dk\, k^2 e^{ikz} J_0 J_0(\cdot) - 2\pi\int dk\, k e^{-ik\Delta z} J_0 J_0(\cdot)\left[-k^2 e^{-k|z-z'|} + 2k\delta(z-z')\right]$$

$$= -4\pi\int dk\, e^{-k|z-z'|} k\,\delta(z-z') J_0 J_0(\cdot) = -4\pi\delta(z-z')\int dk\, k J_0(k\rho)J_0(k\rho') =$$

$$= -4\pi\delta(z-z')\frac{1}{\rho}\delta(\rho-\rho').$$

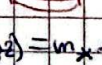
У справа $\sum_\alpha \langle G_{\alpha\alpha}\rangle_\rho = -4\pi\int dk\, k e^{-k|z-z'|} J_0(k\rho)J_0(k\rho')\,\delta(z-z') =$

$$= -4\pi\,\delta(z-z')\int dk\, k J_0(k\rho)J_0(k\rho') = -4\pi\delta(z-z')\frac{1}{\rho}\delta(\rho-\rho') - \text{верное, надо } +4\pi$$

в выражении $\langle G_{zz}\rangle_\rho$.

//В рецензии Kakazu-Aoe'ou - A.W. Julian и его можно, Erzoski и Corbil. Julian сказал написать to Director of dept. "Agustin E. Dgt. Scientific position и напишите отд next week. INTERMAG-26 Egger в Manchester, U.K. Agustin Etxeberria {agustin.etxeberria@ehu.es}

→ Axisymmetrically Symmetric SW in cylindrical dot: 2D SW



$$\frac{\omega}{\gamma} m_+ = \left[H_z + H_z^0 - \ell\nabla^2\right] m_+ - M_s\int d\rho'\rho' \int dz' g(\rho\rho', zz')\, m_+(\rho' z'), \quad m_+(\rho z) = m_{x+iy}$$

kernel $g(\rho\rho', zz') = -\pi\int dk\, k^2 e^{-k|z-z'|} J_0(k\rho)J_0(k\rho')$, $\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{\partial^2 f}{\partial z^2}$.

$$H_z^0(\rho,\rho) = -2\pi M_s \frac{\partial}{\partial z}\int_0^L dz' \frac{\partial}{\partial z'}\int dk\, e^{-k|z-z'|}\frac{R}{k} J_0(k\rho) J_1(kR) =$$

$$= -2\pi M_s R\frac{\partial}{\partial z}\left[\int\frac{dk}{k} e^{-k|z+z'|} J_0(k\rho)J_1(kR)\right]_0^L = -2\pi M_s R\frac{\partial}{\partial z}\int\frac{dk}{k} J_0(k\rho)J_1(kR) \times$$

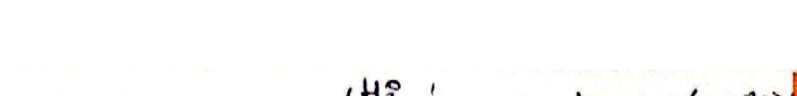
$$\times\left(e^{-k|z-L|} - e^{-k|z|}\right) = -2\pi M_s R\frac{\partial}{\partial z}\int\frac{dk}{k} J_0(k\rho)J_1(kR)\left[e^{k(z-L)} - e^{kz}\right]$$

$$H_z^0(\rho,z) = -2\pi M_s R\int dk\, J_0(k\rho) J_1(kR)\left[e^{k(z-L)} + e^{-kz}\right]. \quad z \text{ в eq. 4. } kR\to k$$

$$H_z^0(\rho,z) = -2\pi M_s\int_0^\infty dk\, J_0(k\rho) J_1(k)\left[e^{\beta k(z-1)} + e^{-\beta kz}\right]. \quad \text{в eq. 4.}$$

и.т.ч. $\rho=0$, $\beta = 1-2$

$H_z^0(\rho, z)$ довольно плавная функция z and moderate $\beta \sim 1$



Дип. dipolar field $h_\bullet^z = h_z(\rho, z) = -M_s \int d\rho' \rho' \int dz' g(\rho\rho', zz') m_+(\rho' z')$

I $m_+(\rho, z) = Q(\rho) F(z)$, $Q(\rho) = J_0(x\rho)$, $x = x_n$ root of $J_0(x) = 0$ – хорошее приближение at least at small β.

$h_z(\rho, z) = -M_s \int d\rho' \rho' \int dz' g(\rho\rho', zz') m_+(\rho' z') = -M_s \int d\rho' \rho' \int dz' g(\rho\rho', zz') J_0(x\rho') F(z')$

$\Gamma(\rho, zz') = \int d\rho' \rho' g(\rho\rho', zz') J_0(x\rho')$, $h_z(\rho, z) = -M_s \int dz' \Gamma(\rho, zz') F(z')$

$\Gamma(\rho, zz') = -\pi \int d\rho' \rho' J_0(x\rho') \int dk\, k^2 e^{-k|z-z'|} J_0(k\rho) J_0(k\rho') = \left| \text{по } \rho' \text{ инт.} \right| =$

$= -\pi \int dk\, k^2 e^{-k|z-z'|} J_0(k\rho) \mathcal{J}_n(k)$, $\mathcal{J}(k) = \int_0^R d\rho' \rho' J_0(x_n \rho') J_0(k\rho')$

$\Gamma(\rho, zz')$ имеет резкий max при $z = z'$, поэтому $h_z(\rho, z) \simeq -M_s F(z) \int dz' \Gamma(\rho, zz')$

$G(\rho, z) = \int dz' \Gamma(\rho, zz')$, $h_z(\rho, z) = -M_s F(z) G(\rho, z)$

УД: $\frac{\omega}{\gamma} m_+ = \left[H_z + H_z^0 - q\nabla^2 \right] Q(\rho) F(z) - M_s F(z) G(\rho, z)$.

II $\nabla^2 Q(\rho) F(z) = \left[\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial}{\partial \rho} \right) + \frac{\partial^2}{\partial z^2} \right] Q(\rho) F(z)$, $\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial Q}{\partial \rho} \right) = \frac{\partial^2 Q}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial Q}{\partial \rho}$

$\frac{\partial^2 J_0(x\rho)}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial J_0}{\partial \rho} + \left(1 - \frac{d^2}{\rho^2}\right) J_0(x\rho) = 0$, $\frac{\partial^2 J_0(x)}{\partial x^2} + x^2 \frac{1}{x} \frac{\partial J_0}{\partial x} =$

$= x^2 \left[\frac{\partial^2 J_0}{\partial x^2} + \frac{1}{x} \frac{\partial J_0}{\partial x} \right] = -x^2 J_0$, $y'' + \frac{1}{x} y = -\left(1 - \frac{d^2}{x^2}\right) y(x) \underset{d=0}{=} -y(x)$

$\nabla^2 Q(\rho) F(z) = \left[\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial Q}{\partial \rho} \right) F(z) + Q(\rho) \frac{\partial^2 F}{\partial z^2} \right] = \left[-x^2 QF + Q \frac{\partial^2 F}{\partial z^2} \right]$, $Q = J_0(x\rho)$

$\frac{\omega}{\gamma} QF = (H_z + H_z^0) QF - q \left[-x^2 QF + Q \frac{\partial^2 F}{\partial z^2} \right] - M_s F(z) G(\rho, z)$,

$G(\rho, z) = \int dz' \Gamma(\rho, zz') = \int dz' (-\pi) \int dk\, k^2 e^{-k|z-z'|} J_0(k\rho) \int d\rho' \rho' Q(\rho') J_0(k\rho')$

$\frac{\omega}{\gamma} QF = (H_z + H_z^0) QF + qx^2 QF - qQF''(z) - M_s F(z) G(\rho, z)$

$\left[\frac{\omega}{\gamma} - H_z - H_z^0 - qx^2 \right] QF = -qQF'' - M_s F(z) G(\rho, z)$, инт. $\int d\rho\, \rho (\dots) Q(\rho)$

$\left[\frac{\omega}{\gamma} - H - qx^2 \right] F(z) \int d\rho\, \rho Q^2(\rho) = -qF'' \int d\rho\, \rho Q^2(\rho) - M_s F(z) \int d\rho\, \rho G(\rho, z) Q(\rho)$,

$\int d\rho\, \rho Q^2(\rho) = \int d\rho\, \rho J_0(x_n \rho)^2 = N_n$, $\left[\frac{\omega}{\gamma} - H - qx^2 \right] N F(z) = -qNF'' - M_s F(z) \int d\rho\, \rho G(\rho z) Q(\rho)$

$\left[\frac{\omega}{\gamma} - H - qx^2 \right] F(z) = -qF'' - M_s F(z) \frac{1}{N} \int d\rho\, \rho G(\rho, z) Q(\rho)$. $F(z) = \psi(z)$.

Def. $\frac{1}{N} \int d\rho\, \rho G(\rho z) Q(\rho) = f(z)$

$\left[\frac{\omega}{\gamma} - H - qx^2 \right] \psi(z) = -q\psi'' - M_s \psi(z) f(z)$. $\left[\frac{\omega}{\gamma} - H - qx^2 + M_s f(z) \right] \psi(z) = -q\psi''(z)$.

$\psi''(z) + \frac{1}{q} \left[\frac{\omega}{\gamma} - H_z - H_z^0(z) - qx^2 + M_s f(z) \right] \cdot \psi(z) = 0$, $H_z^0(z) = \int d\rho\, \rho H_z^0(\rho z) Q^2(\rho)$

$\psi''(z) + \frac{1}{q} \left[\frac{\omega}{\gamma} - H_z - H_z^0(z) - qx^2 + M_s f(z) \right] \psi(z) = 0$ – в точности из magnetostatics импеданса и $m_+(\rho, z) = Q(\rho) F(z)$ assumption получили что $\psi(z) = F(z)$

$\frac{\omega}{\gamma} - H_z - qx^2 = \frac{\delta\Omega(x)}{\gamma}$ $\psi'' + \frac{1}{q} \left[\frac{1}{\gamma} \delta\Omega - H_z^0(z) + M_s f(z) \right] \psi(z) = 0$.

Посчитать $f(z)$. Насколько она плавная и где ее экстремумы?

$f(z) = \frac{1}{N} \int d\rho\, \rho G(\rho z) J_0(x\rho) = \frac{1}{N} \int d\rho\, \rho J_0(x\rho) \int dz' (+\pi) \int dk\, k^2 e^{-k|z-z'|} J_0(k\rho) \mathcal{J}_n(k) =$

$= \left| \mathcal{J}_n(k) = \int d\rho' \rho' J_0(x_n \rho') J_0(k\rho') \right| = -\frac{\pi}{N} \int d\rho\, \rho J_0(x\rho) \int dz' \int dk\, k^2 e^{-k|z-z'|} J_0(k\rho) \int d\rho' \rho' J_0(x\rho') J_0(k\rho')$

$f(z) = -\frac{\pi}{N} \int dz' \int dk\, k^2 e^{-k|z-z'|} \mathcal{J}_n^2(k)$. $N_n = N = \int d\rho\, \rho J_0^2(x_n \rho)$.

Escaneado con CamScanner

29/IX Исправка SW - dot equations, бълнув.

30/IX ← А. В. „Lidl" и „Carrefour" Oleberica. Фенс Венцеслав.

В dyn. dipolar coupling $f(z)$ можело $\int dz'(\dots)$. $\int_0^L dz' e^{-k|z-z'|} =$

$= |z \text{ beg. } L,\ kR \to k| = L\int_0^1 d\zeta' e^{-\beta k|z-z'|} = L\left(\int_0^z d\zeta' e^{-\beta k(z-z')} + \int_z^1 d\zeta' e^{-\beta k(z'-z)}\right) =$

$= e^{-\beta k z} L\int_0^z d\zeta' e^{\beta k z'} + e^{\beta k z} L\int_z^1 d\zeta' e^{-\beta k z'} = e^{-\beta k z}\frac{L}{\beta k} e^{\beta k z'}\Big|_0^z + e^{\beta k z}\frac{L}{\beta k}(-1) e^{-\beta k z'}\Big|_z^1 =$

$= \frac{L}{\beta k} e^{-\beta k z}\left(e^{\beta k z} - 1\right) - \frac{L}{\beta k} e^{\beta k z}\left(e^{-\beta k} - e^{-\beta k z}\right) = \frac{L}{\beta k}\left(1 - e^{-\beta k z}\right) -$

$- \frac{L}{\beta k}\left(e^{\beta k(z-1)} - 1\right) = \frac{L}{\beta k}\left[2 - e^{-\beta k z} - e^{\beta k(z-1)}\right] = \frac{L}{\beta k}\left[2 - e^{-\frac{\beta k}{2}} e^{-\beta k z} + \frac{1}{2}\beta k -\right.$

$\left. - e^{-\frac{\beta k}{2}} e^{\beta k z} e^{-\beta k/2}\right] = \frac{L}{\beta k}\left[2 - e^{-\beta k/2}\left\{e^{-\beta k\left(z-\frac{1}{2}\right)} + e^{\beta k\left(z-\frac{1}{2}\right)}\right\}\right] =$

$= \frac{L}{\beta k}\left[2 - e^{-\beta k/2} 2\,\mathrm{ch}\,\beta k\left(z-\tfrac{1}{2}\right)\right] = \frac{2L}{\beta k}\left[1 - e^{-\beta k/2}\,\mathrm{ch}\,\beta k\left(z-\tfrac{1}{2}\right)\right] = L\int_0^1 dz' e^{-\beta k|z-z'|}$

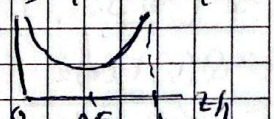
$f(z) = -\frac{\pi}{\int_0^1 d\varsigma\varsigma J_0^2(\varkappa\varsigma)} \int dk k^2 \left[\int_0^1 d\varsigma\varsigma J_0(\varkappa\varsigma) J_0(k\varsigma)\right]^2 \cdot \left(\frac{2L}{\beta k}\right)\left[1 - e^{-\beta k/2}\,\mathrm{ch}\,\beta k\left(z-\tfrac{1}{2}\right)\right] =$

$= -2\pi h \frac{1}{R^2\int_0^1 d\varsigma\varsigma J_0^2(\varkappa\varsigma)} \int dk \underbrace{k^2 R^4}_{k^2}\left[\int_0^1 d\varsigma\varsigma J_0 J_0\right]^2 \left(\frac{1}{\beta k}\right)\left[1 - e^{-\beta k/2}\,\mathrm{ch}(\dots)\right] =$

$= -2\pi\beta \frac{1}{N}\int_0^\infty dk \frac{k^2}{\beta k} J_n^2(k)\left[1 - e^{-\beta k/2}\,\mathrm{ch}\,\beta k\left(z-\tfrac{1}{2}\right)\right],\quad f(z) = -\frac{2\pi}{N}\int_0^\infty dk k J_n^2(k)\left[1 - e^{-\frac{\beta k}{2}}\,\mathrm{ch}\,\beta k\left(z-\tfrac{1}{2}\right)\right]$

Dimensionless $N = \int_0^1 d\varsigma\varsigma J_0^2(\varkappa\varsigma)$, $J_n(k) = \int_0^1 d\varsigma\varsigma J_0(\varkappa\varsigma) J_0(k\varsigma)$, $k \to kR$, $z \to z/L$.

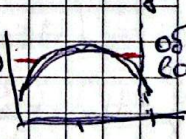
$f(z) = -\frac{2\pi}{N}\int_0^\infty dk k J_n^2(k)\left[1 - e^{-\beta k/2}\,\mathrm{ch}\,\beta k\left(z-\tfrac{1}{2}\right)\right]$



Dynamic dipolar field $h_d(z) = M_s f(z)$.

Ур-ие $q\psi''_{zz} + \left[\frac{\Omega}{\gamma} - H_z^0(z) + M_s f(z)\right]\psi = 0$, $q\psi'' + \left[E - U(z)\right]\psi(z) = 0$. $\frac{\Omega}{\gamma} \to E$.

Potential $U(z) = H_z^0(z) - M_s f(z)$. $\frac{\Omega}{\gamma} = \frac{\omega}{\gamma} - H_z - q\varkappa^2$



File SW_thick_Cylinder.mcd

Potential ур-ие $U(z)$ хар-а имаме min при $z = 0, 1$, $\int^{dot}$ faces, $U(z)$ задължително трябва да е $\beta = 1, 2$.

$f(z)$ получаваме в local dip. field approximation. Търсим low-$\frac{\omega}{\gamma}$ SW localization at the dot faces $z = 0, 1$.

Неотчитане при работа $H_z^0(z)$, вече $H_z^0 = H_z^0(s, z)$.

$\left[\frac{\omega}{\gamma} - H_z^0 + \dots\right] QF = -qQF'' - M_s F G(s, z)$, умн. на $Q(s)$ и $\int \frac{ds\varsigma}{N}(\dots)$, $N = \int ds\varsigma Q^2$

$\left[\frac{\omega}{\gamma} - \frac{1}{N}\int ds\varsigma Q^2(s) H_z^0(z, s) + \dots\right] F = -qF'' - M_s F \frac{1}{N}\int ds\varsigma G(s, z) Q(s)$

Нагл. $H_z^0(z) = \frac{1}{N}\int ds\varsigma Q^2(s) H_z^0(s, z)$, $Q(s) = J_0(\varkappa s)$.

1/X E-mails Адриан Етхеберия (Distinq. Scientist appl.), Mag. Frontiers 2025 proposal и среща (MS on FMR in cyl. dots).

2-3/X Update documents Santander. Кореспонд. с Patricia Juhan, UAV/EHU, Distinguished Scientist contract. File Strelkov - WKB для SW. Отсъства рецензия Романенко. El. Al. Mat. - AM-manuscript.

4/X Старт с Сорокин Телеграфъ: SW thick cyl. dot. $m(z)$ работа с dipolar static и dynamic fields. WKB для ур. Schroedinger eq. Ако FMR - beyond the Kittel mode не е добре localized и mode Fig. 4c even w/ 2 с общо zeros $\neq 0$. Разпечатване на APL paper < FMR Ескеровски.

Hopf index

C.D. Parmee et al. Optical Excitations of Skyrmions, knotted Solitons... Comm. Phys. 2022, 5:54. Stokes vector $\vec{S} = \hat{P}^+ \vec{\sigma} \hat{P}$, $\hat{P} = \begin{pmatrix} n_2+in_1 \\ n_4+in_3 \end{pmatrix}$ Spinor

Hopf charge (Baryon number) $B = \int d^3\vec{x}\, b(\vec{x}) = -\int \frac{d^3\vec{x}}{2\pi^2} \varepsilon_{ijk} \varepsilon_{abcd} n_a \frac{\partial n_b}{\partial x_i} \frac{\partial n_c}{\partial x_j} \frac{\partial n_d}{\partial x_k}$

$b(\vec{x}) = -\frac{1}{4\pi^2}(\vec{j} \cdot \mathrm{rot}\, \vec{j})$, $\vec{j} = \frac{1}{2i}[P^+ \nabla P - \nabla P^+ P] = \vec{A}$ – transverse polarization density current (emergent field potential $\vec{A}$)

Gis' Chaos' 2023 $\vec{A} = 2\,\mathrm{Im}(Z^+ \nabla Z) = -i[Z^+ \nabla Z - \nabla Z^+ Z]$, $\vec{j} = -\frac{i}{2}[P^+ \nabla P - \nabla P^+ P]$

Vortex vector potential $\vec{A} = 2(\vec{P})\vec{j}$, eqw $Z = P$, $\vec{j} = \vec{A}/2$.

$Q_H = B = \int dV\, b(\vec{x}) = -\frac{1}{4\pi^2} \int dV (\vec{j} \cdot \mathrm{rot}\, \vec{j}) = -\frac{1}{4\pi^2}\frac{1}{4} \int dV \vec{B} \cdot \vec{A} = -\frac{1}{(4\pi)^2} \int dV \vec{B} \cdot \vec{A}$

$Q_H = B$ is defined via $\vec{m} = \frac{\partial n_a}{\partial p}$, $m_1 = 2(n_1 n_3 + n_2 n_4)$, $m_2 = 2(n_2 n_3 - n_1 n_4)$, $m_3 = n_1^2 + n_2^2 - n_3^2 - n_4^2$

$\vec{m} = (m_1, m_2, m_3)$ magnetization unit vector.

т.е. Q_H через преобразование через спинор $Z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$ components, но не через $\vec{m}$ – components.

Обоснование eq. for $Q_H = B$ см. S.B. Gudnason, M. Nitta. Ph. Rev. D 101, 065020 (2020)

Linking number of vortices as Baryon number.

$$Q_H = B = -\int \frac{d^3\vec{x}}{2\pi^2} \varepsilon_{ijk} \varepsilon_{abcd} n_a \frac{\partial n_b}{\partial x_i} \frac{\partial n_c}{\partial x_j} \frac{\partial n_d}{\partial x_k},$$ spinor $\hat{P} = \begin{pmatrix} n_2+in_1 \\ n_4+in_3 \end{pmatrix}$, $\vec{m} = \hat{P}^+ \vec{\sigma} \hat{P}$

Hopf index via spinor components n_i of Hopf map.

т.к. deposited cash 5.820€ на Caixa bank see CONSTANTO.

(5/X) E-mails to Patricia, Agustin (Director dept.) и Ruben.

$Q_H = B = \int d^3\vec{x}\, b(\vec{x})$, $b(\vec{x}) = \frac{1}{4\pi^2} \varepsilon_{ijk} Z^+ \partial_i Z\, \partial_j Z^+ \partial_k Z$, spinor $Z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$ Gudnason 2020

Hopf density $q_H = b(\vec{x}) = \frac{1}{4\pi^2} Z^+ \nabla Z \cdot (\nabla Z^+ \times \nabla Z)$, Hopf density via Z. $\partial_i = \partial x_i$, $(\nabla Z)_i := \partial_i Z$

Two-component BEC, $\Psi(\vec{x}) = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \sqrt{\rho(\vec{x})} \begin{pmatrix} z_1(\vec{x}) \\ z_2(\vec{x}) \end{pmatrix}$, $|z_1|^2 + |z_2|^2 = 1$, $|\psi_1|^2 + |\psi_2|^2 =$

Есть сверх. BEC-theory and Hopf mapping $Z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$.

q_H via ∇Z – красивое преобразование!

Gus' Chaos' 2023: $q_H = \frac{1}{(4\pi)^2} \vec{A} \cdot \vec{B}$, $B_\lambda = \frac{1}{2} \varepsilon_{\lambda\mu\nu} F_{\mu\nu}$, $F_{\mu\nu} = -2i[\partial_\mu Z^+ \partial_\nu Z - \partial_\nu Z^+ \partial_\mu Z]$

Из него q_H defined via $\vec{A} \cdot \vec{B}$ можно показать q_H (Gudnason) as triple dot product

$A_\mu = -i[Z^+ \partial_\mu Z - \partial_\mu Z^+ Z]$

$q_H = \frac{1}{(4\pi)^2} A_\mu B_\mu = \frac{1}{(4\pi)^2}(-i)[Z^+ \partial_\mu Z - \partial_\mu Z^+ Z] \frac{1}{2} \varepsilon_{\mu\lambda\nu} (-2i)[\partial_\lambda Z^+ \partial_\nu Z - \partial_\nu Z^+ \partial_\lambda Z]$

$Q_H = -\int \frac{d^3\vec{x}}{2\pi^2} \cdot \varepsilon_{abcd} n_a \nabla n_b \cdot (\nabla n_c \times \nabla n_d)$ через n_i, $Z = \begin{pmatrix} n_2+in_1 \\ n_4+in_3 \end{pmatrix}$

→ Остаток cash на 5/X: 3.530€, Ипп – 200€, А. – 20€. = 3.580€ – 200 for +150€ (ноя)

(6/X) Обзор papers по 3D magnetic textures. Roma DMI in their stability

(7/X) Начинаем Introduction к „Hopfion stability" MS. С А. в среду, обсудим

(8/X) Вернемся к расчету 3D-texture – toroidal hopfion stability 24

$E(\vec{m}) = E_{ex} + E_D + E_m$. E_D – DMI. $\vec{m}(\vec{x}) \to \vec{m}(\vec{x}/a)$, a – (hopfion radius).

$E_{ex} = A \int dV \sum_{i=1}^{3} (\nabla m_i)^2 = A \int dV [(\nabla\Theta)^2 + \sin^2\Theta (\nabla\Phi)^2]$ where $\vec{m} = \vec{m}(\Theta, \Phi)$

$\Theta(\vec{x}) = \Theta(\rho, z)$, $m_z = \cos\Theta = f(y)$ только. $\Phi(\vec{x}) = n\varphi + \gamma(\rho, z) = n\varphi + m\beta(\rho z) \to \Phi(\gamma\beta\varphi) = n\varphi +$

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$\nabla\Theta = \nabla(\cos\Theta)(-1) = -\frac{1}{\sin\Theta}\nabla(m_z)$, $(\nabla\Theta)^2 = \frac{(\nabla m_z)^2}{\sin^2\Theta} = \frac{(\nabla m_z)^2}{(1-m_z^2)}$, $\Phi = n\varphi + m\beta$.

Hopfion Stability

$\nabla\Phi(\gamma,\beta,\varphi) = \nabla(n\varphi+m\beta) = \frac{\tau}{a}\left[\hat e_\gamma\frac{\partial}{\partial\gamma} + \hat e_\beta\frac{\partial}{\partial\beta} + \hat e_\varphi\frac{\partial}{\sin\gamma\,\partial\varphi}\right]\Phi = \frac{\tau}{a}\left[\hat e_\beta m + \hat e_\varphi\frac{n}{\sin\gamma}\right]$, $(\nabla\Phi)^2 = \frac{\tau^2}{a^2}\left[m^2 + \frac{n^2}{\sin^2\gamma}\right]$. $\sinh\gamma \to \sh\gamma$

$E_{ex} = A\int dV\left[\frac{(\nabla m_z)^2}{(1-m_z^2)} + (1-m_z^2)\frac{\tau^2}{a^2}\left(m^2 + \frac{n^2}{\sh^2\gamma}\right)\right]$, $m_z = m_z(\gamma)$, $(\nabla m_z)^2 = \frac{\tau^2}{a^2}\left(\frac{\partial m_z}{\partial\gamma}\right)^2$.

$E_{ex} = A\int dV\frac{\tau^2}{a^2}\left[\frac{1}{(1-m_z^2)}\left(\frac{\partial m_z}{\partial\gamma}\right)^2 + (1-m_z^2)\left(m^2 + \frac{n^2}{\sh^2\gamma}\right)\right]$, $dV = \frac{a^3}{\tau^3}\sh\gamma\,d\gamma d\beta d\varphi$

$E_{ex} = A\int_0^{\infty}\frac{a^3}{\tau^3}\sh\gamma\,d\gamma\int_{-\pi}^{\pi}d\beta\int_0^{2\pi}d\varphi\,\frac{\tau^2}{a^2}[\dots] = Aa\int d\gamma\frac{\sh\gamma}{\tau}\int d\beta\int d\varphi\,[\dots] = \left|\text{инт. по }\varphi\right| =$

$= 2\pi Aa\int d\gamma\frac{\sh\gamma}{\tau}\int d\beta\left[\frac{1}{(1-m_z^2)}\left(\frac{\partial m_z}{\partial\gamma}\right)^2 + (1-m_z^2)\left(m^2 + \frac{n^2}{\sh^2\gamma}\right)\right]$, $\tau = \ch\gamma - \cos\beta$.

$m_z(\gamma)$ дает $\forall m,n$ избежать, но проинтегрировать. Пусть $n=1$, $m=1$, $Q_H = nm = 1$.

Тогда $m_z(\gamma) = 1 - 2\th^2\gamma$, $1-m_z^2 = (1-m_z)(1+m_z) = 2\th^2\gamma\,2(1-\th^2\gamma) = 4\th^2\gamma\frac{1}{\ch^2\gamma}$

$\frac{\partial m_z}{\partial\gamma} = \frac{\partial}{\partial\gamma}(1-2\th^2\gamma) = (-2)\frac{\partial}{\partial\gamma}\th^2\gamma = (-2)\cdot 2\th\gamma\cdot\frac{1}{\ch^2\gamma}$.

$E_{ex} = 2\pi aA\int d\gamma\frac{\sh\gamma}{\tau}\int d\beta\left[\frac{1}{4}\frac{\ch^2\gamma}{\th^2\gamma}16\frac{\th^2\gamma}{\ch^4\gamma} + 4\th^2\gamma\frac{1}{\ch^2\gamma}\left(1+\frac{1}{\sh^2\gamma}\right)\right]$

$\frac{1}{\sh^2\gamma} + 1 = \cth^2\gamma$, $E_{ex} = 8\pi a\int d\gamma\int d\beta\frac{\sh\gamma}{(\ch\gamma-\cos\beta)}\left[\frac{1}{\ch^2\gamma} + \th^2\gamma\frac{1}{\ch^2\gamma}\cth^2\gamma\right] =$

$= 16\pi aA\int d\gamma\int d\beta\frac{\sh\gamma}{(\ch\gamma-\cos\beta)}\frac{1}{\ch^2\gamma}$. $\int_{-\pi}^{\pi}d\beta\frac{1}{(b-\cos\beta)} = 2\int_0^{\pi}d\beta\frac{1}{(b-\cos\beta)} = \frac{2}{b}\int_0^{\pi}\frac{d\beta}{\left(1-\frac{1}{b}\cos\beta\right)} =$

$= \frac{2}{b}\frac{\pi}{\sqrt{1-1/b^2}} = \frac{2\pi}{\sqrt{b^2-1}} = \frac{2\pi}{\sqrt{\ch^2\gamma-1}} = \frac{2\pi}{\sh\gamma}$

$E_{ex} = 16\pi aA\int d\gamma\sh\gamma\frac{1}{\ch^2\gamma}\frac{2\pi}{\sh\gamma} = 32\pi^2 aA\int_0^{\infty}d\gamma\frac{1}{\ch\gamma^2} = 32\pi^2 aA\th\gamma\Big|_0^{\infty} = 32\pi^2 aA$

$E_{ex}(m=1,n=1) = 32\pi^2 aA$ – simplest hopfion с $Q_H=1$, $m=1$, $n=1$.

DMI $E_D = D\int dV(\vec m\cdot\operatorname{rot}\vec m)$. $\operatorname{rot}\vec m$ in toroid. coordinates (γ,β,φ) см. 28/IV.

$\operatorname{rot}\vec m = \hat\gamma\frac{\tau^2}{a^2\sh\gamma}\left[\frac{\partial}{\partial\beta}\left(a\frac{\sh\gamma}{\tau}m_\varphi\right) - \frac{\partial}{\partial\varphi}\left(\frac{a}{\tau}m_\beta\right)\right] + \hat\beta\frac{\tau^2}{a^2\sh\gamma}\left[\frac{\partial}{\partial\varphi}\left(\frac{a}{\tau}m_\gamma\right) - \frac{\partial}{\partial\gamma}\left(\frac{a\sh\gamma}{\tau}m_\varphi\right)\right] +$

$+ \hat\varphi\frac{\tau^2}{a^2}\left[\frac{\partial}{\partial\gamma}\left(\frac{a}{\tau}m_\beta\right) - \frac{\partial}{\partial\beta}\left(\frac{a}{\tau}m_\gamma\right)\right]$, $\vec m = (m_\gamma, m_\beta, m_\varphi)$ и $\vec r = (\gamma,\beta,\varphi)$

Но $m_z = m_z(\gamma)$ и $m_x + im_y = \sqrt{1-m_z^2}\,e^{i(n\varphi+m\beta)}$, $\vec m = (m_x, m_y, m_z)$ – декартовы.

Перейдем к цилиндрическим компонентам $\vec m(m_\rho, m_\varphi, m_z)$ и цил. $\vec r(\rho,\varphi,z)$.

$m_\rho + im_\varphi = (m_x + im_y)e^{-i\varphi} = (m_x + im_y)(\cos\varphi - i\sin\varphi)$, $m_\rho = m_x\cos\varphi + m_y\sin\varphi$

$m_\rho + im_\varphi = \sqrt{1-m_z^2}\,e^{i[(n-1)\varphi + m\beta]}$. $m_\varphi = m_y\cos\varphi + m_x\sin\varphi$

Далее $m=1$, $n=1$ $m_\rho + im_\varphi = \sqrt{1-m_z^2}\,e^{i\gamma(\rho,z)}$, $\gamma(\rho,z) = m\beta$. $\Phi(\rho,\varphi,z) = n\varphi + \gamma(\rho,z)$.

$m_z = m_z(\gamma) = m_z(\rho,z) \to \gamma = \gamma(\rho,z)$.

$\operatorname{rot}\vec m = \left(\frac{1}{\rho}\frac{\partial m_z}{\partial\varphi} - \frac{\partial m_\varphi}{\partial z}\right)\hat\rho + \left(\frac{\partial m_\rho}{\partial z} - \frac{\partial m_z}{\partial\rho}\right)\hat\varphi + \frac{1}{\rho}\left(\frac{\partial}{\partial\rho}(\rho m_\varphi) - \frac{\partial m_\rho}{\partial\varphi}\right)\hat z$

Выразим уравнение $(\vec m\cdot\operatorname{rot}\vec m)$ в (Θ,Φ) переменных. $\vec m(\Theta,\Phi)$ – spherical angles.

E_D in (Θ,Φ) variables see in Bogdanov-Hubert, pss'1994, Gus'2013 перераб.

Но эти ур. недостаточны, т.к. надо учесть $\partial\Theta/\partial z$, $\frac{\partial\Phi}{\partial z}$, чего не было в 2D – skyrmion ansatz, где $\Theta = \Theta(\rho)$, $\Phi = n\varphi + \Phi_0$.

$(\vec m\cdot\operatorname{rot}\vec m) = m_\rho\left(\frac{1}{\rho}\frac{\partial m_z}{\partial\varphi} - \frac{\partial m_\varphi}{\partial z}\right) + m_\varphi\left(\frac{\partial m_\rho}{\partial z} - \frac{\partial m_z}{\partial\rho}\right) + \frac{m_z}{\rho}\left[\frac{\partial}{\partial\rho}(\rho m_\varphi) - \frac{\partial m_\rho}{\partial\varphi}\right]$

$m_z = m_z(\rho,z)$, $m_\rho = \sin\Theta\cos\gamma$, $m_\varphi = \sin\Theta\sin\gamma$, $\gamma = \gamma(\rho,z)$ – helicity, $m_z = \cos\Theta$.

Axially sym. hopfion $\frac{\partial m_z}{\partial\varphi} = 0$, $\frac{\partial m_\rho}{\partial\varphi} = 0$, $m_\rho = m_\rho(\rho,z)$, $m_\varphi = m_\varphi(\rho,z)$.

$(\vec m\cdot\operatorname{rot}\vec m) = -m_\rho\frac{\partial m_\varphi}{\partial z} + m_\varphi\left(\frac{\partial m_\rho}{\partial z} - \frac{\partial m_z}{\partial\rho}\right) + \frac{m_z}{\rho}\frac{\partial}{\partial\rho}(\rho m_\varphi)$. $\gamma(\rho z) = \beta(\rho z) = \operatorname{arctg}\left[\frac{2az}{\rho^2+z^2-a^2}\right]$

Escaneado con CamScanner

$\frac{\partial}{\partial s}(s m_\varphi) = m_\varphi + s\frac{\partial m_\varphi}{\partial s}$. $(\vec m \cdot \mathrm{rot}\vec m) = m_\varphi \frac{\partial m_\varphi}{\partial z} - m_s \frac{\partial m_\varphi}{\partial z} + m_z \frac{\partial m_\varphi}{\partial s} - m_\varphi \frac{\partial m_z}{\partial s} + \frac{1}{s} m_z m_\varphi$.

$(\vec m \cdot \mathrm{rot}\vec m) = \sin\theta\sin\gamma \frac{\partial}{\partial z}(\sin\theta\cos\gamma) - \sin\theta\cos\gamma \frac{\partial}{\partial z}(\sin\theta\sin\gamma) + \cos\theta \frac{\partial}{\partial s}(\sin\theta\sin\gamma) -$

$- \sin\theta\sin\gamma \frac{\partial}{\partial s}(\cos\theta) + \frac{1}{s}\cos\theta\sin\theta\sin\gamma = \sin\theta\sin\gamma\left[\cos\theta\frac{\partial\theta}{\partial z}\cos\gamma + \sin\theta(-\sin\gamma)\frac{\partial\gamma}{\partial z}\right] -$

$- \sin\theta\cos\gamma\left[\cos\theta\frac{\partial\theta}{\partial z}\sin\gamma + \sin\theta\cos\gamma\frac{\partial\gamma}{\partial z}\right] + \cos\theta\left(\cos\theta\frac{\partial\theta}{\partial s}\sin\gamma + \sin\theta\cos\gamma\frac{\partial\gamma}{\partial s}\right) -$

$- \sin\theta\sin\gamma(-\sin\theta)\frac{\partial\theta}{\partial s} + \frac{1}{s}\cos\theta\sin\theta\sin\gamma = -\sin^2\theta\sin^2\gamma\frac{\partial\gamma}{\partial z} - \sin^2\theta\cos^2\gamma\frac{\partial\gamma}{\partial z} +$

$+ \cos^2\theta\sin\gamma\frac{\partial\theta}{\partial s} + \cos\theta\sin\theta\cos\gamma\frac{\partial\gamma}{\partial s} + \sin^2\theta\sin\gamma\frac{\partial\theta}{\partial s} + \frac{1}{s}\cos\theta\sin\theta\sin\gamma$

$(\vec m \cdot \mathrm{rot}\vec m) = -\sin^2\theta\frac{\partial\gamma}{\partial z} + \sin\gamma\frac{\partial\theta}{\partial s} + \cos\theta\sin\theta\left[\cos\gamma\frac{\partial\gamma}{\partial s} + \frac{1}{s}\sin\gamma\right]$. $\Phi(r) = \varphi + \gamma(s,z)$.

Два 2D skyrmions $\gamma = \mathrm{const}$ и $(\vec m \cdot \mathrm{rot}\vec m) = \sin\gamma\frac{\partial\theta}{\partial s} + \frac{1}{s}\sin\gamma\cos\theta\sin\theta = \sin\gamma\left[\frac{\partial\theta}{\partial s} + \frac{1}{s}\sin\theta\cos\theta\right]$

Два дает вклады с разным знаком к Bloch skyrmions $\gamma = \pm\frac{\pi}{2}$, $(\vec m \cdot \mathrm{rot}\vec m) = \pm\left[\frac{\partial\theta}{\partial s} + \frac{1}{s}\sin\theta\cos\theta\right]$.

$(\vec m \cdot \mathrm{rot}\vec m) = -\sin^2\theta\frac{\partial\gamma}{\partial z} + \cos\theta\sin\theta\cos\gamma\frac{\partial\gamma}{\partial s} + \sin\gamma\left[\frac{\partial\theta}{\partial s} + \frac{1}{s}\sin\theta\cos\theta\right]$

$E_D = D\int dV(\vec m \cdot \mathrm{rot}\vec m) = D\int_0^\infty ds\, s\int_0^{2\pi} d\varphi\int_{-\infty}^{\infty} dz\,(\vec m \cdot \mathrm{rot}\vec m)$. $\gamma(s,z) = \mathrm{arctg}\left[\frac{2a^2 z}{s^2+z^2-a^2}\right]$

$\frac{\partial\gamma}{\partial z}$ – is odd on "z", $\sin\gamma$ is odd on "z".

(!) $\theta(s,z)$ is even function "z", т.к. $\cos\theta = m_z(s) = m_z(s,z)$, $\gamma(s,z) = a\cdot\mathrm{th}\left[\frac{2as}{s^2+z^2+a^2}\right]$ is even on "z". Hence

$\frac{\partial\theta}{\partial s} = \frac{1}{\sin\theta}(-\sin\theta)\frac{\partial\theta}{\partial s} = -\frac{1}{\sin\theta}\frac{\partial(\cos\theta)}{\partial s} = -\frac{1}{\sin\theta}\frac{\partial m_z}{\partial s}$, $\cos\theta(s,z)$ is even on "z".

$\theta(s,z) = \arccos[m_z(s,z)]$ we obtain even, since even $m_z(s,z)$ is even on "z".

$\sin\gamma$ is odd on "z", $\cos\gamma(s,z)$ is even on "z". $\theta\in[0,\pi]$, $\sin\theta \geq 0$,

hence $\sin\theta\cos\theta = +\sqrt{1-\cos^2\theta}\cos\theta$ is even function on "z"

$\frac{\partial\theta}{\partial s} = -\frac{1}{\sin\theta}\frac{\partial m_z}{\partial s} = -\frac{1}{\sqrt{1-\cos^2\theta}}\frac{\partial m_z}{\partial s}$ is even on "z".

т.е. $m_z(s)$ определяет только четную часть $(\vec m \cdot \mathrm{rot}\vec m)$, поэтому $\frac{\partial\gamma}{\partial s}$ и $\sin\gamma$ четные.

$(\vec m \cdot \mathrm{rot}\vec m) = -\sin^2\theta\frac{\partial\gamma}{\partial z}$ если m_z имеет только четную часть четную по "z" и нечетная $(\vec m \cdot \mathrm{rot}\vec m)$.

$\frac{\partial\gamma}{\partial z} = \frac{\partial}{\partial z}\mathrm{arctg}\left[\frac{2a^2 z}{s^2+z^2-a^2}\right] = \frac{1}{1+\frac{4a^2z^2}{(s^2+z^2-a^2)^2}}\frac{\partial}{\partial z}\frac{2a^2 z}{(s^2+z^2-a^2)} =$

$= \frac{2a}{1+s^2}\cdot\frac{(s^2+z^2-a^2)\cdot 1 - z(2z)}{(s^2+z^2-a^2)^2}$. $\frac{\partial\gamma}{\partial z} = \frac{2a}{1+s^2}\frac{(s^2-z^2-a^2)}{(s^2+z^2-a^2)^2}$, $s^2 = \frac{4a^2z^2}{(s^2+z^2-a^2)^2}$

$\frac{\partial\gamma}{\partial z} = 2a\frac{(s^2-z^2-a^2)}{(s^2+z^2+a^2)^2+4a^2z^2}$.

$E_D = D\int_0^\infty ds\, s\int_0^{2\pi} d\varphi\int_{-\infty}^{\infty} dz\,(\vec m \cdot \mathrm{rot}\vec m) = D\int ds\, s\int d\varphi\int dz\,(-1)\sin^2\theta\frac{\partial\gamma}{\partial z} = -2\pi D\int_0^\infty ds\, s\int_{-\infty}^{\infty} dz\,\sin^2\theta\frac{\partial\gamma}{\partial z}$

$m_z = \cos\theta = f(s)$ где $\gamma = a\cdot\mathrm{th}\left[\frac{2as}{s^2+z^2+a^2}\right]$, $s(z) = \sin^2\theta(s,z)$.

$\int dz\, s(z)\frac{\partial\gamma}{\partial z} = \int dz\frac{\partial\gamma}{\partial z}s(z) = \gamma(z)s(z)\Big|_{-\infty}^{\infty} - \int\gamma(z)s'(z)dz$, $s'(z) = 2\sin\theta\cos\theta\frac{\partial\theta}{\partial z}$.

$\sin^2\theta = 1-\cos^2\theta = 1-m_z^2 = 4\frac{\mathrm{sh}^2 y}{\mathrm{ch}^2 y} = 4\,\mathrm{th}^2 y\,(1-\mathrm{th}^2 y)$. $\mathrm{th}\left(\frac{b}{a}\right) = \frac{2sa}{(z^2+s^2+a^2)}$

$y = a\cdot\mathrm{arcth}\left[\frac{2as}{s^2+z^2+a^2}\right]$. $\sin^2\theta = 4\cdot\frac{4s^2a^2}{(z^2+s^2+a^2)^2}\left[1-\frac{4s^2a^2}{(z^2+s^2+a^2)^2}\right] =$

$= \frac{16s^2a^2}{(z^2+s^2+a^2)^4}\left[(s^2+z^2+a^2)^2 - 4s^2a^2\right] = \sin^2\theta$

$E_D = -2\pi D\int ds\, s\int dz\,\frac{16s^2a^2}{(z^2+s^2+a^2)^4}\left[(s^2+z^2+a^2)^2 - 4s^2a^2\right]\cdot 2a\frac{(s^2-z^2-a^2)}{[(s^2+z^2-a^2)^2+4a^2z^2]} =$

$= -2\pi D\int ds\, s\int dz\,\frac{16s^2a^2}{(z^2+s^2+a^2)^2}\left[1-\frac{4s^2a^2}{(z^2+s^2+a^2)^2}\right]\cdot 2a\frac{(s^2-z^2-a^2)}{((s^2+z^2-a^2)^2+4a^2z^2]} =$

$= -32\pi D a^3 a^2\int_0^\infty ds\, s\int_{-\infty}^{\infty} dz\,\frac{s^2}{(z^2+s^2+1)^2}\left[1-\frac{4s^2}{(z^2+s^2+1)^2}\right]\frac{2a}{a^2}\frac{(s^2-z^2-1)}{[(s^2+z^2-1)^2+4z^2]}$

$E_D = -64\pi D a^2\int_0^\infty ds\, s\int_{-\infty}^{\infty} dz\,\frac{s^2}{(s^2+z^2+1)^2}\left[1-\frac{4s^2}{(s^2+z^2+1)^2}\right](s^2-z^2-1)/\left[(s^2+z^2-1)^2+4z^2\right]$

Escaneado con CamScanner

Cyl. components $\vec{m}(m_\rho, m_\varphi, m_z)$ ∴ $\operatorname{div}\vec{m} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho m_\rho) + \frac{1}{\rho}\frac{\partial m_\varphi}{\partial\varphi} + \frac{\partial m_z}{\partial z}$, $m_\rho + i m_\varphi = \sqrt{1-m_z^2}\, e^{i\chi}$

$m_\rho = \sin\theta\cos\chi$, $m_\varphi = \sin\theta\sin\chi$, $m_z = m_z(\rho) = m_z(\rho, z)$, $\chi = \chi(\rho, z)$.

One azimuth. symmetric Hopfion $\vec{m} \neq f(\varphi)$ in cylindrical coordinates.

$\operatorname{div}\vec{m} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho m_\rho) + \frac{\partial m_z}{\partial z} = \frac{1}{\rho}m_\rho + \frac{\partial m_\rho}{\partial\rho} + \frac{\partial m_z}{\partial z}$. torsion volume in. charges in ∞ sample.

$E_m = \frac{1}{2}M_s^2 \int dV dV' \operatorname{div}\vec{m}(\vec{r})\operatorname{div}\vec{m}(\vec{r}') \cdot \frac{1}{|\vec{r}-\vec{r}'|} = \sum_\mu \int_0^\infty dk\, e^{-k|z-z'|} J_\mu(k\rho) J_\mu(k\rho') e^{i\mu(\varphi-\varphi')}$

$\operatorname{div}\vec{m}(\vec{r}) = f(\rho, z)$ torsion $\int_0^{2\pi} d\varphi \int_0^{2\pi} d\varphi' e^{i\mu(\varphi-\varphi')} = \delta_{\mu 0}(2\pi)^2$.

$E_m = \frac{1}{2}M_s^2 \int d\rho\rho \int d\varphi \int dz \int d\rho'\rho' \int d\varphi' \int dz' \operatorname{div}\vec{m}\cdot\operatorname{div}\vec{m}(\ldots) \sum \int dk e^{-k|z-z'|} J_\mu J_\mu(\ldots) e^{i\mu(\varphi-\varphi')} =$

$= 2\pi^2 M_s^2 \int d\rho\rho \int dz \int d\rho'\rho' \int dz' \operatorname{div}\vec{m}(\rho z)\operatorname{div}\vec{m}(\rho' z') \int dk e^{-k|z-z'|} J_0(k\rho) J_0(k\rho') =$

$= 2\pi^2 M_s^2 \int dk \int dz \int dz' e^{-k|z-z'|} J^2(z, z', k)$, $J(k, zz') = \int d\rho\rho J_0(k\rho)\operatorname{div}\vec{m}(\rho, z)$, torsion

$E_m = 2\pi^2 M_s^2 \int dk \int dz \int dz' e^{-k|z-z'|} J(z, k) J(z', k)$, $J(z, k) = \int d\rho\rho J_0(k\rho)\operatorname{div}\vec{m}(\rho, z)$.

$E_m = 2\pi^2 M_s^2 \int \frac{d(ka)}{a} a^2 \int d\left(\frac{z}{a}\right) \int d\left(\frac{z'}{a}\right) e^{-|\ldots|} J(z, k) J(z', k)$, $J(z, k) = a^2 \int \frac{d\rho}{a}\left(\frac{\rho}{a}\right) J_0\left(ka\frac{\rho}{a}\right)\operatorname{div}\vec{m}(\rho z)\frac{1}{a}$

$E_m(\vec{m}) = 2\pi^2 M_s^2 a^3 \int dk \int dz \int dz' e^{-k|z-z'|} J(z, k) J(z', k)$, $J(z, k) = \int d\rho\rho J_0(k\rho)\operatorname{div}\vec{m}(\rho z)$ — ρ, z in units of a

$\operatorname{div}\vec{m}(\rho, z) = \frac{1}{\rho}m_\rho + \frac{\partial m_\rho}{\partial\rho} + \frac{\partial m_z}{\partial z} = \frac{1}{\rho}\sin\theta\cos\chi + \frac{\partial}{\partial\rho}(\sin\theta\cos\chi) + \frac{\partial\cos\theta}{\partial z} =$

$= \frac{1}{\rho}\sin\theta\cos\chi + \cos\theta\frac{\partial\theta}{\partial\rho}\cos\chi + \sin\theta(-\sin\chi)\frac{\partial\chi}{\partial\rho} + \frac{\partial m_z}{\partial z}$, $\frac{\partial\theta}{\partial\rho} = -\frac{1}{\sin\theta}\frac{\partial m_z}{\partial\rho}$

Uniform $\vec{m}$ $E_m(\vec{m})$ analytical, MathCad module ero не брать.

$E[\vec{m}] = 32\pi^2 A a - 64\pi D I_D a^2 + 2\pi^2 M_s^2 I_m a^3$, где — Hopfion energy. ∞-sample

$I_D = \int_0^\infty d\rho\rho \int_{-\infty}^\infty dz \frac{\rho^2}{(\rho^2+z^2+1)^2}\left[1 - \frac{4\rho^2}{(\rho^2+z^2+1)^2}\right]\frac{(\rho^2-z^2-1)}{\left[(\rho^2+z^2-1)^2 + 4z^2\right]}$

$I_m = \int_0^\infty dk \int_{-\infty}^\infty dz \int_{-\infty}^\infty dz' e^{-k|z-z'|} J(z, k) J(z', k)$, $J(z, k) = \int_0^\infty d\rho\rho J_0(k\rho)\operatorname{div}\vec{m}(\rho, z)$.

24/IX $\delta/\partial a$ $E[\vec{m}] = E(a) = C_1 a - \frac{1}{2}C_2 a^2 + \frac{1}{3}C_3 a^3$, $C_1 = 32\pi^2 A$, $C_2 = 128\pi D I_D$

MathCad „Hopfion_stability.mcd" даёт $I_D \approx 0{,}260$. $C_3 = 6\pi^2 M_s^2 I_m$

Здесь a — in absolute units (nm).

9/X Компьютерный эксперимент. $E_m[\vec{m}]$ через dipolar field и FFT.

$H_m^\alpha(\vec{r}) = \int d^3\vec{r}'\, G_{\alpha\beta}(\vec{r}\vec{r}')\, M_\beta(\vec{r}')$. $\alpha\beta = \rho, \varphi, z$ cylindrical components.

$H_m^\rho(\vec{r}) = M_s \int d^3\vec{r}' \left[G_{\rho\rho} m_\rho + G_{\rho\varphi} m_\varphi + G_{\rho z} m_z\right]$

$H_m^\varphi(\vec{r}) = M_s \int d^3\vec{r}' \left[G_{\varphi\rho} m_\rho(\vec{r}') + G_{\varphi\varphi} m_\varphi(\vec{r}') + G_{\varphi z}(\vec{r}') m_z(\vec{r}')\right]$

$H_m^z(\vec{r}) = M_s \int d^3\vec{r}' \left[G_{z\rho} m_\rho + G_{z\varphi} m_\varphi + G_{zz} m_z(\vec{r}')\right]$

$\vec{m} = (m_\rho, m_\varphi, m_z)$. Hopfion $m_z = m_z(\rho z)$, $m_\rho + i m_\varphi = \sqrt{1-m_z^2}\, e^{i\chi(\rho, z)}$

мы не забыли об φ, $g_{\alpha\beta}(\rho\rho', zz') = \int_0^{2\pi}\frac{d\varphi}{2\pi}\int_0^{2\pi} d\varphi' G_{\alpha\beta}(\vec{r}\vec{r}')$.

$G_{\alpha\beta}(\vec{r}\vec{r}') = -\frac{\partial^2}{\partial x_\alpha \partial x_\beta'}\frac{1}{|\vec{r}-\vec{r}'|} = \frac{\partial^2}{\partial x_\alpha \partial x_\beta'}\sum_\mu \int dk e^{-k|z-z'|} J_\mu(k\rho) J_\mu(k\rho') e^{i\mu(\varphi-\varphi')}(-1)$

$G_{\varphi\beta} \sim \sum_\mu \mu\, e^{i\mu(\varphi-\varphi')}$ ∴ $\int d\varphi' \exp(-i\mu\varphi') = 2\pi\delta_{\mu 0}$, $\langle G_{\varphi\beta}\rangle_{\varphi'} \sim \sum_\mu \mu\delta_{\mu 0} = 0$

$\langle G_{\alpha\varphi}\rangle_{\varphi'} \sim \sum_\mu \delta_{\mu 0}\mu = 0$. $\langle G_{\varphi\varphi}\rangle_{\varphi'} \sim \sum \mu^2\delta_{\mu 0} = 0$.

Взаимодействуют только $g_{\alpha\beta}$: $g_{\rho\rho}$, $g_{\rho z}$, $g_{z\rho}$, g_{zz} где $\alpha, \beta \neq \varphi$.

$H_m^\rho(\rho z) = M_s \int d\rho'\rho' dz' \left[g_{\rho\rho}(\rho\rho', zz') m_\rho + g_{\rho z} m_z\right]$, $H_m^\varphi(\rho z) = 0$.

$H_m^z(\rho z) = M_s \int d\rho'\rho' dz' \left[g_{z\rho}(\rho\rho', zz') m_\rho + g_{zz}(\rho\rho', zz') m_z\right]$

$g_{\alpha\beta}(\rho\rho', zz') = \int\frac{d\varphi}{2\pi}\int d\varphi'(-1)\frac{\partial^2}{\partial x_\alpha \partial x_\beta'}\sum_\mu \int dk e^{-k\Delta z} J_\mu J_\mu(\ldots) e^{i\mu(\varphi-\varphi')} =$

$= -2\pi \frac{\partial^2}{\partial z_1 \partial z_2} \int dk e^{-k|z-z'|} J_0(k\rho) J_0(k\rho') \quad \nabla_m \, d\beta \, d\varphi \cdot \frac{\partial}{\partial \rho} = \frac{\partial}{\partial \rho'} \frac{\partial}{\partial z}$

$E_m[\vec{m}] = -\frac{1}{2}\mu_0 \int dV \vec{m} \cdot \vec{H}_m(\vec{r}) = -\frac{1}{2}\mu_0 \int dV \left[m_\rho H_m^\rho + m_z H_m^z(\vec{r}) \right]$

$E_m[\vec{m}] = -\frac{1}{2}\mu_0 \cdot 2\pi \int d\rho \rho \int dz \left[m_\rho M_s \int d\rho' \rho' \int dz' \left[g_{\rho\rho} m_\rho(\cdot) + g_{\rho z} m_z(\cdot) \right] + m_z M_s \left[d\rho' \rho' \int dz' \left[g_{z\rho} m_\rho(\cdot) + g_{zz} m_z(\cdot) \right] \right] \right], \quad m_\rho(\cdot) = m_\rho(\rho' z')$.

$E_m[\vec{m}] = -\pi \mu_0 M_s^2 \left[\left(\int d\rho \rho \int dz \right) \int d\rho' \rho' \int dz' g_{\rho\rho} m_\rho m_\rho(\cdot) + \dots \right] =$

$= -\pi \mu_0 M_s^2 \int d\rho \rho \int dz \int d\rho' \rho' \int dz' \left[g_{\rho\rho} m_\rho m_\rho(\cdot) + g_{\rho z} m_\rho m_z(\cdot) + g_{z\rho} m_z m_\rho(\cdot) + g_{zz} m_z m_z \right]$

$g_{\alpha\beta}(\rho\rho', zz') = -2\pi \frac{\partial^2}{\partial z_1 \partial z_2} \int dk e^{-k|z-z'|} J_0(k\rho) J_0(k\rho'), \quad m_\rho = \sin\theta \cos\gamma$.

$E_{\rho\rho} = -\pi \mu_0 M_s^2 \int d\rho \rho \int d\rho' \rho' \int dz \int dz' g_{\rho\rho}(\rho\rho', zz') m_\rho(\rho z) m_\rho(\rho' z')$

$E_{\rho\rho} = -\pi \mu_0 M_s^2 \int d\rho \rho \int d\rho' \rho' \int dz \int dz' m_\rho(\rho z) m_\rho(\rho' z') (-2\pi) \frac{\partial^2}{\partial z \partial z'} \int dk e^{-k|z-z'|} J_0(k\rho) J_0(k\rho')$

Def. $J(z,k) = \int d\rho \rho \, m_\rho(\rho z) \frac{\partial J_0(k\rho)}{\partial \rho}$. $E_{\rho\rho} = 2\pi^2 \mu_0 M_s^2 \int dk \int dz \int dz' e^{-k|z-z'|} J(z,k) \times J(z',k)$

$E_{\rho\rho}$ содержит резкие интегралы поэтому $E_m[\mathrm{div}\vec{m}]$.

Можно не вычислять, Намагниченность $H_m^\varphi(\rho z) = 0$ для тороидальной

$E_{\rho\rho} = 2\pi^2 \mu_0 M_s^2 \int dk \int dz \int dz' e^{-k|z-z'|} J_\rho(zk) J_\rho(z'k), \quad J_\rho(zk) = \int d\rho \rho \, m_\rho(\rho z) \frac{\partial J_0}{\partial \rho}$

Local approximation $\int dz' e^{-k|z-z'|} J(z'k) \approx J(zk) \int dz' e^{-k|z-z'|}$, не учитывает это явление.

Вывод: Исследуем функцию $E_m = -\frac{1}{2}\mu_0 \int dV \vec{m}(\vec{r}) \vec{H}_m(\vec{r})$, что не

можем тем $E_m(\mathrm{div}\vec{m}) = \frac{1}{2}\mu_0 M_s^2 \int dV \int dV' \frac{\mathrm{div}\vec{m}(\vec{r}) \mathrm{div}\vec{m}(\vec{r}')}{|\vec{r}-\vec{r}'|}$

Можно ли разложить $\frac{1}{|\vec{r}-\vec{r}'|}$ в тороидальных координатах и $\int dy \int d\beta (\dots)$?

$\frac{1}{|\vec{r}-\vec{r}'|} = \frac{1}{2\pi} \int d\vec{k} \frac{1}{k} e^{-k|z-z'|} e^{i\vec{k}(\vec{\rho}-\vec{\rho}')}$ $\rho = \frac{a}{\tau} \sin\beta, z = \frac{a}{\tau} \sin\beta, \varphi = \varphi, \tau = \mathrm{ch}y - \cos\beta$

$e^{i\vec{k}\vec{\rho}} = e^{ik\rho \cos(\varphi - \varphi_k)}$, $z - z' = a\left[\frac{\sin\beta}{\tau} - \frac{\sin\beta'}{\tau'} \right]$.

$\mathrm{div}\vec{m} = \frac{1}{\rho} \sin\theta \cos\gamma + \cos\theta \cos\gamma \frac{\partial\theta}{\partial\rho} - \sin\theta \sin\gamma \frac{\partial\gamma}{\partial\rho} + \frac{\partial m_z}{\partial z}$, $\theta(\rho, z)$, $\gamma = \gamma(\rho z)$.

$\mathrm{div}\vec{m}(y,\beta,\varphi) = \frac{\tau}{a \mathrm{sh}y} \sin\theta \cos\beta + \frac{\partial m_\rho}{\partial \rho} + \frac{\partial m_z}{\partial z}$. $\gamma(\rho z)$, $\gamma = \beta$, $m=1$.

$\frac{\partial m_\rho}{\partial \rho} = \frac{\partial}{\partial \rho} m_\rho(y,\beta) = \frac{\partial m_\rho}{\partial y} \frac{\partial y}{\partial \rho} + \frac{\partial m_\rho}{\partial \beta} \frac{\partial \beta}{\partial \rho} = \frac{\partial m_\rho}{\partial y} \frac{\partial y}{\partial \rho} = \cos\beta \frac{\partial \sin\theta(y)}{\partial y} \frac{\partial y}{\partial \rho}$ $m_\rho = \sin\theta \cos\gamma = \sin\theta(y) \cos\beta$

$y = a \cdot \mathrm{arth}\left(\frac{2a\rho}{\rho^2 + z^2 + a^2} \right)$. $\frac{y}{2} = \mathrm{Re} \ln\left[\frac{z + i(\rho + a)}{z + i(\rho - a)} \right]$, $\frac{1}{a} \frac{\partial y}{\partial \rho} = \frac{\partial}{\partial \rho} \mathrm{Re}\left[\ln(z + i(\rho + a)) - \right.$

$\left. - \ln(z + i(\rho - a)) \right] = \mathrm{Re}\left[\frac{1}{z + i(\rho + a)} i - \frac{1}{z + i(\rho - a)} i \right] = f(z + i\rho)$, можно $f(y, \beta$

Далее если $F(\rho\rho', zz') = \int d\varphi \int d\varphi' \frac{1}{|\vec{r}-\vec{r}'|}$ как и, то, $\mathrm{div}\vec{m} = f(\rho z)$,

$E_m[\mathrm{div}\vec{m}] = \frac{1}{2}\mu_0 M_s^2 \int d\rho \rho \int d\rho' \rho' \int dz \int dz' F(\rho\rho', zz') \mathrm{div}\vec{m}(\rho z) \cdot \mathrm{div}\vec{m}(\rho' z')$ 4-fold integral.

3D магнитостатика сложна в осевой симметрии $\vec{m} = \vec{m}(\rho z)$ не сказано.

$E_m[\mathrm{div}\vec{m}] = 2\pi^2 \mu_0 M_s^2 a^3 I_m$, $I_m = \int dk \int dz \int dz' e^{-k|z-z'|} J(z,k) J(z'k)$, $J(z,k) = \int d\rho \rho J_0(k\rho) \mathrm{div}\vec{m}(\rho z)$

$I_z = \int dz \int dz' e^{-k|z-z'|} F(zz') = \int dz \left[\int_{-\infty}^{z} dz' e^{-k(z-z')} + \int_{z}^{\infty} dz' e^{-k(z'-z)} \right] F(zz') =$

$= \int dz \left[e^{-kz} \int_{-\infty}^{z} dz' e^{kz'} F(zz') + e^{kz} \int_{z}^{\infty} dz' e^{-kz'} F(zz') \right]$, $F(zz') = \int dk J(z,k) J(z'k)$.

Local approx. $\int dz' e^{-k|z-z'|} J(z'k) \approx J(zk) \int dz' e^{-k|z-z'|}$

$\tilde{I}_z = \int_{-\infty}^{\infty} dz' e^{-k|z-z'|} = \int_{-\infty}^{z} dz' e^{-k(z-z')} + \int_{z}^{\infty} dz' e^{-k(z'-z)} = e^{-kz} \int_{-\infty}^{z} e^{kz'} dz' + e^{kz} \int_{z}^{\infty} e^{-kz'} dz' =$

$= e^{-kz} \frac{1}{k} e^{kz'} \Big|_{-\infty}^{z} + e^{kz} (-\frac{1}{k}) e^{-kz'} \Big|_{z}^{\infty} = e^{-kz} \frac{1}{k} e^{kz} - 0 + 0 - e^{kz} (-\frac{1}{k}) e^{-kz} = 2/k$

Escaneado con CamScanner

Local approx. $\int dz' e^{-k|z-z'|} J(z',k) \cong J(z,k) \cdot \tilde{J}_2(z) = J(z,k) \cdot \frac{2}{k}$. Local approx. $z \approx z'$

$I_m = \int dk \int dz \int dz' e^{-k|z-z'|} J(z,k) J(z',k) = \int dk \int dz \, J(z,k) \underbrace{\int dz' e^{-k|z-z'|} J(z',k)}_{\text{}} =$

$= \int dk \int dz \, J(z,k) \frac{2}{k} J(z,k) = 2 \int \frac{dk}{k} \int dz \, J^2(z,k)$, $J(z,k) = \int d\rho_s \, J_0(k\rho_s) \, div\vec{m}(\rho_s z)$

$I_m = 2 \int \frac{dk}{k} \int dz \, J^2(z,k) = 2 \int \frac{dk}{k} \int dz \int d\rho_s J_0(k\rho_s) div\vec{m}(\rho_s z) \int d\rho_s' \rho_s' J_0(k\rho_s') div\vec{m}(\rho_s' z) =$

$= 2 \int dz \int d\rho_s \, div\vec{m}(\rho_s z) \, div\vec{m}(\rho_s z) \int_0^\infty \frac{dk}{k} J_0(k\rho_s) J_0(k\rho_s') \, d\rho_s'$, $J_0(\rho\rho') = \int_0^\infty \frac{dk}{k} J_0(k\rho) J_0(k\rho')$ расходится при $k \to 0$.

Local approx. Уже $E_m[div\vec{m}]$ не работает.

Можно оценить $E_{\rho\rho}, E_{\rho z}, E_{zz}$ через ФТ?

Видимо $E_m(div\vec{m})$ local $\to \infty$ из-за ∞-sample. Может $\frac{1}{V} E_m$ будет конечно

$E_{\rho z} = E_{z\rho} = -2\pi M_s^2 \int d\rho_s \int d\rho_s' \rho_s' \int dz \int dz' g_{\rho z} m_\rho(\rho z) m_z(\rho' z') =$

$= -2\pi M_s^2 \int d\rho_s \int d\rho_s' \rho_s' \int dz \int dz' (-2\pi) \frac{\partial}{\partial \rho_s} \frac{\partial}{\partial z'} \int dk e^{-k|z-z'|} J_0(k\rho_s) J_0(k\rho_s') m_\rho(\rho z) m_z(\rho' z') =$

$= 2\pi^2 M_s^2 \int d\rho_s \int d\rho_s' \rho_s' \int dz \, dz' \left[\frac{\partial}{\partial \rho_s} \int dk e^{-k|z-z'|} \frac{\partial J_0(k\rho_s)}{\partial \rho_s} J_0(k\rho_s') \right] m_\rho(\rho z) m_z(\rho' z')$

$\int dz' \frac{\partial}{\partial z'} e^{-k|z-z'|} = e^{-k|z-z'|} \Big|_{-\infty}^{\infty}$ — не δ.

$\int dz' \left[\frac{\partial}{\partial z'} e^{-k|z-z'|} \right] m_z(\rho' z') = e^{-k|z-z'|} m_z(\rho' z') \Big|_{z=-\infty}^{z=\infty} = 0$, если z-finite $m_z(\rho, |z| \to \infty) = \text{const}$ (как?)

$E_{zz} = -2\pi M_s^2 \int d\rho_s \int d\rho_s' \rho_s' \int dz \int dz' g_{zz}(\rho\rho', zz') m_z(\rho z) m_z(\rho' z') =$

$= 2\pi^2 M_s^2 \int d\rho_s \int d\rho_s' \rho_s' \int dz \, dz' \left[\frac{\partial}{\partial z} \frac{\partial}{\partial z'} \int dk e^{-k|z-z'|} J_0(k\rho_s) J_0(k\rho_s') \right] m_z(\rho z) m_z(\rho' z')$

Свойства $\frac{\partial}{\partial z} \frac{\partial}{\partial z'} e^{-k|z-z'|} = -k^2 e^{-k|z-z'|} + 2k\delta(z-z')$

Фурье-образ: $\Gamma_{zz}(\vec{r}-\vec{r}') = -\frac{\partial^2}{\partial z \partial z'} \frac{1}{|\vec{r}-\vec{r}'|} = -\frac{\partial^2}{\partial z \partial z'} \frac{1}{2\pi} \int d^2k \frac{1}{k} e^{-k|z-z'|} e^{i\vec{k}(\vec{\rho}-\vec{\rho}')}$

$\Gamma_{zz}(\vec{r}-\vec{r}') = \frac{1}{2\pi} \int d^2\vec{k} \frac{1}{k} \left[-\frac{\partial^2}{\partial z \partial z'} e^{-k|z-z'|} \right] e^{i\vec{k}(\vec{\rho}-\vec{\rho}')} = \frac{1}{2\pi} \int d^2\vec{k} \frac{1}{k} \left[+k^2 e^{-k|z-z'|} - 2k\delta(z-z') \right] \times$

$\times e^{i\vec{k}(\vec{\rho}-\vec{\rho}')} = \frac{1}{2\pi} \int d^2\vec{k} \, k \, e^{-k|z-z'|} e^{i\vec{k}(\vec{\rho}-\vec{\rho}')} - \frac{1}{2\pi} \int d^2\vec{k} e^{i\vec{k}(\vec{\rho}-\vec{\rho}')} \delta(z-z') =$

$= \frac{1}{2\pi} \int d^2\vec{k} \, k \, e^{-k|z-z'|} e^{i\vec{k}(\vec{\rho}-\vec{\rho}')} - 4\pi\delta(\vec{\rho}-\vec{\rho}')\delta(z-z')$

$E_{zz} = 2\pi^2 M_s \int d\rho_s \int d\rho_s' \rho_s' \int dz \int dz' \int dk J_0(k\rho_s) J_0(k\rho_s') \left[-k^2 e^{-k|z-z'|} + 2k\delta(z-z') \right] m_z(\rho z) m_z(\rho' z')$

$G_{zz}(\vec{r}\vec{r}') = \frac{1}{2\pi} \int d^2\vec{k} \, k \, e^{-k|z-z'|} e^{i\vec{k}(\vec{\rho}-\vec{\rho}')} - 4\pi\delta(\vec{\rho}-\vec{\rho}')\delta(z-z')$, $\delta(\vec{\rho}-\vec{\rho}') = \frac{1}{\rho}\delta(\vec{\rho}-\vec{\rho}')\delta(\varphi-\varphi')$

$g_{zz}(\rho\rho', zz') = \int \frac{d\varphi}{2\pi} \int d\varphi' G_{zz}(\vec{r}\vec{r}') = \frac{1}{2\pi} \int d^2\vec{k} \, k \, e^{-k|z-z'|} \int d\varphi' e^{i\vec{k}(\vec{\rho}-\vec{\rho}')} - 4\pi \int d\varphi' \frac{1}{\rho} \delta(\rho-\rho')\delta(\varphi-\varphi') \delta(z-z') =$

$= \frac{1}{2\pi} \int dk \, k \int d\varphi_k \, k \, e^{-k|z-z'|} \int d\varphi' e^{i\vec{k}(\vec{\rho}-\vec{\rho}')} - 4\pi \int \frac{d\varphi}{2\pi} \int d\varphi' \delta(\varphi-\varphi') \frac{1}{\rho}\delta(\rho-\rho')\delta(z-z')$

$\int \frac{d\varphi}{2\pi} \int d\varphi' e^{i\vec{k}(\vec{\rho}-\vec{\rho}')} = 2\pi J_0(k\rho) J_0(k\rho')$ см. выше. $\int d\varphi_k = 2\pi$

$g_{zz}(\rho\rho', zz') = \frac{1}{2\pi} \int dk \, k^2 \cdot 2\pi e^{-k|z-z'|} \cdot 2\pi J_0(k\rho) J_0(k\rho') - \frac{4\pi}{\rho}\delta(\rho-\rho')\delta(z-z')$

$g_{zz}(\rho\rho', zz') = 2\pi \int dk \, k^2 e^{-k|z-z'|} J_0(k\rho) J_0(k\rho') - \frac{4\pi}{\rho}\delta(\rho-\rho')\delta(z-z')$

$E_{zz} = -2\pi M_s^2 \int d\rho_s \int d\rho_s' \rho_s' \int dz dz' \, g_{zz}(\rho\rho', zz') m_z(\rho z) m_z(\rho' z') =$

$= -2\pi M_s^2 \int d\rho_s \int d\rho_s' \rho_s' \int dz dz' \left[2\pi \int dk \, k^2 e^{-k|z-z'|} J_0 J_0(\cdot) - \frac{4\pi}{\rho} \delta(\rho-\rho')\delta(z-z') \right] m_z m_z(\cdot) =$

$= 2\pi^2 M_s^2 \int d\rho_s \int d\rho_s' \rho_s' \int dz dz' \left[-\int dk \, k^2 e^{-k|z-z'|} J_0 J_0(\cdot) + \frac{2}{\rho} \delta(\rho-\rho')\delta(z-z') \right] m_z m_z(\cdot)$.

С учетом $\int_0^\infty dk \, k \, J_0(k\rho) J_0(k\rho') = \frac{1}{\rho}\delta(\rho-\rho')$ это совп. с E_{zz} Бекмана.

Escaneado con CamScanner

$$E_{zz} = -2\pi^2 M_s^2 \int d\varphi\, s\, \int d\varphi'\, s' \int dz\, dz' \int dk\, k^2 e^{-k|z-z'|} J_0 J_0(\cdot)\, m_z(sz) m_z(s'z') +$$

$$+ 4\pi^2 M_s^2 \int d\varphi\, s \int d\varphi'\, s' \int dz \int dz' \frac{1}{s} \delta(s-s') \delta(z-z') m_z(sz) m_z(s'z')$$

$$E_{zz} = -2\pi^2 M_s^2 \int dz \int dz' \int dk\, k^2 e^{-k|z-z'|} J_z(z,k) J_z(z',k) + 4\pi^2 M_s^2 \int ds\, s \int dz \times m_z^2(s,z)$$

$$J_z(z,k) = \int ds\, s\, J_0(ks)\, m_z(sz)$$

Local approx. $\int dz'\, e^{-k|z-z'|} J_z(z',k) \approx J_z(z,k) \int dz'\, e^{-k|z-z'|} = J_z(z,k) \frac{2}{k}$?

$$E_{zz}(loc) = -2\pi^2 M_s^2 \int dz \int dk\, k^2 J_z(z,k) \cdot J_z(z,k) \frac{2}{k} + 4\pi^2 M_s^2 \int ds\, s \int dz\, m_z^2(sz)$$

$$E_{zz}(loc) = -4\pi^2 M_s^2 \int dz \int dk\, k\, J_z^2(z,k) + 4\pi^2 M_s^2 \int ds\, s\, dz\, m_z^2(s,z)$$

$$\int dk\, k\, J_z^2(k) = \int dk\, k \int ds\, s\, J_0(ks)\, m_z(sz) \int ds'\, s'\, J_0(ks')\, m_z(s'z) =$$

$$= \int ds\, s\, m_z(sz) \int ds'\, s'\, m_z(s'z) \int dk\, k\, J_0(ks) J_0(ks') = \int ds\, s\, m_z(sz) \int ds'\, s'\, m_z(s'z) \times$$

$$\times \frac{1}{s} \delta(s-s') = \int ds\, s\, m_z^2(s,z) \quad \text{и} \quad E_{zz}(loc) = -4\pi^2 M_s^2 \int dz \int ds\, s\, m_z^2(sz) + 4\pi^2 M_s^2 \int ds\, s \int dz\, m_z^2 = 0$$

Dip.energy E_{zz} (local approx.) $\equiv 0$ (!), не расходится (!)

$$E_{ss}(loc) = 2\pi^2 M_s^2 \int dk \int dz \int dz' e^{-k|z-z'|} J_s(zk) J_s(zk) = 2\pi^2 M_s^2 \int dk \int dz\, J_s(zk) \times$$

$$+ J_s(zk) \frac{2}{k} = 4\pi^2 M_s^2 \int dz \int dk\, J_s^2(zk), \quad J_s(zk) = \int ds\, s\, m_s(sz) \frac{\partial J_0(ks)}{\partial s} =$$

$$= \int ds\, s\, m_s(sz)(-1) k\, J_1(ks), \quad E_{ss}(loc) = 4\pi^2 M_s^2 \int dz \int dk \left[\int ds\, s\, m_s(sz) J_1(ks)\right]^2 k^2 =$$

$$= 4\pi^2 M_s^2 \int dz \int dk\, k \int ds\, s\, m_s(sz) J_1(ks) \cdot \int ds'\, s'\, m_s(s'z) J_1(ks') =$$

$$= 4\pi^2 M_s^2 \int dz \int ds\, s\, m_s(sz) \int ds'\, s'\, m_s(s'z) \underbrace{\int dk\, k\, J_1(ks) J_1(ks')}_{\frac{1}{s}\delta(s-s')} = 4\pi^2 M_s^2 \int ds\, s \int ds'\, s' \int dz \times$$

$$\times m_s(sz) m_s(s'z) \frac{1}{s} \delta(s-s') = 4\pi^2 M_s^2 \int dz \int ds\, s\, m_s^2(s,z), \quad \underline{E_{ss}(loc) = 4\pi^2 M_s^2 \int dz \int ds\, s \times m_s^2(s,z)}$$

Lo. E_{ss} is finite если $m_s(z)$ достаточно быстро $\to 0$ при $|z| \to \infty$.

Near $E_{sz} = E_{zs}$ в local approximation.

$\frac{\partial}{\partial z} e^{-k|z-z'|}$, $k_y < 0$ $\frac{\partial}{\partial x} e^{-k|x|}$, a) $x > 0$ $\frac{\partial}{\partial x} e^{-kx} = -k e^{-kx}$, б) $x < 0$, $\frac{\partial}{\partial x} e^{kx} = k e^{kx}$

$$\frac{\partial}{\partial x} e^{-k|x|} = -\operatorname{sign}(x)\, k e^{-k|x|}, \quad \frac{\partial}{\partial z} e^{-k|z-z'|} = -\operatorname{sign}(z-z') k e^{-k|z-z'|}$$

$$\frac{\partial}{\partial z'} e^{-k|z-z'|} = \operatorname{sign}(z-z') k e^{-k|z-z'|}$$

$$E_{sz}(loc) = 2\pi^2 M_s^2 \int ds\, s \int d\varphi'\, s' \int dz \int dz' \left[\int dk\, e^{-k|z-z'|} k \operatorname{sign}(z-z') \frac{\partial J_0(ks)}{\partial s} J_0(ks')\right] \times$$

$$+ m_s(sz) m_z(s'z') = 2\pi^2 M_s^2 \int dz \int dz' \int dk\, k\, s(z-z') e^{-k|z-z'|} J_s(z,k) J_z(z',k) =$$

$$= 2\pi^2 M_s^2 (-) \int dz'\, s(z-z') e^{-k|z-z'|} J_z(z',k) \approx s(z-z') J_z(z,k)\big|_{z=z'} \cdot \int dz' e^{-k|z-z'|}$$

$E_{sz}(loc) = 0$ из-за $\int dz'(\cdots) \sim s(z-z')\big|_{z=z'} = 0$, $\operatorname{sign}(z) = 0$ при $z=0$ by definition.

$E_{sz}(loc) = E_{zs}(loc) = 0$, будучи они малы уже в предыдущем local approx.

T.к. $E_{zz} = 0$, $E_{sz} = 0$, $E_{zs} = 0$ in local approximation, остается $E_{ss}(loc)$.

Есть то S/m

$$E_{ss}(loc) = 4\pi^2 M_s^2 \int dz \int ds\, s\, m_s^2(s,z), \text{ подобна anisotropy с } \hat{s} \text{ hard axis}$$

$$m_s(s,z) = \sin\Theta(sz) \cos\gamma(s,z), \quad \sin\Theta = \sqrt{1 - m_z^2}, \quad \gamma(s,z) \underset{m=1}{=} \beta(s,z) = \operatorname{arctg}\left[\frac{2az}{s^2+z^2-a^2}\right]$$

$$\sin^2\Theta = \frac{16 s^2 a^2}{(s^2+z^2+a^2)^2}\left[1 - \frac{4 s^2 a^2}{(s^2+z^2+a^2)^2}\right], \quad \cos\gamma = \frac{1}{\sqrt{1+\operatorname{tg}^2\gamma}}, \quad \operatorname{tg}\gamma = \frac{2az}{(s^2+z^2-a^2)}$$

$m_s^2 = \sin^2\Theta \cos^2\gamma$.

Чтобы $E_{ss}(loc)$ расходилась $\sim \ln s_m$ нужно $\int_0^{s_m} ds\, (\cdots)$, B_z - вихрь Хопфиона в цилиндре. Чтобы $s_m = R$, чтобы? и $R \gg a$, вообще это - Хопфион ansatz Белавина-Поляков. Тогда E_{ss} станет конечно.

10/I Demag new law Hypoteca. Tensor magnetostatic E_{ss} (hopfion).

$$E_{ss}[\vec m] = 2\pi^2 M_s^2 \int dz \int dz' \int dk\, e^{-k|z-z'|} J_s(z,k) J_s(z';k), \quad J_s(z,k) = \int_{ss} m_s(s,z) \times \frac{\partial J_0(ks)}{\partial s}$$

$$J_s(z,k) = -k \int_{ss} J_1(ks)\, m_s(s,z), \quad m_s(s,z) = \sin\theta(s,z)\cos\chi(s,z), \quad \cos\chi = \frac{1}{\sqrt{1+tg^2\chi}}.$$

$$E_{ss} = 2\pi^2 M_s^2 \int dz \int dz' \int dk\, e^{-k|z-z'|} k^2 \int ds\, s\, J_1(ks)\, m_s(s,z) \int ds'\, s'\, J_1(ks')\, m_s(s',z')$$

$\int_0^\infty dk\, e^{-k|z-z'|} k^2 J_1(ks) J_1(ks')$ see Conway '2001.

$\int dz \int dz' m_s(s,z) m_s(s',z') = B(ss')$. $E_{ss} = (\cdot) \int dk\, e^{-k|z-z'|} k^2 \int ds\, s J_1(ks) \int ds'\, s' J_1(ks') \times B(ss')$

$E_{ss} = 2\pi^2 M_s^2 \int dk\, e^{-k|z-z'|} k^2 \int ds\, s \int ds'\, s' B(ss') J_1(ks) J_1(ks')$

Math Cad gave "Error" when parsing $E_{ss}[\vec m]$.

Note that $E_a = K_1 \int dV (1-\cos^2\theta) = 2\pi K_1 \int_0^{s_m} ds\, s \int dz \sin^2\theta$ taking passage when $s_m \to \infty$,

div m - magnetic: $I_m = \int dk \int dz \int dz' e^{-k|z-z'|} \int ds\, s J_0(ks)\, div\vec m(s,z) \int_0^\infty ds'\, s' J_0(ks')\, div\vec m(s',z')$

$\int_0^\infty dk\, e^{-k|z-z'|} J_0(ks) J_0(ks') = \frac{2}{\pi\sqrt{ss'}} K(x)$, $x^2 = \frac{4ss'}{(s+s')^2 + |z-z'|^2}$

$K(x) = \int_0^{\pi/2} \frac{d\alpha}{\sqrt{1-x^2\sin^2\alpha}}$. $\int dk\, e^{-k|z-z'|} J_0(ks) J_0(ks') = \Gamma(ss', zz')$

$I_m = \int dz \int dz' \int ds\, s \int ds'\, s'\, div\vec m(s,z)\, div\vec m(s',z')\, \Gamma(ss',zz')$. $\Gamma(ss',zz') = \frac{2}{\pi\sqrt{ss'}} K(x)$.

Если E_a расходится, то будем нормализовать $E[\vec m]$ только в ∞-sample.

Здесь мультиплет ∞ в E_m для ∞-film Kiselev, J. Ph. D '2011 Стандартный скачок $\Delta E_m(\theta) = E_m(\theta) - E_m(\theta=0)$, $\theta=0$ ⊥-magnetized film, $\theta = \theta(s)$ is skyrmion profile, см. 4/II 2014, 11/II 2015 – 14/II 2015. Ошибки в Kiselev 2011.

Для hopfion $E_m(\vec m = \vec m_0 = uniform) = 0$, т.к. $div\vec m_0 = 0$ или $\vec m_0 = const$.

Может вместо $E_m[\vec m]$ E_m (transverse magnetized wire)? Где $m_s = 1$? или $m_x = 1$?

$E_m(m_x=1) = 2\pi M_s^2 \int dV m_x^2 \to E_m[m_x=1] = \frac{1}{2}\int dV [N_{xx} m_x^2] M_s^2$

$m_x = \sin\theta\cos\varphi$, $\varphi = n\varphi + m\beta \to \varphi = 0$ даже $m_x = 1$.

⇒ Попробуем $E_{ss}(loc)$, $s_m \to R/l_e$, wire radius. Есть ли $\min E(\vec m)$?

$E[\vec m] = 32\pi^2 A a - 64\pi D I_D a^2 + 2\pi^2 M_s^2 I_m a^3$, $I_m(loc) = 2\int_0^\infty dz \int_0^R ds\, s\, m_s^2(s,z) = I_m(R)$.

$E(a) = C_1 a + \frac{1}{2} C_2 a^2 + \frac{1}{3} C_3 a^3$, $C_1 = 32\pi^2 A$, $C_2 = 128\pi D I_D$, $C_3 = 6\pi^2 M_s^2 I_m(R)$.

Сначала 24/II equilibrium $a_0 = \frac{1}{2C_3}\left[C_2 + \sqrt{C_2^2 - 4C_1C_3}\right]$, условие real a_0

$|C_2| \ge 2\sqrt{C_1C_3}$, $128\pi D I_D \ge 2\sqrt{32\pi^2 A 6\pi^2 M_s^2 I_m(R)} = 2\cdot 8\pi^2 \sqrt{3 A M_s^2 I_m(R)}$

l_e сделать dimensionless, а б eg. $l_e = \sqrt{A/2\pi M_s^2}$

$E[\vec m] = 32\pi^2 A l_e a - 64\pi D I_D l_e^2 a^2 + 2\pi^2 M_s^2 I_m l_e^3 a^3$

$\frac{1}{4\pi M_s^2 l_e^3} E[\vec m] = \frac{32\pi^2 A}{4\pi M_s^2 l_e^2} a - \frac{64\pi D I_D}{4\pi M_s^2 l_e} a^2 + \frac{2\pi^2 M_s^2 I_m l_e^3}{4\pi M_s^2} a^3$, $l_e^3 = V_e$

$\frac{1}{4\pi M_s^2 l_e} E[\vec m] = \frac{32\pi^2 A}{4\pi M_s^2 (A/2\pi M_s^2)} a - \frac{16 D I_D}{M_s^2 l_e} a^2 + \frac{\pi}{2} I_m a^3$. $\varepsilon = E/(4\pi M_s^2 V_e)$

$\varepsilon(a) = 16\pi^2 a - 16 \frac{D I_D}{M_s^2 l_e} a^2 + \frac{\pi}{2} I_m a^3$, Def. $d = \frac{D}{M_s^2 l_e}$

$\varepsilon(a) = 16\pi^2 a - 16 d I_D a^2 + \frac{\pi}{2} I_m a^3$, $C_1 = 16\pi^2$, $C_2 = 32 d I_D$, $C_3 = \frac{3}{2}\pi I_m$

$C_2^2 - 4C_1C_3 = (32)^2 d^2 I_D^2 - 4\cdot 16\pi^2 \cdot \frac{3}{2}\pi I_m$, $d^2 \ge \frac{1}{(32)^2 I_D^2} 16\cdot 6\pi^3 I_m = \frac{3\pi^3}{32} \frac{I_m}{I_D^2}$

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$d \geq \frac{1}{I_D}\sqrt{\frac{3\pi^3}{32} I_m} = \frac{\pi}{4 I_D}\sqrt{\frac{3}{2}\pi I_m}$, $I_D \approx 0.26$. $I_m(R)$, $p_m = \frac{R}{a} = \frac{R}{l_e}\frac{l_e}{a} = \left(\frac{R}{l_e}\right)\frac{1}{a}$

$F_{gg} = 4\pi^2 M_s^2 \int dz \int_0^R d\rho\rho\, m_s^2 = \left|\frac{\rho}{a}=r,\ \frac{z}{a}\to z\right| = 4\pi^2 M_s^2 \times a^3 \int dz \int_0^{R/a} dr\, r\, m_s^2$

$F_{gg} = 4\pi^2 M_s^2 \int dz \int_0^R d\rho\rho \sin^2\Theta \cos^2\gamma = 4\pi^2 M_s^2 \int dz \int_0^R d\rho\rho \frac{16 z^2 a^2 \rho^2}{(\rho^2+z^2+a^2)^2}\left[1-(\cdot)\right]\cos^2\gamma$

$F_{gg} = 4\pi^2 M_s^2 \int a\, dz \int_0^{R/a} a^2 dr\, r \frac{16 z^2 a^2 a^2}{(r^2+z^2+1)^2 a^4}\left[1-(\cdot)\right]\cos^2\gamma = 4\pi^2 M_s^2 \int dz \int_0^{R/a} dr\, r \times$

$\times \frac{16 z^2}{(r^2+z^2+1)^2}\left[1-(\cdot)\right]\cos^2\gamma \cdot a^3$, т.о. $\int_0^{R/a} dr\, r(\cdot)$ есть a в степени, т.е. $F_{gg} \sim a^3$, но R – finite, $E_m[\vec m] = 2\pi^2 M_s^2 I_m\left(\frac{R}{a}\right) a^3$ – volume m. energy.

$I_m(R/a) = 2\int_{-\infty}^{\infty} dz \int_0^{R/a} dr\, r\, m_s^2$

Reduced energy $\tilde E(a) = 16\pi^2 a - 16\pi I_D a^2 + \frac{\pi}{2} I_m\left(\frac{R}{a}\right) a^3$ – hopfion energy

$E(a) = C_1 a - \frac{1}{2} C_2 a^2 + \frac{1}{3} C_3 \left(\frac{R}{a}\right) a^3$

$\frac{\partial E}{\partial a} \to \frac{\partial E}{\partial a} = a - C_2 a + C_3 a^2 + \frac{1}{3} a^3 \frac{\partial C_3}{\partial a}$, $\frac{\partial C_3}{\partial a} = \frac{3}{2}\pi \frac{\partial}{\partial a} I_m\left(\frac{R}{a}\right) =$

$= \frac{3}{2}\pi \frac{\partial}{\partial a} 2\int dz \int_0^{R/a} dr\, r\, m_s^2 = \frac{3}{2}\pi \cdot 2 \int dz \left(-\frac{R}{a^2}\right)\left(\frac{R}{a}\right) m_s^2\left(\frac{R}{a}\right) = -3\pi \frac{R^2}{a^3} \int dz\, m_s^2\left(\frac{R}{a}, z\right)$

$\frac{\partial E}{\partial a} = C_1 - C_2 a + C_3 a^2 + \frac{1}{3} a^3 (-3\pi)\frac{R^2}{a^3} \int dz\, m_s^2(R/a, z)$

$m_s^2\left(\frac{R}{a}, z\right) = \frac{16 z^2}{(r^2+z^2+1)^2}\left[1 - \frac{4r^2}{(r^2+z^2+1)^2}\right]\cos^2\gamma$, $\frac{\partial E}{\partial a} = C_1 - C_2 a + C_3 a^2 -$

$- \pi R^2 \int dz \frac{16 z^2}{(r^2+z^2+1)^2}\left[1 - \frac{4r^2}{(r^2+z^2+1)^2}\right]_{r=R/a} \cos^2\gamma$, $\frac{R^2+z^2}{(r^2+z^2+1)^2}\Big|_{r=R/a}$ чем не удобно сравнивать с 1 даже при $\frac{R}{a} \gg 1$.

$\partial\tilde E/\partial a = 0$ не сводится к quadratic equation for a_0.

Typical $d = \frac{D}{M_s^2 l_e} = \frac{1\ erg/cm^2}{10^6 \frac{erg}{cm^3} \cdot 5\,nm} = \frac{1}{10^6 \cdot 5 \cdot 10^{-7}} = \frac{10}{5} = 2$. $d \sim 1$.

Выводы:

→ MCad и hopfion-stability mod" дает $\tilde E(a)$ имеет min при $a_0 \approx 0.9R$, $R = 10$–300 и min $\tilde E(a)$ есть даже при DMI $d=0$ за счет magnetostatics $E_m \sim a^3 I_m(R/a)$. Роль DMI лишь углубляет min $\tilde E(a)$. $\tilde E(a) > 0$ всегда, т.е. hopfion a_0 – equilibrium или metastable state. Все это дает local-$E_m[\vec m]$.

a, R in units exch. length $l_e = (A/\mu_0 M_s^2)^{1/2} \approx 5$nm.

(11/I) Обзор проекта Украина STCU (4) best for evaluation.

Для hopfion в cylindrical wire ($L\to\infty$, R – finite) для чего W_m^{vol}, то надо учесть и $W_m^{surf} + W_m^{V\text{-}S}$. Side cylinder $\rho = R$, $\forall\varphi$, $L\to\infty$.

$W_m \to E_m^S = \frac{M_s^2}{2}\int dS \int dS' \frac{(\vec m\cdot\vec n)(\vec m'\cdot\vec n')}{|\vec r - \vec r'|}$, $(\vec m\cdot\vec n) = m_\rho$

$E_m^S = \frac{1}{2} M_s^2 \int dS \int dS' \frac{m_\rho(\vec r)\, m_\rho(\vec r')}{|\vec r - \vec r'|}$, $dS = R\, d\varphi\, dz$. $m_\rho = m_\rho(\rho, z)$.

$E_m^S = \frac{1}{2} M_s^2 \int R d\varphi dz\, R d\varphi' dz'\, m_\rho(\rho z) m_\rho(\rho' z') \sum_\mu \int dk\, e^{-k|z-z'|} e^{i\mu(\varphi-\varphi')} J_\mu J_\mu(\cdot) =$

$= \frac{1}{2} M_s^2 R^2 (2\pi)^2 \int dz \int dz' m_\rho(R,z) m_\rho(R,z') \int dk\, e^{-k|z-z'|} J_0^2(kR)$, $\rho,\rho' = R$

$E_m^S = 2\pi^2 M_s^2 R^2 \frac{a^2}{a} \int dz \int dz' m_\rho(Rz) m_\rho(Rz') \int dk\, e^{-k|z-z'|} J_0^2\left(k\frac{R}{a}\right)$, $z/a \to z$, $ka \to k$ $\sim a F(R/a)$.

$E_m^S(a) = 2\pi^2 M_s^2 R^2 a \int dz \int dz' m_\rho(Rz) m_\rho(Rz') \int dk\, e^{-k|z-z'|} J_0^2\left(k\frac{R}{a}\right) \times \frac{1}{4\pi M_s^2 l_e}$

$E_m^S(a) = 2\pi^2 M_s^2 R^2 a\, I_m^S(R/a)$, $I_m^S(R/a) = \int dz \int dz' m_\rho(Rz) m_\rho(Rz') \int dk\, e^{-k|z-z'|} J_0^2(kR/a)$

$E_m^S(a) = \frac{\pi}{2} R^2 a\, I_m^S(R/a)$, где R и a в l_e units – side surface magn. energy.

Volume-Surface charge energy $E_m^{VS} = \int dV \int ds' \frac{\operatorname{div}\vec{m}(\vec{r})\, m_s(\vec{r}')}{|\vec{r}-\vec{r}'|} M_s^3$.

Два вклада дает $E_m[\vec{m}]$ total: 1) вклад $\operatorname{div}\vec{m}$, $(\vec{m}\cdot\vec{n})$ volume & surface charges; 2) вклад Φ г. $E_m[\vec{m}] = -\frac{1}{2} M_s \int dV\, \vec{m}(\vec{r})\, \vec{H}_m(\vec{r})$, где все volume и surface charges содер в $\vec{H}_m(\vec{r}) = \int dV' \hat{G}(\vec{r}-\vec{r}')\, \vec{m}(\vec{r}') M_s$

Если считать $E_m[\vec{m}]$ как E_{ss}, E_{zz} (=0 in local approx), то E_{ss} учитывает все charges, и $E(a)$-10/X забирает расчет.

typical scale of hopfion energy $4\pi M_s^2 l_e^3 = 10 \cdot (1000)^2 G^2 5^3 10^{-21} cm^3 = 10^4 \cdot 10^2 10^{-21} erg = 10^{-12} erg$. $k_B T = 1.4 \cdot 10^{-16} erg/K \cdot 3 \cdot 10^2 K \approx 4.2 \cdot 10^{-14} erg$.

$4\pi M_s^2 l_e^3 / k_B T \sim 10^{-12} erg / 4 \cdot 10^{-14} erg \approx 25 K$.

Если считать R-limit или расчет E_m, то R надо включать и в E_{ex} и E_{DMI}.

Hopfion stability - cyl. wire

(12/X) E_{ex} 8/X считается в (η, φ, β) toroidal coordinates.

$E_{ex} = A \int dV [(\nabla\theta)^2 + \sin^2\theta (\nabla\Phi)^2]$, $\eta = \operatorname{arth}\left(\frac{2as}{s^2+z^2+a^2}\right)$, $\beta = \operatorname{arctg}\left(\frac{2az}{s^2+z^2-a^2}\right)$

$\theta = \theta(s,z)$, $\Phi = n\varphi + \gamma(s,z)$. $(\nabla\theta)^2 = \frac{(\nabla m_z)^2}{(1-m_z^2)}$, $m_z = 1 - 2\operatorname{th}^2\eta = \cos\theta$.

$s = \frac{a}{c}\operatorname{sh}\eta$, $z = \frac{a}{c}\sin\beta$, $\varphi = \varphi$. $E_{ex} = 16\pi a A \int d\eta \operatorname{sh}\eta \int d\beta \frac{1}{\operatorname{ch}^2\eta}$

$m_z(s,z) = \cos\theta = 1 - \frac{8s^2a^2}{(a^2+s^2+z^2)^2}$. $\sin^2\theta = \frac{16 s^2 a^2}{(s^2+z^2+a^2)^2}\left[1 - \frac{4s^2a^2}{(s^2+z^2+a^2)^2}\right]$

$1 - m_z^2 = \sin^2\theta = 4\operatorname{th}^2\eta (1 - \operatorname{th}^2\eta)$, $\nabla\Phi = \left(\hat{s}\frac{\partial}{\partial s} + \frac{\partial}{\partial z}\hat{z} + \frac{1}{s}\frac{\partial}{\partial\varphi}\hat{\varphi}\right)\Phi = \nabla\gamma + \hat{\varphi}\frac{1}{s}$

In cylindrical coordinates, Hubert, Eq. (3.2a) $\vec{m} = \vec{m}(s,z)$

$\sum_\alpha (\nabla m_\alpha)^2 = \left(\frac{\partial m_s}{\partial s}\right)^2 + \left(\frac{\partial m_\varphi}{\partial s}\right)^2 + \left(\frac{\partial m_z}{\partial s}\right)^2 + \left(\frac{\partial m_s}{\partial z}\right)^2 + \left(\frac{\partial m_\varphi}{\partial z}\right)^2 + \left(\frac{\partial m_z}{\partial z}\right)^2 +$

$+ \frac{1}{s^2}[m_\varphi^2 + m_s^2]$, $m_s = \sin\theta\cos\gamma$, $m_\varphi = \sin\theta\sin\gamma$, $m_z = m_z(s,z)$.

$(\nabla\Phi)^2 = (\nabla\gamma)^2 + \frac{1}{s^2}$. $(\nabla\theta)^2 + \sin^2\theta(\nabla\Phi)^2 = \frac{1}{\sin^2\theta}(\nabla m_z)^2 + \sin^2\theta\left[(\nabla\gamma)^2 + \frac{1}{s^2}\right]$

$E_{ex} = 2\pi A \int_0^R ds\, s \int_{-\infty}^{\infty} dz \left[\frac{(\nabla m_z)^2}{\sin^2\theta} + \sin^2\theta\left((\nabla\gamma)^2 + \frac{1}{s^2}\right)\right]$, $(\nabla m_z)^2 = \left(\frac{\partial m_z}{\partial s}\right)^2 + \left(\frac{\partial m_z}{\partial z}\right)^2$

$E_{ex}(a) = 2\pi a A \int_0^{R/a} d\eta\, s \int_{-\infty}^{\infty} dz \left[\frac{(\nabla m_z)^2}{\sin^2\theta} + \sin^2\theta\left((\nabla\gamma)^2 + \frac{1}{s^2}\right)\right]$

$\frac{2\pi A a}{4\pi M_s^2 l_e^3} = \frac{2\pi A a}{4\pi M_s^2 \frac{A}{2\pi M_s^2} l_e} = \pi a$, a в ед. l_e. $l_e = \sqrt{A/2\pi M_s^2}$, $l_e \approx 5 nm$.

Reduced $E_{ex}(a) = \pi a \int_0^{R/a} d\eta\, s \int_{-\infty}^{\infty} dz \left[\frac{(\nabla m_z)^2}{\sin^2\theta} + \sin^2\theta\left((\nabla\gamma)^2 + \frac{1}{s^2}\right)\right] = \pi a\, I_{ex}(R/a)$ - имеет max на a'.

when $\frac{R}{a} \to \infty$, $E_{ex}(a) = \pi a\, I_{ex}(\infty) = 16\pi^2 a$ в согласии с 8/X.

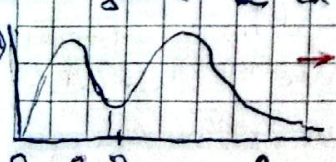
DMI. $E_D = -2\pi D \int_0^R ds\, s \int_{-\infty}^{\infty} dz \sin^2\theta \frac{\partial\gamma}{\partial z} = -2\pi D a^2 \int_0^{R/a} d\eta\, s \int_{-\infty}^{\infty} dz \sin^2\theta \frac{\partial\gamma}{\partial z}$

$\frac{2\pi D a^2}{4\pi M_s^2 l_e^3} = \frac{2\pi D}{4\pi M_s^2 l_e}\left(\frac{a}{l_e}\right)^2 = \frac{1}{2} d a^2$, $E_D(a) = -\frac{1}{2} d a^2 \int_0^{R/a} d\eta\, s \int_{-\infty}^{\infty} dz \sin^2\theta \frac{\partial\gamma}{\partial z} = \frac{4\pi}{2} d a^2 I_D(R/a)$

$E_D(a, R) = -\frac{1}{2} a^2 d\, I_D(R/a)$, $I_D(R/a) = \int_0^{R/a} d\eta\, s \int_{-\infty}^{\infty} dz \sin^2\theta \frac{\partial\gamma}{\partial z}$

$I_D(\infty) = 32\, I_{Dodd}(\infty)$ - в работе 2/X.

Учет finite R in E_{ex}, DMI - E_D меняет $E(a)$, но на min $E(a)$ не влияет сильно.



Мой расчет дает $a_0 \approx 0.9R$, energy $E(a) \to 0$ при $a \to 0$ и $a \to \infty$ и $E(a)$ имеет max $E(a)$.

$a=0$ и $a=\infty$ соответ. 1 SD state с $m_z = 1$. Barriers $a \to \begin{matrix} a=0 \\ a=\infty \end{matrix}$

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(13/X) Получил new Law Hipoteca и переслал Антону разобраться. Comments referees „Vortex GH" Adv. El. Materials.

(14-15/X) Travel report Pakaraci. Рецензия Smalyukh Sci. Advances. Reminder Petrecca - „Distinguished Scientist" Contract.

(16/X) Magnetic Vortex Stability MS. IVA II Feb 2023.

(17/18/X) Расчет IVA „Constanta" III Feb. 2023. E-mails o IEEE membership Mag. Frontiers 2025 application.

(19/X) Замысел Оксаны. Fischer, Hoffmann, Smalyukh на предмет в Org. Committee Mag. Frontiers – 2025. С А. Carrejour Oleberia, Mercedona. Машина Киев, N.D. Тана – 48 € 10 kg 11 kg (53-5)

(20/X) Прислали Alex у нас дома. 10 из 1000 € (800 € transfer + 200 € cash).

(20-31/X) Trip to Porto. Ответы referees of Artem's paper no vortex strings, Adv. El. Mat. Radial Vortex GH, аномально large W_G для thin dot. Сделал теор. описание. С А. переговоры. Program Org. / Program Committees для „Magnetic Frontiers – 2025".

(1/XI) Revised Mag. Frontiers – 2025 Proposal и новая Оксана.

(2/XI) Order no Trip to Porto. переписка с Gleb, Michal – бились

(3/XI) Zoom session с Michal Mruczkiewicz, Глеб, Артем о non-reciprocity of $W_G(k_z)$ in long cylindrical wire. Invited Artem to Donostia to discuss его аналитич. расчет W_G в cylindrical dots в figure Gus' Magn. 2022. By Michal немонотонна (W_G - wire, PRL). У меня в Vortex string расчет cyl. wire с $\partial\vec{S}/\partial z$ членом, W_G^n не зависел от знака $\partial\vec{S}/\partial z$, конечно и Cp (C - vortex chirality, p - polarization).

(4/XI) Задача Глеба: gyrotropic dynamics of radial vortex W_G Files „report-1.pdf", „report 2.pdf" и ppt - presentation. Radial Vort



Stray field of hard layer с $\vec{M} = M_z\hat{z}$ создает в radial vortex in thin soft layer. ① $W_G \uparrow$ с L-thickness увеличением (0.5 - 3.0 nm) от 2.7 to 4.2 GHz (!) ② W_G не зависит от R при $R \geq 20$ nm (!). ③ W_G растет с L_1-hard layer thickness ④ W_G довольно сильно зависит от d, spacer thickness, падая от 4.3 до 3.6 GHz при $d \in [0.5, 3.00]$ nm (!).

Взяли hard layer stray field $\vec{H}_d = (H_\rho, 0, H_z) = \vec{H}_d(\rho, z)$ axial sym., определяет W_G через $W_H(\vec{X}) = -M_S \int dV\, \vec{m}(\vec{r}, \vec{X}) \cdot \vec{H}_d(\vec{r})$.

Есть вклады $W_{k_z}(\vec{X})$, $W_m(\vec{X})$ (Soft layer), но они, видимо малы — надо посмотр.

Сильная зависимость $W_G(L_1)$ (1.5 - 4.5 GHz) при $L = 10$-40 nm под вопросом.

Важность $W_H(\vec{X})$ stray field.

Published Wegrzyn et al. PRB 108, 174403 (2023), где 13 citations of my vortex papers, включая magnetism '2022. Typical vortex profile $m_z(\rho)$ radial vortex при $\delta M I = 0$.

$m_z(\rho)$ radial vortex похож на $m_z(\rho)$ - vortex in small R-dot.

$\vec{H}_d(\rho, z)$ использовано из замысла для recent - MS.

$W_H(\vec{X}) = -M_S \int dV \left[m_z(\vec{r}, \vec{X}) H_z(\rho, z) + m_\rho(\vec{r}, \vec{X}) H_\rho(\rho, z) \right]$, можно $\int_0^{2\pi} d\varphi (\ldots)$

$\vec{m}(\vec{r}, \vec{X}) = \vec{m}(z, \vec{\rho}, \vec{X})$. Rigid model $\vec{m}(z, \vec{\rho}, \vec{X}) = \vec{m}(z, \vec{\rho} - \vec{X})$.

В eq. thiele $\vec{F}_H = -\frac{\partial W_H}{\partial \vec{X}} = -\nabla_{\vec{X}} W_H = M_S \int dV \left[\nabla_{\vec{X}} m_z \cdot H_z + \nabla_{\vec{X}} m_\rho \cdot H_\rho \right]$

$\nabla_{\vec{X}} m_d = -\nabla_{\vec{\rho}} m_d$, в rigid model. $\nabla_{\vec{\rho}} f = \hat{\rho} \frac{\partial f}{\partial \rho} + \frac{1}{\rho} \hat{\varphi} \frac{\partial f}{\partial \varphi}$

$\nabla_{\vec{X}} m_d = -\left[\hat{\rho} \frac{\partial m_d}{\partial \rho} + \frac{1}{\rho} \hat{\varphi} \frac{\partial m_d}{\partial \varphi} \right]$, $d = z, \rho$. $\vec{m}(z, \vec{\rho}, \vec{X}) = \vec{m}_0(z, \vec{\rho} - \vec{X})$, где

$\vec{m}_0(z,\vec{s})$ is static centered vortex profile. T.k. soft layer thickness $L = 0.5 - 3.0$ нм, то $\vec{m}(z,\vec{s},\vec{X})$ не зависит от z.

$W_H(\vec{X}) = -M_S \int ds\, s\, d\varphi \int dz \left[\vec{m}(\vec{s},\vec{X})\cdot\vec{H}_d(\vec{s},z)\right] = -M_S \int d^2\vec{s}\, \vec{m}(\vec{s},\vec{X})\,\vec{H}'_d(\vec{s})$,

где $\vec{H}'_d(\vec{s}) = \int dz\, \vec{H}_d(\vec{s},z)$. $\vec{F}_H(\vec{X}) = -\frac{\partial W_H}{\partial \vec{X}} = M_S \int d^2\vec{s}\left[\nabla_{\vec{X}} m_s H'_s(\vec{s}) + \nabla_{\vec{X}} m_\varphi H'_\varphi(\vec{s})\right]$.

$\vec{m}(\vec{s},\vec{X}) = \vec{m}_0(\vec{s}-\vec{X})$, $\nabla_{\vec{X}} m_\alpha = -\nabla_{\vec{s}} m^0_\alpha(\vec{s}-\vec{X}) = -\left[\frac{\partial}{\partial s}\hat{s} + \frac{1}{s}\hat{\varphi}\frac{\partial}{\partial\varphi}\right] m^0_\alpha(\vec{s}-\vec{X})$

$\vec{F}_H(\vec{X}) = -M_S \int d^2\vec{s}\left[\nabla_{\vec{s}} m^0_s(\vec{s}-\vec{X})\cdot H'_s(\vec{s}) + \nabla_{\vec{s}} m^0_\varphi(\vec{s}-\vec{X})\cdot H'_\varphi(\vec{s})\right]$

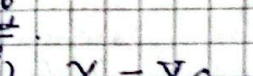
$m^0_\alpha(\vec{s}-\vec{X}) = m^0_\alpha(\vec{s}) - \vec{X}\cdot\frac{\partial m^0_\alpha}{\partial \vec{s}}\Big|_{\vec{X}=0} + O(\vec{X}^2)$. В $F_H(\vec{X})$ достаточно члены $\sim \vec{X}$.

$\vec{F}_H(\vec{X}) = -M_S \int d^2\vec{s}\left[\nabla_{\vec{s}}\left\{m^0_s(\vec{s}) - \vec{X}\cdot\frac{\partial m^0_s}{\partial \vec{s}}\Big|_{\vec{X}=0}\right\} H'_s(\vec{s}) + \nabla_{\vec{s}}\left\{m^0_\varphi(\vec{s}) - \vec{X}\cdot\frac{\partial m^0_\varphi}{\partial \vec{s}}\Big|_{\vec{X}=0}\right\} H'_\varphi\right]$

$\vec{F}_H(\vec{X}) = M_S(\vec{X}) \int d^2\vec{s}\left[\nabla_{\vec{s}}\left\{\vec{X}\cdot\frac{\partial m^0_s}{\partial \vec{s}}\Big|_{\vec{X}=0}\right\} H'_s(\vec{s}) + \nabla_{\vec{s}}\left\{\vec{X}\cdot\frac{\partial m^0_\varphi}{\partial \vec{s}}\Big|_{\vec{X}=0}\right\} H'_\varphi(\vec{s})\right]$

$\vec{X}\cdot\frac{\partial m^0_\alpha}{\partial \vec{s}} = \vec{X}\cdot\nabla_{\vec{s}} m^0_\alpha = \vec{X}\cdot\left(\hat{s}\frac{\partial m^0_\alpha}{\partial s} + \frac{1}{s}\hat{\varphi}\frac{\partial m^0_\alpha}{\partial \varphi}\right)$.

Static radial vortex $\vec{m}_0(\vec{s}) = \vec{m}_0(s)$ в силу симметрии $\frac{\partial m^0_\alpha}{\partial \varphi} = 0$ и $\vec{X}\cdot\frac{\partial m^0_\alpha}{\partial \vec{s}} = (\vec{X}\cdot\hat{s})\frac{\partial m^0_\alpha}{\partial s} = X_s \frac{\partial m^0_\alpha}{\partial s}$.



$\nabla_{\vec{s}}\left\{\vec{X}\cdot\frac{\partial m^0_\alpha}{\partial \vec{s}}\right\} = \nabla_{\vec{s}}\left\{X_s \frac{\partial m^0_\alpha}{\partial s}\right\}$, $\vec{X} = (X, Y, 0)$, $X_s = X\cos\varphi + Y\sin\varphi$

$\nabla_{\vec{s}}\left\{X_s \frac{\partial m^0_\alpha}{\partial s}\right\} = \hat{s}\frac{\partial}{\partial s}\left(X_s \frac{\partial m^0_\alpha}{\partial s}\right) + \frac{1}{s}\hat{\varphi}\frac{\partial}{\partial\varphi}\left(X_s \frac{\partial m^0_\alpha}{\partial s}\right) =$ $X_s = X_s(\varphi)$ только.

$= \hat{s} X_s \frac{\partial^2 m^0_\alpha}{\partial s^2} + \frac{1}{s}\hat{\varphi}\frac{\partial m^0_\alpha}{\partial s}\frac{\partial X_s}{\partial \varphi}$. $\frac{\partial X_s}{\partial \varphi} = -X\sin\varphi + Y\cos\varphi = X_\varphi$

$\nabla_{\vec{s}}\left\{X_s \frac{\partial m^0_\alpha}{\partial s}\right\} = \hat{s} X_s \frac{\partial^2 m^0_\alpha}{\partial s^2} + \frac{1}{s}\hat{\varphi} X_\varphi \frac{\partial m^0_\alpha}{\partial s}$. Будем $\int_0^{2\pi} d\varphi (\dots)$.

$\hat{s} X_s = (\cos\varphi\hat{x} + \sin\varphi\hat{y})(X\cos\varphi + Y\sin\varphi)$, $\int_0^{2\pi}\frac{d\varphi}{2\pi}\hat{s} X_s = \frac{1}{2}(X\hat{x} + Y\hat{y}) = \frac{1}{2}\vec{X}$

$\int_0^{2\pi} d\varphi \cos^2\varphi = \int_0^{2\pi} d\varphi \sin^2\varphi = \pi$. $\hat{\varphi} X_\varphi = (-\sin\varphi\hat{x} + \cos\varphi\hat{y})(-X\sin\varphi + Y\cos\varphi)$.

$\int_0^{2\pi}\frac{d\varphi}{2\pi} X_\varphi \hat{\varphi} = \frac{1}{2}(X\hat{x} + Y\hat{y}) = \frac{1}{2}\vec{X}$.

$\vec{F}_H(\vec{X}) = M_S \int ds\, s\, d\varphi \left[\left(\hat{s} X_s \frac{\partial^2 m^0_s}{\partial s^2} + \frac{\hat{\varphi}}{s} X_\varphi \frac{\partial m^0_s}{\partial \varphi}\right) H'_s(s) + \left(\hat{s} X_s \frac{\partial^2 m^0_\varphi}{\partial s^2} + \frac{\hat{\varphi}}{s} X_\varphi \frac{\partial m^0_\varphi}{\partial \varphi}\right) H'_\varphi\right] =$

$= \pi M_S \vec{X} \int ds\, s\left[\left(\frac{\partial^2 m^0_s}{\partial s^2} + \frac{1}{s}\frac{\partial m^0_s}{\partial s}\right) H'_s(s) + \left(\frac{\partial^2 m^0_\varphi}{\partial s^2} + \frac{1}{s}\frac{\partial m^0_\varphi}{\partial s}\right) H'_\varphi(s)\right]$

Static vortex $\vec{m}_0(\vec{s}) = \vec{m}_0(s)$ и $\vec{m}_0 = (m^0_s, 0, m^0_z)$, $m^0_\varphi = 0$ для радиального вортекса.

$W(\vec{X}) = \frac{1}{2}\varkappa\vec{X}^2$, $\vec{F}(\vec{X}) = -\varkappa\vec{X}$, $\varkappa_H = -\pi M_S \int ds\, s\left[\left(\frac{\partial^2 m^0_s}{\partial s^2} + \frac{1}{s}\frac{\partial m^0_s}{\partial s}\right) H'_s(s)\right]$ Σ_t

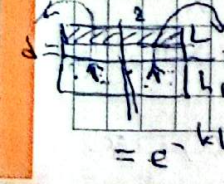
$\varkappa_H = -\pi M_S \int ds\, s\left(\frac{\partial^2 \vec{m}_0}{\partial s^2} + \frac{1}{s}\frac{\partial \vec{m}_0}{\partial s}\right)\cdot\vec{H}'(s)$, $\vec{H}'(s) = \int_{\text{soft layer}} dz\, \vec{H}_d(s,z)$

Energy hard layer stray field & stiffness $\varkappa$ of radial vortex energy $W(\vec{X}) = \frac{\varkappa}{2}\vec{X}^2$,
rigid soliton model.

3/IX 2019 я вновь ещё раз кольцо $\vec{M}$ stray field $H_s(s,z)$, $H_z(s,z)$. $H_\varphi = 0$.
Там через $r_1 \to 0$ (inner ring radius) и $z \to -z$ (можно сказать, что кольцо над ним с $z' > 0$)

$H_s(s,z) = -2\pi M_S \frac{\partial}{\partial s}\int ds' s' \int dz' \frac{\partial}{\partial z'}\int dk\, e^{-k|z-z'|} J_0(ks) J_0(ks')$

$H_z(s,z) = -2\pi M_S \frac{\partial}{\partial z}\int ds' s' \int dz' \int dk \frac{\partial}{\partial z'} e^{-k|z-z'|} J_0(ks) J_0(ks')$



Сумма hard m. dot m_z in $z \le 0$, Spacer $0 \le z \le d$, Soft dot - $d \le z \le d+L$.

$\int dz' \frac{\partial}{\partial z'} e^{-(\dots)} = \int dz' \frac{\partial}{\partial z'} e^{-k|z-z'|} \Big|_{z'=-L_1}^{z'=0} =$

$= e^{-k|z-z'|}\Big|_{z'=-L_1}^{z'=0} = e^{-k|z|} - e^{-k|z+L_1|} = e^{-kz} - e^{-k(z+L_1)} = e^{-kz}(1 - e^{-kL_1})$

Hard m. dot при $z' \le 0$. M_{s1} – hard layer

$$H_\rho(\rho,z) = -2\pi M_s \int dk\, e^{-kz}(1-e^{-kL_1}) \frac{\partial}{\partial \rho} J_0(k\rho) \int_0^R d\rho' \rho' J_0(k\rho') =$$

$$= -2\pi M_s \int dk\, e^{-kz}(1-e^{-kL_1})(-k) J_1(k\rho) \frac{R}{k} J_1(kR), \quad \text{Soft dot } d \le z \le d+L.$$

$$H_\rho(\rho,z) = 2\pi M_s R \int dk\, e^{-kz}(1-e^{-kL_1}) J_1(k\rho) J_1(kR)$$

$$H_z(\rho,z) = -2\pi M_s \frac{\partial}{\partial z} \int dk\, e^{-kz}(1-e^{-kL_1}) \int d\rho' \rho' J_0(k\rho) J_0(k\rho') =$$

$$= -2\pi M_s \frac{\partial}{\partial z} \int dk\, e^{-kz}(1-e^{-kL_1}) J_0(k\rho) \frac{R}{k} J_1(kR). \quad \text{см. 3/IX 2019.}$$

$$H_z(\rho,z) = 2\pi M_s R \int dk\, e^{-kz}(1-e^{-kL_1}) J_0(k\rho) J_1(kR)$$

Надо $\int dz(\dots)$, $H'_\rho(\rho) = \int_d^{d+L} H_\rho(\rho,z) = \int dz\, (2\pi M_s R) \int dk\, e^{-kz}(1-e^{-kL_1}) \times$

$\times J_1(k\rho) J_1(kR)$, $\int_d^{d+L} dz\, e^{-kz} = -\frac{1}{k} e^{-kz}\Big|_d^{d+L} = -\frac{1}{k}\left[e^{-k(d+L)} - e^{-kd}\right] =$

$= -\frac{1}{k} e^{-kd}\left[e^{-kL} - 1\right] = \frac{1}{k} e^{-kd}(1-e^{-kL})$

$$H'_\rho(\rho) = 2\pi M_s R \int dk\, \frac{1}{k} e^{-kd}(1-e^{-kL})(1-e^{-kL_1}) J_1(k\rho) J_1(kR)$$

$$H'_z(\rho) = \int_d^{d+L} H_z(\rho,z) = \int dz\, (2\pi M_s R) \int dk\, e^{-kz}(1-e^{-kL_1}) J_0(k\rho) J_1(kR)$$

$$H'_z(\rho) = 2\pi M_s R \int_0^\infty dk\, \frac{1}{k} e^{-kd}(1-e^{-kL})(1-e^{-kL_1}) J_0(k\rho) J_1(kR)$$

Здесь R – dot radius, L – soft layer thickness, d – spacer thickness, L_1 – hard layer thickness. Это $\vec{H}'(\rho) = (H'_\rho(\rho), 0, H'_z(\rho))$ надо иметь в definition of stiffness $\varkappa_H$. $W_H(\vec{x}) = \frac{1}{2}\varkappa_H \vec{x}^2$.

Лучше определить $\vec{H}'_d = \frac{1}{L}\int dz\, \vec{H}_d(\rho,z)$; т.е. $H'_d \to \frac{1}{L} H'_d$

$$H'_\rho(\rho) = 2\pi M_s \frac{R}{L} \int \frac{dk}{k} e^{-kd}(1-e^{-kL})(1-e^{-kL_1}) J_1(k\rho) J_1(kR), \quad kR \to k, \ \rho \text{ в ед. } R$$

$$H'_\rho(\rho) = 2\pi M_s \frac{1}{\beta} \int \frac{dk}{k} e^{-kd}(1-e^{-\beta k})(1-e^{-\beta_1 k}) J_1(k\rho) J_1(k) \qquad \beta = L/R,\ \beta_1 = \frac{L_1}{R},\ d = \frac{d}{R}$$

$$H'_z(\rho) = 2\pi M_s \frac{1}{\beta} \int \frac{dk}{k} e^{-kd}(1-e^{-\beta k})(1-e^{-\beta_1 k}) J_0(k\rho) J_1(k)$$

$$\varkappa_H = -\pi M_s L \int_0^\infty d\rho\rho \left[\frac{\partial^2 \vec{m}_0}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \vec{m}_0}{\partial \rho}\right] \cdot \vec{H}'(\rho), \quad [\varkappa_H] = \text{erg/cm}^2 \text{ Stiffness}$$

Stray field hard layer $\vec{H}'(\rho)$ рассчитано, $\varkappa_H$ задано within rigid Skyrmion motion model of static radial vortex profile $m_z(\rho)$ и $m_\rho = \sqrt{1-m_z^2} = m_\rho(\rho)$. W_G – Radial Vortex

The simplest approximation $m_z(\rho) = 1 - a\rho^2$, other one $m_z(\rho) = \frac{\rho_c^2 - \rho^2}{\rho_c^2 + \rho^2}$ –

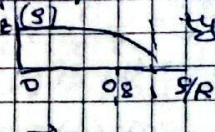
– BP ansatz, с $\rho_c > R$. (a, ρ_c) are fitting parameters $= f(L, L_1, d, R)$.

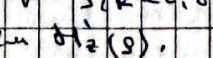
BP ansatz $m_z = \cos\Theta$, $m_\rho = \sin\Theta$, $\operatorname{tg}\frac{\Theta}{2} = \rho/\rho_c$.

Gyrovector $G_z = 2\pi \frac{M_s L}{\gamma}[m_z(0) - m_z(\rho=R)]$. Gyrofrequency $\omega_G = \frac{\varkappa_H}{G_z}$.

Задание radial vortex profile даёт $\omega_G(L, d, L_1, R)$, $\varkappa_H(L, d, L_1, R)$ также задано от $m_z(\rho)$, но более сложно, чем G_z.

5/XI Определение ω_G radial vortex сделать с тем, задавая – вид $m_z(\rho)$ profile для разных L, L_1, d, R. Будем, в первом приближении $m_z(\rho)$ подгонять profile $H'_z(\rho) = \frac{1}{L}\int dz\, H_z(\rho,z)$. $H'_z(\rho)$ резко падает при $\rho/R \approx 0.8$. $m_z(\rho)$ должно быть чем $H'_z(\rho)$.





Можно найти $m_z(\rho)$ аналитически из $W(\vec{m})$ – soft layer in $\vec{H}_d(\vec{r})$ stray field.

При этом $W_m(\vec{m}) \approx 2\pi m_z^2$, т.к. soft layer dot is very thin (1-3 nm)

6/XI Travel report Porto Oct. 2023. Переписка с Оксаной о Mag. Frontiers – 2025 и MS „Block point DW in nanowires" G-dynamics simulations.

7/XI M. Modic прислал simulated профили $m_z(s)$, $H_m^2(s)$ в soft mag. dot. Запрос C. Donnelly о Mag. Frontiers-2025. Обещал MS to Journal Frontiers in Physics (Subm. deadline 3/IV 2024, P. Fischer), the topic issue „High speed spintronics from MHz to THz": BP DW dynamics in Cyl. nanowires, transverse domain wall stability MS.

8/XI Сдал Eli travel reports Porto – Oct. 2023 и Taiwan Domagoj June 2023. Анализ $m_z(s)$ Soft layer Simulations Modic. Запрос Аркадию trip Madrid. Запрос Оксане new PID 2022 project и Irashun – % deposit Santander.

9/XI Simul. Modic: 1) увеличение soft layer thickness L, расширяет $m_z(s)$ профиль;
2) увеличение dot radius R от 15 to 50 nm резко сужает профиль $m_z(s)$;
3) увеличение spacer thickness d от 0.5 до 30 nm слабо сужает профиль $m_z(s)$.

Необходимо найти $m_z(s)$ Soft layer аналитически, stray field hard layer $H_d(s,z)$ будет много inhomogeneous external field. $W_m(\vec m) = 2\pi M_s^2 m_z^2$ – thin layer limit. Slonczewski 'JMMM'1979:

$W = \int dV w$, $w = A[(\nabla\theta)^2 + (\nabla\Phi)^2 \sin^2\theta] + K\sin^2\theta + \frac{1}{2\pi}\vec H^2$, $\vec H$ is mag. field including both applied and demagnetizing terms. $\vec H = \vec H_d + \vec H_{ext}$ у нас $W_m = 2\pi M_s^2 m_z^2 = 2\pi M_s^2 \cos^2\theta = 2\pi M_s^2(1-\sin^2\theta)$, $K = -2\pi M_s^2$ – easy dot/layer plane anisotropy. $\vec H_d$ создает уменьшение $W_m = 2\pi M_s^2 m_z^2$, $H_{ext} = \vec H_d(s,z)$ – stray field of hard layer. $\vec m = \vec m(\theta,\Phi)$.

$$\begin{cases} \frac{\delta W}{\delta\theta} = [K + A(\nabla\Phi)^2]\sin 2\theta - 2A\nabla^2\theta - \vec H\cdot\frac{\partial\vec m}{\partial\theta} = 0 \\ \frac{\delta W}{\delta\Phi} = -2A(\nabla^2\Phi)\sin^2\theta - 2A(\nabla\Phi\cdot\nabla\theta)\sin 2\theta - \vec H\cdot\frac{\partial\vec m}{\partial\Phi} = 0 \end{cases}$$

– static equilibrium eqs.

Cyl. coordinates, azimuthal symmetry $\theta = \theta(s)$, thin dot $\theta = \theta(s)$, $\vec s(s,\varphi,z)$. $\Phi(\vec r) = q\varphi + \Phi_0$, $\vec m = (\sin\theta\cos\Phi, \sin\theta\sin\Phi, \cos\theta)$ (m_x, m_y, m_z).

$\nabla\Phi = \left(\hat s\frac{\partial}{\partial s} + \frac{1}{s}\hat\varphi\frac{\partial}{\partial\varphi} + \hat z\frac{\partial}{\partial z}\right)\Phi = \frac{1}{s}\hat\varphi\, q$, $(\nabla\Phi)^2 = q^2/s^2$, $q = 1$.

$\nabla^2\Phi = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial\Phi}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2\Phi}{\partial\varphi^2} + \frac{\partial^2\Phi}{\partial z^2} = 0$ (кроме special points).

$\nabla\theta = \nabla\theta(s) = \hat s\frac{\partial\theta}{\partial s}$, $\nabla\theta\cdot\nabla\Phi = \frac{\hat s}{s}\frac{\partial\theta}{\partial\varphi}\cdot\hat\varphi\frac{q}{s} = 0$. $\vec m = (m_s, m_\varphi, m_z)$

$m_s = m_x\cos\varphi + m_y\sin\varphi = \sin\theta\cos\Phi\cos\varphi + \sin\theta\sin\Phi\sin\varphi = \sin\theta\cos(\Phi-\varphi)$

$m_\varphi = -m_x\sin\varphi + m_y\cos\varphi = -\sin\theta\cos\Phi\sin\varphi + \sin\theta\sin\Phi\cos\varphi = \sin\theta\sin(\Phi-\varphi)$

Два радиальных вортекса $q=\pm1$ $\Phi = \varphi + \Phi_0$, $\Phi_0 = 0, \pi$, $m_s = \sin\theta$, $m_\varphi = 0$, $m_z = \cos\theta$.

$M_s\vec H\frac{\partial\vec m}{\partial\theta} = M_s\left[H_s\frac{\partial m_s}{\partial\theta} + H_z\frac{\partial m_z}{\partial\theta}\right] = M_s[H_s\frac{\partial\sin\theta}{\partial\theta} + H_z\frac{\partial\cos\theta}{\partial\theta}] =$

$= M_s[H_s\cos\theta - H_z\sin\theta]$. $M_s\vec H\cdot\frac{\partial\vec m}{\partial\Phi} = M_s\left[H_s\frac{\partial m_s}{\partial\Phi} + H_z\frac{\partial m_z}{\partial\Phi}\right] \equiv 0$, т.к. $\frac{\partial\vec m}{\partial\Phi} = 0$.

$\frac{\delta W}{\delta\theta} = \left[K + \frac{A}{s^2}\right]\sin 2\theta - 2A\nabla^2\theta - M_s[H_s\cos\theta - H_z\sin\theta] = 0$.

Ур-е на вортекс профиль $\theta(s)$: $2A\nabla^2\theta - \left[K + \frac{A}{s^2}\right]\sin 2\theta + M_s[H_s\cos\theta - H_z\sin\theta] = 0$.

Здесь $\vec H = (H_s, 0, H_z) = \vec H(s,z)$ stray fields of hard layer.

$\nabla^2\theta = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial\theta}{\partial s}\right)$. Надо $\frac{1}{L}\int d(\dots) \to 2A\nabla^2\theta - \left[K + \frac{1}{s^2}\right]\sin 2\theta + M_s(H_s\cos\theta - H_z\sin\theta) = 0$

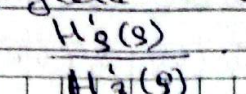
$\vec H(\theta) = \frac{1}{L}\int_0^L \vec H(s,z)\,dz$ обещано 4/XI. $\theta(s)$ из нелинейного диф. уравнения.

$\theta(s) \sim m_z(s) = \cos\theta(s)$ аналитически не получается. Надо $\min W(\vec m)$ на пробных функциях (trial function), $\theta(s)$ или $m_z(s)$.

Escaneado con CamScanner

Если stray field >> всех остальных вкладов, то $H'_\rho \cos\Theta - H'_z \sin\Theta = 0$

$\operatorname{tg}\Theta(\rho) \approx \frac{H'_\rho(\rho)}{H'_z(\rho)}$. $H'_{\rho,z}(\rho)$ посчитано $\Rightarrow \cos\Theta(\rho) = \frac{1}{\sqrt{1+\operatorname{tg}^2\Theta(\rho)}}$

такой $m_z(\rho) = \cos\Theta(\rho)$ профиль 

ориентационный фазовый typical / dot parameters.

$W(\vec m) = A\left[(\nabla\Theta)^2 + \sin^2\Theta\right] + K\sin^2\Theta - M_s \vec m \cdot \vec H$, $\nabla\Theta = \frac{-1}{\sin\Theta}\nabla(\cos\Theta)$

$W(\vec m) = A\left[\frac{(\nabla m_z)^2}{1-m_z^2} + (1-m_z^2)\frac{1}{\rho^2}\right] + 2\pi M_s^2 m_z^2 - M_s \vec m \cdot \vec H$.

$\vec m \cdot \vec H = m_\rho H_\rho + m_z H_z$. $W(\vec m) = \int dV\, w(\vec m) = 2\pi L \int_0^R d\rho\, \rho\, \tilde w(\vec m)$

$\tilde w(\vec m) = A\left[\frac{(\nabla m_z)^2}{1-m_z^2} + (1-m_z^2)\frac{1}{\rho^2}\right] + 2\pi M_s^2 m_z^2 - M_s \vec m \cdot \vec H$, $\vec H' = \frac{1}{L}\int dz\, \vec H$.

$w(\vec m)$ зависит $m_z(\rho)$, можно зависит $\Theta(\rho)$.

$w(\vec m) = A\left[(\nabla\Theta)^2 + \frac{1}{\rho^2}\sin^2\Theta\right] + 2\pi M_s^2 \cos^2\Theta - M_s\left[H'_\rho \sin\Theta + H'_z \cos\Theta\right] = w(\Theta)$.

$w(\Theta) = A\left[\left(\frac{\partial\Theta}{\partial\rho}\right)^2 + \frac{1}{\rho^2}\sin^2\Theta\right] + 2\pi M_s^2 \cos^2\Theta - M_s\left[H'_\rho \sin\Theta + H'_z \cos\Theta\right]$

$W_{[\Theta]} = 2\pi L \int_0^R d\rho\, \rho\, w(\Theta)$. $W[\Theta(\rho)] = 2\pi L \int_0^R d\rho\, \rho\, w(\Theta)$. $\operatorname{tg}\frac{\Theta}{2} = \rho/\rho_c$ – BP ansatz.

$W[\Theta] = 2\pi R^2 L M_s^2 \int_0^1 d\rho\,\rho \left[\frac{A}{M_s^2}(\dots) + 2\pi\cos^2\Theta - \frac{1}{M_s}\left(H'_\rho \sin\Theta + H'_z\cos\Theta\right)\right]$

$W[\Theta] = (2\pi)^2 R^2 L M_s^2 \int_0^1 d\rho\,\rho[\dots]$, $(2\pi)^2 R^2 L = 4\pi^2 R^2 L = 4\pi V$

$W[\Theta(\rho)] = 4\pi M_s^2 V \int_0^1 d\rho\,\rho \left[\frac{l_e^2}{R^2}\left(\Theta'^2 + \frac{1}{\rho^2}\sin^2\Theta\right) + \cos^2\Theta - \frac{1}{2\pi M_s}\left(H'_\rho \sin\Theta + H'_z\cos\Theta\right)\right]$

$l_e^2 = A/2\pi M_s^2$.

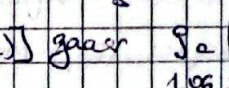
BP = trial function $\Theta(\rho, \rho_c) = 2\arctg(\rho/\rho_c)$. $H'_\rho, H'_z = f(\rho, \beta, \beta_1, d)$, $\beta = L/R$

In $\pi M_s^2 V$ units $W[\Theta(\rho)] = \int_0^1 d\rho\,\rho \left[\left(\frac{l_e}{R}\right)^2\left(\Theta'^2 + \frac{1}{\rho^2}\sin^2\Theta\right) + \cos^2\Theta - 2\left(H'_\rho \sin\Theta + H'_z\cos\Theta\right)\right]$

$d \to d/R$, $\beta_1 = \frac{L_1}{R}$

$\vec H'$ is in πM_s units.

Модель: $L_1 = 30$ nm, $2R = 50$ nm, $R = 25$ nm, $d = 1$ nm $\longrightarrow$ L изменяется от 0.5 до 3 nm.

$\beta = L/R \in [0.02, 0.12]$. BP ansatz из min $W[\Theta(\rho, \rho_c)]$ даёт ρ_c 

Sim. $\rho_c(L)$ ... с ростом L, χ растёт и сжато.

У Модель $m_z(\rho=R) = 0.43 - 0.53$ – у меня в BP ansatz $m_z(\rho=R) \approx 0.1$ даже $L = 0.5 - 3$ nm. BP ansatz даёт ~~больше~~ меньше $m_z(\rho=R)$.

Модель: $L_1 = 30$ nm, $L = 1$ nm, $d = 1$ nm, увеличение R резко сужает $m_z(\rho)$, уменьшает ρ_c in simulations. В BP ansatz у меня $\rho_c(R)$ растёт, от 1.4 до 0.9 in reduced units $\frac{\rho_c}{R}$. У Модель $m_z(\rho)$ сужается при ρ в reduced units $x \in [0, 100]$ nm, т.к. x это ρ/R в %.

Модель: $L_1 = 30$ nm, $R = 25$ nm, $L = 1$ nm, d увел. от 0.5 до 2 nm, расс. d сильно сужает профиль $m_z(\rho)$. BP ansatz даёт $\rho_m(d)$ падает сильно от 1.12 до 1.06 при $d = 0.5 - 2$ nm в согласии с Модель – sim. Но BP – $m_z(\rho=R) \approx 0.1$ Сильно мало по сравн. с Simulated Модель $m_z(\rho=R) \approx 0.5$.

Вывод: BP ansatz даёт качественно верное зависимость сжатия radial vortex $\rho_c(R)$ и $\rho_c(d)$, но неверную $\rho_c(L)$, $\beta = \frac{L}{R}$. Но BP ansatz всегда сильно занижает $m_z(\rho=R) \approx 0.1$ по сравн. с simulations, где Модель $m_z(\rho=R) \approx 0.5$. BP ansatz плох для расчёта/подгонки $m_z(\rho)$ radial vortex.

Сравнить $H_{ex}(\rho)$, $H_m(\rho)$ Модель при $\rho \approx R$, dot edge. Проверить. Обсудить. с Prof. Ed. и.

Л. написать Рутгерс Нуланд "это для 2π в Zeeman. Constant"

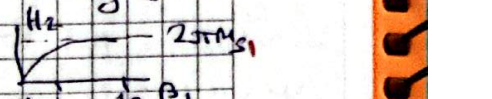
10/Fi) Сравнение analytic dipolar fields с simulations Moalic.

Conv. 4/Fi $\begin{cases} H_\rho(\rho,z) = 2\pi M_{s_1} R \int dk\, e^{-kz}(1-e^{-kL_1}) J_1(k\rho) J_1(kR) \\ H_z(\rho,z) = 2\pi M_{s_1} R \int dk\, e^{-kz}(1-e^{-kL_1}) J_0(k\rho) J_1(kR) \end{cases}$ Hard $\vec{M}_1 = M_{s_1}\hat{z}$

Это stray field components of hard layer с $M_1 = M_{s_1}\hat{z}$ (L_1 m $z<0$)

Calculated for $z>0$.

$\begin{cases} H_\rho(\rho,z) = 2\pi M_{s_1} \int dk\, e^{-k z \beta_1}(1-e^{-\beta_1 k}) J_1(k\rho) J_1(k) \\ H_z(\rho,z) = 2\pi M_{s_1} \int dk\, e^{-\beta k z}(1-e^{-\beta_1 k}) J_0(k\rho) J_1(k) \end{cases}$ z - b eg. Soft layer thickness L, $\rho \to \rho/R$. - stray field.



При $\rho=0, z=0$ $H_z(0,0) = 2\pi M_{s_1} \int dk (1-e^{-\beta_1 k}) J_1(k)$

$\to 2\pi M_{s_1}$ при $\beta_1 \gg 1$. Скачок на $z=0$ p-ле

$\Delta H_z = 4\pi M_{s_1} = H_z(z+0) - H_z(-0)$

При $L_1 = 30$ nm, $2R = 50$ nm, $\beta_1 = 30/25 = 1.2$, $H_z(\rho, z=\frac{L}{2}) \approx 0.4\,(4\pi M_{s_1})$

Hard layer $M_{s_1} = 10^3$ kA/m $= 1000$ G, $4\pi M_{s_1} = 12.57 \cdot 10^3$ G $= 1.26$ T

Soft layer $M_s = 10^3 \cdot 0.81$ kA/m $= 810$ G, $4\pi M_s = 10.18 \cdot 10^3 = 1.02$ T

H_m (Soft layer) $\simeq -4\pi m_z(\rho) M_s$ при $W_m = 2\pi M_s^2 m_z^2$, при $L = 0.5$–3 nm Soft layer.

In the dot center $\rho=0$, $H^z_{dem} = H^z_m + H^z_d = -4\pi M_s m_z(\rho=0) +$

$+ H_z(0,0) = -1.02$ T $+ (0.40 \div 0.38) \cdot 1.26$ T $= -0.53$ T ($L=3$ nm, $d=$(nm, $R=25$ nm)

у Moalic ($L=3$ nm, $d=1$ nm, $R=25$ nm) $H^z_{dem}(0) \approx -0.65$ T. Можно Moalic дал H_{dem} averaged по z in soft layer?

At the dot edge $H_m(\rho=R) = -4\pi M_s \cdot m_z(\rho=R) = -1.02 \cdot 0.53$ T $= -0.54$ T

$H^z_d(\rho=R, z=1/25, \beta=1.2) = 0.172 \cdot 4\pi M_{s_1} = 0.172 \cdot 1.26$ T $= 0.217$ T

Calc. $H^z_{dem}(\rho=R) = H^z_m(R) + H^z_d(R) = -0.54$ T $+ 0.217$ T ≈ -0.32 T, у Moalic $H^z_{dem}(R) \approx 0$ ($+0.05$ T). Откуда такое различие H^z_{dem} - calc. и H^z_{dem} - simulated при $\rho = R$ dot edge? Ими $2R = 50$ nm, at least.

Проверка $\vec{H}_{ex}$ - Moalic. $\vec{H}_{ex} = -\frac{2A}{M_s}\nabla^2\vec{m}$, $H^z_{ex}(\rho) = -\frac{2A}{M_s}(\nabla^2 m)^z = -\frac{2A}{M_s}\nabla^2 m_z(\rho) =$

$= -\frac{2A}{M_s}\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial m_z}{\partial\rho}\right) = -\frac{2A}{M_s}\left[\frac{\partial^2 m_z}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial m_z}{\partial\rho}\right]$. В центре dot center $\rho=0$

$m_z(\rho) \approx 1 - a\rho^2$, ($a>0$) $\frac{\partial m_z}{\partial\rho} = -2a\rho$, $\frac{\partial^2 m_z}{\partial\rho^2} = -2a$, $H^z_{ex}(\rho=0) = -\frac{2A}{M_s}(-2a) = \frac{8A}{M_s}a > 0$

Мое $H^z_{ex}(\rho=0) = \frac{8A}{M_s}a > 0$, у Moalic $H^z_{ex}(\rho=0) \approx -0.6$ T <0 (?)

// $\vec{H}_{ex} = -\frac{2A}{M_s}\nabla^2\vec{m}$ это дает для $\vec{M} = \vec{M}_0 + \vec{m}$. Для общего LLG

$\vec{H}_{ex} = +\frac{2A}{M_s^2}\nabla^2\vec{M}$ при $\vec{M}\times\left[\frac{2A}{M_s^2}\nabla^2\vec{M} + M_s^0\vec{H}\right] = \frac{1}{\delta}\frac{d\vec{M}}{dt}$

$H^z_{ex}(\rho=0) \approx -0.6$ T Moalic вычислено безусловно.

Moalic сбрасывает $\vec{H}_{ex}(\rho)$, $\vec{H}_{dem}(\rho)$ в самом верхнем слое нанопиллар, т.е. при $z = d+L$ в наших обозначениях, $z=0$ hard layer/spacer interface.

Тогда $\frac{z}{L} = \frac{d}{L} + 1 = \frac{1}{3} + 1 = \frac{4}{3}$. $d=1$ nm, $L=3$ nm. $H_z\left(\rho=0, \frac{4}{3}, \frac{3}{25}, 1.2\right) = 0.324 \cdot 4\pi M_{s_1}$

$H^z_{dem}(\rho=0) = -1.02$ T $+ 0.324 \cdot 1.26$ T $= -0.612$ T, что хорошо согласуется с simulation Moalic $H^z_{dem}(0) = -0.65$ T

На dot edge $\rho=R$ $H^z_d\left(\rho=R, z=\frac{4}{3}, \frac{3}{25}, 1.2\right) = 0.134 \cdot 4\pi M_{s_1} = 0.169$ T

Calc. $H^z_{dem}(\rho=R) = -0.54$ T $+ 0.17$ T $= -0.37$ T. У Moalic $H^z_{dem}(R) \approx 0$.

Итог Moalic расчета при $z = d+L$ (top layer) обрезает $H_{dem}(\rho=0)$, но не $H^z_{dem}(R)$.

Escaneado con CamScanner

(11/XI) УД на $\theta(\rho)$: $2A\nabla^2\theta - \left[K + \frac{A}{\rho^2}\right]\sin 2\theta + M_s[H_\rho' \cos\theta - H_z' \sin\theta] = 0$

Можно предположить $\theta \ll 1$, т.е. при $\rho/R \ll 1$, где $m_z = \cos\theta \approx 1$.

$\sin\theta \approx \theta$, $\cos\theta \approx 1$. $2A\nabla^2\theta - 2\left(K + \frac{A}{\rho^2}\right)\theta - M_s H_z' \theta = -M_s H_\rho'$

$2A\nabla^2\theta - 2\left[\frac{A}{\rho^2} + K + \frac{1}{2}H_z'(\rho)\right]\theta = -M_s H_\rho'(\rho)$. $H_z'(\rho) \approx$ const при $\rho/R \ll 1$.

при $\rho \to 0$ $\nabla^2\theta - \frac{1}{\rho^2}\theta(\rho) = 0$. $\frac{\partial^2\theta}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\theta}{\partial\rho} - \frac{1}{\rho^2}\theta(\rho) = 0$.

$\theta(\rho) = C J_1(\rho)$. $\rho \to \rho/R$. $J_1(x) \simeq \frac{1}{\Gamma(2)} \cdot \frac{x}{2} = \frac{1}{2}x$ при $x \ll 1$.

$m_z(\rho) = \cos\theta = 1 - \frac{1}{2}\theta^2 = 1 - \frac{1}{2}C^2 J_1^2(\rho) = 1 - \frac{1}{2}C^2\frac{1}{4}\rho^2 = 1 - \frac{1}{8}C^2\rho^2$. ($\rho/R \ll 1$)

BP ansatz $m_z(\rho) = -\frac{\rho^2 - \rho_c^2}{(\rho^2 + \rho_c^2)}$, согласуется с расчетом $m_z(\rho)$ - exchange при $\rho \ll 1$.

BP $m_z(\rho) = 1 - 2\rho^2/\rho_c^2$ и $m_z(\rho) = |\text{exch. approx}| = 1 - \frac{1}{8}C^2\rho^2$,

$\rho_c^2 = 16/C^2$, т.е. BPolyakov ansatz хорошо при $\rho/R \ll 1$. При ρ, $\rho/R \sim 1$, точность BP - ansatz падает.

Точнее при $\rho/R \ll 1$: $\nabla^2\theta - \theta(\rho) = -\frac{M_s}{2A} H_\rho'(\rho)$. $\theta'' + \frac{1}{\rho}\theta' - \frac{1}{\rho^2}\theta = -\frac{M_s R^2}{2A}\frac{H_\rho}{\rho}$

$H_\rho'(\rho) = 2\pi M_s \frac{1}{\beta}\int_0^\infty \frac{dk}{k} e^{-kd}(1 - e^{-\beta k})(1 - e^{-\beta_1 k}) J_1(k) J_1(k\rho) \sim \rho$ при $\rho \ll 1$.

при $\beta \ll 1$, $H_\rho'(\rho) = 2\pi M_s \int dk e^{-kd}(1 - e^{-\beta_1 k}) J_1(k) J_1(k\rho)$.

$\theta(\rho) = C J_1(\rho) = \frac{1}{2}C\rho$, $\theta' = \frac{1}{2}C$, $\theta'' = 0$. $\frac{1}{\rho}\frac{1}{2}C - \frac{1}{\rho^2}\frac{1}{2}C\rho = -\frac{M_s R^2}{2A}\frac{H_\rho}{\rho}$

можно уточнить $\theta(\rho)$ как решение Н.Н.D.У. с учетом $H_\rho'(\rho)$.

→ Найти radial vortex gyrofrequency при $m_z(\rho)$ как BP - ansatz.

$\omega_G = \varkappa_H / G$. $\varkappa_H$ через $\frac{\partial^2 \vec{m}}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\vec{m}}{\partial\rho}$, $\vec{m} = \vec{m}_0(\rho)$ static vortex profile.

$\frac{\partial^2 m_\perp}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial m_\perp}{\partial\rho} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho m_\perp) = \nabla^2 m_\perp$, при $m_\perp = m_\perp(\rho)$.

(12-14/XI) Trip to Madrid (Alcobendas), подача appl. на new passport Arc. (120,90 €) и время ожидания. Two appl submitted (18 €) — 13/XI 2023. Проверяем site – gov.ua" о готовности паспорта.

(15/XI) "Mag. Frontiers 2025 appl.", Travel report Madrid.

(16/XI) Submitted Mag. Frontiers - 2025 application. Полная Madrid report to Arcady. Оплачен "Tuberona" 768,60 € за клобасы (resto pedido).

// Gyrotropic frequency of radial vortex in soft/hard layer nanopillar (4/XI)

stiffness из-за dipolar field generated by hard layer in soft layer:

$\varkappa_H = -\pi M_s L \int_0^\infty d\rho\rho \left[\frac{\partial^2 \vec{m}_0}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\vec{m}_0}{\partial\rho}\right]\vec{H}_1'$, где $\vec{m}_0(\rho) = (m_\rho, 0, m_z)$ static vortex profile. $\vec{H}' = \frac{1}{L}\int dz \vec{H}_d(\rho, z)$, $\vec{H}_d$ is the stray field.

Gyrovector $G_z = 2\pi\frac{M_s}{\gamma} L\left[m_z(\rho = R) - m_z(\rho = 0)\right]$. Для $m_z^0(\rho)$ используем BP ansatz $m_z^0(\rho) = (\rho_c^2 - \rho^2)/(\rho^2 + \rho_c^2)$ и находим $\rho_c(L, L, d, R)$ из min $W(\vec{m})$.

Но если еще учесть stiffness $\varkappa$ из-за $W(\vec{x})$ soft layer, не только $W_H(\vec{x})$ stray field (4/XI). $W = W_{soft} + W_H = \int dV\left[W_s + W_H\right] = \int dV\, w$

$w(\vec{m}) = A(\nabla\vec{m})^2 + K\sin^2\theta - M_s\vec{H}_d\cdot\vec{m}$, local magnetostatics $W_m = 2\pi m_z^2$ of thin soft layer, $L = 1$–3 nm. $K = -2\pi M_s^2$ - easy plane anisotropy, см. 9/XI.

$m_z(\rho = 0) = 1$, $m_z(\rho = R) = (\rho_c^2 - R^2)/(\rho_c^2 + R^2) = \frac{(\rho_c^2 - 1)}{(\rho_c^2 + 1)}$ при $R = 1$ radius ρ_c in R units.

$G_z = 2\pi\frac{M_s}{\gamma} L\left[(\rho_c^2 - 1)/(\rho_c^2 + 1) - 1\right] = 2\pi\frac{M_s}{\gamma} L(-2)\frac{1}{1 + \rho_c^2}$, $G_z = G_z(\rho_c)$, ρ_c из min $W(\vec{m})$.

Escaneado con CamScanner

(17/XI) $W(\vec{X}) = \frac{1}{2}\varkappa \vec{X}^2$. Vortex stiffness within rigid soliton model. 4/XI

Dynamic $\vec{m}(z,\vec{s},\vec{X}) = \vec{m}_0(z,\vec{s}-\vec{X})$ within rigid model, $\vec{X} = \vec{X}(t)$.

Thin dot, $\vec{m}(\vec{s},\vec{X}) = \vec{m}_0(\vec{s}-\vec{X})$, $\vec{m}_0(\vec{s})$ is static vortex profile.

$W(\vec{X}) = \int dV\, w[\vec{m}(z,\vec{s},\vec{X})] = L\int d^2\vec{s}\, w[\vec{m}(\vec{s},\vec{X})] = L\int d^2\vec{s}\, w[\vec{m}_0(\vec{s}-\vec{X})]$

$\vec{F}(\vec{X}) = -\nabla_{\vec{X}} W(\vec{X}) = -\nabla_{\vec{X}} L\int d^2\vec{s}\, w[\vec{m}_0(\vec{s}-\vec{X})]$, $\nabla_{\vec{X}} w[\vec{m}_0(\vec{s}-\vec{X})] =$

$= \frac{\partial w}{\partial \vec{m}}\cdot\frac{\partial \vec{m}}{\partial \vec{X}}$. $F_\alpha = -\frac{\partial}{\partial X_\alpha} L\int d^2\vec{s}\, w[\vec{m}]$, $\frac{\partial}{\partial X_\alpha} w(\vec{m}) = \frac{\partial w}{\partial \vec{m}}\cdot\frac{\partial \vec{m}}{\partial X_\alpha}$.

$\frac{\partial \vec{m}}{\partial X_\alpha} = \frac{\partial}{\partial X_\alpha}\vec{m}_0(\vec{s}-\vec{X})$, $\nabla_{\vec{X}} w = \frac{\partial w}{\partial m_\alpha}\frac{\partial m_\alpha}{\partial \vec{X}}$ где также $w = w(\vec{m})$ это

зависит только от $\vec{m}$, но не от $\partial\vec{m}/\partial x_\beta$.

$\nabla_{\vec{X}} m_\alpha = \frac{\partial m_\alpha}{\partial \vec{X}} = \frac{\partial}{\partial \vec{X}}\cdot(\vec{m}_0)_\alpha(\vec{s}-\vec{X}) = -\nabla_{\vec{s}}\, m_\alpha$, $\nabla_{\vec{s}} m_\alpha = \hat{s}\frac{\partial m_\alpha}{\partial s} + \frac{1}{s}\hat{\varphi}\frac{\partial m_\alpha}{\partial \varphi}$

$\vec{F} = -\nabla_{\vec{X}} W(\vec{X}) = -L\int d^2\vec{s}\, \nabla_{\vec{X}} w[\vec{m}_0(\vec{s}-\vec{X})] = -L\int d^2\vec{s}\frac{\partial w}{\partial m_\alpha}\frac{\partial m_\alpha}{\partial \vec{X}} = -L\int d^2\vec{s} \times$

$\times \frac{\partial w}{\partial m_\alpha}(-1)\nabla_{\vec{s}} m_\alpha = L\int d^2\vec{s}\frac{\partial w}{\partial m_\alpha}\left[\hat{s}\frac{\partial m_\alpha}{\partial s} + \frac{1}{s}\hat{\varphi}\frac{\partial m_\alpha}{\partial \varphi}\right]$.



$\vec{F}(\vec{X}) = L\int d^2\vec{s}\frac{\partial w}{\partial m_\alpha}\nabla_{\vec{s}} m_\alpha$, $\nabla_{\vec{s}}\cdot m_\alpha(\vec{s}-\vec{X}) = m_\alpha^0(s) - \vec{X}\cdot\frac{\partial m_\alpha^0}{\partial \vec{s}}\Big|_{\vec{X}=0}$

$m_\alpha = m_\alpha^0(\vec{s}-\vec{X})$. $\vec{F}(\vec{X}) = L\int d^2\vec{s}\frac{\partial w}{\partial m_\alpha}\nabla_{\vec{s}}\left[m_\alpha^0 - \vec{X}\cdot\frac{\partial m_\alpha^0}{\partial \vec{s}}\right]$, если $\frac{\partial w}{\partial m_\alpha}$ не зависит

от $\vec{m}$ (как в Зеемановской энергии), то $\vec{F}(\vec{X}) = -\vec{X}\cdot L\int d^2\vec{s}\frac{\partial w}{\partial m_\alpha}\nabla_{\vec{s}}\left(\frac{\partial m_\alpha^0}{\partial \vec{s}}\right)$

$\vec{F}(\vec{X}) = -L\int d^2\vec{s}\frac{\partial w}{\partial m_\alpha}\nabla_{\vec{s}}\left[\vec{X}\cdot\frac{\partial m_\alpha^0}{\partial \vec{s}}\right] \sim \vec{X}$, where $\frac{\partial w}{\partial m_\alpha} = f_\alpha(\vec{m})$, $\vec{m} = \vec{m}_0(\vec{s}-\vec{X})$ зависит от $\vec{X}$.

$f(\vec{m}) = f[\vec{m}_0(\vec{s}-\vec{X})] = f\left[\vec{m}_0(\vec{s}) - \vec{X}\cdot\frac{\partial}{\partial \vec{s}}\vec{m}_0\right]$

$\neq f(m_\alpha) = f\left[m_\alpha^0 - \vec{X}\cdot\frac{\partial m_\alpha^0}{\partial \vec{s}}\right] = f(m_\alpha^0) + \frac{\partial f}{\partial m_\alpha}\left(\vec{X}\cdot\frac{\partial m_\alpha^0}{\partial \vec{s}}\right)$

$f(\vec{m}) = f(\vec{m}_0 - \vec{m}_1) = f(\vec{m}_0) - \frac{\partial f}{\partial \vec{m}}\cdot\vec{m}_1$, $\vec{m}_1 = \left(\vec{X}\cdot\frac{\partial}{\partial \vec{s}}\right)\vec{m}_0$

$\vec{F}(\vec{X}) = L\int d^2\vec{s}\frac{\partial w}{\partial m_\alpha}\nabla_{\vec{s}}\left[m_\alpha^0 - \left(\vec{X}\cdot\frac{\partial}{\partial \vec{s}}\right)m_\alpha^0\right]$, $\frac{\partial w}{\partial m_\alpha} = f_\alpha(\vec{m})$, т.к. $W = W(\vec{m})$.

$\frac{\partial w}{\partial m_\alpha} = f_\alpha(\vec{m}) = f_\alpha(\vec{m}_0) - \frac{\partial f_\alpha(\vec{m})}{\partial \vec{m}}\cdot\left(\vec{X}\cdot\frac{\partial}{\partial \vec{s}}\right)\vec{m}_0$, $f_\alpha(\vec{m}) = \frac{\partial w(\vec{m})}{\partial m_\alpha}$.

$\vec{F}(\vec{X}) = L\int d^2\vec{s}\left[f_\alpha(\vec{m}_0) - \frac{\partial f_\alpha(\vec{m})}{\partial \vec{m}}\cdot\left(\vec{X}\cdot\frac{\partial}{\partial \vec{s}}\right)\vec{m}_0\right]\nabla_{\vec{s}}\left[m_\alpha^0 - \left(\vec{X}\cdot\frac{\partial}{\partial \vec{s}}\right)m_\alpha^0\right] =$

$= L\int d^2\vec{s}\left\{+f_\alpha(\vec{m}_0)\nabla_{\vec{s}}\left[\left(\vec{X}\cdot\frac{\partial}{\partial \vec{s}}\right)m_\alpha^0\right] + \frac{\partial f_\alpha}{\partial \vec{m}}\cdot\left(\vec{X}\cdot\frac{\partial}{\partial \vec{s}}\right)\vec{m}_0\nabla_{\vec{s}} m_\alpha^0\right\}$, здесь $\vec{F}(\vec{X}) \sim \vec{X}$ только

$\vec{F}(\vec{X}) = -L\int d^2\vec{s}\left\{f_\alpha(\vec{m}_0)\nabla_{\vec{s}}\left[\left(\vec{X}\cdot\frac{\partial}{\partial \vec{s}}\right)m_\alpha^0\right] + \frac{\partial f_\alpha}{\partial \vec{m}}\cdot\left(\vec{X}\cdot\frac{\partial}{\partial \vec{s}}\right)\vec{m}_0\nabla_{\vec{s}} m_\alpha^0\right\}$, если $W = W(\vec{m})$

4/XI если $\frac{\partial f_\alpha}{\partial \vec{m}}$ не зависит от $\vec{m}$, т.к. $W(\vec{m}) = -M_s\vec{m}\cdot\vec{H}_d$, $\frac{\partial w}{\partial \vec{m}} \sim \vec{H}_d$

Трансляция $\vec{X} = X_\beta\hat{\beta}$, т.к. $X_\theta = 0$ by definition (?). не зависит от $\vec{m}$, $\partial f_\alpha/\partial\vec{m} = 0$

$\vec{X}\cdot\frac{\partial}{\partial \vec{s}} = X_\beta\hat{\beta}\cdot\left[\hat{s}\frac{\partial}{\partial s} + \frac{1}{s}\hat{\varphi}\frac{\partial}{\partial \varphi}\right] = X_\beta\frac{\partial}{\partial s}$.

$\vec{F}(\vec{X}) = -L\int d^2\vec{s}\left\{f_\alpha(\vec{m}_0)\nabla_{\vec{s}}\left[X_\beta\frac{\partial}{\partial s}m_\alpha^0\right] + \frac{\partial f_\alpha}{\partial \vec{m}}\cdot X_\beta\frac{\partial}{\partial s}\vec{m}_0\cdot\nabla_{\vec{s}} m_\alpha^0\right\}$

$\nabla_{\vec{s}}\left[X_\beta\frac{\partial m_\alpha^0}{\partial s}\right] = \left[\hat{s}\frac{\partial}{\partial s} + \frac{1}{s}\hat{\varphi}\frac{\partial}{\partial \varphi}\right]\left(X_\beta\frac{\partial m_\alpha^0}{\partial s}\right)$, $X_\beta = X\cos\varphi + Y\sin\varphi = X_\beta(\varphi)$

$\nabla_{\vec{s}}\left[X_\beta\frac{\partial m_\alpha^0}{\partial s}\right] = \hat{s}\frac{\partial}{\partial s}(\dots) + \frac{1}{s}\hat{\varphi}\frac{\partial}{\partial \varphi}(\dots) = \hat{s}X_\beta\frac{\partial^2 m_\alpha^0}{\partial s^2} + \frac{1}{s}\hat{\varphi}\frac{\partial X_\beta}{\partial \varphi}\frac{\partial m_\alpha^0}{\partial s}$, $m_\alpha^0 = m_\alpha^0(s)$. $\frac{\partial}{\partial \varphi}X_\beta$

$\left(\vec{X}\cdot\frac{\partial}{\partial \vec{s}}\right) = (X_s\hat{s} + X_\varphi\hat{\varphi})\cdot\left(\hat{s}\frac{\partial}{\partial s} + \frac{1}{s}\hat{\varphi}\frac{\partial}{\partial \varphi}\right) = X_s\frac{\partial}{\partial s} + \frac{1}{s}X_\varphi\frac{\partial}{\partial \varphi}$. Но оператор $\left(\vec{X}\cdot\frac{\partial}{\partial \vec{s}}\right)$ действует только

на $m_\alpha^0 = m_\alpha^0(s)$ и $\vec{m}^0 = \vec{m}_0(s)$, $\partial/\partial\varphi = 0$ или $X_\varphi \neq 0$.

$$\vec{F}(\vec{X}) = -L\int ds\, s\left\{ f_\alpha(\vec{m}_0)\left[\hat{s}X_s\frac{\partial^2 m_\alpha^0}{\partial s^2} + \frac{\hat{\varphi}}{s}X_\varphi\frac{\partial m_\alpha^0}{\partial s}\right] + X_s\frac{\partial f_\alpha}{\partial \vec{m}}\cdot\frac{\partial \vec{m}_0}{\partial s}\hat{s}\frac{\partial m_\alpha^0}{\partial s}\right\}$$

$$\frac{1}{2\pi}\int_0^{2\pi} d\varphi\, \hat{s}X_s = \frac{1}{2}\vec{X}, \quad \frac{1}{2\pi}\int_0^{2\pi} d\varphi\, \hat{\varphi}X_\varphi = \frac{1}{2}\vec{X}, \quad \vec{X} = (X, Y)$$ Сим. 4/XI.

$\frac{\partial f_\alpha}{\partial \vec{m}}\Big|_{\vec{m}=\vec{m}_0} = P(\vec{m}_0)$ не зависит от φ, т.к. $\vec{m}_0 = \vec{m}_0(s)$.

$$\vec{F}(\vec{X}) = -\pi L\vec{X}\int ds\, s\left\{ f_\alpha(\vec{m}_0)\left[\frac{\partial^2 m_\alpha^0}{\partial s^2} + \frac{1}{s}\frac{\partial m_\alpha^0}{\partial s}\right] + \frac{\partial f_\alpha}{\partial \vec{m}}\cdot\frac{\partial \vec{m}_0}{\partial s}\frac{\partial m_\alpha^0}{\partial s}\right\}$$

$\int dz\, w(\vec{m}) = L\, w(\vec{m})$ поэтому мысль в сомнение

$$\int dz\, w(\vec{m}) = \int dz\left[W_m(\vec{m}) - M_s\vec{m}\vec{H}_d(z s)\right] = LW_m - LM_s\int \frac{dz}{L}\vec{m}\vec{H}_d(s z) =$$

$$= LW_m - LM_s\vec{m}\cdot\vec{H}' = L\left[W_m - M_s\vec{m}\vec{H}'\right] = L\,W(\vec{m}),$$ где $\vec{H}' = \frac{1}{L}\int dz\,\vec{H}_d(sz)$ averaged stray fields of hard layer. $\vec{F}(\vec{X}) = -\pi\vec{X}$, $W(\vec{X}) = \frac{1}{2}\pi\vec{X}^2$.

$$\pi = \pi L\int ds\, s\left\{ f_\alpha(\vec{m}_0)\left(\frac{\partial^2 m_\alpha^0}{\partial s^2} + \frac{1}{s}\frac{\partial m_\alpha^0}{\partial s}\right) + \frac{\partial m_\alpha^0}{\partial s}\frac{\partial f_\alpha}{\partial \vec{m}}\cdot\frac{\partial \vec{m}_0}{\partial s}\right\}, \quad f_\alpha(\vec{m}) = \frac{\partial W}{\partial m_\alpha}$$

где $W = W_H = -M_s\vec{m}\cdot\vec{H}'$, $f_\alpha(\vec{m}) = -M_s H'_\alpha$ это и сбирается к π_H 4/XI.

Note: в общем случае $W = W(\vec{m})$, т.е. без W_{ex} вклада. π_{ex} надо считать отдельно.

$W_m = 2\pi M_s^2 m_z^2$, $f_\alpha = \frac{\partial}{\partial m_\alpha}\left[2\pi M_s^2 m_z^2 - M_s\vec{m}\vec{H}'\right] = 2\pi M_s^2\cdot 2m_z\delta_{\alpha z} -$

$- \frac{\partial}{\partial m_\alpha}M_s m_\beta H'_\beta = 4\pi M_s^2 m_z\delta_{\alpha z} - M_s H'_\alpha$. Магнитостатика дает вклад

в f_α и в $\frac{\partial f_\alpha}{\partial \vec{m}}\cdot\frac{\partial f_\alpha}{\partial m_\beta}$? $W_m = 2\pi M_s^2(\vec{m}\cdot\hat{z})^2$, $\vec{f} = \frac{\partial W_m}{\partial \vec{m}} = 2\pi M_s^2 \times$

$\times\frac{\partial}{\partial \vec{m}}(\vec{m}\cdot\hat{z})^2 = 2\pi M_s^2(\vec{m}\cdot\hat{z})\cdot 2\cdot\hat{z} = 4\pi M_s^2(\vec{m}\cdot\hat{z})\hat{z} = f_z\hat{z}$, $f_\alpha = f_z\delta_{\alpha z}$

$$\frac{\partial f_\alpha}{\partial \vec{m}} = \frac{\partial}{\partial \vec{m}}f_z\delta_{\alpha z} = \frac{\partial}{\partial \vec{m}}4\pi M_s^2(\vec{m}\cdot\hat{z})\delta_{\alpha z} = 4\pi M_s^2\hat{z}\delta_{\alpha z}$$

$$\frac{\partial f_\alpha}{\partial \vec{m}}\cdot\frac{\partial \vec{m}_0}{\partial s} = 4\pi M_s^2\hat{z}\delta_{\alpha z}\cdot\frac{\partial \vec{m}_0}{\partial s} = 4\pi M_s^2\delta_{\alpha z}\frac{\partial m_z^0}{\partial s}$$

$$\pi_m = \pi L\int ds\, s\left\{\delta_{\alpha z}\cdot 4\pi M_s^2(\vec{m}\cdot\hat{z})\left(\frac{\partial^2 m_\alpha^0}{\partial s^2} + \frac{1}{s}\frac{\partial m_\alpha^0}{\partial s}\right) + 4\pi M_s^2\delta_{\alpha z}\frac{\partial m_z^0}{\partial s}\frac{\partial m_\alpha^0}{\partial s}\right\}$$

$$\pi_m = 4\pi^2 L M_s^2\int ds\, s\left[m_z^0\left(\frac{\partial^2 m_z^0}{\partial s^2} + \frac{1}{s}\frac{\partial m_z^0}{\partial s}\right) + \left(\frac{\partial m_z^0}{\partial s}\right)^2\right]$$ – магнитостатический вклад в мягкий слой (толщина мала) жесткость, жесткий vs модель.

$$W_{ex} = A\int dV(\nabla m_\alpha)^2 = 8AL\int d^2\vec{s}\,\frac{1}{(1+w\bar{w})^2}\left[\frac{\partial w}{\partial z}\frac{\partial \bar{w}}{\partial \bar{z}} + \frac{\partial w}{\partial \bar{z}}\frac{\partial \bar{w}}{\partial z}\right]$$, где

Vortex $w(z,\bar{z}) = \begin{cases} f(z), & |f|\le 1 \text{ in core} \\ 1/f(z)\,/|f(z)|, & |f|>1, \text{ вне core} \end{cases}$, $m_x + im_y = \frac{2w}{1+w\bar{w}}$, $m_z = \frac{1-|w|^2}{1+|w|^2}$

$f(z)$ is analytic function on $z = x+iy$. Rigid soliton model $f(z) = \frac{e^{i\Phi_0}}{c}(z-s)$

$f(z) = \frac{1}{z}e^{i\Phi(x,y)}$, $\Phi(\vec{s}) = \arctg\frac{y}{x} + \Phi_0 = \varphi + \Phi_0$, $\Phi_0 = \pm\pi/2$ – vortex (Bl. Sk.)

$c = 0$, $\Phi_0 = (0,\pi)$ – radial vortex (weak skyrmion).

$c = s_c/R$ – soliton radius, $m_z(s_c) = 0$. Computer simulations Moriya $m_z(s) > 0$ $\forall s$, $s < R$, т.е. $s_c > R$, $c = s_c/R > 1$

Domain skyrmion – $w(z,\bar{z}) = f(z)$ и vortex conditions $|f|\le 1$, $|f| > 1$ не надо. и vortex с $s_c > R$

$\partial w/\partial\bar{z} = 0$, $\frac{\partial w}{\partial z} = \frac{\partial f}{\partial z}$ и $W_{ex} = 8AL\int d^2\vec{s}\,\frac{1}{(1+f\bar{f})^2}\left|\frac{\partial f}{\partial z}\right|^2$

$\partial f/\partial z = \frac{1}{c}e^{i\Phi_0}$, $W_{ex}(c,s) = 8AL\,\frac{1}{c^2}\int d^2\vec{s}\,\frac{1}{\left[1+\frac{1}{c^2}|z-s|^2\right]^2}$, $|z-s|^2 = (\vec{s}-\vec{s})^2$

$$W_{ex}(c,s) = 8AL\,c^2\int d^2\vec{s}\,\frac{1}{\left[c^2+s^2+s^2-2ss\cos\varphi\right]^2}$$ Интегрируя для $W_{ex}(c,s)$ можно получить 7/XI 2016 г. для skyrmion BP rigid model $\forall c, s$.

We $\varepsilon_{ex}(c,s)$ на забавит ос ѕима skyrmion (Bloch or Neel) с ѕим vortex (Bloch or radial vortex), а ε_m не зависи ос Φ_0.
Можно разширить $W_{ex}(c,s)$ по $s<1$. Такое разложение published in Gus - IEEE Mag. Lett. 2016

$$\varepsilon_{ex}(c) = -32\pi AL\frac{c^2}{(1+c^2)^3}, \quad W(\vec{s}) = W(0) + \frac{1}{2}\varepsilon_{ex}|s|^2.$$

В наших единицах $[\varepsilon] = erg/cm^2$, $\varepsilon_{ex}(c) = -32\pi\frac{AL}{R^2}\frac{c^2}{(1+c^2)^3}$ для rigid BP model.

Gyrovector $|G| = 4\pi\frac{M_s L}{\gamma}\frac{1}{(c^2+1)}$ см в Gus'2016.

Total $\varepsilon = \varepsilon_{ex} + \varepsilon_m + \varepsilon_H$ – function of the nanopillar parameters.

$$\frac{1}{R^2} = \frac{l_e^2}{R^2}\frac{1}{l_e^2}, \quad l_e^2 = A/2\pi M_s^2, \quad \frac{AL}{R^2}\cdot 32\pi = 32\pi LA\frac{l_e^2}{R^2}\frac{2\pi M_s^2}{A} = 64\pi^2\frac{l_e^2}{R^2}M_s^2 L$$

$$\varepsilon_{ex}(s_c) = -64\pi^2 M_s^2 L\left(\frac{l_e}{R}\right)^2\frac{c^2}{(1+c^2)^3}, \quad c = s_c/R.$$

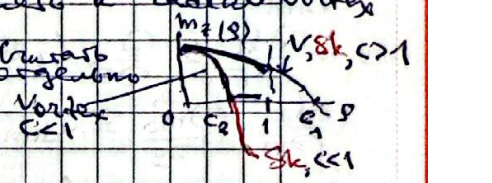
$$\varepsilon_{ex}+\varepsilon_m = 16\pi M_s^2 L\frac{c^2}{(1+c^2)^3}\left[2\pi(1-c^2) - \frac{1}{n^2}\right]$$ нормировано в Gus' IEEE Mag. Lett. 2016

$$n^2 = \frac{R^2}{L_e^2} = \frac{R^2}{2A/M_s^2} = \frac{R^2\cdot(4\pi)^{-1}}{(A/2\pi M_s^2)} = \frac{1}{4\pi}\frac{R^2}{l_e^2}, \quad \frac{1}{n^2} = \frac{l_e^2}{R^2}\cdot 4\pi$$

$$\varepsilon_{ex}+\varepsilon_m = 16\pi^2 M_s^2 L\frac{c^2}{(1+c^2)^3}\left[(1-c^2) - 4\frac{l_e^2}{R^2}\right], \quad \varepsilon_m = 16\pi^2 M_s^2 L\frac{c^2(1-c^2)}{(1+c^2)^3}.$$

Очевидно $\varepsilon_{ex}+\varepsilon_m < 0$ $\forall R$ или $c>1$, radial vortex is unstable without external stray field $\vec{H}_d$, $\varepsilon_H > 0$.

// $\varepsilon_{ex}(c)$, $\varepsilon_m(c)$ from Gus' IEEE Mag. Lett. 2016 derived for skyrmion in rigid BP ansatz $f(z) = \frac{1}{c}e^{i\Phi_0}(z-c)$ можно применить к radial vortex если $c = s_c/R > 1$. Если $c<1$, то область $c < s < 1$ (см. рис.) относится к vortex core.



(18/XI) $W(\vec{m}) = W[\Theta(s)]$ для нашего equilibrium s_c берем в $4\pi M_s^2 V$ units, см. 9/XI. ($s_c > 1$).

$$W = \frac{1}{2}\varepsilon|\vec{x}|^2, \quad \frac{W}{4\pi M_s^2 V} = \frac{1}{2}|\vec{x}|^2\left(\frac{\varepsilon c}{4\pi M_s^2 V}\right), \quad [\varepsilon] = \frac{erg}{cm^2}, \text{ reduced } [\tilde{\varepsilon}] = \frac{1}{cm^2}$$

$$\varepsilon_H = -\pi M_s L\int_0^1 ds\, s\left[\frac{\partial^2\vec{m}_0}{\partial s^2} + \frac{1}{s}\frac{\partial\vec{m}_0}{\partial s}\right]\cdot\vec{H}'(s), \quad \frac{\varepsilon_H}{4\pi M_s^2 V} = -\frac{\pi M_s L}{4\pi M_s^2 V}\int_0^1 ds\, s[\dots]\cdot\vec{H}' =$$

$$= -\frac{\pi M_s L}{4\pi M_s^2\pi R^2 L}\int_0^1 ds\, s[\dots]\cdot\vec{H}' = -\frac{1}{R^2}\int_0^1 ds\, s[\dots]\cdot\frac{\vec{H}'}{(4\pi M_s)}; \quad |G| = 4\pi M_s L\frac{1}{\gamma(1+s_c^2)}$$

$$\omega_G = \frac{\dot{\varepsilon}}{|G|} = \frac{1}{|G|}\left[\dot{\varepsilon}_{ex} + \dot{\varepsilon}_m + \dot{\varepsilon}_H\right] = \omega_G^0 + \frac{\dot{\varepsilon}_H}{|G|} = \omega_G^0 + \omega_G^H$$

$$\omega_G^0 = \omega_M\frac{c^2}{(1+c^2)^2}\left[(1-c^2) - 4\frac{l_e^2}{R^2}\right] = 16\pi^2 M_s^2 L\frac{c^2}{(1+c^2)^3}[\dots]\frac{\gamma}{4\pi M_s L}(1+c^2)$$ из Gus'2016.

В Gus'2016 опечатка в ω_G^0, Eq (11), надо $\frac{1}{(1+c^2)^2}$ умножить и не $\frac{1}{(1+c^2)}$.

$$\omega_G^H = \frac{\dot{\varepsilon}_H}{|G|} = \left(\frac{\dot{\varepsilon}_H}{4\pi M_s^2 V}\right)\frac{4\pi M_s^2 V}{4\pi M_s L}\gamma(1+s_c^2) = \tilde{\dot{\varepsilon}}_H(\text{reduced})\frac{4\pi M_s^2\pi R^2 L}{4\pi M_s L}\gamma(1+s_c^2) =$$

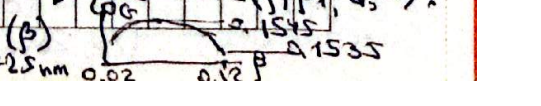
$$= \tilde{\dot{\varepsilon}}_H(1+s_c^2)\gamma\pi M_s R^2 = \frac{1}{4}(1+s_c^2)\omega_M R^2\tilde{\dot{\varepsilon}}_H, \quad R^2\tilde{\dot{\varepsilon}}_H = \dot{\varepsilon}'_H, \quad \tilde{\dot{\varepsilon}}_H = \frac{\dot{\varepsilon}_H}{4\pi M_s^2 V}$$

$$\dot{\varepsilon}'_H = -\int_0^1 ds\, s\left[\frac{\partial^2\vec{m}_0}{\partial s^2} + \frac{1}{s}\frac{\partial\vec{m}_0}{\partial s}\right]\cdot\vec{H}', \quad \omega_G^H = \frac{1}{4}(1+s_c^2)\omega_M\dot{\varepsilon}'_H.$$

s_c in units of R, $\dot{\varepsilon}'_H$ – dimensionless, $\vec{H}'$ in units $(4\pi M_s)$. Assume $s_c \geq 1$.

$\omega_G(c) = \omega_G^0(c) + \omega_G^H(c)$ надо построить в районе $c \in [1.00, 1.20]$ для разных L, t, R, d и найти в 2 раза: В $\omega_G(c)$ надо подставить $c_{eq} = c_0 = s_c(\beta, \beta_1, d, R)$.

Mathcad „Radial vortex - gyrofrequency.mcd" посчитал $\omega_G(\beta)$ $\beta = L/R$, $R = 25$ nm



$\omega_G(\beta)$ Слабая зависимость $\beta = L/R$, можно max при $\beta \approx 0{,}06$, $\omega_G \approx 0{,}13\,\omega_M$

$\omega_M/2\pi \approx 30$ GHz, $\omega_G/2\pi \approx 0{,}154 \cdot 30 = 4{,}62$ GHz (!), $R = 25$ nm, $\beta = 1{,}2$, $a \approx 0{,}02\,R$

$\frac{\omega_M}{2\pi} = \frac{\gamma}{2\pi} 4\pi M_s = 2{,}95 \cdot 4\pi \cdot 0{,}810\ \text{kG} \cdot \frac{\text{GHz}}{1\text{kG}} = 30{,}03$ GHz. $\frac{\omega_M}{2\pi} \approx 30$ GHz

(19/X) Hopfion stability? MS и публикация Figures. Hopfion magnetostatic energy.

Обсуждение с Гео о Figures, ссылка на Vortex GMR Artem's paper

Решение до re-submit в журнал (Nanoscale or APL Mat.).

Вопрос, local approximation для magnetostatic energy of L.hopfion

$W_m(a)$ т.е. $z = z'$ в $e^{-k|z-z'|}$, зависимость $W_m(a)$. Это важно для

больших R ($R/l_e > 50$), т.к. $W_m \sim \ln R$ при $R \to \infty$ и $W_m(a) \gg W_{ex}, W_{DMI}$.

Reduced hopfion energy (в $\pi\mu_0 M_s^2 l_e^3$ units) $\mathcal{E}(a,R) = \pi a I_{ex}(R/a) - \frac{1}{2} d a^2 I_D(R/a) + \underbrace{\frac{\pi}{2} a^2 I_m(R/a)}_{\text{dipolar en.}}$

$I_m(x) = 2\int_{-\infty}^{\infty} dz \int_0^x d\rho\,\rho\,[m_\rho(\rho,z)]^2$ in local approximation.

$\mathcal{E}_m(a)$ при больших $R/l_e \Rightarrow E_{ex}, E_{DMI}$ и выражается вид $E(a)$

Ибо при $R/l_{ex} = R/l_e \to \infty$ $\mathcal{E}_m(a,R)$ расходится как $\ln R$. 0 a

Magnetost. energy $E_{\rho\rho} = 2\pi^2 M_s^2 \int dk \int dz \int dz' e^{-k|z-z'|} J_\rho(z,k) J_\rho(z',k)$

$J_\rho(z,k) = \int d\rho\,\rho \frac{\partial J_0(k\rho)}{\partial \rho} m_\rho(\rho,z) = -k \int d\rho\,\rho\, J_1(k\rho)\, m_\rho(\rho,z)$

$E_{\rho\rho}(a) = 2\pi^2 M_s^2 a^3 \frac{1}{R} \int dk\, k^2 \int dz \int dz' e^{-\beta k|z-z'|} \frac{k^2}{R^2} R^2 \int d\rho\,\rho\, J_1(k\rho) m_\rho(\rho z) \times$

$\times R^2 \int d\rho\,\rho\, J_1(k\rho) m_\rho(\rho z')$. Здесь надо ρ, z в ед. hopf. radius a.

$F_{\rho\rho}(a) = 2\pi^2 M_s^2 a \int dk \int dz \int dz' e^{-k|z-z'|} \frac{k^2}{a^2} a^2 \int d\rho\,\rho\, J_1(k\rho) m_\rho(\rho z) a^2 \int d\rho\,\rho\, J_1(k\rho) m_\rho(\rho z') =$

$= 2\pi^2 M_s^2 a^3 \int dk \int dz \int dz' e^{-k|z-z'|} k^2 \int d\rho\,\rho\, m_\rho(\rho z)\, J_1(k\rho) \int d\rho'\rho'\, m_\rho(\rho' z') J_1(k\rho')$

Если $J_z(z) = \int_{-\infty}^{\infty} dz' e^{-k|z-z'|} = 2/k$, если local approx. $z \approx z'$ в $m_\rho(\rho z')$.

$E_{\rho\rho}(a,R) = 2\pi^2 M_s^2 a^3 \int dk \int dz \int dz' k^2 e^{k|z-z'|} I(kz) I(kz')$, $I(k,z) = \int_0^{R/a} d\rho\,\rho\, J_1(k\rho) m_\rho(\rho z)$.

$\mathcal{E}_m(a,R) = \frac{1}{4\pi M_s^2 l_e^3} E_{\rho\rho}(a,R) = \frac{\pi}{2}\left(\frac{a}{l_e}\right)^3 \int dk \int dz \int dz' k^2 e^{-k|z-z'|} I(kz) I(kz')$

Если local approximation $I(kz') \to I(kz)$, $\mathcal{E}_m(a,R) = \frac{\pi}{2} a^3 \int dk\, k^2 \int dz \int dz' \times$

$\times e^{-k|z-z'|} I^2(k,z) = \frac{\pi}{2} a^3 \int dk\, k^2 \int dz \frac{2}{k} I^2(kz) \left(\int dz\right)$. $(a/l_e) \to a$.

$\int dk\, k\, I^2(kz) = \int dk\, k \int d\rho\,\rho\, m_\rho(\rho z) J_1(k\rho) \int d\rho'\rho'\, m_\rho(\rho' z) J_1(k\rho') =$

$= \int d\rho\,\rho\, m_\rho(\rho z) \int d\rho'\rho'\, m_\rho(\rho' z) \int dk\, k\, J_1(k\rho) J_1(k\rho') = \int d\rho\,\rho\, m_\rho(\rho z) m_\rho(\rho' z) \frac{1}{\rho} \delta(\rho - \rho') =$

$= \int d\rho\,\rho\, m_\rho^2(\rho z)$ и $\mathcal{E}_m(a,R) = \frac{\pi}{2} a^3 \cdot 2 \int d\rho\,\rho\, dz\, m_\rho^2(\rho z) = \frac{\pi}{2} a^3 I_m\left(\frac{R}{a}\right)$ (local approx.),

где $I_m(x) = 2\int_{-\infty}^{\infty} dz \int_0^x d\rho\,\rho\, m_\rho^2(\rho z)$ как в manuscript draft, а a в ед. l_e.

Можно оставить local - $\mathcal{E}_m(a,R) = \pi a^3 \int dz \int dk\, k\, I^2(kz)$, $I(kz) = \int_0^{R/a} d\rho\,\rho\, J_1(k\rho) m_\rho(\rho z)$

$\mathcal{E}_m(a,R) = \frac{\pi}{2} a^3 I_m(R/a)$, $I_m(R/a) = \int dk \int dz \int dz' k^2 e^{-k|z-z'|} I(kz) I(kz')$

Именно $I_m(R/a)$ MathCad не считает, если не будет local approximation.

$I_m(x) = \int dk \int dz \int dz' k^2 e^{-k|z-z'|} \int_0^x d\rho\,\rho\, J_1(k\rho) m_\rho(\rho z) \int_0^x d\rho'\rho'\, J_1(k\rho') m_\rho(\rho' z')$.

$m_\rho = \sin\theta \cos\gamma = \sqrt{1 - m_z^2}\cos\gamma$. $\cos\gamma = \frac{1}{\sqrt{1+\operatorname{tg}^2\gamma}}$, $\operatorname{tg}\gamma = \operatorname{tg}\beta = \underset{(m=1)}{\frac{2az}{(\rho^2 + z^2 - a^2)}}$.

Escaneado con CamScanner

$1+tg^2\gamma = 1 + \frac{4a^2z^2}{(s^2+z^2-a^2)^2} = \frac{(s^2+z^2+a^2)^2 - 4s^2a^2}{(s^2+z^2-a^2)^2}$. $\gamma = m\beta = \beta$ для $m=1$ тороида.

$m_s(\gamma,\beta,\varphi) = \sin\theta\cos\beta = \sqrt{1-m_z^2(\gamma)}\cos\beta$ и не зависит от φ.

$s = a\frac{sh\gamma}{\tau}$, $z = a\frac{\sin\beta}{\tau}$, $\varphi = \varphi$, $\tau = ch\gamma - cos\beta$. $\vec{r}(\gamma,\beta,\varphi)$ – тороидальные коорд.

$\vec{r}(s,\varphi,z)$ – цилиндр. коорд.

$E_{ss} = -\pi m_s^2 \int ds\, s\, ds' \int_0 dz' \, g_{ss}(ss',zz')\, m_s(s,z)\, m_s(s',z')$, см. 9/2.

$g_{ss}(ss',zz') = -2\pi\frac{\partial^2}{\partial s\partial s'}\int dk\, e^{-k|z-z'|} J_0(ks) J_0(ks')$

$\frac{1}{|\vec r - \vec r'|} = \frac{1}{2\pi}\int d\vec k' \frac{1}{k} e^{-k|z-z'|} e^{i\vec k(\vec s - \vec s')}$, $\int_0^{2\pi} d\varphi\, e^{iks\cos\varphi} = 2\pi J_0(ks)$.

$J_0'(x) = -J_1(x)$.

$g_{ss}(ss',zz') = -2\pi\cdot\int dk\, k^2 e^{-k|z-z'|} J_1(ks) J_1(ks')$. $g_{\mu\rho}(ss',zz') = \int \frac{d\varphi}{2\pi} dk\, k\, G_{\mu\rho}(\vec r\vec r')$

$E_{ss} = -\frac{1}{2}\int m_s\, dV\, m_s H_m^s(\vec r) = -\frac{1}{2} m_s \int dV\, m_s(ss) \int_s dV'\, G_{ss}(\vec r\vec r')\, m_s(\vec r') =$

$= -\frac{1}{2} m_s^2 \int dV \int dV'\, G_{ss}(\vec r\vec r')\, m_s(\vec r)\, m_s(\vec r')$. $m_s(\vec r') = m_s(s,z) = m_s(\gamma,\beta)$.

$\int d^3\vec r = \int dV = \int ds\, s \int d\varphi \int dz = \int d\gamma \int d\varphi \int d\beta \frac{a^3}{\tau^3} sh\gamma$ в тороид. координатах.

$E_{ss} = -\frac{1}{2} m_s^2 \int d\gamma \int d\varphi \int d\beta \frac{a^3}{\tau^3} sh\gamma \int d\gamma' \int d\varphi' \int d\beta' \frac{a^3}{\tau'^3} sh\gamma' G_{ss}(\vec r\vec r')\, m_s(\gamma\beta)\, m_s(\gamma'\beta')$

– аксиально симм. тороид, $\tau = ch\gamma - cos\beta$. $\int_0^{2\pi} d\varphi \int_0^{2\pi} d\varphi\, G_{ss}(\vec r\vec r') = 2\pi g_{ss}(ss',zz')$

$E_{ss} = -\pi m_s^2 a^6 \int d\gamma \int d\beta \frac{sh\gamma}{\tau^3} \int d\gamma' \int d\beta' \frac{sh\gamma'}{\tau'^3} g_{ss}(ss',zz')\, m_s(\gamma\beta) m_s(\gamma'\beta')$.

Можно ли $g_{ss}(ss',zz')$ записать как функцию от тороид. координат (γ,β)?

Формально можно $g_{ss}(\gamma\gamma',\beta\beta') = -2\pi\int dk\, k^2 e^{-ka\left|\frac{\sin\beta}{\tau} - \frac{sh\beta'}{\tau'}\right|} J_0\left(ka\frac{sh\gamma}{\tau}\right) J_0\left(ka\frac{sh\gamma'}{\tau'}\right)$.

$ka \to k$. $E_{ss}(a) = -\pi m_s^2 a^3 \int d\gamma \int d\beta \frac{sh\gamma}{\tau}\int d\gamma' \int d\beta' \frac{sh\gamma'}{\tau'} g_{ss}(\gamma\gamma',\beta\beta') \sin\theta(\gamma)\sin\theta(\gamma')\cos\beta\cos\beta'$

Wik: $(\gamma,\beta,\varphi) \to (\tau,\theta,\varphi)$.

$\nabla^2\Phi = 0$ допуск. разделение переменных. $\Phi = U\sqrt{ch\gamma - cos\beta}$, $U(\gamma,\beta,\varphi) \sim e^{i\nu\beta} e^{im\varphi} \times P_{\nu-1/2}^{m}(ch\gamma)$.

$\frac{1}{|\vec r|}$ удовл. уравн. Лапласа $\nabla^2\left(\frac{1}{r}\right) = -4\pi\delta(\vec r)$, $\frac{1}{|\vec r|}$ можно $\sim P_{\nu-1/2}^{m}(ch\gamma)$ (?)

Записан ряд по тороид. гармоникам $U_{m\nu}(\gamma,\beta,\varphi) \sim e^{i\nu\beta} e^{im\varphi} P_{\nu-\frac12}^{m}(ch\gamma)$.

$tg^2\beta = \frac{4a^2z^2}{(s^2+z^2-a^2)^2} \to 0$ при $s\to\infty$, $\beta(s=\infty) = 0$, $\cos\beta \approx 1$, $\sin\beta \approx 0$

Тороид $m_s = \sin\theta\cos\beta \approx \sin\theta$, $m_\varphi = \sin\theta\sin\beta \to 0$ при $s\to\infty$.

Зависимость от $W_m(R) \to \infty$ при $R\to\infty$ расходимость сингулярности local approximation в E_{ss}? Или расходимость есть и для тороид $W_m(R)$?

(20/IV) Численная тороид stability MS. Тороид. обл. dot

(21/IV) Reading: Jackson (Library Cent.), Otegui Eli travel report Madrid. C.N. 6 и Carrefour market – Toledo. Buy.

(22/IV) Тороидальная тороид магнитостатическая энергия, E_m (см. 8-9/IV).

$div\vec m = \frac{1}{s}\sin\theta\cos\beta + \cos\theta\cos\beta\frac{\partial\theta}{\partial s} - \sin\theta\sin\beta\frac{\partial\beta}{\partial s} + \frac{\partial m_z}{\partial z}$.

$m_z = \cos\theta = f(\gamma)$, $m_s = \sin\theta\cos\beta$, $m_\varphi = \sin\theta\sin\beta$. $m_z = 1 - \frac{8s^2a^2}{(a^2+s^2+z^2)^2}$

$m_z = 1 - 2th^2\gamma$. $\gamma = \operatorname{arcth}\left[\frac{2as}{s^2+z^2+a^2}\right]$, $\beta = \operatorname{arctg}\left[\frac{2az}{s^2+z^2-a^2}\right]$

$\sin\theta = \frac{4sa}{(s^2+z^2+a^2)}\sqrt{1 - \frac{4s^2a^2}{(s^2+z^2+a^2)^2}}$. $\sin^2\theta = 1 - m_z^2 = 4th^2\gamma(1-th^2\gamma) = 4\frac{th^2\gamma}{ch^2\gamma}$

$\operatorname{div}\vec{m}(\rho,z) = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho m_\rho) + \frac{\partial m_z}{\partial z} = \frac{1}{\rho}m_\rho + \frac{\partial m_\rho}{\partial\rho} + \frac{\partial m_z}{\partial z}$ | Нужен $\rho\to\infty$.

$\frac{1}{\rho}m_\rho(\rho\to\infty) = \frac{1}{\rho}\sin\theta\cos\beta \approx \frac{1}{\rho}\frac{4a}{\rho}\cos\beta$, $1+\operatorname{tg}^2\beta \equiv 1+\frac{4a^2z^2}{\rho^4}$

$\cos\beta = \frac{1}{\sqrt{1+\operatorname{tg}^2\beta}} = \frac{1}{\sqrt{1+\frac{4a^2z^2}{\rho^4}}} \approx 1 + O(1/\rho^4)$. $\frac{1}{\rho}m_\rho(\rho\to\infty) \approx \frac{4a}{\rho^2}$.

$\frac{\partial m_\rho}{\partial\rho} = \frac{\partial}{\partial\rho}(\sin\theta\cos\beta) = \cos\beta\frac{\partial\sin\theta}{\partial\rho} + \sin\theta\frac{\partial\cos\beta}{\partial\rho} \approx \frac{\partial}{\partial\rho}\left(\frac{4a}{\rho}\right) +$

$+\sin\theta\,[0+O(1/\rho^5)] \approx -\frac{4a}{\rho^2}$. $\frac{\partial m_z}{\partial z} = \frac{\partial}{\partial z}\left[1-\frac{8\rho^2a^2}{(a^2+\rho^2+z^2)^2}\right] =$

$= -8\rho^2a^2\frac{\partial}{\partial z}\frac{1}{[a^2+\rho^2+z^2]^2} = -8\rho^2a^2\frac{(-2)}{(a^2+\rho^2+z^2)^3}\cdot 2z$.

$\frac{\partial m_z}{\partial z} = \frac{32\rho^2a^2z}{(a^2+\rho^2+z^2)^3} \underset{\rho\to\infty}{\approx} \frac{32\rho^2a^2z}{\rho^6} = \frac{32a^2z}{\rho^4}$.

$m_\rho = \sin\theta\cos\beta = \frac{4a}{\rho}\sqrt{1+\frac{4a^2}{\rho^2}}\left[1+O(1/\rho^4)\right] = \frac{4a}{\rho}\left(1-\frac{2a^2}{\rho^2}\right)$

$\rho m_\rho = 4a\left(1-\frac{2a^2}{\rho^2}\right) \Rightarrow \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho m_\rho) = \frac{1}{\rho}\frac{\partial}{\partial\rho}4a\left(1-\frac{2a^2}{\rho^2}\right) = \frac{4a}{\rho}\left[0-2a^2\frac{(-2)}{\rho^3}\right]$

$\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho m_\rho) \equiv 16\frac{a^3}{\rho^4}$, $\operatorname{div}\vec{m}(\rho\to\infty) \approx \frac{16a^3}{\rho^4} + \frac{32a^2z}{\rho^4} \sim \frac{1}{\rho^4}$. Можно ли $\rho\gg z$?

$W_m^{vol} = \frac{1}{2}M_s^2\int dV dV' \frac{\operatorname{div}\vec{m}(\vec{r})\operatorname{div}\vec{m}(\vec{r}')}{|\vec{r}-\vec{r}'|}$. $dV = \rho d\rho d\varphi dz$.

$\operatorname{div}\vec{m}(\rho\to\infty) \sim 1/\rho^4$ хочется знать где сходимость W_m^{vol}, но есть еще W_m^S и W_m^{SV}. // ФГ – метод, где учитывают all m. charges.

$E_{\rho\rho} = -\pi M_s^2\int d\rho\rho\int d\rho'\rho'\int dz\int dz' g_{\rho\rho}(\rho\rho',zz')\,m_\rho(\rho z)\,m_\rho(\rho' z')$

Standard $g_{\rho\rho}(\rho\rho',zz') = -2\pi\frac{\partial^2}{\partial\rho\partial\rho'}\int dk e^{-k|z-z'|}J_0(k\rho)J_0(k\rho')$ надо отнять

ρ – нужен для члена $\Gamma, k\cdot(k\rho)\to\infty$ не нужен при $k=0$.

$\Gamma(\rho,\rho') = \int_0^\infty dk e^{-k|z-z'|}J_0(k\rho)J_0(k\rho') = \frac{2}{\pi\sqrt{\rho\rho'}}K(\kappa)$, $\kappa^2 = \frac{4\rho\rho'}{(\rho+\rho')^2+(z-z')^2}$.

$F_{\rho\rho} = -\pi M_s^2\int d\rho\rho\int d\rho'\rho'\int dz\int dz' m_\rho(\rho z)\,m_\rho(\rho' z')\,(-2\pi)\frac{\partial^2}{\partial\rho\partial\rho'}\Gamma(\rho\rho',zz')$

$F_{\rho\rho} = 2\pi^2M_s^2\int d\rho\rho\int d\rho'\rho'\int dz\int dz' m_\rho(\rho z)\,m_\rho(\rho' z')\frac{\partial^2}{\partial\rho\partial\rho'}\Gamma(\rho\rho',zz')$

$\int d\rho\rho\frac{\partial\Gamma}{\partial\rho}(\rho\rho')\cdot m_\rho(\rho) = \int d\rho\frac{\partial\Gamma}{\partial\rho}\cdot\underbrace{\rho m_\rho(\rho)}_{u} = \left|dv = \frac{\partial\Gamma}{\partial\rho}d\rho,\ v=\Gamma\right| =$

$= \rho m_\rho\Gamma(\rho)\Big|_0^R - \int\Gamma(\rho)\,d(\rho m_\rho)$. $d(\rho m_\rho) = m_\rho + \rho\frac{\partial m_\rho}{\partial\rho}$

$\int d\rho\rho\frac{\partial\Gamma}{\partial\rho}m_\rho(\rho) = \rho m_\rho\Gamma(\rho)\Big|_0^R - \int\Gamma(\rho)m_\rho d\rho - \int\Gamma(\rho)\rho\frac{\partial m_\rho}{\partial\rho}d\rho$, ρ_z – в ед. a.

$E_{\rho\rho} = 2\pi^2M_s^2a^3\int d\rho\rho\int d\rho'\rho'\int dz\int dz' m_\rho m_\rho(\cdot)\frac{\partial^2}{\partial\rho\partial\rho'}\Gamma(\rho\rho',zz')$. Делим на $4\pi M_s^2\ell_e^3$.

$\varepsilon_m(a,R) = \frac{E_{\rho\rho}}{4\pi M_s^2\ell_e^3} = \frac{\pi}{2}a^3\int_0^{R/a}d\rho\rho\int_0^{R/a}d\rho'\rho'\int dz\int dz' m_\rho(\rho z)m_\rho(\rho' z')\frac{\partial^2}{\partial\rho\partial\rho'}\Gamma(\rho\rho',zz')$,

нужно ли $\rho\to\infty$ если $\int dk(\dots)$ от $1/z$? $E_m(a,R) = \frac{\pi}{2}a^3J_m(R/a)$.

$J_m(x) = \int_0^x d\rho\rho\int_0^x d\rho'\rho'\int dz\int dz' m_\rho(\rho z)m_\rho(\rho' z')\frac{\partial^2}{\partial\rho\partial\rho'}\Gamma(\rho\rho',zz')$

$m_\rho(\rho,z) = \sin\theta(\rho,z)\cos\beta = \sin\theta(\rho z)\frac{1}{\sqrt{1+\operatorname{tg}^2\beta}}$. $\beta = \beta(\rho,z)$ $K(\alpha) = \int_0^{\pi/2}\frac{d\varphi}{\sqrt{1-\kappa^2\sin^2\varphi}}$

нужно ли $J_m(x)\sim K(x)$. Надо посчитать.

$J_m(x) = \int dk\int dz\int dz' k^2e^{-k|z-z'|}I(k,z)I(kz')$. $I(kz) = \int_0^x d\rho\rho\,J_1(k\rho)m_\rho(\rho z)$.

$E_{cm} = \int_{A(k)}dz\int dz' F(zz')e^{-k|z-z'|} = \int dz\left[\int_0^z dz' F e^{-k(z-z')} + \int_z^1 dz' F e^{-k(z'-z)}\right] =$

$= \int dz \left[e^{-kz} \int_0^z dz' F(zz') e^{kz'} + e^{kz} \int_z^2 dz' F(zz') e^{-kz'} \right]$ $F(zz') = \int dk k^2 I(kz) I(kz')$

$F(zz')$ – плавная по z, плавная по z'. $F(zz') = F(z'z)$.

$\int_0^2 dz F(zz') e^{kz'} = |z' = -z| = \int_0^{-2} dz F(z,-z) e^{-kz}$ плавно.

Ищем энергию E_{ss} ос 19/11. MCad считает при $x = R/a = 0.1$, не считает $R/a = 10$ и очень считает долго $x = R/a = 1$.

E_{ss} вычисляем через $I(k,z,x) = \int_0^x ds_s J_1(ks) m_s(s,z)$, $x = R/a$.

(23/11) Найдем $I(k,z,x)$ by Simpson's or Gauss' quadrature.

Simpson: $\int_a^b f(x)dx = \frac{1}{3}h \left[f(x_0) + 4 \sum_{j=1}^{N/2} f(x_{2j-1}) + 2 \sum_{j=1}^{N/2-1} f(x_{2j}) + f(x_N) \right]$

Gauss (used for the integral eqs): $\int_a^b f(x)dx = h \left[\frac{1}{2} f(x_0) + \sum_{j=1}^{N-1} f(x_j) + \frac{1}{2} f(x_N) \right]$

$h = \frac{b-a}{N}$ – шаг. $x_j = a + jh$, $x_0 = a$, $x_N = b$.

MCad считает $I(k,z,x)$ очень быстро. Нет смысла не использовать Simpson/Gauss approx.

$E_{ss} \to E_m(a,R) = \frac{\pi}{2} a^3 J_m(R/a)$. $J_m(R/a) = \int dk \int dz \int dz' e^{-k|z-z'|} k^2 I(kz) I(kz')$

$I(k,z) = I(k,z,x) = \int_0^x ds_s J_1(ks) m_s(s,z)$, $x = R/a$.

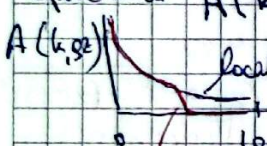
$J_m(x) = \int dk \int dz \int dz' k^2 e^{-k|z-z'|} I(k,z,x) I(k,z',x)$, $\tilde{J}_z(k,z) = \int_{-\infty}^{\infty} dz' e^{-k|z-z'|} = \frac{2}{k}$.

$J_z = \int dz\, dz' e^{-k|z-z'|} m_s(s,z) m_s(s',z') \approx$ |local appr.| $\leq$

$(ss'k) = \int dz \int dz' e^{-k|z-z'|} m_s(sz) m_s(s'z') = \int dz\, m_s(sz) m_s(s'z') \int dz' e^{-k|z-z'|} =$

$= \frac{2}{k} \int dz\, m_s(sz) m_s(s'z)$, $\int dz' e^{-k|z-z'|} m_s(sz') \simeq m_s(sz) \int_{-\infty}^{\infty} dz' e^{-k|z-z'|} \leq \frac{2}{k} m_s(sz)$

Сравним $A(k,s,z) = \int dz' e^{-k|z-z'|} m_s(sz')$ с $A'(k,s,z) = \frac{2}{k} m_s(s,z)$ – local appr.



local $A'(k) \approx A(k)$ при малых k, но $A'(k)$ завышено при больших k. Т.е. local approximation завышает энергию E_{ss} за счет области больших k (нет, see below, A_1 – неверно)

Насколько точно $\tilde{J}_z(kz) = \int_{-z_m}^{+z_m} dz' e^{-k|z-z'|} = \frac{2}{k}$ при конечных z_m?

$J_z(k,z,z_m) = \int_{-z_m}^{+z_m} dz' e^{-k|z-z'|} = |\text{см. } g|_z^z| = e^{-kz} \frac{1}{k} e^{kz} \Big|_{-z_m}^{z} + e^{kz} \left(-\frac{1}{k}\right) e^{-kz'} \Big|_z^{z_m} =$

$= e^{-kz} \frac{1}{k} \left[e^{kz} - e^{-kz_m} \right] - \frac{1}{k} e^{kz} \left[e^{-kz_m} - e^{-kz} \right] = \frac{1}{k} \left[1 - e^{-k(z+z_m)} - e^{k(z-z_m)} + \frac{1}{k} \right] =$

$= \frac{2}{k} - \frac{1}{k} e^{-kz_m} \left(e^{-kz} + e^{+kz} \right) = \frac{2}{k} - \frac{2}{k} e^{-kz_m} \operatorname{ch}(kz) = \frac{2}{k} \left[1 - e^{-kz_m} \operatorname{ch}(kz) \right]$

$J_z(k,z,z_m) = \int_{-z_m}^{z_m} dz' e^{-k|z-z'|} = \frac{2}{k} \left[1 - e^{-kz_m} \operatorname{ch}(kz) \right]$

Странно, но MathCad неверно считает $\int_{-z_m}^{z_m} dz' e^{-k|z-z'|} m(sz')$, $\int_{-z_m}^{z_m} dz' e^{-k|z-z'|}$ при больших z_m (50, 100, 300, ...). Аналитич. $J_z(kz z_m) = \frac{2}{k} \left[1 - e^{-kz_m} \operatorname{ch} kz \right]$ гораздо точнее, чем численное $\int_{-z_m}^{z_m} dz' e^{-k|z-z'|} \cdot A(k,s,z)$ счет. Вернусь к этому

// Используем local approx., но будем использовать $J_z(kz z_m) = \frac{2}{k} \times$

$\times \left[1 - e^{-kz_m} \operatorname{ch} kz \right]$ вместо $\tilde{J}_z(kz) = 2/k$ от J/z.

$J_m(x) = \int dk \int dz \int dz' k^2 e^{-k|z-z'|} I(kz) I(kz') =$ |local appr., $I(kz') = I(kz)$| $=$

$= \int dk \int dz \int dz' k^2 e^{-k|z-z'|} I(kz) I(kz) \neq \int dk \int dz\, k^2 \int e^{-k|z-z'|} dz' I(kz) I(kz) =$

$= \int dk\, J_2 k^2 \frac{2}{k}\left[1-e^{-kz_m}ch(kz)\right] I(kz) I(kz) = \int dk \int dz\, k^2 \frac{2}{k}\left[1-e^{-kz_m}ch(kz)\right] \times$

$\times \int_0^x d\rho\rho\, J_1(k\rho) m_\rho(\rho z) \cdot \int_0^x d\rho'\rho'\, J_1(k\rho') m_\rho(\rho' z) = 2\int dk\, k \int dz \int d\rho\rho\, J_1(k\rho) m_\rho \int d\rho'\rho' J_1(k\rho') m_\rho(\rho' z)$

$- 2\int dk \int dz\, k\, e^{-kz_m} ch(kz) \int d\rho\rho J_1(k\rho)\, m_\rho(\rho z) \int d\rho'\rho' J_1(k\rho') m_\rho(\rho' z) =$

$= \int_0^\infty dk\, k\, J_1(k\rho) J_1(k\rho') = \frac{1}{\rho}\delta(\rho-\rho') \Big| = 2\int dz \int_0^x d\rho\rho\, m_\rho^2(\rho, z) -$

$- 2\int dk \int dz\, k e^{-kz_m} ch(kz) \int d\rho\rho J_1(k\rho) m_\rho(\rho z) \int d\rho'\rho' J_1(k\rho') m_\rho(\rho' z)$

Рассм. $J_2(k z, z_m) = \int_{-z_m}^{z_m} dz' e^{-k|z-z'|} = \frac{2}{k}\left[1-e^{-kz_m}ch(kz)\right] \to (2/k)$ при $\underset{z_m\to\infty}{z\le z_m}$

т.е. $\int_{-z_m}^{z_m} dz (\dots)$ и берем $z = z_m$, получим $J_2(k, z_m) \sim 2/k \ \forall z$.

Т.е., работа в рамках local approximation, и применен local appr. правильно, поскольку $J_2(k z_m) = J_2(kz) = 2/k$.

Тогда $J_m(x) = 2\int dk \int dz\, k \left[1-e^{-kz_m}ch(kz)\right] \int_0^x d\rho\rho J_1(k\rho) m_\rho(\rho z) \int_0^x d\rho'\rho' J_1(k\rho') m_\rho(\rho' z)$

$B(k z_m \rho\rho') = \int_{-z_m}^{z_m} dz \left[1-e^{-kz_m}ch(kz)\right] m_\rho(\rho z) m_\rho(\rho' z)$

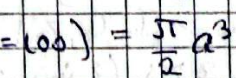
$J_m(x) = 2\int dk\, k \int_0^x d\rho\rho J_1(k\rho) \int_0^x d\rho'\rho' J_1(k\rho') B(k z_m \rho\rho')$ или

$J_m(x) = 2\int_0^\infty dk \int_{-z_m}^{z_m} dz\, k \left[1-e^{-kz_m}ch(kz)\right] I^2(k, z, x)$, $I(k, z, x) = \int_0^x d\rho\rho\, J_1(k\rho) m_\rho(\rho z)$.

Мсад считает $J_m(x)$ также при малых $z_m \le 5$, т.е. для cylindrical dot ($x = R/a$, R, $z_m(L)$ are in units of $a \approx 5$ nm). $\leftarrow$ Ca: 25/27

$J_m(x)$ меняется примерно, чем $J_m(x)$-old с $J_2 = 2/k$.

При $x = 50$, $z_m = 5$ $J_m = 288.4$, $J_{m\text{-old}} = 733.0$. Min $E_m(a)$ при $a \approx 0.9R$, $x \approx 1$.

$E_m(a, R=100) = \frac{\pi}{2} a^3 J_m(R/a)$  gaas min $(z_m = \frac{L}{2} = 5)$ $R = 100$.

Но $\frac{L}{2} = 5$, $L = 10$, $L = 50$ nm — это thin cyl. dot, $R \approx 500$ nm.

Предел thin dot $\frac{L}{R} \to 0$ — $\vec{m}(\rho, z)$ слабо зависит от толщины. Но проверим. $W_m = \int dW\, 2\pi M_s^2 m_z^2$, $m_z(\rho, z) = 1 - \frac{8\rho^2 a^2}{(a^2+\rho^2+z^2)^2}$

$m_z(\rho, z) = 1 - \frac{8\rho^2}{(1+\rho^2+z^2)^2}$ — в cgs, $a = 1$ ρ, z. Угол $z=0$ при $L/R = \frac{2z_m}{R} \to 0$.

$W_m = 2\pi M_s^2 \int dW\, m_z^2$. $\operatorname{tg}\beta = \frac{2az}{(\rho^2+z^2-a^2)} \to 0$ при $z=0$, $\gamma = \beta$.

При $\rho \to 0$, $m_z = \cos\theta$, $m_\rho = \sin\theta$, $m_\varphi = 0$. $m_z^2 = 1 - m_\rho^2$

$W_m = 2\pi M_s^2 \int dW \left[1 - m_\rho^2\right] = 2\pi M_s^2 V - 2\pi M_s^2 \int dW\, m_\rho^2$, $m_\rho^2 = \sin^2\theta = \left|z=0\right| =$

$= \frac{4\rho}{(\rho^2+1)\sqrt{1+\frac{4\rho^2}{(\rho^2+1)^2}}} = \frac{4\rho}{(\rho^2+1)^2}\sqrt{(\rho^2+1)^2 - 4\rho^2} = \frac{4\rho}{(\rho^2+1)^2}(\rho^2-1)$.

$\int dW\, m_\rho^2 = \int d\rho\rho \int dz \int dz \frac{4\rho}{(\rho^2+1)^2}(\rho^2-1) = 2\pi L \cdot 4\int_0^{R/a} d\rho\rho \frac{\rho(\rho^2-1)}{(\rho^2+1)^2}$ расходится как $\propto R$ при $R \to \infty$. Нормировка для $\pi M_s^2 k^2$, а не для $\pi M_s^2 V$.

(24/xi) Тартаковская написала, что она выиграла Opus-project (420 k€ initial proposal), Начало Jan. 1, 24 → May 1, 2024. Дата 3/мм. Go KPLM. Обещана travel money ≈ 4 k€. Тема: Curvilinear Geometry 4 years

Local magnetostatics $E_m(a, R)$ считается max при $z_m = 7$ всего. Можно $\int dz (\dots)$ по трапеции Simpson или Gauss? [N. Carrefour, Cor. Marketing

25/IV

Gauss: $\int_a^b f(z)dz = h\left[\frac{1}{2}f(z_0) + \sum_{j=1}^{N-1} f(z_j) + \frac{1}{2}f(z_N)\right]$, $z_j = a + jh$, $h = \frac{b-a}{N}$, $j = 0, 1, \dots N$

Local appr. $\rightarrow J_m(x) = 2\int dk \int_{-z_m}^{z_m} dz\, k\left[1 - e^{-kz_m}\mathrm{ch}(kz)\right] I^2(kz, x)$, $I(kzx) = \int_0^x t^{m+1} J_1(kt)\, t^m g(t)$.

$J_G(k,x) = \int_{-z_m}^{z_m} dz \left[1 - e^{-kz_m}\mathrm{ch}(kz)\right] I^2(kzx)$ — more nearly for 300€

$f(z) = \left[1 - e^{-kz_m}\mathrm{ch}(kz)\right] I^2(kzx)$, $a = -z_m$, $b = z_m$, $h = \frac{2z_m}{N}$

$J_z(k,x) = \int_{-z_m}^{z_m} dz\, f(z,k,x)$ — через квадратуру Гаусса.

$J_G(k,x) = h\sum_{j=0}^{N} A_j f(z_j, k, x)$, $f(z)$ is even function of z.

Надо будет $J_z(k,z,z_m) = \frac{1}{k}\left[2 - e^{-k(z+z_m)} - e^{+k(z-z_m)}\right]$, т.к. MCad $\mathrm{ch}(kz)$ при больших (kz) не считает, находит число $> 10^{307}$.

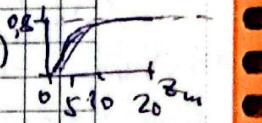
Зачем ей $J_z(k,z,z_m) = \frac{2}{k}\left[1 - e^{-kz_m}\mathrm{ch}(kz)\right]$ Вдруг при каких-то z_m MCad считать больше z_m, $z_m > 7$ 23/IV. // Численно считает

$J_G(k,x) = \int_{-z_m}^{z_m} dz\, \frac{1}{2^0}\left[1 - e^{-kz_m}\mathrm{ch}(kz)\right] I^2(kzx) = \int_{-z_m}^{z_m} dz\, \frac{1}{2}\left[2 - e^{-k(z+z_m)} - e^{k(z-z_m)}\right] I^2$

уже не надо.

MCad считает $J_m(x, z_m) = 2\int dk \int_{-z_m}^{z_m} dz\, k\,\frac{1}{2}\left[2 - e^{-k(z+z_m)} - e^{k(z-z_m)}\right] I^2(kzx)$ — Local

За разумное время. $J_m(x, z_m)$ и old $J_m(x, z_m) = 2\int dk \int_{-z_m}^{z_m} dz\, I^2(kzx)$ практически совпадают (ошибка $< 1\%$) при $z_m > 5$. $J_m(x=1, z_m)$ быстро насыщается с ростом z_m (перерезов от dot и wire) при $z_m \geq 5$. При $z_m < 5$ old $J_m(x, z_m)$ немного $> J_m(x, z_m)$. Физика: быстрый спад $m_s(s, z)$ при росте z при $z > 1$ (z in units of hopf. radius a'') — good hopfion localization вдоль оси Oz.



Note: z в единицах a'', $x = R/a$, a'' в единицах l_e. z_m в единицах a''.

Min $E_m(a, R)$ при $a_0 \approx 0.5R$. При $R \approx 100$, $a_0 \approx 100$. $z_m/a_0 = 1$

Соответствует $L = 2z_m = 2a_0 = 2 \cdot 100 \cdot l_e \approx 200.5$ нм < 1000 нм, это wire.

$z_m = 1$ соответ $L \approx 1000$ нм при $R \approx a = a_0 = 100$, в абс. единицах $R \approx 100 \approx a_0$, $a_0 = 100 \rightarrow 100 \cdot l_e = 500$ нм, т.к. $l_e \approx 5$ нм.

$z_m = 5$, $L = 2z_m \cdot a_0 = 2 \cdot 5 \cdot 100 \cdot l_e = 1000 \cdot l_e \approx 5000$ нм при $R \approx a_0 = 100$

т.е. при $R = 100$ (в ед. l_e), $R = 500$ нм, $\begin{cases} z_m = 1 \rightarrow L = 1000 \text{ нм} \\ z_m = 5 \rightarrow L = 5000 \text{ нм} \end{cases}$ $R = 100 l_e \approx 500$ нм.

при $R = 10$, $R = 50$ нм, $a_0 \approx 50$ нм $\begin{cases} z_m = 1 \rightarrow L = 100 \text{ нм} \\ z_m = 5 \rightarrow L = 500 \text{ нм} \end{cases}$

$z_m = 1$, thick dot $< \frac{L}{R} = 2$.

$z_m \geq 5$, $\approx$ wire $< L/R \geq 10$. // $z_m < 1$ ($z/a < 1$) нет смысла расчет. т.к. тогда toroidal hopfion не помещается в dot, и не-hopfion anzatz (vortex).

// $z_m > 5$ предел wire, $E_m(a, R) \sim J_m(\frac{R}{a}, z_m)$ перестает

Зависит от z_m. Но если $z_m = \frac{L}{2a} < 5$, то зависимость $E_m(a, R)$ от z_m (и от L есть). Надо при $\int_{-z_m}^{z_m} dz(\dots)$ явно учесть зависимость от a'', т.е $\int_{-L/2a}^{+L/2a} dz(\dots) = \int_0^{L/2a} dz(\dots)$, и $E_m(a, R) \rightarrow E_m(a, R, L)$. Не надо. — Зоя

(26/XI) Брем $z_m = \frac{L}{2a} = \frac{\beta}{2} \times$ и/ли $\int_{-z_m}^{z_m} dz\,(m)$ и E_{ex}, E_{DMI}, E_m.
Качественно ничего не изменилось; z_m
$a_0 \approx 0{,}8R$. E_m уменьшается и пропорционально $\min_a E(a)$ $\forall R, L$ уменьшается.

$$l_e = \sqrt{\frac{A}{2\pi M_s^2}} = \sqrt{\frac{1{,}1\cdot 10^{-6}\ \text{erg/cm}}{2\pi (0{,}2_4\cdot 10^3)^2\,G^2}} = \sqrt{\frac{1{,}1}{2\pi\cdot(0{,}8)^2}}\cdot 10\ \text{nm} = 4{,}98\ \text{nm} \approx 5{,}0\ \text{nm}.$$

$A = 1{,}1\cdot 10^{-6}$ erg/cm, $M_S = 830 - 840$ G, далее $l_e \simeq 5{,}0$ nm. $M_S = 837$ G.

$4\pi M_s^2 l_e^3 = 4\pi (0{,}837)^2\cdot 10^8\,G^2\,(5{,}0\ \text{nm})^3 = 4\pi(0{,}837)^2 10^6\ \frac{\text{erg}}{\text{cm}^3}\cdot 5^3\cdot 10^{-21}\,\text{cm}^3 =$
$= 1{,}1\cdot 10^3\cdot 10^{-15}$ erg $= 1{,}1\cdot 10^{-12}$ erg, $k_B T = 4{,}2\cdot 10^{-14}$ erg при $T = 300$ K.
$4\pi M_s^2 l_e^3 / k_B T = 1{,}1\cdot 10^{-12}\ \text{erg} / (4{,}2\cdot 10^{-14}\ \text{erg}) = 26{,}2$.

$E_m(a,R) = \frac{\pi}{2} a^3 J_m\left(\frac{R}{a},\beta\right)$. Даже при $R = 20$ $\left(\frac{R}{l_e} = 20\right)$, $E_m, E_{tot} \sim 10^4$
Barrier ΔE $(a = a_0 \to a = 0)$ for hopfion to SD transition $\Delta E \sim 10^4$, or
in abs. units $\Delta E = (4\pi M_s^2 l_e^3)\cdot 10^4 = 10^{-12}\,\text{erg}\cdot 10^4 = 10^{-8}$ erg, $\frac{\Delta E}{k_B T} = 26\cdot 10^4$ is giant (!)
Если нет ошибки, то hopfions в cylindrical bars/wires
очень глубокий min $E(a)$, $\Delta E(\text{hopf.} \to \text{SD}) / k_B T \times 10^4 \gg 1$.
ΔE(hopf. → SD) нет смысла считать, насколько barriers ΔE велики.
$a_0(R=20, \beta=2) = 18{,}30$, $a_0/R = 0{,}915$.
MCad очень медленно считает $a_0(R,\beta,d)$ из $\frac{\partial E_{tot}}{\partial a} = 0$.
Change переменных

(27/XI) $E_m(a,R) = \frac{1}{4\pi M_s^2 l_e^3} E_{ss}(a,R) = \frac{\pi}{2}\left(\frac{a}{l_e}\right)^3 \int dk \int dz \int dz'\, k^2 e^{-k|z-z'|} I(k,z) I(k,z')$
$I(k,z) = \int_{R/a}^{\infty} d\rho_s\, J_1(k\rho_s)\, m_s(\rho_s, z)$, если ρ, z, R в единицах "a".
Т.к. $a_0/R \approx$ const, можно брать ρ, z, а считать в единицах R, см. 19/XI.
$E_{ss} = 2\pi^2 M_s^2 \int dk \int dz \int dz'\, e^{-k|z-z'|} J_0(zk) J_0(z'k)$, $J_0(z,k) = -k\int d\rho_s J_1(k\rho_s) m_s(\rho_s z)$
Local approx. $\int dz'\, e^{-k|z-z'|} J_0(z'k) \approx J_0(zk) \int dz' e^{-k|z-z'|}$
$\int_{-z_m}^{z_m} dz'\, e^{-k|z-z'|} = \frac{2}{k}\left[1 - e^{-kz_m} ch(kz)\right] = J_z(k,z)$, $z_m = \frac{L}{2}$. $(kR) \to k$.
$E_{ss}(loc) = 2\pi^2 M_s^2 \int dk \int dz\, J_z(kz)\, J_0^2(zk)$. $J_z(kz) = \frac{2R}{kR}\left[1 - e^{-kR\frac{z_m}{R}} ch\left(kR\frac{z}{R}\right)\right] =$
$= |kR \to k| = \frac{2R}{k}\left[1 - e^{-k\beta\frac{z_m}{L}} ch\left(k\beta\frac{z}{L}\right)\right] = \frac{2R}{k}\left[1 - e^{-\frac{1}{2}k\beta} ch(\beta k z)\right]$, z
в единицах L, $z \in [-\frac{1}{2}, \frac{1}{2}] \to [-\frac{1}{2}, \frac{1}{2}]$. $z/R = \beta(z/L)$.
$J_0(kz) = -k\int_0^R d\rho_s\, \rho\, J_1(k\rho_s)\, m_s(\rho, z) = -\frac{(kR)}{R} R^3 \int_0^1 d\rho_s J_1(kR\rho_s) m_s(\rho, z) = -kR\int_0^1 d\rho_s \rho J_1(k\rho_s) m_s$
В $m_s(\rho, z)$ надо ρ в ед. R, а z в ед. R, z в ед. L.
$m_s = \sin\theta\cos\gamma$, $\sin^2\theta = \frac{16\rho^2 a^2}{(\rho^2+z^2+a^2)^2}\left[1 - \frac{4\rho^2 a^2}{(\rho^2+z^2+a^2)^2}\right] = \frac{16\rho^2 a^2}{(\rho^2+\beta^2 z^2+a^2)^2}\left[1 - \frac{4\rho^2 a^2}{(\rho^2+\beta^2 z^2+a^2)^2}\right]$
$\cos\gamma = \frac{1}{\sqrt{tg^2\gamma + 1}}$, $tg\gamma = tg\beta(\rho, z) = \frac{2az}{(\rho^2+z^2-a^2)} = \frac{2a\beta z}{(\rho^2+\beta^2 z^2 - a^2)}$
$E_{ss}(loc) = 2\pi^2 M_s^2 \int_0^{\infty} d(kR) L \int_{-1/2}^{1/2} dz \cdot \frac{2R}{k}\left[1 - e^{-\frac{1}{2}\beta k} ch(\beta k z)\right] k^2 R^2 \left[\int_0^1 d\rho_s J_1(k\rho_s) m_s\right]^2 =$
$E_{ss}(loc) = 2\pi^2 M_s^2 R^2 L \int_0^{\infty} dk \int_{-1/2}^{1/2} dz\, k\left[2 - e^{-\beta k(z+\frac{1}{2})} - e^{\beta k(z-\frac{1}{2})}\right]\left[\int_0^1 d\rho_s \rho J_1(k\rho_s)\, m_s(\rho z)\right]^2$.
$W_m(a) = \frac{E_{ss}(loc)}{4\pi M_s^2 V} = \frac{1}{2}\int_0^{\infty} dk \int_{-1/2}^{1/2} dz\, k\left[2 - e^{-\beta k(z+\frac{1}{2})} - e^{\beta k(z-\frac{1}{2})}\right]\left[\int_0^1 d\rho_s \rho J_1(k\rho_s) m_s(\rho z a)\right]^2$
$V = \pi R^2 L$, $W_m = W_m(a,\beta)$, $\xrightarrow{-1/2} \frac{\partial W_m(a,\beta)}{\partial a} = 0$ дает $a_0 = a_0(\beta)$.
т.е. при учете только dominating magnetostatic energy, equilibrium
hopfion radius $a_0 = a_0(\beta)$ только, $\beta = L/R$.

Escaneado con CamScanner

$$\frac{\partial W_m(a,\beta)}{\partial a} = \int dk \int_{-1/2}^{1/2} dz\, k\left[2 - e^{-\beta k\left(z+\frac{1}{2}\right)} - e^{\beta k\left(z-\frac{1}{2}\right)}\right] \int_0^{\cdot} d\rho\rho J_1(k\rho) m_z(\rho,z) \cdot \int_0^{\cdot} d\rho'\rho' J_1(k\rho') \frac{\partial m_z(\rho',z)}{\partial a}$$

Расчет $\frac{\partial E_m(a,R)}{\partial a} = 0$ дал, что $\beta = 2$, $a_0(R=20) = 0{,}915$ и $a_0(R=50) = 0{,}901$.
$a_0(\beta, R)$ слабо зависит от R, падает с $R\uparrow$. Это из-за E_{ex}, E_{DMI}?

При $\beta = 10$ $a_0(R=20) = 0.905$, $a_0(R=50) = 0.908$, $a_0(\beta=10, R)$ растет с $R\uparrow$?

Scaling $E_m(a,R,\beta) = \frac{\pi}{2} a^3 \gamma_m\left(\frac{R}{a},\beta\right)$, $\gamma_m(x,\beta) = \int_0^{\infty} dk\, k \int_{-1/2}^{1/2} dz \left[2 - e^{-k\left(z+\frac{1}{2}\beta x\right)} - e^{k\left(z-\frac{1}{2}\beta x\right)}\right] \times$

$\times I^2(k,z,x)$, $I[k,z,x] = \int_0^{x} d\rho\rho J_1(k\rho) m_z(\rho,z)$, $\frac{R}{a} = x$

$E_m(x,\beta) = \frac{\pi}{2} R^3 \frac{1}{x^3} \gamma_m(x,\beta)$, $\gamma_m(x,\beta) = \int dk\, k \int_{-1/2}^{1/2} dz \left[2 - e^{-k(\cdot)} - e^{k(\cdot)}\right] I^2(k,z,x)$

Если $E_m \gg E_{ex}, E_D$, то $\min E_z(a,R,\beta,d)$ определяется $E_m(x,\beta)$, $x_0 = \frac{R}{a_0} = f(\beta)$

Отсюда $\frac{a_0}{R} = f(\beta)$ только, это не следует из $W_m = W_m(a,\beta)$ нормировки на R.

$E_m(x,\beta) = \frac{R}{\left(\frac{\pi}{2}R^3\right)} \frac{1}{x^3} \gamma_m(x,\beta)$.

Вопрос: как нарисовать metastable hopfion in thick cyl. dot as wire? (создать)

Может с помощью поля H_z, как задать hysteresis loop $\langle M_z\rangle(H_z)$.

$m_z = 1 - \frac{8\rho^2 a^2}{(a^2+\rho^2+z^2)^2}$. $\langle m_z(\rho,z)\rangle = \frac{1}{V}\int d^3\vec{r}\, m_z(\rho,z) = \frac{2\pi}{\pi R^2 L}\int d\vec{\rho}\, m_z =$

$= \frac{2}{R^2 L}\int_0^R d\rho\rho \int_{-L/2}^{L/2} dz\, m_z(\rho,z)$. $\int d\rho\rho \int dz\, m_z = \int_0^R d\rho\rho \int_{-\frac{1}{2}L}^{L/2} dz \left[1 - \frac{8\rho^2 a^2}{(a^2+\rho^2+z^2)^2}\right] =$

$= \frac{1}{2}R^2 L - 8\int d\rho\rho \int dz \frac{\rho^2 a^2}{(a^2+z^2+\rho^2)^2} = (\cdot) - 8a^2 \frac{1}{2}\int dt \frac{t\, dz}{(t+a^2+z^2)^2} = \left|t = \rho^2\right|$

$\int_{-L/2}^{L/2} dz \frac{1}{(b^2+z^2)^2} = \int dx \frac{1}{x^2} = \left|x = b^2+z^2\right| = \left[\frac{z}{2b^2(b^2+z^2)} + \frac{1}{2b^3}\arctg\left(\frac{z}{b}\right)\right]_{-L/2}^{L/2} =$

$= \frac{L}{2b^2(b^2+(L/2)^2)} + \frac{1}{2b^3} 2\arctg\left(\frac{L}{2b}\right)$, $b^2 = a^2+\rho^2$.

$\int d\rho\rho \int dz \frac{\rho^2}{(\rho^2+z^2+a^2)^2} = \int d\rho\rho\, \rho^2 \left[\frac{L}{2(a^2+\rho^2)\left[a^2+\rho^2+\frac{L^2}{4}\right]} + \frac{1}{(a^2+\rho^2)^{3/2}}\arctg\left(\frac{L}{2\sqrt{a^2+\rho^2}}\right)\right]$

$\langle m_z(a)\rangle = \frac{2}{R^2 L}\left[\frac{1}{2}R^2 L - 8a^2 \int d\rho\rho \int dz \frac{\rho^2}{(\rho^2+z^2+a^2)^2}\right]$

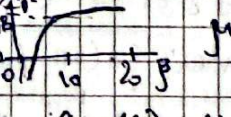
$\langle m_z(a)\rangle = 1 - \frac{16a^2}{R^2 L}\int d\rho\rho \int dz \frac{\rho^2}{(\rho^2+z^2+a^2)^2}$. $\frac{16a^2}{R^2 L}\int_0^R d\rho\rho \int_{-L/2}^{L/2} dz \frac{\rho^2}{(\rho^2+z^2+a^2)^2} =$

$= \left|\rho \to \rho/R, z \to z/L, a \to a/R\right| = \frac{16a^2R^2}{R^2L} R^2 \int \frac{d\rho\rho}{R^2} L \int \frac{dz}{L} \frac{\rho^2 R^2}{R^4\left[\rho^2+\beta^2 z^2+a^2\right]^2} =$

$= \frac{16a^2R^4}{R^2L}\frac{1}{R^2}\int_0^{\cdot} d\rho\rho \int_{-1/2}^{1/2} dz\, L \frac{1}{R^2} \frac{\rho^2}{\left[\rho^2+\beta^2 z^2+a^2\right]^2} = 16a^2 \int_0^{\cdot} d\rho\rho \int_{-1/2}^{1/2} dz \frac{\rho^2}{(\rho^2+\beta^2 z^2+a^2)^2}$

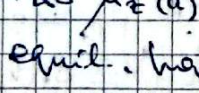
$\langle m_z(a)\rangle = 1 - 32a^2 \int_0^{1} d\rho\rho \int_0^{1/2} dz \frac{\rho^2}{(\rho^2+\beta^2 z^2+a^2)^2} = \langle m_z(a,\beta)\rangle = \mu_z(a,\beta)$.

$\mu_z(a_0,\beta) \to 1$ при $\beta \to \infty$. Берем $a_0 = 0.9$



$\mu_z(\beta=9.5) = 1$, $\mu_z(\beta=1) = 0$, $a_0 = 0.9$

Интересно как $\mu_z(a)$ имеет



вид вблизи и вдали equil. hopfion радиуса $a_0(\beta)$. min при при $a_m(\beta)$ и $a_m(\beta)$-value, $a_0 \approx (0.90 - 0.92)$. $a_m(\beta=2) = 0.86$, $a_m(\beta=\infty) = 1.06$

(28/XI) Написание hopfion-stability MS, подготовка Figs. Перед ACS переписка с Materials/Nanomaterials об оплате APC. review.

(29/XI) Написание radial vortex GD paper на основе simulations by Modic. 18/XI при расчете ω_G^H, χ_H в сферу ошибку, не учел 2πR

Hard layer stray field $\vec{H}_d \sim M_{s1}$ (hard layer magnetization). $M_{s1} = 1000$ G, M_s (Soft layer) = 810 G (Fe-V). В eqs. give stray field H_d', H_d' от 4/XI надо $M_s \to M_{s1}$.

4/XI: $\chi_{11} = -\pi M_s L \int_0^1 ds\, s \left(\frac{\partial^2 \vec{m}_0}{\partial s^2} + \frac{1}{s}\frac{\partial \vec{m}_0}{\partial s} \right) \cdot \vec{H}_d'(s)$, $\vec{H}_d'(s) = \frac{1}{L}\int dz\, \vec{H}_d(s,z)$, $\vec{H}_d \sim M_{s1}$.

18/XI: Zeeman Stiffness $\chi_H = \frac{\chi_{11}}{4\pi M_s^2 V} = -\frac{\pi M_s L}{4\pi M_s^2 \pi R^2 L} \int_0^1 ds\, s \left(\frac{\partial^2 \vec{m}_0}{\partial s^2} + \frac{1}{s}\frac{\partial \vec{m}_0}{\partial s} \right) \cdot \vec{H}_d(s) =$

$= -\frac{1}{4\pi M_s R^2} \int ds\, s \left(\frac{\partial^2 \vec{m}_0}{\partial s^2} + \frac{1}{s}\frac{\partial \vec{m}_0}{\partial s} \right) \vec{H}_d' = -\frac{M_{s1}}{M_s}\frac{1}{R^2} \int_0^1 ds\, s \left(\frac{\partial^2 \vec{m}_0}{\partial s^2} + \frac{1}{s}\frac{\partial \vec{m}_0}{\partial s} \right) \left(\frac{\vec{H}'(s)}{4\pi M_{s1}} \right)$

$\chi_H' = R^2 \chi_H = -\frac{M_{s1}}{M_s} \int_0^1 ds\, s \left(\frac{\partial^2 \vec{m}_0}{\partial s^2} + \frac{1}{s}\frac{\partial \vec{m}_0}{\partial s} \right) \cdot \left(\frac{\vec{H}(s)}{4\pi M_{s1}} \right)$

MCad - Radial vortex gyrofrequency mcd. Считает $\vec{H}'$ - stray field in $(4\pi M_{s1})$ units.

Устранение ошибки в нормировке 18/XI кривая $\chi_{11} \to \frac{M_{s1}}{M_s}\chi_{11}'$, т.е.

$\omega_G^H = \frac{1}{4}(1+\beta_c^2)\omega_M \chi_{11}'(old)\cdot\left(\frac{M_{s1}}{M_s}\right)$ - перенормируется, $\chi_{11}'(old) = -\int ds\, s\,(\cdot)\cdot\frac{\vec{H}'}{4\pi M_{s1}}$ (18/XI)

Домножим ω_G^H в MCad-file на (M_{s1}/M_s). $\omega_G^2 = \omega_G^{H\,2} + \omega_G^{M\,2}$, ω_G вернулся на ω_G(Calc) заметно больше ω_G(Sim) из-за rigid V-model 20-30%.

и BP ansatz for static dot magnetization soft layer $\vec{m}_d(s)$.

ω_G можно поправить, если в $\omega_M = \gamma \cdot 4\pi M_s$ брать $\gamma/2\pi = 2.95$ GHz/kOe вместо $\gamma_e/2\pi = 2.80$ GHz/kOe $\to \omega_M = 2.80 \cdot 4\pi \cdot 0.81 = 2.85$ GHz.

$(\omega_M/2\pi = 30.0$ GHz), а $\omega_M/2\pi = 28.5$ GHz. $M_s = 810$ G, M_{s1}(hard) = 1000 G.

30/XI Note that Sam [Gus' IEEMagLett'2016] не учитывал give ultrathin dot (face surface charges only, $N_m = 2\pi M_s^2 m_z^2$).

$\chi_m = 16\pi M_s^2 L \frac{c^2(1-c^2)\pi}{(1+c^2)^3}$. $\chi_{ex} = -6\pi^2 M_s^2 L \left(\frac{l_e}{R}\right)^2 \frac{c^2}{(1+c^2)^3}$, Cu (7/XI).

Для MCad-расчетов $\omega_G(L, L_x, d, R)$ а брали $l_e = 5$ nm.

Для параметров Modic $M_s = 810$ G, $A = 1.05\cdot10^{-6}$ erg/cm, $l_e = \sqrt{\frac{A}{2\pi M_s^2}} = 5.05$ nm.

Domcan Word file „Radial vortex gyromodes", broadband type Figs. из MathCad $\omega_G(L, L_x, d, R)$ и насчет to Mathieu, Gleb, Maciej.

1/XII Написание description stability MS, Materials - MDPI, Materials Physics Section, сопроводительное письмо. my description MS free of charge (APC ~ 3000 €). С а.и.ю. - Lidl", „Carrefour" Olaberria.

2/XII Как Ksin²θ energy (m. anisotropy) влияет на description stability?

$E_a = 2\pi K \int dz \int ds\, s \sin^2\theta = 2\pi K \cdot 2\int_0^{z_m} dz \int_0^{R/a} ds\, s \cdot a^3 \sin^2\theta$, $z_m = \frac{L}{2a} = \beta\left(\frac{R}{a}\right)$, $x = \frac{R}{a}$

$\varepsilon_{an}(a,R) = \frac{1}{4\pi M_s^2 l_e^3} = \frac{2\pi K}{4\pi M_s^2}\left(\frac{a}{l_e}\right)^3 \cdot 2\int_0^{z_m} dz \int_0^{R/a} ds\, s \sin^2\theta$, $\frac{K}{2\pi M_s^2} = Q$ (quality factor).

$\varepsilon_{an}(a,R) = 2\pi Q\, a^3 I_{an}(a, x, \beta)$, $I_{an}(x,\beta) = 2\int_0^{\frac{1}{2}\beta x} dz \int_0^{R/a} ds\, s \sin^2\theta$, $\frac{R}{a} = x$.

$\varepsilon_{an}(a,R) = \pi Q a^3 I_{an}(x,\beta)$ - uniaxial m. anisotropy.

$E_a = \frac{2\pi K}{(2\pi M_s^2)} 2\pi M_s^2 \int ds\, s \int dz \sin^2\theta = 4\pi^2 M_s^2 Q \int ds\, s \int dz \sin^2\theta$

$E_a = 4\pi^2 M_s^2 Q\, a^3 I_{an}(x,\beta)$, $\varepsilon_{an}(a,R) = \frac{E_a}{4\pi M_s^2 l_e^3} = \pi Q \left(\frac{a}{l_e}\right)^3 I_{an}(x,\beta)$.

3-4/XII Написание description stability paper. переписка с E. Martinez (Nanomaterials, отменили APC) и Оксана BP-DW Simulations.

Skype обсуждение с Глебом идеи ω_G + Calcs. и Modic - Radial vortex project.

(5/XII) При finite $p = L/R$ надо изменить $E_m(\perp SD)$, она конечна.

$E(\perp SD) = \pi\mu_0 M_s^2 N_{zz}(\beta) V$, $N_{zz}(\beta) = \frac{2}{\beta}\int_0^\infty \frac{dk}{k^2} J_1^2(k)(1-e^{-\beta k})$ Joseph

$N_{xx}(\beta) = \int_0^\infty \frac{dk}{k} f(\beta k) J_1^2(k)$, $f(x) = 1 - \frac{1}{x}(1-e^{-x})$ & сов. с Gus 'JAP' 2002.

Transition $\perp SD \to \parallel SD$ state при $\beta = 1{,}813$ (Aharoni).

Для кобальта Sample $\beta \gg 2$, $E(\perp SD) > E(\parallel SD)$.

$$E(\perp SD) = \frac{E(\perp SD)}{4\pi M_s^2 l_e^3} = \frac{2\pi M_s^2 \pi R^2 L}{4\pi M_s^2 l_e^3} N_{zz}(\beta) = \frac{\pi}{2}\left(\frac{R}{l_e}\right)^3 \beta N_{zz}(\beta)$$

Г.-Рыжик 6.575 $\int_0^\infty \frac{J_\nu(x) J_\mu(x)}{x^{\nu+\mu}} dx = \frac{\sqrt{\pi}\,\Gamma(\nu+\mu)}{2^{\nu+\mu}\Gamma(\nu+\mu+\frac{1}{2})\Gamma(\nu+\frac{1}{2})\Gamma(\mu+1/2)}$

$\mu=\nu=1$, $\int_0^\infty dx \frac{J_1^2(x)}{x^2} = \frac{\sqrt{\pi}}{4}\frac{\Gamma(2)}{\Gamma(5/2)\Gamma^2(3/2)} = \frac{\sqrt{\pi}}{4}\frac{1}{\frac{\sqrt{\pi}}{4}\cdot 1\cdot 3\cdot\left[\frac{\sqrt{\pi}}{2}\cdot 1\right]^2} = \frac{1}{3}\frac{4}{\pi}$

$\Gamma(n+\frac{1}{2}) = \frac{\sqrt{\pi}}{2^n}(2n-1)!!$ $N_{zz}(\beta\gg 1) = \frac{2}{\beta}\int \frac{dk}{k^2} J_1^2(k) = \frac{2}{\beta}\frac{4}{3\pi} = \frac{8}{3\pi\beta}$ как у Joseph 'JAP' 1966. $N_{zz}(\beta\gg 1) = \frac{8}{3\pi\beta} - \frac{1}{2\beta^2} + \dots$ (Joseph '66).

$E(\perp SD) = \frac{\pi}{2}\left(\frac{R}{l_e}\right)^3 \beta N_{zz}(\beta) \xrightarrow[\beta\gg 1]{} \frac{\pi}{2}\left(\frac{R}{l_e}\right)^3 \frac{8}{3\pi} = \frac{4}{3}\left(\frac{R}{l_e}\right)^3$, т.к. $8/3\pi$

$R/l_e = 20{,}50$, то $E_1(SD)$ is large number even at $\beta \gg 1$.

При $\beta = 10$, $R/l_e = 20$ $E(\perp SD) = E(SD) = \frac{4}{3}(20)^3 = \frac{4}{3}2^3\cdot 10^3 = \frac{32}{3}\cdot 10^3 \simeq$

Max hopfion $E(a,R)$ при $R/l_e = 20$, $\beta = 10$ есть $\simeq 1{,}35\cdot 10^4 \simeq 10{,}7\cdot 10^4$ (10 ro).

$m_z(\rho,z) = 1 - \frac{8\rho^2 a^2}{(a^2+\rho^2+z^2)^2}$, при $a\to 0$ $m_z(\rho,z) \simeq 1 - \frac{8\rho^2 a^2}{(\rho^2+z^2)^2} = 1 - O(a^2) = (\perp SD)$.

Должно $E(a,R) \to E(\perp SD)$ при $a\to 0$, т.к. $m_z(\text{hopf}) = 1 - O(a^2) \to$ может при $a\to 0$ нельзя использ. ρ/a и z/a? $\to m_z = 1\,(SD)$.

$x = R/a$ и $a\to 0$ эквивалентно $(R\to\infty)$ $x\to\infty$ at const R.

$\varepsilon_{ex}(x\to\infty) = 16\pi^2 a$, $\varepsilon_D(x\to\infty) = -16 d J_s a^2$, $\varepsilon_{ex}(a)\to 0$, $\varepsilon_D(a)\to 0$ при $a\to 0$.

$I_{an}(x,\beta) = I_{an}(x)$ и $I_m(x) \to \infty$ при $x\to\infty$, расходится. Но $a^3 I_{an}(x)$, $a^3 I_m(x)$ при $x\to\infty$ $a\to 0$ нельзя

Без перенормировки ρ/a, z/a:

$E_a = 2\pi K \int_0^R d\rho\rho \int_{-L/2}^{L/2} dz \sin^2\theta$, $\sin^2\theta(\rho,z) = \frac{16\rho^2 a^2}{[\rho^2+z^2+a^2]^2}\left[1 - \frac{4\rho^2 a^2}{(\rho^2+z^2+a^2)^2}\right] = O(a^2)$ при $a\to 0$.

$E_a(a,a\to 0) = 2\pi K a^2 \int d\rho\rho \int dz \frac{16\rho^2}{(\rho^2+z^2+a^2)^2}\left[1 - \frac{4\rho^2 a^2}{(\rho^2+z^2+a^2)^2}\right]$, и нельзя $a\to 0$ в знаменателе, т.к. если $\rho=0$, $z=0$. $a \ll |z|, \rho$

$m_z(\rho,z) = 1 - \frac{8\rho^2 a^2}{(\rho^2+z^2+a^2)^2}$, и нельзя $a \ll \rho, |z|$, $a\to 0$, т.к. если $z=0$, $\rho=0$, $m_z\to 1$ (вблизи $z=0$ ось $\hat z$)

Нормировка ρ, z, a на l_e: $E(a) = 2\pi K l_e^3 \int_0^{R/l_e} d\rho\rho \int_{-L/2l_e}^{L/2l_e} dz \frac{16\rho^2 a^2}{(\rho^2+z^2+a^2)^2}\left[1 - \frac{4\rho^2 a^2}{(\rho^2+z^2+a^2)^2}\right]$

$\frac{2\pi K l_e^3}{4\pi M_s^2 l_e^3} = \pi Q$, $Q = \frac{K}{2\pi M_s^2}$.

$\varepsilon_{an}(a) = \pi Q \int_0^{R/l_e} d\rho\rho \int_{-L/2l_e}^{L/2l_e} dz \sin^2\theta(\rho,z)$. $\frac{L}{2l_e} = \frac{\beta}{2}\frac{R}{l_e}$, a "сидит" в $\sin^2\theta(\rho,z,a)$.

$\varepsilon_{an}(a,R,\beta) = 2\pi Q \int_0^R d\rho\rho \int_{-\beta R/2}^{\beta R/2} dz \sin^2\theta(\rho,z,a)$, R,ρ,z,a в единицах l_e.

Не очевидно, что $a\to 0$ это переход к $\vec m = m_z\hat z = \hat z$, $\perp SD$.

Cad. – hopfion stability – 2 меры: показали, что $\varepsilon_{an}(a,R,\beta)\to 0$ при $a\to 0$ и $\varepsilon_{an}(a) \sim a^2$ при достаточно малых $a \neq 0$. Hopfion $\varepsilon_{an}(a)\to 0$ при $a\to 0$ как и $\frac{\perp SD}{\varepsilon_{an}=0}$ <

$\lim_{a\to 0} a^3 I_{an}(R/a) \simeq Ca^2$, где порядок на "a" дает g, z, R.

Как ведет себя $E_{an}(a)$ при $a \to \infty$, в пределе g, z, R а где l?

Важно, что Hopfion $E_m \sim E_{ss}$, а где 1-SD будет нужна E_{zz}.

См. 27/xi Magnetostatic energy $E_{ss}(R) = 2\pi^2 M_s^2 \int dk \int_{-L/2}^{L/2} dz\, J_z(kz) J_s^2(zk)$.

$J_z(kz) = \frac{2}{k}\left[1 - e^{-kz_m} ch(kz)\right] = \frac{1}{k}\left[2 - e^{-k(z+z_m)} - e^{-k(z-z_m)}\right]$, $z_m = \frac{L}{2}$

$J_s(zk) = -k \int_0^R dg\, g\, J_1(kg)\, m_s(gz)$. можно перенормировать на l,

а как 27/xi g в ед. R, z в ед. L, "a" в ед. R и использовать $W_m(a)$.

$m_s(g, z, a) = \sin\theta \cos\gamma$, $\sin^2\theta$ и $\cos\gamma$ как 27/xi.

MCd показал, что $W_m(a,\beta) \sim a^2$ при $a \to 0$, т.е. дает Hopfion $\mathcal{E}(a) \to 0$ (tot) $a \to 0$.

Здесь только E_{ss} magnetostatic energy.

Пределы $a \to \infty$ исследовать трудно, т.к. можно брать $I_a(x)$, $I_m(x)$

$x = \frac{R}{a} \to 0$. $I_a(x) = 2\int_0^x dg\, g \int_0^{\frac{1}{2}\beta x} dz\, \sin^2\theta$, $\sin^2\theta = \frac{16g^2}{(g^2+z^2+1)^2}\left[1 - \frac{4g^2}{(g^2+z^2+1)^2}\right]$

β - finite, $x \to 0$ $I_a(x) = 2\int_0^x dg\, g \int_0^{+\frac{1}{2}\beta x} dz\, (16 g^2) = 32 \int_0^x dg\, g\, g^2 \int_0^{\frac{1}{2}\beta x} dz =$

$= 32 \cdot \frac{1}{4} g^4 \Big|_0^x \cdot \frac{1}{2}\beta x = 4\beta x^5$

$E_{an}(R,R) = \pi Q a^3 I_a(x) = \pi Q a^3 \cdot 4\beta x^5 = \pi Q a^3 \cdot 4\beta \frac{R^5}{a^5} = 4\pi\beta R^5 \frac{1}{a^2} \sim \frac{1}{a^2}$ при $a \to \infty$.

Такое $W_m(a,\beta) \sim 1/a^2$ при $a \to \infty$. $\mathcal{E}(a)$ $\sim 1/a^2$ Hopfion

И именно асимптотика hopfion $\mathcal{E}(a) \sim a^2$ при $a \to 0$ и $\mathcal{E}(a) \sim \frac{1}{a^2}$ – и нужна в MCd-расчете.

Пусть $a \to 0$, $a^{-1} \to \infty$, $m_z \to 1$ возбуждает 1-SD вместе $m_z = 1$

$\mathcal{E}(\text{1-SD}) = \frac{\pi}{2}\left(\frac{R}{a}\right)^3 \beta N_{zz}(\beta)$ is finite, hopfion $\mathcal{E}(a) \to 0$ при $a \to 0$, $a \to \infty$.

Вопрос в hopfion $\mathcal{E}(a)$ пропорц. какой-то степ. Это E_{zz}-магн.?

Вернемся к расчету магнитостатики E_{zz}, E_{sz}, E_{ss} от g, z.

ФГ $\Gamma_{d\beta}(\vec{r}\vec{r}') = -\frac{\partial^2}{\partial x_\alpha \partial x'_\beta}\frac{1}{|\vec{r}-\vec{r}'|}$, $\Gamma_{zz}(\vec{r}\vec{r}') = \frac{1}{2\pi}\int d\vec{k}\, k e^{-k|z-z'|} e^{i\vec{k}(\vec{g}-\vec{g}')} - 4\pi\delta(\vec{r}-\vec{r}')$

$\Gamma_z[\Gamma_{d\beta}(\vec{r}\vec{r}')] = -4\pi\delta(\vec{r}-\vec{r}')$, $\delta(\vec{r}-\vec{r}') = \delta(\vec{g}-\vec{g}')\delta(z-z')$

При расчете E_{zz} g/z применим local approximation и $\int dz' e^{-k|z-z'|} = \frac{2}{k}$.

Local approximation допустима, хотя точность ее неясна.

Но надо $\int dz' e^{-k|z-z'|} = \int_{-z_m}^{z_m} dz' e^{-k|z-z'|} = J_z(k, z, z_m) = \frac{2}{k}\left[1 - e^{-kz_m} ch(kz)\right] =$

$= \frac{2}{k}\left[1 - \frac{1}{2}e^{-k(z+z_m)} - \frac{1}{2}e^{+k(z-z_m)}\right]$ для MCd расчета.

Сама $\int dz' e^{-k|z-z'|} = \frac{2}{k}$ сокращалась, но здесь $-\frac{2}{k}e^{-kz_m} ch(kz)$ остается

$E_{zz}(loc) = -2\pi^2 M_s^2 \int dz \int dk\, k^2 I_z^2(zk)\left(-\frac{2}{k}\right) e^{-kz_m} ch(kz) =$ finite $\beta = L/R$.

$= 4\pi^2 M_s^2 \int_{-z_m}^{z_m} dz \int dk\, k\, I_z^2(zk) \times e^{-kz_m} ch(kz)$, $I_z(g,k) = \int_0^g dg\, g J_0(kg) m_z(gz)$.

$E_{zz}(loc) = 4\pi^2 M_s^2 \int_{-z_m}^{z_m} dz \int dk\, k e^{-kz_m} ch(kz)\, I_z^2(z,k)$. При $a \to 0$ hopfion $m_z \to 1$.

$I_z(z,k) = \int_0^R dg\, g J_0(kg) = \frac{1}{k^2}\int_0^{kR} dx\, x J_0(x) = \frac{1}{k^2}\int_0^{kR} dx\, x J_0(x) = \frac{1}{k^2} x J_1(x)\Big|_0^{kR} =$

$= \frac{R}{k} J_1(kR)$. $E_{zz} = 4\pi^2 M_s^2 \int_{-z_m}^{z_m} dz \int dk\, k e^{-kz_m} ch(kz) \frac{R^2}{k^2} J_1^2(k) = 4\pi^2 M_s^2 R \int dz \int \frac{dk}{k} e^{-kz_m} ch(kz) J_1^2(k)$

$\int_{-z_m}^{z_m} dz\, ch(kz) = \frac{1}{k} sh(kz)\Big|_{-z_m}^{z_m} = \frac{2}{k} sh(kz_m)$, сведется ли к N_{zz}?

$E_{zz}(a\to 0) = 4\pi^2 M_s^2 R^2 \int \frac{dk}{k} e^{-kz_m} \frac{2}{k} \sinh(kz_m) J_1^2(kR)$, $e^{-kz_m} \cdot 2\sinh(kz_m) = e^{-\frac{kL}{2}}(e^{\frac{kL}{2}} - e^{-\frac{kL}{2}}) = 1 - e^{-kL}$.

$E_{zz}(a\to 0) = 4\pi^2 M_s^2 R^2 \int \frac{dk}{k^2} J_1^2(kR)(1-e^{-kL}) = 4\pi^2 M_s^2 R^2 \frac{R}{R} \int \frac{dk}{k^2} J_1^2(k)(1-e^{-\beta k})$

$E_{zz}(a\to 0) = 4\pi^2 M_s^2 R^3 \int \frac{dk}{k^2} J_1^2(k)(1-e^{-\beta k}) = 4\pi M_s^2 V \frac{R}{L} \int \frac{dk}{k^2} J_1^2(k)(1-e^{-\beta k})$

$E_{zz}(a\to 0) = 2\pi M_s^2 V N_{zz}(\beta)$, $N_{zz}(\beta) = \frac{2}{\beta} \int \frac{dk}{k^2} J_1^2(k)(1-e^{-\beta k})$, вариант интеграл $m_z \to 1$.

Local $E_{zz}(a,R,\beta) = 8\pi^2 M_s^2 \int_0^{L/2} dz \int dk\, k e^{-kL/2} \cosh(kz) I_z^2(R,k)$

$\int_0^{L/2} dz \int k\,dk \left[\int_0^R d\varrho\, \varrho(\cdot)\right]^2 = \left|ka\to k,\ \frac{\varrho}{a}\to\varrho,\ \frac{z}{a}\to z\right| = a\int_0^{L/2a} dz \frac{1}{a^2} \int_0^\infty dk\,k \left[a^2 \int_0^{R/a} d\varrho\,\varrho\right]^2 =$

$= a^3 \int_0^{z_m} dz \int dk\,k \left[\int_0^x d\varrho\,\varrho\right]^2$ $\quad I_z(z,k) = \int_0^R d\varrho\, \varrho J_0(k\varrho) m_z(\varrho z)$

$E_{zz}(a,R,\beta) = 8\pi^2 M_s^2 a^3 \int_0^{z_m} dz \int dk\,k e^{-kz_m} \cosh(kz) I_z^2(x,k,z)$, $I_z(x,k,z) = \int_0^x d\varrho\,\varrho J_0(k\varrho) m_z(\varrho z)$

$\varepsilon_z = \frac{E_{zz}}{(a,R,\beta)\ 4\pi M_s l_e^3} = 2\pi a^3 \int_0^{z_m} dz \int_0^\infty dk\,k\, e^{-kz_m} \cosh(kz) I_z^2(x,k,z)$, $z_m = \frac{L}{2a} = \frac{1}{2}\beta x$.

$x = R/a$.

$E_z(a,R,\beta) = \pi a^3 \int_0^{z_m} dz \int_0^\infty dk\,k \left[e^{-k(z+z_m)} + e^{k(z-z_m)}\right] I_z^2(x,k,z)$, $I_z(x,kz) = \int_0^x d\varrho\,\varrho J_0(k\varrho) \times$ $\times m_z(\varrho z)$

Зависи ε, R, z в ед. a^n, a^n в ед. l_e.

$\Delta E_z = E_z(\text{hopf}) - E_z(SD) = E_z(a,R,\beta) - \frac{3}{2} R^3 \beta N_{zz}(\beta)$, $\Delta E_z\left(\substack{a\to 0 \\ a\to\infty}\right) = 0$.

(a)

Local $E_{zz}(a,R,\beta) = 4\pi^2 M_s^2 a^3 \int_{-z_m}^{z_m} dz \int dk\,k\, e^{-kz_m} \cosh(kz) I_z^2(x,k,z)$

$E_{zz}(a,R,\beta) = 2\pi^2 M_s^2 a^3 I_m^{zz}(x)$, $I_m^{zz}(x) = 2\int_0^{z_m} dz \int dk\,k\, e^{-kz_m} \cosh(kz) I_z^2(x,k,z)$

$I_z(x,k,z) = \int_0^x d\varrho\,\varrho J_0(k\varrho) m_z(\varrho z)$. $\quad \frac{2\pi^2 M_s^2 a^3}{4\pi M_s^2 l_e^3} = \frac{\pi}{2}\left(\frac{a}{l_e}\right)^3$

6/XII С п.и Ю. в нек. Oiartzun за числами.

Написание Hopfion stability MS, построение Figs. $\varepsilon(a)$.

Wang PRL '2019, Liu-Zhang PRB '2018 говорят, что для стабилизации toroidal hopfion in cylindrical dot нужна strong (Neel) surface 1-anisotropy at $z = \pm L/2$ faces. Так ли это?

$E_s = K_s \int dS\, m_z^2 \left(z = \pm\frac{L}{2}\right)$. $\int dS\, m_z^2(\varrho,z) = \int d\varrho\,\varrho \int d\varphi\, m_z^2 = 2\pi \int_0^R d\varrho\,\varrho\, m_z^2$

$E_s(a) = 2\pi K_s \int_0^R d\varrho\,\varrho\, m_z^2(\varrho,z,a) = 2\pi K_s R^2 \int_0^1 d\varrho\,\varrho\, m_z^2(\varrho,z,a)$, ϱ в ед. R, a в ед. R

$z/R = \beta\frac{z}{L}$. $m_z(\varrho,z,a) = 1 - 8\frac{\varrho^2 a^2}{(a^2+\varrho^2+z^2)^2} = 1 - \frac{8\varrho^2 a^2}{(a^2+\varrho^2+\beta^2 z^2)^2}$, z в ед. R.

$z = \pm 1/2$.

$\varepsilon_s(a) = \frac{E_s(a)}{4\pi M_s^2 l_e^3} = \frac{2\pi K_s R^2}{4\pi M_s^2 l_e^3} \int_0^1 d\varrho\,\varrho\, m_z^2 = \left(\frac{R}{l_e}\right)^2 \frac{K_s}{2M_s^2 l_e} \int_0^1 d\varrho\,\varrho\, m_z^2$.

$\varkappa_s = K_s/(2M_s^2 l_e) = 1\ \text{erg/cm}^2 \Big/ 2 \cdot (0.237)^2 \cdot 10^6 \frac{\text{erg}}{\text{cm}^3} \cdot 5\cdot 10^{-7}\text{cm} = \frac{1}{(0.237)^2} = 1.43$ для $K_s = 1\ \text{erg/cm}^3$,

$\varepsilon_s(a,\beta) = \left(\frac{R}{l_e}\right)^2 \varkappa_s \int_0^1 d\varrho\,\varrho\, m_z^2\left(\varrho, \frac{1}{2}, a\right)$, $\frac{a}{R} = \frac{a}{l_e}\frac{l_e}{R}$, a в ед. l_e, если R в ед. l_e.

$|\varkappa_s| \gtrsim 1$. $\varkappa_s < 0$ для easy axis anisotropy.

MCad, Hopfion stability-2.mcd: получен $\varepsilon_s(a,R,\beta) = -200$ для $R = 20$, $\beta = 2.10$ при $\varkappa_s = -1\ \text{erg/cm}^2$. Т.е., вклад $\varepsilon_s(a,R,\beta)$ в hopfion energy $-1.2\cdot 10^3$ для $R = 30$ при выключенном ман. Surface

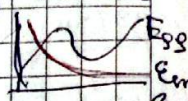
7/XII Построение Figs. с учетом E_{zz} и Fan. MCad перестал считать $E_{zz}(a,R,\beta) = \frac{\pi}{2} a^3 I_m(x,\beta)$ для $R = 20$, $\beta = 10$. Нарушен вклад в E_{zz} и пересчитано $27/E_1$ через $w(a) = E_{zz}/(4\pi M_s^2 V)$.

Escaneado con CamScanner

$E_{ss} = 2\pi^2 M_s^2 J_m(x,\beta) a^3 = 4\pi M_s^2 V \cdot W(a)$.

$\varepsilon_m^{ss} = \frac{1}{4\pi M_s^2 l_e^3} E_{ss} = \frac{4\pi M_s^2 \cdot \pi R^2 L \cdot W}{4\pi M_s^2 \cdot l_e^3} = \pi \left(\frac{R}{l_e}\right)^3 \beta\, W(a) = \varepsilon_m(a,R,\beta)$.

$\varepsilon_m(a,R,\beta) = \pi R^3 \beta\, W(a,\beta)$ и $E_{ss}(a,R,\beta) = \frac{\pi}{2} a^3 J_m(x,\beta)$ сильно



различаются как $f(a)$, $\varepsilon_m(a)$ не имеет min по a?

качественно их вариация. Видно, E_{ss} чувствительнее к пределам интегр. по k и/или по z?

$\varepsilon_m \sim W(a)$ слабо зависит от k_m, $\int_0^{k_m} dk (\dots)$, E_{ss} очень сильно.

$J_m(x,\beta,k_m)$ растет от 0.769 до 0.784 при $k_m = 100 \to 300$, при $x=1$, β. $J_m(x,\beta)$ - old и $J_m(x,\beta)$ не различаются при $\beta = 0$.

поэтому посчитаем $E_{ss}(a)$ через $J_m^{old}(x,\beta)$. (see Hopf-stability_1.mcd)

MCad не считает $R=20, \beta=10$ даже с $J_m^{old}(x,\beta) = 2\int_{-k_m}^{k_m} dz \int_0^{k_m} \frac{dk}{k} J_0^2(z,k,x)$,

$J_0(z,k,x) = -k \int_x^{\infty} ds\, s\, J_1(ks)\, m_s(sz)$.

Radial vortex W_G

3/XII) On-line session with Maciej, Mathieu, Gleb. Обсуждали свои расчеты $W_G(L, L_1, d, R)$ radial vortex in soft/hard nanodisks. Mathieu использовал для них mm-field $H_0 = 100$ Oe. Это non-linear regime. Mathieu сделал a) image $\vec{m}(\vec{r}, t)$, чтоб убедиться, что $\frac{\Delta x}{R} \approx 0.3$. $\Delta x / R \approx 0.3$

б) начинает H_0 до 10 Oe и посмотрел как это повлияет на $|\vec{x}|$ и W_G. Для узкого DMI с $|D| \le 2$ mJ/m^2.

Т.к. $R=20, \beta=10$ MCad не считает, поставим $\beta = 8$. Для Fig.3 (вместо K) возьмем $R=20, \beta=2$ и $K/\pi M_s^2 = +1, -0.5$.

ISD energy at finite K.

$\varepsilon(1SD) = \frac{\pi}{2} R^3 \beta N_{zz}(\beta)$ при $K=0$. При $K \neq 0$ надо пересчитать.

$E(SD) = \int dV \left[-K m_z^2 - \frac{1}{2} M_s \vec{m} \vec{H}_m \right] = 2\pi M_s^2 V N_{zz}(\beta) - KV$

($m_z = \pm 1$)

$\frac{KV}{4\pi \cdot M_s^2 l_e^3} = \frac{K \cdot \pi R^2 L}{4\pi M_s^2 l_e^3} = \frac{1}{2} \pi Q \beta \frac{R^3}{l_e^3}$, $Q = (2\pi M_s^2)^{-1} K = \frac{K}{2\pi M_s^2}$.

$\varepsilon(SD) = \frac{\pi}{2} R^3 \beta N_{zz} - \frac{\pi}{2} Q \beta R^3 = \frac{\pi}{2} R^3 \beta [N_{zz}(\beta) - Q]$.

Finished and submitted "Hopfion stability" MS to Nanomaterials.

9/XII) переписка с E. Navarro + W. Tong о замене Fig. 3 in the MS.

11/XII) Отказ с другими в рег. "Lukas", Словакия. Грант от Аркадия о "Maria Zambrano" post doc fellowship for foreigners. Посчитали порядка 54 k€

12/XII) Наши с другими solicited to UPV-grant на проведение conferences (6,000 €). Написали e-mail Зарко to Julian.

12/XII – 13/XII) переписка с Maciej о visit to Poznan. Feb. 4 – Feb. 15, 2024. Maciej reserved room #4 и обещал оплатить. позвонили.

14/XII) Взял себе и Лене aircrafticket to Poznan. 4–5 Feb. Lufthansa. Booked hotel in Bilbao на 15/II ночи. Послал standards info to Lyuba.

15/XII) Revised "Vortex EM in thick disks" MS и послал Аркадию и Гневу. Они MS сохранили для Nanoscale, но боялись wrong interpretation Or eigenmodes coupling, hybridization and anti-Crossing.

16/XII) Search and booking "Savoya" hotel in Bologna, ICM, June 30 – July 6, 24. Сл. "Eroski".

(17/XII) Замсаг и писмо to Nanomaterials referee's report на MS Hussain - Cottam „Spin waves in nano-rings with DMI". Classical spins. Кудов. решетка. Ур. Holstein-Primakoff. Черемисов - Отсъдил о BP-DW nanowires MS и Abstracts, ICM, Bologna.

(18/XII) Подготовка docs на пребиване TIE, Permiso de Residencia. Form EX-17, passport, и TIE copies.

(19/XII) Отваряне сейф TIE-appl. в Iolora „Santander", Photo в Urbil, нов пребиване. Обсъждане „GM thick dots" MS с Artem и Theo. T.к. coupling, hybridization of vortex modes в линейно и приближение sin, то Artem очаква hybridization of „uniform" mode Gus'JAP'02 с $\omega_G^n \sim 1/L^\beta$ exchange modes, to explain Fig. 1, $\omega_G^n(L)$ - simulated by MuMax, submit on Jan. 8

(20/XII) Замсаване „Vortex GM thick dots" в Overleaf. New Sec. по описание hybridization на собствените мод $\omega_0(L)$ - Gus'2002 и $\omega_G^n(L)$ - exch. approxim. Нови revised MS to Artem и G-BB. Неофициални comments Theo.

(21/XII) Visit to Policia, San Sebastian, подаване на New TIE-tarjetas. Завръщане през Зо Дунав. Cita para recogidas de tarjetas, 30 Jan. 2024 13:45 - 13:55.

Пристигане Referees reports от Nanomaterials на „Hopfion stability" MS: major revision, new Fig - image of hopfion и сравнение ξ с hopfion - $\vec{m}(\vec{r})$ взето от Gus'Chaos'2023.

Info „passport.mfa.gov.ua" за сделки в Киев passport Ангора.

(22/XII) С.П.В „Carrefour" Olaberria.

(22-24/XII) преработка „Hopfion stability" MS Nanomaterials.

S. Li et al. Phys. Rev. B 105, 174407 (2022). Exp. energy barrier преход hopfion to „toron" (нара BP). Hopfion in mag. film is stable ($L = 15$ nm, $w = 128$ nm, square dot), ecc. $K_0 = 1\ mJ/m^2$ is large. ΔE(hopfion → toron) $= 0.4 \cdot 10^{-18}$ J, при $M_S = 150$ G и $A = 0.16$ pJ/m.

$k_B T$ (room) $= 1.38 \cdot 10^{-23}$ J/K $\cdot$ 300 K $= 4.14 \cdot 10^{-21}$ J

ΔE(hopf → toron) $/ k_B T = 0.4 \cdot 10^{-18}$ J $/ 4.14 \cdot 10^{-21}$ J $= 0.097 \cdot 10^3$ K $= 97$.

Две toroidal hopfion en. barrier ΔE(hopf → SD) при $a \to 0$

при $R = 20 l_e$, $\Delta E = (1.5 - 0.6) \cdot 10^4$ $(R=20, \beta=2)$, $\xi = \frac{1}{4\pi M_S^2 l_e^3} E(\text{cm})$.

En. Scale $4\pi M_S^2 l_e^3 = 4\pi \cdot (0.837)^2 \cdot 10^6\ G^2 (5\ nm)^3 = 12.56 \cdot 0.7006 \cdot 10^6 \frac{erg}{cm^3} \cdot 125 \cdot 10^{-21} cm^3 =$

$= 1099.9 \cdot 10^{-15}$ erg $= 1.10 \cdot 10^{-12}$ erg.

$\Delta E = \Delta E \cdot 1.10 \cdot 10^{-12}$ erg $= 0.9 \cdot 10^4 \cdot 1.10 \cdot 10^{-12}$ erg $= 0.99 \cdot 10^{-8}$ erg $=$

$= 0.99 \cdot 10^{-2} \cdot 10^{-7}$ J $= 0.99 \cdot 10^{-15}$ J. $\frac{\Delta E}{k_B T} = \frac{0.99 \cdot 10^{-15}\ J}{4.14 \cdot 10^{-21}\ J} = 0.24 \cdot 10^6 = 2.4 \cdot 10^5$.

(giant, Сн. 26/XI)

Точно за $R = 20 l_e$, $\beta = 2$, $\Delta E / k_B T = 2.39 \cdot 10^5$.

$R = 20 l_e = 100$ nm, $\beta = 2 \to L = 200$ nm. У 15-PRB'2022 square dot $L = 15$ nm, $w = 128$ nm, значително по-малко обхващ за сравнение.

Тъпичка. За $R = 20 l_e$, $\beta = 8$, $\Delta E = (2.62 - 1.38) \cdot 10^4 = 0.24 \cdot 10^4$ и

$\frac{\Delta E}{k_B T}(R = 8\,l_e, \beta = 8) = \frac{0.24}{0.9} \cdot 2.39 \cdot 10^5 = 1.97 \cdot 10^5$

(25/XII) Отговор to referees. Bloch-like Néel-hopfion? Exp. на разлика при пресмятането E_{ex}, DMI и W_m? Сн. 8/X.

Hopfion $\Theta(\vec{r}) = \Theta(\rho, z)$, $\Phi(\vec{r}) = m\varphi + m\beta$, $\beta = \beta(\rho, z)$ poloidal angle.

$m_x + i m_y = \sqrt{1 - m_z^2}\, e^{i(m\varphi + m\beta)} = \sin\Theta\, e^{i\Phi}$

$k = 1$, $k = 1$, $\Phi(\vec{r}) = \Phi(\varphi, \beta) = \varphi + \beta$. normal $\varphi \to \varphi + \varphi_0$?

Toroidal φ_0 $\Phi = m\varphi + \varphi_0 + m\beta$?

$m_x + i m_y = \sqrt{1 - m_z^2}\, e^{i(m-1)\varphi + \varphi_0 + m\beta}$, $m_x + i m_y = \sqrt{1 - m_z^2}\, e^{i(\beta + \varphi_0)}$

a phase $\varphi_0 = 0$. in the Hopfion stability MS (for $n = 1$)

$m_x = \sin\Theta \cos(\beta + \varphi_0)$, $m_y = \sin\Theta \sin(\beta + \varphi_0)$, $\chi = m\beta = \beta$, $m = 1$.

$E_{ex} \propto (\nabla\Theta)^2 + \sin^2\Theta (\nabla\Phi)^2$, $\nabla\Phi = \nabla(\varphi + \varphi_0 + \beta) = \nabla(\varphi + \beta)$, $\nabla\varphi_0 = 0$,

E_{ex} не зависит от φ_0. А E_{DM1} и $E_m = E_m^{33} + E_m^{22}$ вырожд будут зависеть от φ_0

Chui-Chaos' 2023: $w = \frac{Z_2}{Z_1}$, $m_x + i m_y = \frac{2w}{1 + |w|^2}$, $m_z = \frac{1 - |w|^2}{1 + |w|^2}$. $Z = \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}$ спинор

$Z_1 = X_1 + i X_2$, $Z_2 = X_3 + i X_4$. $Z_1 = -\frac{1}{\mathrm{ch}\chi} e^{-i\beta}$, $Z_2 = \mathrm{th}\chi\, e^{i\varphi}$

$w = -\mathrm{th}\chi\, e^{i(\varphi + \beta)}$. Замена $\beta + \varphi \to \beta + \varphi + \varphi_0$ эквивалентна замене $Z_2 \to Z_2 e^{i\varphi_0}$

$(X_1 + i X_2) \to (X_1 + i X_2) e^{i\varphi_0}$, $X_1 \to X_1 e^{i\varphi_0}$, $X_2 \to X_2 e^{i\varphi_0}$, т.е.

дублет X_1, X_2. Но по определению hyperspherical S_4 X_i coordinates

$m_i = \frac{2X_i}{(1 + X^2)}$, $X_i = x, y, z$ и X_j не могут быть комплексными.

т.е. standard Hopfion formula + mapping $R_3(xyz)$ на $S_2(m^2 = 1)$ не позволяет φ_0.

(26/XII) Отобрал References и revision, Hopfion stability + MS.

По магнитным свойствам reference на Nanomaterials web-site.

• Lin-Zhang, PRB '2018: a) Есть phase diagram hopfion stability для cylindrical dot $\frac{L}{L_D}$ [stable], b) MAP (monopole antimonopole pair) state between hopfion & uniform

c) есть ен. барьер для перехода hopfion → MAP

$\Delta E \cong 0.1 \cdot 10^{-18}$ J, $\Delta E / kT = 0.1 \cdot 10^{-18}$ J $/ (4.14 \cdot 10^{-21}$ J$) = \frac{10^{-18}}{4.14 \cdot 10^{-21}} \approx 0.242 \cdot 10^3$

$\frac{\Delta E(\text{hopf} \to \text{MAP})}{k_B T} = 242$. У Liu et al PRB 2022 $\frac{\Delta E(\text{hopf} \to \text{toron})}{k_B T} = 97$.

$L_D = 4\pi \frac{A}{D} = 4\pi \frac{2.2\ \text{pJ/m}}{0.395\ \text{mJ/m}^2} = 4\pi \frac{2.2}{0.395} \frac{10^{-12}}{10^{-3}}$ m ≈ 70 nm.

$\Delta E / k_B T$ Lin-Zhang подходит для $L \approx 70$ nm, $R = 100$ nm ($\beta = 0.7$).

У меня это при $R = 100$ nm, $\beta \geq 2$, т.е. $L \geq 200$ nm, $\Delta E \sim U_{dot}$, поэтому разница dot и infinite не роль ΔE.

(27/XII) Отобрал Reference #3 Nanomaterials, re-submitted "Hopfion Stability" MS to Nanomaterials (обзвон возврат ~ the 20th).

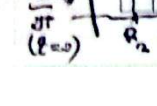
(28/XII) • F. Zheng, N.S. Kiselev, F.N. Rybakov et al. "Hopfion rings in cubic chiral magnet" Nature 623, 718–723 (2023). Hopfion rings + skyrmion strings.

FeGe, $A = 4.35$ pJ/m, $D = 0.853$ mJ/m^2, $M_S = 384$ G. Experiment на cylindrical dot $L = 180$ nm, $R = 250$ nm.

• Hopfion ring ansatz, Eqs (4)–(5): $\Theta(\vec{r}) = \pi\left[1 - \frac{1}{R_1}\sqrt{(R_2 - s)^2 + z^2}\right]$, R_1 – minor, R_2 – major torus radius

$\Phi(\vec{r}) = \mathrm{arctg}\left(\frac{y}{x}\right) + \mathrm{arctg}\left(\frac{z}{R_2 - s}\right) - \frac{\pi}{2}$. Как это? Разбей это toroidal hopfion?

$m_z = \cos\Theta$, $m_z = \mathrm{const} \to R_1 = \sqrt{(R_2 - s)^2 + z^2} = \mathrm{const}$.



Escaneado con CamScanner

Two toroidal hopfion $m_z = m_z(\gamma)$, $m_z = \text{const}$, $\gamma = \arctan\left(\frac{2a\rho}{\rho^2+z^2+a^2}\right)$

$\gamma = \text{const} \to 2a\rho = c\cdot(\rho^2+z^2+a^2)$. У Zheng 2023 $R_1 = \sqrt{(R_2-\rho)^2+z^2} = c$ или $(R_2-\rho)^2 + z^2 = c_1$. $\frac{1}{c}\,2a\rho = (\rho-a)^2 + z^2 + 2a\rho$. // m_z - Zheng '23 и Gus 'chaos '23 близко обозначения.

Zheng '23 $\Phi(\vec{r}) = \varphi - \arctg\left(\frac{z}{R_2-\rho}\right) - \pi/2$

Gus' Chaos '23: $\Phi(\vec{r}) = \varphi + \beta(\rho,z) = \varphi + \text{arctg}\left(\frac{2az}{\rho^2+z^2-a^2}\right)$ (m=1, n=1) — big difference

Emergent field: $B_\lambda = \frac{1}{2}\varepsilon_{\lambda\mu\nu}F_{\mu\nu} = \frac{1}{2}\varepsilon_{\lambda\mu\nu}\vec{m}\cdot(\partial_\mu\vec{m}\times\partial_\nu\vec{m})$

$B_1 = \frac{1}{2}\varepsilon_{1\mu\nu}\vec{m}(\partial_\mu\vec{m}\times\partial_\nu\vec{m}) = \frac{1}{2}\varepsilon_{123}\vec{m}(\partial_2\vec{m}\times\partial_3\vec{m}) + \frac{1}{2}\varepsilon_{132}\vec{m}(\partial_3\vec{m}\times\partial_2\vec{m}) =$

$= |\varepsilon_{123}=+1, \varepsilon_{132}=-1| = \vec{m}\cdot(\partial_2\vec{m}\times\partial_3\vec{m})$

Hopf index: „for a magnetic texture $\vec{m}(\vec{r})$ localized within 3D domain $\Omega \subseteq R^3$, with a fixed magnetization $\vec{m}(\partial\Omega) = \vec{m}_0$ at the domain boundary $\partial\Omega$ obey $\pi_3(S_2, \vec{m}_0) = Z$, i.e. $R^3 \to S_2$ mapping degree Z is integer, Hopf index is integer, H is integer.

Two $\Omega = R^3$ это неверно, но если Ω is restricted, fixed domain, to задав $\vec{m}(\partial\Omega) = \vec{m}_0$ получим integer Hopf index Q_H - это новое (!?)

Ref. V. Arnold, The geometry of vector fields, Routledge, 2013 - check.

Our toroidal hopfion $\vec{m}_0 = \pm\hat{z}$ и выполнялся на ∞ $(\rho,z)\to\infty$!

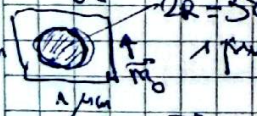
Реализуется и $\vec{m}(\rho=R) = \vec{m}_0$, $\vec{m}(z=\pm L/2) = \vec{m}_0$ для цилиндра dot ???

boundary conditions

$Q_H(\vec{m}_0) = -\frac{1}{16\pi^2}\int_\Omega dV\,\vec{B}\cdot\vec{A}$, $\vec{A} = (\nabla\times)^{-1}\vec{B}$, $\vec{B} = \text{rot}\,\vec{A} = \nabla\times\vec{A}$ - Eq. (16).

(20) $\vec{A} = (\nabla\times)^{-1}\vec{B} = \int dx\,\vec{B}\times\hat{e}_x$. Верно ли такое определение of emergent field vector potential (Gus 'Chaos' 2023)? // $\vec{m}_0 = \hat{e}_z$ (Zheng '23)

~ на границах of $\mu m \times \mu m \times 180$ nm computation rectangular box (dot size $\Rightarrow$ $2R = 250$ nm, $L = 180$ nm). т.е. Zheng '23 вычисл. для Q_H ~ $\int dV(-)$ по Ω - rectangular box вблизи где нет magnetic material.



1 μm × 1 μm in-plane computation box, где $\vec{m}(\partial\Omega) = \vec{m}_0$

$-\text{rot}\,\vec{A} = \vec{B} = \text{rot}\left(\int dx\,\vec{B}\times\hat{e}_x\right)\cdot\vec{B}\times\vec{e}_x = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ B_x & B_y & B_z \end{vmatrix} = \vec{e}_y B_z - \vec{e}_z B_y$

$B_x = \text{rot}_x\left(\int_x \vec{B}\times\vec{e}_x\right) = \frac{\partial}{\partial y}A_z - \frac{\partial}{\partial z}A_y$, $A_z = \hat{z}\cdot\int dx\,\vec{B}\times\hat{x} = \int dx\,\hat{z}\cdot(\vec{B}\times\hat{x}) =$

$= \int dx\,\hat{z}\cdot(\hat{y}B_z - \hat{z}B_y) = -\int dx\,B_y$, $A_y = \hat{y}\int dx\,\vec{B}\times\hat{x} = \int dx\,B_z$

$B_x = \frac{\partial}{\partial y}(-)\int dx\,B_y - \frac{\partial}{\partial z}\int dx\,B_z$ - это не совпадает с $B_x = B_x$.

→ Hansen (LibGen) ссылка Arnold, The geometry of vector fields, 2020.

// Там нет ур. $\vec{A} = \int dx\,\vec{B}\times\hat{e}_x$, $\vec{B} = \text{rot}\,\vec{A}$, но есть интересные equations for Hopf mapping $S_3 \to S_2$ (pp. 45-48). // Проверить $\vec{B}$ и $\text{div}\,\vec{B} = 0$

28/XII Оставил Cash 2 600 € (см. 5/I). Эти ангаты Zheng '2023.

С.Л. и Ю. в San Sebastian: Cafe „La Kontxa", Eroski, Urbil.

30/XII С.Л. „Mercadona", „Lidl", Carrefour Market. E-mails to Andersen и Ming (sponsor). Под спец IVA XII ub. и приобрели CONSTANTO '23. Aramburu 39€

1/I 2024 План работы (Сч. 19/IX)

1) Vortex string GM nanowires, hopfcrystals ω_n, Osc. simulations
2) BP-DW $\vec{G}$ - dynamics nanowires, Osc. simulations (Feb 29, 2024)
3) distinguished scientist UPV/EHU appl.
4) El. Signature application - restore
5) MS - Gypenosh thick $\frac{1}{3}$ dots, localized SW mode - Subm. 23/05/24 PRA
6) MS Zelent, FM ring over SK nanodot, $\vec{H}_{inj}(t)$
7) CV - soft/hard nanopillar Glob/Moalic
8) % bozbyar contender, new Hipoteca 1,22% fixed, new law Hipoteca
9) Direc Iberbasque, Dorer UPV/EHU za 2023
10) Hopf. stability, Nanomaterials, waive APC
11) Abstracts ICM'24 Bologna: hopfions - hopfion $\vec{G}$
12) Guss police R.N.A. - 30/IZuz., NIE tarjetas
13) A. passport status # 10334779
14) Artem's MS vortex GM thick dots → Nanoscale Horizons, subm. 5/IV
15) Ommont Zheng 'Nature' Hopfions '2023, sent to authors 10/III, Subm. 19/III
16) IVA Constanso" za 29/I 2024
17) Биметь в Poznań, Oct. 2024.

(1/I) Найдем $\vec{B}$ emergent field по ansatz Zheng '23 и проверим $\operatorname{div}\vec{B}=0$.

$$\Theta(\vec{r})/\pi = 1 - \frac{1}{R_1}\sqrt{(R_2-\rho)^2+z^2},\quad \Phi(\vec{r}) = \varphi - \operatorname{arctg}\left(\frac{z}{R_2-\rho}\right) - \pi/2$$

Standard $\Theta(\vec{r}) = \Theta(\rho,z)$, $\Phi(\vec{r}) = N\varphi + \gamma(\rho,z) + \Phi_0$ - Popadiuk 'RRL' 2023

$N=1$, $\gamma(\rho,z) = -\operatorname{arctg}\left(\frac{z}{(R_2-\rho)}\right)$ берем. Gus' chaos $\gamma(\rho,z) = \beta(\rho,z) = \operatorname{arctg}\left(\frac{2az}{\rho^2+z^2-e^2}\right)$ give toroidal hopfion

$\vec{B} = (B_\rho, B_\varphi, B_z)$, $B_\rho = -\frac{1}{\rho}\sin\Theta\frac{\partial\Theta}{\partial z}$, $B_\varphi = \sin\Theta\left[\frac{\partial\Theta}{\partial z}\frac{\partial\gamma}{\partial\rho} - \frac{\partial\Theta}{\partial\rho}\frac{\partial\gamma}{\partial z}\right]$, $B_z = \frac{\sin\Theta}{\rho}\frac{\partial\Theta}{\partial\rho}$

$$\operatorname{div}\vec{B} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho B_\rho) + \frac{1}{\rho}\frac{\partial B_\varphi}{\partial\varphi}^{=0} + \frac{\partial B_z}{\partial z} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[-\sin\Theta\frac{\partial\Theta}{\partial z}\right] + \frac{\partial}{\partial z}\left[\frac{\sin\Theta}{\rho}\frac{\partial\Theta}{\partial\rho}\right] =$$

$$= \frac{1}{\rho}\left\{-\frac{\partial\rho\sin\Theta}{\partial\rho}\frac{\partial\Theta}{\partial z} - \rho\sin\Theta\frac{\partial^2\Theta}{\partial\rho\partial z} + \frac{\partial\rho\sin\Theta}{\partial z}\frac{\partial\Theta}{\partial\rho} + \rho\sin\Theta\frac{\partial^2\Theta}{\partial z\partial\rho}\right\} =$$

$$= \frac{1}{\rho}\left\{-\rho\cos\Theta\frac{\partial\Theta}{\partial\rho}\frac{\partial\Theta}{\partial z} + \rho\cos\Theta\frac{\partial\Theta}{\partial z}\frac{\partial\Theta}{\partial\rho}\right\} = 0,\ \operatorname{div}\vec{B}\equiv 0$$ given

any $\Theta(\rho,z)$, $\Phi(\vec{r}) = N\varphi + \gamma(\rho,z)$, but $\Theta(\rho,z)$, helicity $\gamma(\rho,z)$ не важно

Важно, что $\frac{\partial B_\varphi}{\partial\varphi} = 0$, axially symmetric $\vec{B}(\vec{r}) = \vec{B}(\rho,z)$.

Ur $\vec{A} = \int dx\,\vec{B}\times\hat{e}_x$ следует $A_z = -\int dx B_y$, $A_y = \int dx B_z$, $\int dx(\cdot) = \int_0^x (\cdot)$

Check $B_x = \frac{\partial}{\partial y}A_z - \frac{\partial}{\partial z}A_y = \frac{\partial}{\partial y}(-)\int dx B_y - \frac{\partial}{\partial z}\int dx B_z = -\int dx\left[\frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}\right] =$

$= \left|\operatorname{div}\vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0\right| = +\int dx\frac{\partial B_x}{\partial x} = B_x(x,y,z)\Big|_{x_0}^{x} = B_x(x,y,z) - B_x(x_0,y,z)$

$B_x(x,y,z) = B_x(x,y,z) - B_x(x_0,y,z)$, надо $B_x(x_0,y,z) = 0$

$B_x = B_\rho\cos\varphi - B_\varphi\sin\varphi = -\frac{1}{\rho}\sin\Theta\frac{\partial\Theta}{\partial z}\cos\varphi - \sin\Theta\left[\frac{\partial\Theta}{\partial z}\frac{\partial\gamma}{\partial\rho} - \frac{\partial\Theta}{\partial\rho}\frac{\partial\gamma}{\partial z}\right]\sin\varphi$

$B_x(x_0) \to 0$, какое x_0? Будем $x_0\to\infty$. См. VIII.I, $x_0\to\infty$ на и near спасибо.

$$\frac{\partial\Theta}{\partial z} = \pi\frac{\partial}{\partial z}\left[1 - \frac{1}{R_1}\sqrt{(R_2-\rho)^2+z^2}\right] = \pi(-1)\frac{1}{R_1}\frac{2z}{2\sqrt{(R_2-\rho)^2+z^2}} = -\frac{\pi}{R_1}\frac{z}{\sqrt{(R_2-\rho)^2+z^2}} \to 0,\ x_0\to\infty$$

$$\frac{\partial\Theta}{\partial\rho} = \pi\frac{\partial}{\partial\rho}\left[1 - \frac{1}{R_1}\sqrt{(R_2-\rho)^2+z^2}\right] = -\frac{\pi}{R_1}\frac{(\rho-R_2)}{\sqrt{(R_2-\rho)^2+z^2}} \to \frac{\pi}{R_1}\sin(\rho-R_2)$$ finite

$$\frac{\partial\gamma}{\partial z} = (-1)\frac{\partial}{\partial z}\operatorname{arctg}\left(\frac{z}{R_2-\rho}\right) = (-1)\frac{1}{1+z^2/(R_2-\rho)^2}\frac{1}{(R_2-\rho)} = -(R_2-\rho)/[(R_2-\rho)^2+z^2] \to 0,\ x_0\to\infty$$

$B_x(x_0) \to 0$ при $x_0 \to \infty$. No Simul. area square dot $1 \times 1 \mu m$.

И где взять $x_0 \to \infty$? $B_x(x, y_2) = B_x(x, y_2) - B_x(x_0, y_2)$.

Проверим B_z. $B_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$, $\vec{A} = \int dx \vec{B} \times \hat{e}_x = \int dx [\hat{e}_y B_z - \hat{e}_z B_y]$

$A_y = \hat{y} \cdot \vec{A} = \hat{e}_y \cdot \vec{A} = \int dx B_z$, $A_x = \hat{e}_x \cdot \vec{A} = \hat{e}_x \cdot \int dx [\hat{e}_y B_z - \hat{e}_z B_y] = 0$

$B_z = \frac{\partial A_y}{\partial x} = \frac{\partial}{\partial x} \int dx B_z = \frac{\partial}{\partial x} \int_{x_0}^{x} d\xi B_z(\xi, y_2) = B_z(x, y_2)$ — Ok

$A_x = 0$

Проверим B_y. $B_y = \frac{\partial A_x}{\partial z} - \partial A_z / \partial x$, $A_z = -\int dx B_y$

$B_y = -\frac{\partial}{\partial x} \int dx (-1) B_y = \frac{\partial}{\partial x} \int^{x} d\xi B_y(\xi, y_2) = B_y(x, y, z)$ - Ok.

Выводы из Zhong'2023 $\vec{A} = \int_{x_0}^{x} dx \vec{B} \times \hat{e}_x$:

1) $A_x = 0$, $\vec{A} = (0, A_y, A_z)$; 2) B_y и B_z хорошо определены, а вот с $B_x(x, y_2)$ сложности; B_x undefined в finite sample (!) Вопрос Zheng Eq. 8?

3) $Q_H \sim \int dV \vec{A} \cdot \vec{B}$ и B_x не дает вклада в Q_H, т.к. $A_x = 0$.

4) B_x можно найти одно из $\sin\Theta \nabla\Theta \times \nabla\Phi = \vec{B}$ и определить $B_x(x_0, y_2)$ — как.

$A_y = \int dx B_z$, $A_z = -\int dx B_y$.

$\int dV \vec{A} \cdot \vec{B} = \int dV [A_y B_y + A_z B_z] = \int dV \left[B_y \int_{x_0}^{x} d\xi B_z(\xi, y_2) - B_z \int_{x_0}^{x} d\xi B_y(\xi, y_2)\right]$

$\int dV \vec{A} \cdot \vec{B} = \int [A_s B_s + A_\varphi B_\varphi + A_z B_z] dV$, $A_s = A_x \cos\varphi + A_y \sin\varphi = A_y \sin\varphi$

$A_\varphi = -A_x \sin\varphi + A_y \cos\varphi = A_y \cos\varphi$

$\int dV \vec{A} \cdot \vec{B} = \int dV [A_y \sin\varphi B_s + A_y \cos\varphi B_\varphi + A_z B_z]$, $B_s, B_\varphi, B_z = f(s, z)$.

? Как A_y, A_z зависят от φ, чтоб $\int dV = \int ds\, s\, d\varphi\, dz$ брался. $A_y, A_z = F(s, z)$?

$A_y = \int^{x} d\xi B_z(\xi, y_2)$, $B_z(\xi, y_2) = B_z(s, z) = B_z(\sqrt{\xi^2 + y^2}, z)$.

$A_y = \int_{x_0}^{x} d\xi B_z(\sqrt{\xi^2 + y^2}, z)$, $B_z = \frac{1}{s} \sin\Theta \frac{\partial\Theta}{\partial s}$, $\frac{\partial\Theta}{\partial s} = -\frac{\pi}{A_1} \frac{(s - R_2)}{\sqrt{(s - R_2)^2 + z^2}}$

$\frac{\partial A_y}{\partial \varphi} = \frac{\partial}{\partial \varphi} \int_{x_0}^{x} d\xi B_z(\sqrt{\xi^2 + y^2}, z) = \leftarrow$

$\hookrightarrow = \frac{\partial x}{\partial \varphi} B_z(\sqrt{x^2 + y^2}, z) - \frac{\partial x_0}{\partial \varphi} B_z(\sqrt{x_0^2 + y^2}, z)$, $x = s\cos\varphi$, $\frac{\partial x}{\partial \varphi} = -\sin\varphi \cdot s$.

Должно $\frac{\partial A_y}{\partial \varphi} = 0$, реально $\frac{\partial A_y}{\partial \varphi} \neq 0$, т.к. $\frac{\partial x}{\partial \varphi} = -s\sin\varphi \neq 0$.

$B_y = B_\varphi \cos\varphi + B_s \sin\varphi$

$\int dV \vec{A} \cdot \vec{B} = \int dV B_y(\vec{r}) \int_{x_0}^{x} d\xi B_z(\xi, y_2) - \int dV B_z(s, z) \int_{x_0}^{x} d\xi B_y(\xi, y_2) =$

$= \int dV [B_\varphi(s_2) \cos\varphi + B_s(s_2) \sin\varphi] \int_{x_0}^{x} d\xi B_z(\sqrt{\xi^2 + y^2}, z) -$

$- \int dV B_z(s, z) \int_{x_0}^{x} d\xi B_y(\xi, y_2)$, $F(x, x_0) = F(\varphi, s) = \int_{x_0}^{x} d\xi B_y(\xi, y_2)$, $x = s\cos\varphi$, $x_0 = s_0 \cos\varphi$

$\int_0^{2\pi} d\varphi F(\varphi, s) = \int_0^{2\pi} d\varphi F(\cos\varphi)$, $F(\cos\varphi) = F(d)$, $F(-d) = F(-\cos\varphi) =$

$= F(-x, -x_0) = \int_0^{-x} d\xi B_y(\xi, y_2) = \int_{x_0}^{x} d\xi B_y(-\xi, y_2)$, если $B_y(-\xi) = B_y(\xi)$ тог.

то $F(-d) = F(d)$ — такие тоже. $F(d) = F(\cos\varphi)$ — тог. $\Rightarrow \int_0^{2\pi} d\varphi F(\cos\varphi) \neq 0$.

$B_y(x, y_2) = B_\varphi(s_2) \cos\varphi + B_s(s_2) \sin\varphi$. $x \to -x$, сим. бог. $\varphi \to \varphi + \pi$

$\cos(\varphi + \pi) = -\cos\varphi$, $\sin(\varphi + \pi) = -\sin\varphi$

$B_y(-x, y_2) = B_\varphi(s_2) \cos(\varphi + \pi) + B_s(s_2) \sin(\varphi + \pi) = -B_\varphi \cos\varphi - B_s \sin\varphi = -B_y(x, y_2)$

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$B_y(-x) = -B_y(x)$ нечет. по x. $F(-d) = -\int_{x_0}^{x} d\xi\, B_y(\xi, y_z) = -F(d)$ нечет.

$F(d) = F(\cos\varphi)$ — нечет. $\int_0^{2\pi} d\varphi\, F(\cos\varphi) \equiv 0$, $\int dV\, B_z \int_{x_0}^{x} d\xi\, B_y(\xi, y_z) \equiv 0$.

$$\int dV\, \vec{A}\cdot\vec{B} = \int dV\, B_y(\vec{r}) \int_{x_0}^{x} B_z(\xi, y_z)\, d\xi =$$

$$= \int dV \left[B_\varphi(\rho_z)\cos\varphi + B_\rho(\rho_z)\sin\varphi \right] \int_{\rho_0\cos\varphi}^{\rho\cos\varphi} d\xi\, B_z\left(\sqrt{\xi^2+y^2}, z\right).$$

$J(\alpha) = \int_{\rho_0\cos\varphi}^{\rho\cos\varphi} d\xi\, B_z\left(\sqrt{\xi^2+y^2}, z\right) = J(\cos\varphi) = J(\alpha)$. $J(-\alpha) = \int_{-\rho_0\cos\varphi}^{-\rho\cos\varphi} d\xi\, B_z(\xi) =$

$= -\int_{\rho_0\cos\varphi}^{\rho\cos\varphi} d\xi\, B_z(-\xi)$, $B_z(\xi) = B_z\left(\sqrt{\xi^2+y^2}\right)$ зависит по ξ. $B_z(-\xi) = B_z(\xi)$

$J(-\alpha) = -\int_{\rho_0\cos\varphi}^{x} d\xi\, B_z(\xi) = -J(\alpha)$. нечет. $\int dV \left[B_\varphi\cos\varphi + B_\rho\sin\varphi\right] F(\cos\varphi)$ нечет.

$\int_0^{2\pi} d\varphi \cos\varphi\, F_{odd}(\cos\varphi) \neq 0$, $\int_0^{2\pi} d\varphi \sin\varphi\, F_{odd}(\cos\varphi) \equiv 0$.

$\int dV(\vec{A}\cdot\vec{B}) = \int dV\, B_\varphi(\rho_z)\cos\varphi \int_{\rho_0\cos\varphi}^{\rho\cos\varphi} d\xi\, B_z\left(\sqrt{\xi^2+y^2}, z\right)$. $\int dz\, B_\rho(\rho_z) B_z\left(\sqrt{\xi^2+y^2}, z\right)$ — зависит по z?

- $B_z(\rho,z) = \frac{1}{\rho}\sin\theta\frac{\partial\theta}{\partial\rho}$, $\theta(\rho,z)$ зав. по "z" $\to B_z(\rho,z)$ — зав. по z.

$B_\varphi = B_\varphi(\rho,z) = \sin\theta\left[\frac{\partial\theta}{\partial z}\frac{\partial\psi}{\partial\rho} - \frac{\partial\theta}{\partial\rho}\frac{\partial\psi}{\partial z}\right]$, $\frac{\partial\theta}{\partial z}$ нечет. по z, $\frac{\partial\psi}{\partial\rho}$ нечет. по z.

$\frac{\partial\theta}{\partial\rho}$ — зав. по "z", $\frac{\partial\psi}{\partial z}$ — зав. по "z". $B_\varphi(\rho,z)$ — зав. по "z". $\int dz\, B_\varphi(z) B_z(z) \neq 0$

$Q_H \sim \int dV(\vec{A}\cdot\vec{B}) = \int dV\, B_\varphi(\rho_z)\cos\varphi \int_{x_0}^{x} d\xi\, B_z\left(\sqrt{\xi^2+y^2}, z\right)$, $x = \rho\cos\varphi$, $x_0 = \rho_0\cos\varphi$. (!).

$B_\varphi(\rho_z)$, $B_z(\rho_z)$ — чувствительны. $Q_H \sim \int dV(\vec{A}\cdot\vec{B})$ — можно найти в MCad.

(2/I) Mathieu начинает new simulations soft/hard nanopillars. Among (Landmarks Matters arising to Zheng "Nature" 2023. Emus.

(3/I) начинает Hopfion stability "MS Nanomaterials, по ВРС. Modic

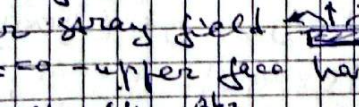
Modic simulations: • 1) profiles $m_z(\rho)$ soft layer для $2R = 40 \div 54$ nm

- $m_z(\rho = R) \approx 0.98 - 0.99$. Это не radial vortex, а 1 SD dot.

Можно расчет WG 3rd не 3D-зависимость, а 1SD-radial SW?

• 2) $H_z(t) = A\sin(\omega_0 t)$, $A = 5 \div 455$ Oe (!). Н/м $A = H_0 = 5$ Oe мала $F(\omega)$ нет, н/м $A = 80 \div 455$ Oe, мало н/м $\omega_G/2\pi \approx 3.76$ GHz. $\omega_G(A)$ сильно растет с насыщением. $A > 100$ Oe это граница, non-linear regime. check $H_z(\omega)$.

• 3) hard layer stray field $H_d^z(\rho)$, $z=0$ — upper face hard layer. нет hard layer для $2R = 30 \div 60$ nm.



$H_d^z(\rho,z) = \frac{\pi}{2} M_{S1} \int dk\, e^{-\beta k z}(1 - e^{-\beta_1 k}) J_0(k\rho) J_0(k)$, z - в ед. soft layer thickness L, $\beta = L/R$, $\beta_1 = L_1/R$, L_1 - hard layer thickness.

M_{S1} (hard) $= 1000$ G, $\mu_0 M_{S1} = 1.26$ T. $H_d^z(\rho=0, z=0) = 2\pi M_{S1} \int dk (1 - e^{-\beta_1 k}) J_1(k)$

$H_d^z(0,0)/\mu_0 M_{S1} = \frac{1}{2}\int dk (1 - e^{-\beta_1 k}) J_1(k)$ растет с $\beta_1 = L_1/R$ (много ур. Кажется L_1

• у Modic? Возможно $L_1 = 30$ nm, тогда $R = 15$ nm, $\beta_1 = \frac{30}{15} = 2$. $H_d^z/\mu_0 M_{S1} = 0.464$

или $0.464 \cdot 1.26\,T = 0.585\,T$ (у Modic $H_d^z(0,0) \approx 0.47\,T$); $R = 30$ nm, $\beta_1 = 1$,

$H_d^z(0,0)/\mu_0 M_{S1} = 0.321$, $H_d^z(0,0) = 0.321 \cdot 1.26\,T = 0.404\,T$ (у Modic $H_d(0,0) = 0.39\,T$)

т.е. $H_d^z(\rho, z=0)$ Modic меньше чем $H_d^z(\rho, z=0) \approx$ на 20%

можно в simulations Modic брал $M_S = 810$ G, а не $M_{S1} = 1000$ G.

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2) Sinc-function $H_\omega(t) = H_0 \operatorname{Sinc}(\omega_m t)$, $\operatorname{Sinc}(x) = \frac{\sin(x)}{x}$, $x = \omega_m t$.

24/IX 2022

$f(t) = \operatorname{Sinc}(\omega_m t)$, $\hat{f}(\omega) = \int_{-\infty}^{\infty} dt\, f(t) e^{-i\omega t} = \frac{1}{\omega_m} \operatorname{rect}(\omega/2\pi a)$ - ?

Другое определение Sinc-функции $f(t) = \operatorname{Sinc}(at)$, $\operatorname{Sinc}(x) = \frac{\sin(\pi x)}{\pi x}$

$f(t) = \operatorname{Sinc}(at) = \frac{\sin(\pi a t)}{\pi a t}$ - нормализованная Sinc-функция.

$\hat{f}(\omega) = \int_{-\infty}^{\infty} dt\, f(t) e^{i\omega t}$, $\hat{f}(\omega) = \frac{1}{a} \operatorname{rect}\left(\frac{\omega}{2\pi a}\right)$, $\operatorname{rect}(x)$: 1 на $[-\frac{1}{2}, \frac{1}{2}]$

Max $x = \frac{1}{2}$, $\frac{\omega_m}{2\pi a} = 1/2$, $\omega_m = \pi a$. f - не нормализованная Sinc-функция.

$a = 2\left(\frac{\omega_m}{2\pi}\right) = 2 f_m$, $[a] = $ Гц. У Matlic $\operatorname{Sinc}(\omega_m t)$, $\omega_m = 2\pi f_m$, т.е. Matlic использует не нормализованную Sinc-функцию $\operatorname{Sinc}(x) = \frac{\sin x}{x}$.

$\operatorname{Sinc}_M(\omega_m t) = \frac{\sin(\omega_m t)}{\omega_m t} = \frac{\sin(\pi a t)}{\pi a t} = \operatorname{Sinc}_N(at) = f(t)$

$\hat{f}(\omega) = \int dt e^{-i\omega t} f(t) = \frac{1}{a} \operatorname{rect}\left(\frac{\omega}{2\pi a}\right) = \frac{1}{a} 2a \operatorname{rect}\left(\frac{\omega}{2\omega_m}\right)$

$\operatorname{rect}(x)$: 1 на $[0, 1/2]$, x, $\max x = \frac{\omega}{2\omega_m} = 1/2$, $\omega = \omega_m$

$\hat{f}(\omega)$: 1 на $[0, \omega_m]$, ω

$H_\omega(t) = H_0 \operatorname{Sinc}(\omega_m t)$, $H_\omega(\omega) = \int dt e^{-i\omega t} H_\omega(t)$

$H_\omega(\omega) = \int dt e^{-i\omega t} \operatorname{Sinc}_M(\omega_m t) H_0 = H_0 \hat{f}(\omega) = H_0 \frac{\pi}{\omega_m} \operatorname{rect}\left(\frac{\omega}{2\omega_m}\right)$

$H_\omega(t) = H_0 \operatorname{Sinc}_M(\omega_m t) \to H_\omega(\omega) = H_0 \frac{\pi}{\omega_m} \operatorname{rect}\left(\frac{\omega}{2\omega_m}\right)$ то при $\omega \le \omega_m$.

$H_\omega(\omega) = \frac{\pi H_0}{\omega_m} \cdot \begin{cases} 1, & \omega \in \omega_m \\ 0, & \omega > \omega_m \end{cases}$

Matlic не ответил на вопросы 8/III: а) image $\vec{m}(\vec{s})$ at $t = t_0$, т.е. dynamics вдоль symmetry $\vec{m}(\vec{s}, t)$ и величину $|\vec{X}|$, б) как H_0 влияет на $|\vec{X}|$ и core?

Zheng Nature '2023. Hopfions.

$\Theta(\vec{s}) = \pi\left[1 - \frac{1}{R_1}\sqrt{(s-R_2)^2 + z^2}\right]$, $m_z(s,z) = \cos\Theta(s,z)$, $\sqrt{(s-R_2)^2 + z^2} \in [0, R_1]$

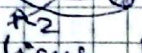
Max s из $(s-R_2)^2 + z^2 = R_1^2$, $s_m = \sqrt{R_1^2 - z^2} + R_2$, $R_1 < R_2$.

Isosurface $m_z = 0$, $\Theta(s,z) = \frac{\pi}{2}$, $1 - \frac{1}{R_1}\sqrt{(\cdot)} = \frac{1}{2}$, $\frac{R_1^2}{4} = (s-R_2)^2 + z^2$.



- кружок радиуса R_1 с центром $(s_0, z_0) = (R_2, 0)$.

В 3D-space $m_z(\vec{r}) = 0$ это тор с большим радиусом R_2 и малым R_1.



Для toroidal hopfions, the torus centers are located at $s = a \cdot \operatorname{ch}\eta$ in xOy plane и тороидальный радиус $= \frac{a}{\operatorname{sh}\eta}$. Т.е. $R_2 = a \operatorname{ch}\eta$, $R_1 = \frac{a}{\operatorname{sh}\eta}$

$\frac{R_2}{R_1} = \frac{a \operatorname{ch}\eta}{a/\operatorname{sh}\eta} = \operatorname{ch}\eta$ не есть arbitrary, $\eta \in [0, \infty]$ toroidal coord.

$\Phi(\vec{r}) = \varphi + \operatorname{arctg}\left(\frac{z}{s - R_2}\right) - \frac{\pi}{2}$, $\Phi(s, z=0, \varphi) = \varphi - \frac{\pi}{2}$, $m_s = 0$, $m_z \neq 0$.

(4/I) "Hopfion stability" MS is published in Nanomaterials with no APC. Послан pdf of the paper to Ricardo Rama Fiorin, Nerea и Maciej.

(5/I) Рассмотрим приглашения invited talks на SolStyMag-2024, June 24-28. Предложены Strelkov и Timofeev написать FMR-exp. по MS на CW localized at the faces of Cyl. dot и написать MS draft in any Editable.

(7/I) Подготовлены и посланы Eli ответы на articles и projects за 2022 год. Предложено Т. написать обзор по Nanomaterials paper для него. Предложены simulations by Sobucki, hopfions in cyl. dots, varying R и R.

(8/I) Вопрос о NAV/EMR. $d_L = S_2$. Updated refs., citations in ResearchGate-CV

Escaneado con CamScanner

- Submitted Ikerbasque report (updated info) to 2023 (9/I).

Приглашение invited speakers SolSkyMag'2024.

(9/I) Submitted Ikerbasque report + e-mail to Miguel Arocena, Ikerbasque General Manager. Разговор над Artem's Vortex GM manuscript. Skype с Yves. Pontoon Oscene. e-mail.

(10/I) Editing „Vortex GM thick dots" MS + переписка с Yves.

(11/I) Написание comments (Matters arising) to Zheng 'Nature' 2023, Hopf. Посещение в Cuelba -1. Ane/Dane 9 kg + Rene-10 kg, 20 kg × 2,1 = 42 € и закупки 5×2 = 10 €. Всего 52 €.

Zheng, Hopfions, 2023

Zheng'2023: FeGe dot $R = 250$ nm, $L = 180$ nm – experiment/simulations

Computational box is square μm × μm dot (Yves exp. cylindrical dot с $2R = 500$ nm), periodic B.c. along Oz, and $\vec{m} = \vec{m}_0 = \hat{z}$ at the dot boundaries. „Bulk" sample is domain $5L_D \times 5L_D \times 10 L_D$, $L_D = 70$ nm, т.е. $350 \times 350 \times 700$ nm.

И нашли для „bulk" sample поочередно Hopf index $H = 1, 2, 5, 6, 10, 12$.

$H = 21$ (Fig. 4) simulated то и для square 1×1 μm dot, то и для „bulk" sample.

- Square dot $180 \times 1000 \times 1000$ nm сравнили с „bulk" sample $350 \times 350 \times 700$ nm.

Написан draft „Hopfions" Matters Arising to Zheng'2023 и послан Оскане.

Skype с Yves: Fig. 4. сравнение $f^{11}(\omega)$ с Fourier-image FMR-Adekunle. Yves прислал для $L = 100$ nm correct simul. с экспериментом почти и Fig. 4a надо убрать. Для $L = 350$ nm (Fig. 4c) есть аномальный экстремум при $\approx 0,1$ GHz, это оказался background не более. И регулярный поведение $M(H)$, только некоторые dots are in vortex state.

(12/I) Переписка invited SolSkyMag'2024: из 18 invitations 5 agreed, 2 – устно agreed, 3 – declined, 8 – молчат. Договор с Обане надо.

(13/I) Gauge of vector potential $\vec{A}$ of emergent magnetic field $\vec{B}$ and gauge invariance of $\vec{A}$ ($\vec{A} \to \vec{A}' = \vec{A} + \nabla f$, arbitrary $f(\vec{r})$, но $\int dV f^2 = const$).

$\vec{A}$ gauge

- Рассуждения о gauge invariance of $Q_H = \int dV \vec{A} \cdot \vec{B}$ для toroidal hopfions and BP in sphere, см. 11/XI 2020г.

$Q_H(\vec{A}') = Q_H(\vec{A})$ если $\vec{A} \to \vec{A}' + \nabla f$: $\oint d\vec{S} \times \vec{B} \to 0$ для поверхности S уравнений $r = 0$ origin, $R \to \infty$ describing S для $R \to \infty$.

Для BP-hopfion f чрезвычайно $\int dV f^2$ – конечен т.к. это non-localized solution.

Для toroidal hopfion, видимо, достаточно $f(\vec{r})_{R\to\infty} \to$ конечна.

$I_S = \oint_{R\to\infty} d\vec{S} \cdot \vec{A}' = (f(R) - \text{конечна}) = f(R) \oint d\vec{S} \cdot \vec{B}$ $\int dS = \int dr r^2 \sin\theta d\theta d\varphi$

$I_S(R) = f(R) \oint d\vec{S} \cdot \vec{B} \to 0$ при $R \to \infty$ если $\vec{B}(\vec{r}) \sim \frac{1}{r^{3+d}}$, $\oint d\vec{S} \vec{B} \sim \int_0^R dr r^2 \frac{1}{r^{3+d}} =$

$= \int_0^R \frac{1}{r^{1+d}} \sim \int_0^R dr r^{-1-d} \sim \frac{1}{-d} \frac{1}{r^d} \Big|_0^R \sim \frac{1}{R^d}$, $I_S(R) \to 0$ при $R \to \infty$, $d > 0$.

Согласно Дорадина RRL'2023 для toroidal hopfion $\vec{B}(\vec{r}) \sim 1/r^4$ и все ОК с gauge invariance of Q_H – Hopf index. $Q_H = 1$

- It follows from the Helmholtz theorem (24/III, 11/XI 2020), что $\vec{B} = \text{rot}\vec{A}$, то $\vec{A}(\vec{r}) = -\frac{1}{4\pi} dV' \frac{\vec{R} \times \vec{B}(\vec{r}')}{R}$, $\vec{R} = (\vec{r} - \vec{r}')$, $div \vec{B} = 0$

Zheng'2023 дает $\vec{A}(\vec{r}) = \int dx' \vec{B}(x', y^2) \times \vec{e}_x$, что вычисляет Эрренхаус Kundu'82 используя Biot–Savart law (Helmholtz) + theorem to show

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Zhe $Q_H \sim \int d^3x ds \vec{B}(\vec{r}) \cdot (\vec{R} \times \vec{B}(\vec{r}))$ $\vec{R} = \vec{r} - \vec{r}'$,

→ A. Zheng '023 upon the Helmholtz theorem in paper Kundu 1982

S. Komineas, N. Papanicolaou. Topology and dynamics in ferromagnetic media, Phys D: Nonlinear Phenomena 99, 81-107 (1996) have the gener integral representation of Hopf index Q via $\int dV dV'$ from $\vec{B}(\vec{r}), \vec{B}(\vec{r}')$ similar to Kundu 1982.

→ Note: zhe Q_H (Kundu '82) bilo konechnym, gdi asymp $\vec{B}(\vec{r}) \sim \frac{1}{r^{2+d}}$, $r \to \infty$, $d > 0$

zhe $Q_H \sim \int dV \vec{A} \cdot \vec{B}$ bilo gauge invariant razo $\vec{B}(\vec{r}) \sim \frac{1}{r^{3+d}}$, $r \to \infty$, $d > 0$, ili $\vec{A} \to \vec{A}' = \vec{A} + \nabla \chi$ u $\chi(r \to \infty)$ is finite.

(14/i) Vernemsia k emergent field $\vec{B}$ iz Zheng '023, if it is calculated by Zheng equations (4)(5) gde $\vec{m}(\vec{r})$ of a "toroidal" hopfion.

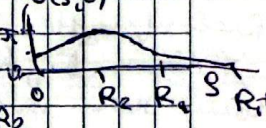
Proverim naskolko bistro $\vec{B}$ - components ugasaet ili $|\vec{r}| \to \infty$. B_x? $\vec{B}(\vec{r})$ Cm. 1/3. $B_x = B_\rho \cos\varphi - B_\varphi \sin\varphi$. $B_\rho = -\frac{1}{\rho} \sin\Theta \frac{\partial \Theta}{\partial z}$,

$B_\varphi = \sin\Theta \left[\frac{\partial \Theta}{\partial z} \frac{\partial \chi}{\partial \rho} - \frac{\partial \Theta}{\partial \rho} \frac{\partial \chi}{\partial z} \right]$, $\Theta(\rho, z)$ u $\Phi(\rho, z, \varphi) = N\varphi + \chi(\rho, z)$ helicity

Zheng $\Theta(\rho, z) = \pi \left[1 - \frac{1}{R_1} \sqrt{(\rho - R_2)^2 + z^2} \right]$, $\chi(\rho, z) = \operatorname{arctg}\left(\frac{z}{\rho - R_2} \right) - \pi/2$. $\vec{m}(\Theta, \Phi)$

$B_\rho = -\frac{1}{\rho} \sin\Theta \left(-\frac{\pi}{R_1} \right) \frac{z}{\sqrt{(\rho - R_2)^2 + z^2}} = \frac{\pi}{R_1} \frac{1}{\rho} \sin\Theta \frac{z}{\sqrt{(\rho - R_2)^2 + z^2}}$

$\Theta(\rho, z) = \pi \left[1 - \frac{1}{R_1} \sqrt{(\rho - R_2)^2 + z^2} \right]$ ili $(\rho - R_2)^2 + z^2 \le R_1^2$



$\Theta(0,0) = \pi \left(1 - \frac{R_2}{R_1} \right)$, $\Theta(R_2, 0) = \pi$, $\Theta(R_1, 0) = \pi \left[1 - \frac{1}{R_1}(R_1 - R_2) \right] = \pi \frac{R_2}{R_1}$

ρ nachina → ∞. $(\rho - R_2)^2 \le R_1^2 - z^2$ ili $\rho \le R_2 + \sqrt{R_1^2 - z^2} \le R_2 + R_1$ ili $z = 0$

$\Theta(R_1 + R_2, 0) = \pi \left[1 - \frac{1}{R_1} \sqrt{R_1^2} \right] = 0$ ili $\rho = R_1 + R_2$

$B_\rho(\rho, z = 0) = 0$. Esli $z \neq 0$, to $B_\rho(R_2 + \sqrt{R_1^2 - z^2}, z) = \frac{\pi}{R_1} \frac{1}{\rho} \sin\Theta \frac{z}{R_1} \sim \sin\Theta$

$\Theta(\rho, z) \to \Theta(R_2 + \sqrt{R_1^2 - z^2}, z) = 0$ u $\sin\Theta = 0 \to B_\rho(\max \rho, z) = 0$

$\rho_{max} = R_2 + \sqrt{R_1^2 - z^2}$, predel $\rho \to \infty$ ne imeet smysla.

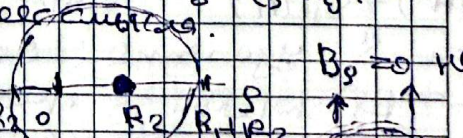
Predel $z \to \infty$. $\rho \le \max_{\pm} \sqrt{(R_2 - \rho)^2 + z^2} \le R_1$, ili rost z $\Theta(\rho, z) \to 0$

ili z_{max} u $\sqrt{(\rho - R_2)^2 + z_m^2} = R_1$

z_m is finite, $\Theta(\rho, z_m) = 0$, u $z_m \to \infty$ ne imeet smysla.

$B_\rho = 0$ na granice $(\rho - R_2)^2 + z^2 = R_1^2$

Kogda $\Theta(\rho, z) = 0$, $(\rho - R_2)^2 + z^2 = R_1^2$, centered at $\rho = R_2$.



ili $(\rho - R_2)^2 + z^2 > R_1$, $\Theta(\rho, z) = 0$ $m_z = \cos\Theta = 1$.

ili uzele φ, $\vec{m}(\vec{r})$ sonapravlen $\vec{m} = m_z \hat{z} = \hat{z}$ Hopf fun

tora $(\rho - R_2)^2 + z^2 = R_1^2$. $B_\rho|_S = 0$ na granice tora S

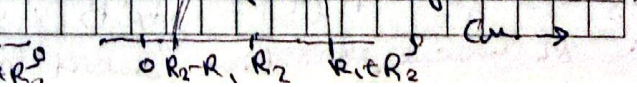


$B_\varphi \sim \sin\Theta \Rightarrow B_\varphi|_S = 0$ na granice tora, $B_x|_S = 0$ na granice tora S

Pozhe. esli ili $B_x(x_0, yz)$ na toka $x_0 \to \infty$, $B_x(x_0, yz) = 0$ ili x_0 na granice tora, $x_0 \in S$. $B_z = \frac{1}{\rho} \sin\Theta \frac{\partial \Theta}{\partial \rho} \to B_z|_S = 0$ iz-za $\Theta_S = 0$ na granice tora $\sqrt{(\rho - R_2)^2 + z^2} = R_1$.

Ansatz Zheng '023 gde $(\Theta, \Phi) \to \vec{m}$ postroen tak, chto $\Theta_S = 0$ u $\vec{B}_S = 0$ na granice tora S, kuda tora, $\vec{m} \to \vec{m}_0 = \hat{z}$ - uniform magnetization.

$R_1 < R_2$ modify $\Theta(\rho, 0)$ π $\Theta(\rho, 0)$



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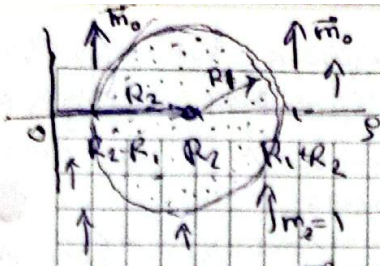

- torus $(\rho-R_2)^2+z^2=R_1^2$, $R_1<R_2$, stop pagnye R_1 - small
Pagnye topa large R_2.

// Вероятност P_H и $\int dV(\vec{A}\cdot\vec{B})$ и матрица Q_H за аналитичен анзац Eq. (4) Zhang. Но P_H и Figures 4, 8 Zhang 2023 получени от simulated configurations $\vec{m}(\vec{r})$ in square / rectangular dot.

(15/I) Кореспонденция с Ортуфлом о Pt/NiO SW manuscript. Мои бележки по ръкописа. Hillebrands / Serga e-mails.

(16/I) Уточняване invited speakers за SolSkyMag 2024. Travel MCIM grant.

(17/I) Разбиране с Travel grant application. E-mails to Cristina y Rubén (invite Tatara to SolSkyMag)

(18/I) Заявяване в електр. UPV/EHU grant validation certificate.

(19/I) Memoria Travel grant. ← Л. и Ю. в „Nam" Ondarreta и „Lidl".

(20/I) Кореспонденция относно Stub Affiliated member of the International Inst. for Sustainability with Knotted chiral Meta Matter (WPI-SKCM2). Coc-лабина и носещо лице T. Hopfion-G abstract за ICMI2, Bologna.

(22/I) Писма до Marina / UPV Memoria travel grant и CV-Dec. 2023. A. напомня за събрание в Madrid / Упр. носещо ядро.
Reminders SolSkyMag 2024 invited speakers и пратени and new inv.

(23/I) Работа по въпроси Т. Yokota (Japan) о vortex models.

(24/I) кореспонденция с Marina / UPV и подготовка Travel application.

(25-27/I) Пътуване в Madrid. А. получи нов passport. UPV/EHU подготвя нов Certificado de validación за Travel appl. 3 mes.

(28/I) IVA „Anstanto" 2023 declaration. Подготвя facturas '2023. Запитване в носещо SKCM2, Japan, info bank account „Caixa Bank". Осигурени 250 k JPY ≈ 1550 € в края на March 2024.

(29/I) До отпътуване в Poznań, 4/II: 1) ~~Submit Travel appl.~~; 2) ~~NIE Police 30/I~~; 3) ~~Maciej - taxi, keys~~; 4) ~~meet Julian~~ / Olarain contract; 5) Намери нужните docs

Submit / publish papers before June 15, 2024: (1) Nature Comment „Hopfions" (2) something in Magnetism + MDPI.

Submitted Ministerio Travel appl. (Portugal, 3 months) on 3/II. Отпуснати общо 150 €: 50 € Л.Д. наемател, 100 € Подкрепа.

(29/I - 1/II) Кореспонденция с Maciej и С. Donnelly. подготовка Travel appl.

(2-3/II) подготовка Travel appl. Запитване пропуски в Л.

(4-16/II) Пътуване в Poznań, room H4. Maciej отпуска 2,400 PLN per diem. Срещи с Mateusz (ring + skyrmion), Mathieu (GM nanopillar), Sobucki / Graczek (hopfions), Kuświk (SW MO / Pt), Jarosław (SC + FM). Връщане Frankfurt plus оверзряд Night + dinner in „Sheraton". 500 € compensation идва в Л. Пътуване от Poznań 3,600 € и 140 PLN.

(17/II) Днес: (1) Подготвя analytic calc. $\vec{H}_d$ и $E(S_k)$ за Mateusz; (2) Получи data files theor. figs. $Q_H(L, t, R, d)$ за Mathieu; (3) Продължи подготовка hopfion R_S и stability Sobucki + Graczek и сравн. с Nmoment. (4) travel reports Madrid и Poznań; (5) Ministerio 2024 travel grant; (6) travel grant - Maciej - A. за кол.; (7) Л. отчет IVA 2019-2022 - Excel (8) Actualizar SW зад. nanowires (FT mesop) с Ренан. (9) Bortolotti, Thales - new European projects on spintronics (in 2025? network)

(18/II) Подготвя IVA 2019. Travel reports Madrid, Poznań.

(19/II) E-mails to Maciej и Yurchuk (postdoc Nano Gune).

(20/II) IVA 2019 подготвя, gondola, facturas.

(21/II) Сдаване Travel reports Madrid, Poznań to Eli (нагласяване на e-signature от PI)

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(24/II) Пристигна Y. Xu, York as invited Colloquium. Изпратих Email (до 27/II).
(22/II) Skype meet. с Alexandra – споделяне на idea. 19/III – meeting Michelotto project participants
(23/III) СЛ.и Email в Carrefour, Oleberria, поискан Alexandra N° Caixa account.
(24/II) IVA проверена (Excel) 2020 и 2021, facturas.
(25/II) Имам 65 лев. с СЛ., 10 г. Email в ресепшън „Kashazwili" Domostik. Email hoja(ima имам + СЛ. 1200 €.
(26/II) неразчетена с Eli (e-signature), запитах Patricia (Investigadora Distinguido с 0% IVA/EIII 3/mo.), помолих A. Ort в hotel „Londres". Offer Gica Ort „Londres" reservation с Booking.com до June 20, 2024. IVA 2019 проверена. (23-26 June)
(27/II) неразчетена с Eli Mikel, Julian о e-signature Travel reports. Предложихме алтернативен e-sign. от Julian. Generated data files WG (4, 4, 1, d, R) soft/hard nanopillars и поискан to Mathieu/Maciej.
(28/II) Отпуск IVA 2022.
(1-2/III) Отведох documents „BP-DW dynamics" Nanoscale. E-mail to P. Fischer. С СЛ. Carrefour „Lidl" Oleberria и Carref. Market".
(3/III) поискан Excel документи PID2019 to Gustina. E-mail Patricia о Inv. Distinguido (неразч. още 200 нагоре). Maciej поискан ANR travel appl. to h PLN give name. Comments к Artemis manuscript „Vortex states". (4/III)
(5/III) работа над Mallik MS „WG in soft/hard nanopillars" Обсъждане с Email Fig. 3, 4 give Artemis manuscript и изпратени Mallik's MS (Email предложения Nature Comm. и библиография изпратен Intzod.) Обобщен spin-torque equations Simulations.
(6/III) MS with Zelent: benchmark dip. field of out-of-plane saturated thg на Skyrmion stability in cylindrical dot.
Refs: Guslienko 'APEX' 2018 и Aranda 'Materials' 2018.
$\tan\frac{\Theta(r)}{2} = \exp[-f(r)]$ и de Bonte ansatz for $f(r) = \ln r/r_s + \xi(r-r_s)$
$\xi^2 = Q^2 - 1$, $Q = K_u/2\pi M_s^2$, $r = s/l_{ex}$, $l_{ex} = \sqrt{2A/\mu_0 M_s^2} = \sqrt{A/2\pi M_s^2}$ ch APEX.
Eq (2) Zelent give $f(r)$, $r = s/l_{ex}$, $l_{ex} = \sqrt{A/2\pi M_s^2}$ взято от APEX'18 и Zelent $\varepsilon^2 = \xi^2 = (Q-1) > 0$.
APEX: $E[\Theta(r)] = 2\pi AL \int_{core}[(\Theta')^2 + (\frac{1}{r^2} + \xi^2)\sin^2\Theta + d(...)]$, $r = s/l_{ex}$, dot R
Materials: $E[\Theta(r)] = 2\pi AL \int_{core}[(\Theta')^2 + (\frac{1}{r^2} + 1)\sin^2\Theta + d(...)]$, $r = s/l$, ∞-film
$l^2 = A/K$, $K = K_u - 2\pi M_s^2$, $= 2\pi M_s^2(Q-1)$, $l_{ex}^2 = A/(2\pi M_s^2)$
$\varepsilon^2 = A/K = A/[2\pi M_s^2(Q-1)] = l_{ex}^2 \frac{1}{(Q-1)}$ – connect. l и l_{ex}.
$E[\Theta]$ – Eq. (3) – Zelent взято от Materials'2018 с $r = s/l$, Zelent има numer $r = s/l_{ex}$, това невярно.
APEX'18: $f(r) = \ln\frac{r}{r_s} + \xi(r-r_s)$, $r = s/l_{ex}$, $\xi^2 = \sqrt{Q-1}$, намира $f(r_s, \xi)$ дава $E(r_s, \xi)$, която минимизирана только по r_s, at $\xi^2 = Q-1$ fixed
Equilibrium $r_s(\Delta, \xi)$ дава MAP $E(\Delta, \xi) = E(\Delta Q)$
Materials'18: $f(r) = \ln r/r_s + \frac{1}{\Delta}(r-r_s)$, $r = s/l$, $l^2 = \sqrt{A/K}$, $K = K_u - 2\pi M_s^2$
→ exchange energy $E(r_s, \delta)$, номер $E(r_s, r_s/\Delta) = E(r_s, \Delta)$.
$E(r_s, \Delta)$ минимизирана по r_s и Δ ($r = r_s/\Delta$). r_s намираме корен r и уравнение $\partial E(r)$ только, в нула $F(r) = 0$, корен то $r_s^0 = r_s(\Delta_0)$, ост. $r_s = r_s(r)$, и $\Delta_0 = r_s^0/20$ equilibrium sk-radius r_s и sk-width Δ, б получаме от $1/\Delta = 1/r_s + 1/\delta$.

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Zelent сделал это на APEX'18, $f(r) = \ln\frac{r}{r_s} + \xi(r-r_s)$, нашел $E(r_s,\xi)$ и минимизировал $E(r_s,\xi)$ только по r_s, B_{crit} $\xi^2 = Q-1$ fixed.

Изобразите 1D stray fields от ring, post * SPM, 2019.

Note: Nanog Materials'18 solved equation (min по r_s и Δ) – finite dot, но infinite film: APEX'18 $\frac{1}{\delta} \to \xi = \sqrt{Q-1}$ is fixed, min по r_s, no finite dot.

APEX: $f(r) = \ln\frac{r}{r_s} + \xi(r-r_s)$, $r = s/l_{ex}$, $\xi = \sqrt{Q-1}$ и min $E(r_s)$

Aerials: $f(r) = \ln r/r_s + \frac{1}{\delta}(r-r_s)$, $r = s/l$, $l = \sqrt{A/K}$ и min $E(r_s,\delta)$ или $E(r_s,\Delta)$

$\frac{1}{\Delta} = \frac{1}{\delta} + \frac{1}{r_s}$.

$(r-r_s) = (\frac{s}{l_{ex}})(\frac{r}{l_{ex}} - \frac{r_s}{l_{ex}}) = \xi'(r-r_s)$ in l_{ex} units

$\xi = \xi' l_{ex} = \sqrt{Q-1}$ минимизировано в APEX'18 и в Overleaf-Zelent.

но минимизировал (more correct min d-term) $E_1(r_s,\xi)$ Overleaf, что не изменило $r_s \to \infty$ это по s/l_{ex} normalization и $\xi \to \xi l_{ex} = \sqrt{Q-1}$ is fixed.

$\frac{1}{\delta}(r-r_s) = \frac{l}{s}(\frac{r}{l} - \frac{r_s}{l}) = (\frac{l}{s})(r-r_s)$ in $l = \sqrt{A/K}$ units, $\delta \to \delta/l = \delta'$.

Materials'18: $\frac{1}{\Delta} = \frac{1}{r_s} + \frac{1}{\delta}$, $\frac{l}{\Delta} = \frac{l}{r_s} + \frac{l}{\delta} = \frac{l}{r_s} + \frac{l}{s} = \frac{l}{r_s} + \frac{1}{\delta'}$,

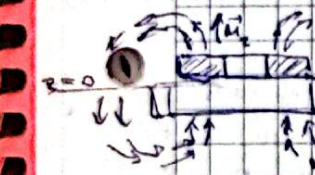
Mat. $f(r) = \ln\frac{r}{r_s} + \frac{1}{\delta'}(r-r_s)$ и $f(r) = \ln\frac{r}{r_s} + \xi'(r-r_s)$ APEX, $\xi' = \sqrt{Q-1}$.

$\frac{1}{\delta'} = \frac{l}{\Delta} - \frac{l}{r_s}$ - Возможно ли $\frac{1}{\delta'} = \xi'$, $\frac{l}{\Delta} - \frac{l}{r_s} = \sqrt{Q-1}$.

$(r_s/l) \gg 1$, $\frac{l}{\Delta} = \sqrt{Q-1}$? $\Delta = \frac{l}{\sqrt{Q-1}} = \frac{l_{ex}}{\sqrt{Q-1}\sqrt{Q-1}} = \frac{l_{ex}}{(Q-1)}$ – нет это странное соотношение.

Однородное $\Delta = \sqrt{A/K} = l$, $\Delta = l$ соответствует $\frac{1}{\delta'} = 1 - \frac{l}{r_s} \approx 1$, $\xi' = 1$, но $\xi' = \sqrt{Q-1}$. /Note: δ (in l units) $= 1$ соответ $\xi = \sqrt{Q-1}$ (in l units).

Ring dipolar field $H_s(s,z)$ и $H_z(s,z)$ Materials Zaman before, сам ring, это dH_s, H_z are in $(-2\pi M_{s\,ring})$ units. $\vec{M}_{ring}(\vec{r}')$ и $z' > 0$. Но вторые $dH_s(s,z)$, $H_z(s,z)$ at $z<0$, где dot z_s located. $\vec{H}_d$ ring, см. 3/IV 2019.



и/м $z<0$ (dot volume) $H_s<0$, $H_z>0$ near $\vec{M}_{ring} = \hat{z} M_{sr}$

$E_{int}(dot\text{-}ring) = -\int_{dot} dV\, \vec{H}_d \cdot \vec{H}_s = -\int_{dot} dV [m_s H_s + m_z H_z] M_{sd}$

$H_s(ring) = (-2\pi M_{sr}) \cdot H_s$, $H_z(ring) = (-2\pi M_{sr}) H_z$,

где H_s, H_z are dimensionless. $E_{int} = 2\pi M_{sd} M_{sr} \int dV [m_s H_s + m_z H_z]$

$dV = \int_0^R ds_s \int_0^{2\pi} d\varphi \int dz\ (z<0) = 2\pi \int_0^R ds_s \int dz\ (z<0)$.

$E_{int} = (2\pi)^2 M_{sd} M_{sr} \int_0^R ds_s \int_{z<0} dz [m_s H_s + m_z H_z]$ сравнить с Eq. (8) Zelent

$\int_0^R ds_s \to \int_0^{R/l} ds_s \cdot l^2$, s в eq. l и $\int dz \to L_z \int_{-L_d/L_z}^{0} d\tilde{z}$, z in L_z units.

L_z thickness ring, L_d – thickness dot.

Note: E_{int} дает $z = s/l$ normalization.

Eq. (12) дает E_1 – Zelent содержит минимум много (2π).

$\Theta_r' = -(\frac{1}{r} + \frac{1}{\delta})\sin\Theta$ дает $f(r) = \ln\frac{r}{r_s} + \frac{1}{\delta}(r-r_s)$, что дает для Sk. energy

$E_1 = 2\pi A_d \int_0^R dr\, r \left[(\frac{1}{r} + \frac{1}{\delta})^2 \sin^2\Theta + (\frac{1}{\delta^2} + 1)\sin^2\Theta + d\left\{ -(\frac{1}{r} + \frac{1}{\delta})\sin\Theta + \frac{1}{2}\sin\Theta\cos\Theta \right\} \right]$

в s/l нормализации, у Zelent в E_1 минимизировано в 6 нормализ. Зачем $E_1(12)$.

Послал to Mateusz и Maciej e-mail о предложениях Materials. Overleaf file "Multistable skyrmions". Reminder Eli о meeting с Julian to put his e-Sign Travel.

8/III) С Л. и Ю. в „Vaim" Domestica. С Л. в Lidl "Hermani и „Aldi" Andorra.

9/III) На среща 9/III за Abstracts chel. 9 invited. Reminders to Fert, Blanter, Gruenler (X2), Fischer, Tatara (Ruben), Mokrousov, Hertel, Garst, Gross, Smalyukh, Murakami. Booked hotel in Braga for Sept. 1-7, остана до Aug. 29.

10/III) Записване и модул „Matters arising" file to Zhong, Kiselev, Rybakov Nature Hopfions' Dec 2023. Информация ICM, Bologna abstracts: max hopfion + poster, Tchernyshyova (Hopfion oral).

11/III) Осигуряване Prof 1000€ с безбрежен до June 30, 2024. Напомняне SolSkyMag. E-mail to Patricia (Investigador Distinguido): UAM/ESM не хонорира нашите им 0% 3/ne + Soc Sec., ни 40% 3/ne. + Soc. Sec. Conf. for ICM-Bologna.

12/III) Напомняне invited sp. SolSkyMag. Оферта: tripe to Oviedo и Bologna.

13/III) 14/III) Запис ECI в travel compensation (Madrid, Poznan)

напомняне SolSkyMag и „SW-localized moiré thick dots" с Спецкоб/Ти Мороков.

15/III) Coordinates SolSkyMag program (55 talks).

16-17/III) Планиране Nature - Matters Arising. Zhong 2023. С Л. Ю. Alex в Zoom и Orsay (office). На Beamer (17/III) - 57 talks SolSkyMag.

18/III) Emergent magnetic field for axisymmetrically sym. hopfion $\vec{m}(\rho,z)$:

$\vec{B}=(B_\rho, B_\varphi, B_z)$, $B_\rho = +\frac{n}{\rho}\sin\Theta\frac{\partial\Theta}{\partial z}$, $B_\varphi = \sin\Theta\left[\frac{\partial\Theta}{\partial\rho}\frac{\partial\Psi}{\partial z}-\frac{\partial\Theta}{\partial z}\frac{\partial\Psi}{\partial\rho}\right]$, $B_z = \frac{n}{\rho}\sin\Theta\frac{\partial\Theta}{\partial\rho}$,

если $\vec{m}(\Theta,\Phi)$ где $\Theta(\vec{r})=\Theta(\rho,z)$, $\Phi_{(\vec{r})}=n\varphi+\gamma(\rho,z)$, $\vec{r}=(\rho,\varphi,z)$ Cylindr. Coordinates.

Также $\vec{B}$ в Wang-Bretaes' 2019 и Ros' Chaos' 2023. B_ρ, B_φ, B_z for any $\Theta(\rho,z)$ $\Phi(\vec{r})=n\varphi+\gamma(\rho,z)$ ansatz, не забравя, че toroidal "hopfions".

Vector potential $(\vec{A})_\rho = -(1+m_z)\frac{\partial\gamma}{\partial\rho}$, $A_\varphi=(1-m_z)\frac{n}{\rho}$, $A_z=(1+m_z)\frac{\partial\gamma}{\partial\rho}$, $m_z=\cos\Theta$ -

- в сферични за toroidal hopfions, $\gamma=0$ е „natural" gauge, Ros' Chaos' 23.

$B_\rho, B_\varphi, B_z \sim \sin\Theta$ for any axially symmetric $\vec{m}(\vec{r})=\vec{m}(\rho,z)$.

Если необходимо $\sin\Theta=0$ на Sample Surface S, то $\vec{B}_S\equiv 0$. $\sin\Theta=0$, $\vec{m}_S=\hat{z}$

$\vec{A}\to\vec{A}'=\vec{A}+\nabla f$, $Q_H'=Q_H+\frac{1}{(4\pi)^2}\oint_S \vec{B}'\cdot d\vec{S}$. $Q_H'=Q_H$ если $\vec{B}_S=0$, и

Hopf index Q_H is gauge invariant в finite Sample (см. Zhang ansatz 14/I).

и не надо $\vec{B}(\vec{r})\sim 1/r^{3+\varepsilon}$ при $|\vec{r}|\to\infty$ как 13/I.

Бибоб: for any axially symmetric $\vec{m}(\vec{r})=\vec{m}(\rho,z)$ the Hopf index будет gauge invariant ($\vec{A}\to\vec{A}'+\nabla f$) ~~even~~ in finite Sample если $\sin\Theta=0$, $\vec{m}_S=\hat{z}$ на Sample Surface S. Это следует из cylindrical coordinates, $B_\rho\sim\sin\Theta$, $d=\rho,\varphi,z$.

Тоже е ясно $\vec{m}(\rho,z)$ axially sym. надо cylindrical Sample S

$B_S=0$ возможно за cylindrical Sample и axially symmetric $\vec{m}(\vec{r})=\vec{m}(\rho z)$ ще-за $\sin\Theta_S=0$ и $B_\rho\sim\sin\Theta=0$, $\vec{m}_0=\hat{z}$ на повърхнината S.

19/III) Записване Hopfion Matters Arising MS, написан Cover Lett, и Submitted to Nature. С Л. в Mercadona "Asador". Оферта Hotels-Bologna и booked apt. in Bologna с отмяна free до June 26 (June 30 - July 1). Оферта Куриер или Ryanair ticket Santander ⇄ Bologna (30/V - 3/VII). А куриер Ryanair tickets 10 пъти и пари на 2 sec pieces.

20/III) Разбиране с compensation Hipoteca Santander, напомняне с Окс. ICM Conf. fee. Обаждане to Zelat о Skyrmion energy calculation.

21/III) Планиране Reclamación to Santander о възстановяване hipoteca fees.

22/III) Editing MS Streltsov „SW-fine thick dots". Записване Izaskun. Оферта Santander debit history 2014 (Връщане на още 674,88€ за hipoteca fees)

23/III) С Л. в Mercadona "Asador" и Carrefour. Market. Skype с Prof Artem's papers - Nanoscale Horizons. Editing Artem's MS in Overleaf.

Escaneado con CamScanner

(24/III) Референция с Zelent: skyrmion energy in nanodot.

(25/III) Вочен. Бъмм. Договор с Olarain".

(26/III) Edited Overleaf „Vortex gyromodes" Bondarenko et al. Обзор. MS с ръбове: добавих new fig. dot image with a vortex

Сл. в Mercadona " Aldain.

(27/III) Harmian „New concepts" дане Artem's MS и normal study, Aprilny.

Проверих gauge invariance ЕЗ Hopf index Q_H у Tejo. Sci. Rep. 2021. Q_H (Tejo sheme) остава $B_{surface}=0$ и за $\sin\Theta_s=0$, azimuthal sym. Проверих $\vec{A}(r\theta)$, $\vec{B}$ по обменната част с 1-ви Helmholtz.

Offers to be sessions chairs SolSkyMag'2024. Program update.

(28/III) Editing/Correcting Artem's Vortex GM manuscript.

(29/III) SolSkyMag'2024 program update. Проверка Zelent Sk-energy. Entitled „Magnetism" OSN. Free article. Предложение Dovion за magnetic topfions (> 4000 words, critical and constructive).

(30/III) Сл. в „Mercadona" Aldain и „Carref. market" товаза. Бъмм.

(31/III) Zelent предаден new comments о Skyrmion energy calc./simul.

Параметри „Zelent_Skyrmion_CGS.mcd" взети от Tejo Sci. Rep. 2018 дане Pt/Co/Pt: $M_S = 580$ G (10^3 kA/m), $A = 15$ pJ/m, $K_u = 0.7\cdot10^6$ J/m^3, $D = 3.0$ mJ/m^2.

DeBonte ansatz $\tan\frac{\Theta}{2} = e^{-f(r)}$, $f(r) = \ln\frac{r}{z_s} + \gamma(r-z_s)$, $z_s^2 = \sqrt{Q-1}$, $Q = \frac{2K_u}{4\pi M_s^2}$,

$s \to r = s/l_{ex}$, $l_{ex}^2 = (A/2\pi M_s^2)$.

Tejo'2018: $L(dot) = 0.6$ nm, variable dot radius R.

Tejo'2018: $E_{sk}(R_s)$ дане Pt/Co/Pt имаме min при $R_s \geq 11$ nm, при $L = 0.6$ nm, $R = 100$ nm и $H = 10$ mT, $E_{min} = -9.2\cdot10^{-18}$ J, Fig. 7.

Zelent успяхме да, но ние нямаме как да reproduce $E_{sk}(R_s)$ при $H=0$ и получих $E_{min} = -9.2\cdot10^{-18}$ J и $R_s^0 \simeq 26$ nm / 2 = 13 nm ($2R = 250$ nm ?). Tejo $2R = 200$ nm

$$d(\text{tejo}) = \frac{D\cdot l}{A} = \frac{3.0\ \text{mJ/m}^2 \cdot 5.54\ \text{nm}}{15\ \text{pJ/m}} = \frac{3\cdot5.54}{15}\ 10^{-3}\ \frac{\text{J}}{\text{m}^2}\ \text{nm}\cdot10^{12}\frac{\text{m}}{\text{J}} =$$

$$= 1.108\cdot10^{-3}\cdot\frac{1}{\text{m}}\cdot10^{-9}\ \text{m}\cdot10^{12} = 1.108$$ ($D = 3.0$ mJ/m^2), $d = 1.108$.

У Tejo'18 DeBonte ansatz с $z_s^2 = \sqrt{Q-1}$, z_{00} coordinate, $\delta = 1$ в $E(R_s, \delta)$ и generized DeBonte ansatz used in Arranz'18 и Zelent_Skyrmion_CGS.mcd"

$E(R_s, \delta=1)$ - analytic/mcad имаме min $E_{min}(r_s) = ?.473$ при $r_s = 1.91$ nm $R_s^0 = r_s\cdot l = 10.6$ nm.

У Tejo'18 есть Zeeman energy $E_Z = -\int dV \vec{m}\cdot\vec{H}$.

$H = H_z$, $E_Z = -M_S\int dV\, m_z(s) H_z = -M_S H_z \int_0^R ds\, s \int_0^{2\pi} d\varphi \int_0^L dz\, m_z(s) = -M_S H_z 2\pi L \times$

$\times \int_0^R ds\, s\, m_z(s) = -2\pi L M_S H_z l^2 \int_0^{R/l} dr\, r\, m_z(r)$

Дане $H_z = 10$ mT $= 100$ Oe, $E_Z = -2\pi\cdot0.6$ nm $\cdot 580$ G $\cdot 10^2$ Oe $(5.54\text{ nm})^2 \int_0^{R/e} dr\, r\, m_z(r)$.

$L = 0.6$ nm

$E_Z/(2\pi AL) = E_H = -\frac{2\pi L M_S H_z l^2}{2\pi AL}\cdot I = -M_S H_z \frac{A}{K\cdot A} I = -M_S\frac{H_z}{K} I$, $I = \int dr\, r\, m_z$

$M_S H_z/K = \frac{580\text{ G}\cdot100\text{ Oe}}{4.886\text{ erg/cm}^3\cdot10^6} = \frac{0.58\cdot10^6\text{ erg/cm}^3}{4.886\cdot10^6\text{ erg/cm}^3} = 0.119\cdot10^{-1}$

При $H_z = 100$ Oe $E_{min}(r_s) = 0.586$ при $r_s = 1.48$, или $R_s^0 = r_s\cdot l = 9.86$ nm.

Abs $E_{min} = 2\pi AL\cdot0.586 = 2\pi\cdot10^{-12}\frac{\text{J}}{\text{m}}\cdot15\cdot0.6\text{ nm}\cdot0.586 = 5.655\cdot10^{-13}\cdot0.586$ J

Точно Abs $E_{min} = 2\pi\cdot1.5\cdot10^{-6}\frac{\text{erg}}{\text{cm}}\cdot0.6\text{ nm}\cdot0.586 = 5.655\cdot10^{-13}\text{ erg}\cdot0.586 = 3.314\cdot10^{-13}$ erg

Нов 1/III Abs $E_{min} = 3.314\cdot10^{-13}$ erg $= 3.314\cdot10^{-20}$ J, close to 0.1 eV $\to E_{min}$ (Tejo).

Για $H_z = 0$ $E_{min}(r_s) = 2.473$ και Abs $E_{min} = 2\pi AL \cdot 2.473 = 1.398 \cdot 10^{-12}$ erg =
Abs E_{min} (calculated) $= -9.255 \cdot 10^{-18}$ J για $H_z = 0$. $= 1.398 \cdot 10^{-19}$ J.

// Τ.ε. analytic $E_{min} > E_{min}$ (Tajo) για $H_z = 100$ Oe (Fig. 7) και
analytic E_{min} ($H=0$) > E_{min} - simul ($H=0$)

Abs E_{min} = $3.314 \cdot 10^{-20}$ J, Abs $E_{min} = -9.1 \cdot 10^{-18}$ J (Tajo) για $H_z = 100$ Oe.
Calculated

Βοηθητικό: παρόμοιες E_{min} (calculated) > E_{min} (Tajo), Fig. 7 για τις παραμέτρους
τους Β Tajo 2018 ο γενικός magnetostatic energy density term, 2ου
$2\pi M_z^2$? Δηλαδή, βάζω στο Tajo $L = 0.6$ nm ο magnetostatic energy
is local $\varepsilon_m = 2\pi M_s^2 m_z^2$ in very good approximation.

// Βεβαιώνω, Fig. 7 Tajo 2018 με εσάς δε βρήκατε analytic
Sk energy, a simulated Sk-energy. Στο δικό μου πρόγραμμα $E_{sk}(r_s)$
Fig. 7 coincides με τα δικά σας simulations, άρα με εσάς με analytic $E_{sk}(r_s)$
για $\delta = 1$.

Exact magnetostatic energy of Neel Skyrmion εσάς Β Ruslienko' JMMM 2017 in units $4\pi M_s^2 V$, Eq. (7').

$4\pi M_s^2 V = 4\pi M_s^2 \pi R^2 L$. $I_z(k, r_s) = \int_0^1 dr\, r\, J_0(kr) m_z(r, r_s)$, r in units of dot radius R

$I_z(k, r_s) = \frac{1}{R^2} \int_0^R d\rho\, \rho\, J_0\left(k\frac{\rho}{R}\right) m_z(\rho, \rho_s)$. παραβεσεις W_m to $4\pi AL$ - units.

$I_z(k, r_s) = \frac{\ell^2}{R^2} \int_0^{R/\ell} d\rho\, \rho\, J_0\left(k\frac{r}{R}\right) m_z(r, r_s) = \frac{\ell^2}{R^2} \int_0^{R/\ell} dr\, r\, J_0\left(\frac{k}{R} r\right) m_z = \left(\frac{k}{R} = x,\ R \to \frac{R}{\ell}\right) =$

$= \frac{\ell^2}{R^2} \int_0^{R/\ell} dr\, r\, J_0(xr)\, m_z$, $W_m^B = \frac{1}{\beta} \int dx\, R\left[1 - e^{-\beta x R}\right] I_z^2(x, r_s) =$

$= \left(\frac{L}{\ell}\right) \int dx \left[1 - e^{-xL/\ell}\right] I^2(x, r_s) = L \int dx \left[1 - e^{-xL}\right] I^2(x, r_s)$, R, ℓ in units of ℓ

$W_m^B = 4\pi M_s^2 V \cdot L \int dx \left[1 - e^{-xL}\right] I_\ell^2(x, r_s) = 4\pi M_s^2 V \cdot L \int dx \left[1 - e^{-xL}\right] \cdot$ $\ell^2 = A/K$.

$\cdot \frac{\ell^4}{R^4} \left[\int_0^{R/\ell} dr\, r\, J_0(xr)\, m_z\right]^2 = 4\pi M_s^2 \pi R^2 L \cdot \frac{L}{\ell} \frac{\ell^4}{R^4} \int dx \left[1 - e^{-xL/\ell}\right] \left[\int_0^{R/\ell} dr\, r\, J_0(xr)\, m_z\right]^2$

$4\pi M_s^2 \pi R^2 L \cdot \frac{L}{\ell} \frac{\ell^4}{R^4} = 4\pi^2 M_s^2 \left(\frac{L}{R}\right)^2 \ell^3 \cdot \frac{1}{(4\pi AL)} (4\pi AL) = (4\pi AL)\, \pi M_s^2 \frac{L}{R^2} \ell^3 \frac{1}{AL} =$

$= \left|\ell^2 = A/K\right| = (4\pi AL)\, \pi M_s^2 \frac{1}{R^2} \ell \frac{A}{K} \frac{1}{A} = (4\pi AL) \frac{\pi M_s^2}{K} \frac{L\ell}{R^2} = (4\pi AL) \frac{1}{2Q} \beta \left(\frac{\ell}{R}\right)$

$W_m^B = (4\pi AL) \frac{\beta}{2Q} \frac{\ell}{R} \int dx \left[1 - e^{-xL/\ell}\right] \left[\int_0^{R/\ell} dr\, r\, J_0(xr)\, m_z(r)\right]^2$, $Q = K/2\pi M_s^2$, $\beta = \frac{L}{R}$

Με γράφουμε κοριμπόλικα magnetostatic en. W_m^B (W_m^M). στο JMMM 2018.

Συν. 2/XII 2017. $E_{sk} = 2\pi AL \int_0^R d\rho\, \rho \left[(\Theta')^2 + \left(\frac{1}{\ell^2} + \frac{1}{\rho^2}\right) \sin^2\Theta - \frac{d}{\ell}\left(\Theta' + \frac{1}{\rho} \sin\Theta\cos\Theta\right)\right]$

$\ell^2 = A/K$, $d = D\ell/A$.

$E_{sk} = 2\pi L \int_0^R d\rho\, \rho \left[A\left\{\Theta'^2 + \frac{1}{\rho^2} \sin^2\Theta\right\} + K\sin^2\Theta - D\left(\Theta' + \frac{1}{\rho} \sin\Theta\cos\Theta\right)\right]$

$E_a = \int dV \left(-K\cos^2\Theta\right)$ μεσοκρυβεν & Aranda 'materials' 2018 & κατατε

$\varepsilon_a = -K\cos^2\Theta = -K(1 - \sin^2\Theta) = -K + K\sin^2\Theta$. Νομιζω δυσβραση

Τελικα $E_s = 2\pi AL \int_0^{R/\ell} dr\, r \left[(\Theta_r')^2 + \left(\frac{1}{r^2} + 1\right) \sin^2\Theta + d\left\{\Theta_r' + \frac{1}{r} \sin\Theta\cos\Theta\right\}\right]$ (-K)

$d = D\ell/A$, $r = \rho/\ell$, $\ell = \sqrt{A/K}$, $K = K_u - 2\pi M_s^2$, κατ & βλε Zeeman - Sk - CGS.mcd

$E_a + E_m = \int dV \left[-K_u \cos^2\Theta + 2\pi M_s^2 \cos^2\Theta\right] = \int dV \left[-K\cos^2\Theta\right] = \int dV \left[-K + K\sin^2\Theta\right] =$

$= -KV + K \int dV \sin^2\Theta$. $\frac{1}{2\pi AL} (E_a + E_m) = -\frac{KV}{2\pi AL} + \frac{K}{2\pi AL} \int dV \sin^2\Theta$.

$\frac{KV}{2\pi AL} = \frac{K\cdot\pi R^2 L}{2\pi AL} = \frac{1}{2}K R^2 = \frac{1}{2}\frac{R^2}{\ell^2}$, $\frac{K}{2\pi AL}\int\int\sin^2\theta = \frac{K}{2\pi AL}\times$

$\times \int_0^R d\rho\,\rho \int_0^L dz \int_0^{2\pi} d\varphi \sin^2\theta = \frac{K}{2\pi AL}\,2\pi L \int_0^R d\rho\,\rho \sin^2\theta = \frac{K}{A}\ell^2 \int_0^{R/\ell} d(r/\ell)\,(r/\ell)\sin^2\theta = \int_0^{R/\ell} dr\,r\sin^2\theta$

$\frac{1}{2\pi AL}(E_a + E_m) = -\frac{1}{2}\frac{R^2}{\ell^2} + \int_0^{R/\ell} dr\,r\sin^2\theta$.

$E_m = 2\pi M_s^2 m_z^2$ здесь. Это $\to 0$ при $\ell \to \infty$ (нет магнитного диполя).

В расчете аналитика $E_1(r_s)$ эллипс при сравнении с Fig.7 Tejo 2018 и Zelent – Simulation ($L = 10$ nm) надо вычесть $-\frac{1}{2}\frac{R^2}{\ell^2}$ из $E_1(r_s)$. ($L = 0.6$ nm)

$\delta E_1 = -\frac{1}{2}\frac{R^2}{\ell^2} = -\frac{1}{2}\frac{(100\text{ nm})^2}{(5.541\text{ nm})^2} = -0.5\left(\frac{100}{5.541}\right)^2 = -162.852$ (in $2\pi AL$ units)

Corrected $E_1 = E_1(\text{old}) + \delta E_1 = E_1(\text{old}) - 162.852$

(1) Tejo 2018, $H_z = 10$ mT $= 100$ Oe: $E_{min}(\text{corr}) = E_{min}(\text{old}) - 162.852$, те же.
$\delta = 1$, $L = 0.6$ nm

Note: у Zelent MCad $m_z(\delta=0) < 0$, у Tejo $m_z(\delta=0) > 0$ и H_z у них имеют разный знак m_z у Zelent-MCad. Спираль $H_z \gtrless 0$ $\uparrow H_z$ – Tejo

$E_{min}(r_s) = 3.746$ при $r_s = 1.41$.

$E_{min}(\text{corr}) = E_{min}(\text{old}) - 162.852 = 3.746 - 162.852 = -159.106$

Abs $E_{min}(\text{corr}) = 2\pi AL \cdot E_{min}(\text{corr}) = 2\pi\cdot 15\cdot 10^{-12}\frac{J}{m}\cdot 0.6\text{ nm}\cdot(-159.106) =$

$= 2\pi\cdot 15\cdot 0.6\cdot 10^{-21}\,J\,(-159.106) = 56.549\cdot 10^{-21}\,J\cdot(-159.106) = 10^{-21}\,J\cdot(-)8.997\cdot 10^3 =$

$= -8.997\cdot 10^{-18}\,J$

Abs $E_{min}(H_z = 100\text{ Oe}) = -8.997\cdot 10^{-18}$ J (!). Это энергия к $E_{min} = -9.1\cdot 10^{-18}$ J [Tejo, Fig. 7]

(2) Zelent simul. $H_z = 0$, $L = 10$ nm; corrected $E_{min}(r_s) = -161.038$ при $r_s \approx 1.40$.
$\delta = 1$

Abs $E_{min} = 2\pi AL \cdot E_{min}(r_s) = 2\pi\cdot 15\cdot 10^{-12}\,J/m\cdot 10\text{ nm}\,(-161.038) = 2\pi\cdot 15\cdot 10\cdot 10^{-19}\frac{J}{m}\cdot 10^{-9}\,m \times$
$\times(-161.038) = 10^{-21}\,J\cdot 2\pi\cdot 15\cdot 10(-161.038) = -10^{-21}\,J\cdot 1.518\cdot 10^5\cdot 10^0 = -1.518\cdot 10^{-16}\,J$ (!?)

MCad дал Abs $E_{min} = -1.518\cdot 10^{-16}$ J, simul. Zelent Abs $E_{min} \approx -9.8\cdot 10^{-18}$ J

Как так? Взял $L = 10$ nm, а $E_{min} \sim -1\cdot 10^{-17}$ J как у Tejo при $L = 0.6$ nm.

(2/IV) Hiroshima Univ. transferred me 1495 € (250.000 ¥) as reward (Japan). Ист. 18,60 € монедеро booking.com до May 17, 2024. Отказан 179,25 € спасение багаж. Заключил и послал в Santander Reclamación на безбрежие Hipoteca gastos (cover lett. + 5 pages facturas). Запросил подпись Iberbasque ген. менеджер на Certificate affiliation.

(3/IV) Writing Strelkov's manuscript Overleaf, sent comments.

(4/IV) trip to Oviedo с Л. Grant meeting PID2022. Оссана Заказала

(6/IV) BP-TM manuscript adding Simulations. Gleb submitted "Vortex Gel thick dots" to Nanoscale Horizons.

(7/IV) Editing Strelkov's MS. Green function и YD cyl. dot. Сн. 27–30/IX 2023.

(8/IV) E-mails to Oksana: Oviedo travel report, PID 2019 final report.
E-mails to Zelent, Strelkov, Timofeev о manuscripts in progress.

(9/IV) Skype с Сергеем Мешковым „thick dot FMR-H_z" и Kokaze – soft/hard Moabic manuscript. Editing Strelkov's MS.

(10/IV) Iberbasque sent the signed certificate of affiliation for Travel grant

Проверка magnetostatic Green functions. Appendix Strelkov's MS.
G_{xx}, G_{yy} выписаны справа, и, видимо неверно. Проверить.
$\langle G_{xx}\rangle_e = \langle G_{yy}\rangle_e$ (A4a) верно. Это показал в Sept. 2023 (27/IX).

Escaneado con CamScanner

$$G_{\alpha\beta}(\vec r,\vec r') = -\frac{\partial^2}{\partial x_\alpha \partial x'_\beta}\,\frac{1}{|\vec r-\vec r'|},\quad \frac{1}{|\vec r-\vec r'|} = \sum_\mu \int_0^\infty dk\, e^{-k|z-z'|} e^{i\mu(\varphi-\varphi')} J_\mu(k\rho) J_\mu(k\rho')$$

Найти G_{xx}, G_{yy}, G_{zz}, хотя natural are $G_{\rho\rho}, G_{\varphi\varphi}, G_{\rho\varphi}, G_{\varphi\rho}$ ... in cylindrical coordinates $\vec\rho = (\rho,\varphi,z)$. $R = |\vec r-\vec r'|$

$$\frac{\partial}{\partial x} = \cos\varphi\frac{\partial}{\partial\rho} - \frac{\sin\varphi}{\rho}\frac{\partial}{\partial\varphi},\quad \frac{\partial}{\partial y} = \sin\varphi\frac{\partial}{\partial\rho} + \frac{\cos\varphi}{\rho}\frac{\partial}{\partial\varphi}.$$

$$\frac{\partial}{\partial x} + i\frac{\partial}{\partial y} = e^{i\varphi}\frac{\partial}{\partial\rho} + ie^{i\varphi}\frac{1}{\rho}\frac{\partial}{\partial\varphi} = e^{i\varphi}\left(\frac{\partial}{\partial\rho} + \frac{i}{\rho}\frac{\partial}{\partial\varphi}\right).$$

$$G_{xx} = -\frac{\partial^2}{\partial x\partial x'}\frac{1}{R},\quad G_{yy} = -\frac{\partial^2}{\partial y\partial y'}\frac{1}{R},\quad \frac{\partial^2}{\partial x\partial x'} + \frac{\partial^2}{\partial y\partial y'} = \mathrm{Re}\,e^{i\varphi}\left(\frac{\partial}{\partial\rho} + \frac{i}{\rho}\frac{\partial}{\partial\varphi}\right)\left(\frac{\partial}{\partial\rho'} + \frac{i}{\rho'}\frac{\partial}{\partial\varphi'}\right) = \mathrm{Re}\,e^{i\varphi}(\dots) = \mathrm{Im}\,e^{i\varphi}(\dots).$$

$$\frac{\partial^2}{\partial x\partial x'} = \left(\cos\varphi\frac{\partial}{\partial\rho} - \frac{\sin\varphi}{\rho}\frac{\partial}{\partial\varphi}\right)\left(\cos\varphi'\frac{\partial}{\partial\rho'} - \frac{\sin\varphi'}{\rho'}\frac{\partial}{\partial\varphi'}\right) =$$

$$= \cos\varphi\cos\varphi'\frac{\partial^2}{\partial\rho\partial\rho'} + \frac{1}{\rho\rho'}\sin\varphi\sin\varphi'\frac{\partial^2}{\partial\varphi\partial\varphi'} - \frac{\sin\varphi}{\rho}\cos\varphi'\frac{\partial}{\partial\varphi}\frac{\partial}{\partial\rho'} - \frac{\sin\varphi'}{\rho'}\cos\varphi\frac{\partial}{\partial\rho}\frac{\partial}{\partial\varphi'}$$

$$G_{xx} = \cos\varphi\cos\varphi' G_{\rho\rho} + \frac{\sin\varphi\sin\varphi'}{\rho\rho'}G_{\varphi\varphi} - \frac{\sin\varphi}{\rho}\cos\varphi' G_{\varphi\rho} - \frac{\sin\varphi'}{\rho'}\cos\varphi G_{\rho\varphi}$$

$$G_{\rho\rho} = -\frac{\partial^2}{\partial\rho\partial\rho'}\frac{1}{R} = -\sum_\mu\int dk\, e^{-k\Delta z} e^{i\mu(\varphi-\varphi')}\frac{\partial J_\mu(k\rho)}{\partial\rho}\frac{\partial J_\mu(k\rho')}{\partial\rho'}$$

$$G_{\varphi\varphi} = -\frac{1}{\rho}\frac{\partial}{\partial\varphi}\frac{1}{\rho}\frac{\partial}{\partial\varphi'}\frac{1}{R} = -\frac{1}{\rho\rho'}\sum_\mu\int dk\, e^{-k\Delta z}\mu^2 e^{i\mu(\varphi-\varphi')} J_\mu(k\rho) J_\mu(k\rho')$$

$$G_{\varphi\rho} = -\frac{1}{\rho}\frac{\partial}{\partial\varphi}\frac{\partial}{\partial\rho'}\frac{1}{R} = -\frac{1}{\rho}\sum_\mu\int dk\, e^{-k\Delta z}(i\mu) e^{i\mu(\varphi-\varphi')} J_\mu(k\rho)\frac{\partial J_\mu(k\rho')}{\partial\rho'}$$

$$G_{\rho\varphi} = -\frac{1}{\rho'}\frac{\partial}{\partial\rho}\frac{\partial}{\partial\varphi'}\frac{1}{R} = -\frac{1}{\rho'}\sum_\mu\int dk\, e^{-k\Delta z}(i\mu) e^{i\mu(\varphi-\varphi')}\frac{\partial J_\mu(k\rho)}{\partial\rho} J_\mu(k\rho')$$

$$\frac{dJ_n}{dx} = \frac{n}{x}J_n - J_{n+1} = -\frac{n}{x}J_n + J_{n-1} = \frac{1}{2}(J_{n-1} - J_{n+1}),\quad x = k\rho,\ \frac{\partial}{\partial\rho} = k\frac{\partial}{\partial x}$$

$$G_{zz} = -\frac{\partial^2}{\partial z\partial z'}\frac{1}{R} = -\sum_\mu\int dk\,\frac{\partial^2}{\partial z\partial z'}e^{-k\Delta z} e^{i\mu(\varphi-\varphi')} J_\mu(k\rho) J_\mu(k\rho')$$

$$\frac{\partial^2}{\partial z\partial z'}e^{-k\Delta z} = \frac{\partial^2}{\partial z\partial z'}e^{-k|z-z'|} = -k^2 e^{-k|z-z'|} + 2k\delta(z-z')$$

Проверим $\mathrm{Tr}(\hat G) = -4\pi\delta(\vec r-\vec r')$. $\mathrm{Tr}(\hat G) = G_{\rho\rho} + G_{\varphi\varphi} + G_{zz} =$

$$= -\sum_\mu\int dk\, e^{-k\Delta z} e^{i\mu(\varphi-\varphi')}\left[\frac{\partial J_\mu}{\partial\rho}\frac{\partial J_\mu(\cdot)}{\partial\rho'} + \frac{\mu^2}{\rho\rho'}J_\mu J_\mu(\cdot) - k^2 J_\mu J_\mu(\cdot)\right] +$$

$$+ (-)\sum_\mu\int dk\int 2k\delta(z-z')\, e^{i\mu(\varphi-\varphi')} J_\mu(k\rho) J_\mu(k\rho') = (\dots) - 2\sum_\mu\int_0^\infty dk\, k\,\delta(z-z')\times$$

$$\times e^{i\mu(\varphi-\varphi')} J_\mu(k\rho) J_\mu(k\rho') = (\dots) - 2\delta(z-z')\sum_\mu e^{i\mu(\varphi-\varphi')}\frac{1}{\rho}\delta(\rho-\rho') =$$

$$= (\dots) - 2\delta(z-z')\cdot 2\pi\delta(\varphi-\varphi')\frac{1}{\rho}\delta(\rho-\rho') = (\dots) - 4\pi\frac{1}{\rho}\delta(z-z')\delta(\varphi-\varphi')\delta(\rho-\rho') = -4\pi\delta(\vec r-\vec r') + (\dots)$$

$$G_{\rho\rho} + G_{\varphi\varphi} + G_{zz} = -\int dk\, e^{-k\Delta z} e^{i\mu(\varphi-\varphi')}\left[\frac{\partial J_\mu}{\partial\rho}\frac{\partial J_\mu(\cdot)}{\partial\rho'} + \frac{\mu^2}{\rho\rho'}J_\mu J_\mu(\cdot) - k^2 J_\mu J_\mu(\cdot)\right] -$$

$$-4\pi\delta(\vec r-\vec r') = |\text{формула}| = -4\pi\delta(\vec r-\vec r')$$

$$\int dk\, e^{-k|z-z'|} e^{i\mu(\varphi-\varphi')}\left[\frac{\partial J_\mu}{\partial\rho}\frac{\partial J_\mu(\cdot)}{\partial\rho'} + \left(\frac{\mu^2}{\rho\rho'} - k^2\right)J_\mu(k\rho) J_\mu(k\rho')\right] = 0 \quad \text{? не очевидно.}$$

$$\frac{\partial J_\mu}{\partial\rho}\frac{\partial J_\mu(\cdot)}{\partial\rho'} = \left(\frac{\mu}{\rho}J_\mu - kJ_{\mu+1}\right)\left(\frac{\mu}{\rho'}J_\mu(\cdot) - kJ_{\mu+1}(\cdot)\right) = \frac{\mu^2}{\rho\rho'}J_\mu J_\mu(\cdot) + k^2 J_{\mu+1}J_{\mu+1}(\cdot) -$$

$$- \frac{k\mu}{\rho}J_{\mu+1}J_\mu(\cdot) - \frac{k\mu}{\rho}J_\mu J_{\mu+1}(\cdot),\quad [\dots]_{J_\mu=0} = k^2 J_1 J_1(\cdot) - k^2 J_0 J_0(\cdot) \neq 0$$

Номер (27/II 2023) $G_{\alpha\beta}(\vec r-\vec r') = -\frac{1}{2\pi}\int d^2\vec k\,\frac{k_\alpha k_\beta}{k^2} e^{-k|z-z'|} e^{i\vec k(\vec\rho-\vec\rho')}$, $\alpha,\beta = x,y$

$$e^{i\vec k\vec\rho} = e^{ik\rho\cos(\varphi-\varphi_k)} = \sum_{n=-\infty}^{\infty} i^n J_n(k\rho) e^{in(\varphi-\varphi_k)},\quad e^{-i\vec k\vec\rho'} = \sum_{n'} i^{n'} J_{n'}(k\rho') e^{in'(\varphi'-\varphi_k)}$$

$$e^{-i\vec k\vec\rho'} = \sum_{n'}(-i)^{n'} J_{n'}(k\rho') e^{-in'(\varphi'-\varphi_k)}$$

$$G_{\alpha\beta}(\vec r-\vec r') = -\frac{1}{2\pi}\int d^2\vec k\,\frac{k_\alpha k_\beta}{k^2} e^{-k|z-z'|}\sum_{nn'} i^n(-i)^{n'} J_n(k\rho) J_{n'}(k\rho') e^{in\varphi - in'\varphi'} e^{-i(n-n')\varphi_k}$$

$\sim 3\pi c_0$ $1/k$

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Comp. xx, $\frac{k_x^2}{k^2} = \cos^2\varphi_k$, $\int_0^{2\pi} d\varphi_k \cos^2\varphi_k\, e^{i(n'-n)\varphi_k} = \int d\varphi_k e^{i(n'-n)\varphi_k} \frac{1}{2}[1+\cos 2\varphi_k]$

$= \int d\varphi_k e^{in\varphi_k} \frac{1}{2}\left[1 + \frac{1}{1}(e^{i2\varphi_k} + e^{-2i\varphi_k})\right] = \frac{1}{2}\delta_{m=0}\cdot 2\pi + \frac{1}{4}\int d\varphi_k e^{in\varphi_k} \times$

$\times (e^{2i\varphi_k} + e^{-2i\varphi_k}) = \pi\delta_{nn'} + \frac{1}{4} 2\pi \left(\delta_{m,-2} + \delta_{m,2}\right) = \pi\delta_{nn'} + \frac{\pi}{2}\left[\delta_{n,n+2} + \delta_{n,n-2}\right]$

$G_{xx}(\vec{r}\vec{r}') = -\frac{1}{2\pi}\int dk\, k^2 e^{-k|z-z'|} \sum_{nn'} i^n (-i)^{n'} J_n(k\rho) J_{n'}(k\rho') e^{in\varphi - in'\varphi'} \times$

$\times \left[\pi\delta_{nn'} + \frac{\pi}{2}(\delta_{n',n+2} + \delta_{n',n-2})\right] = -\frac{1}{2\pi}\int dk\, k^2 e^{-k|z-z'|} \sum_n i^n J_n(k\rho) e^{in\varphi} \times$

$\times \left[(-i)^n J_n(k\rho') e^{in(\varphi-\varphi')} + \frac{1}{2}(-i)^{n+2} J_{n+2}(k\rho') e^{-i(n+2)\varphi'} + \frac{1}{2}(-i)^{n-2} J_{n-2}(k\rho') e^{-i(n-2)\varphi'}\right] =$

$= -\frac{1}{2}\int dk\, k^2 e^{-k|z-z'|} \sum J_n(k\rho) e^{in(\varphi-\varphi')} \left[J_n(k\rho') - \frac{1}{2} J_{n+2}(k\rho') e^{-2i\varphi'} - \right.$

$\left. - \frac{1}{2} J_{n-2}(k\rho') e^{2i\varphi'}\right]$, обобщаем G_{xx} - работа 27/IV 2023.

$G_{xx}(\vec{r}\vec{r}') = -\frac{1}{2}\int dk\, k^2 e^{-k|z-z'|} \sum_n J_n(k\rho) e^{in(\varphi-\varphi')} \left[J_n(k\rho') - \frac{1}{2} J_{n+2}(k\rho') e^{-2i\varphi'} - \frac{1}{2} J_{n-2}(k\rho') e^{2i\varphi'}\right]$

$\frac{1}{2\pi}\int d\varphi\, G_{xx} = -\frac{1}{2}\int dk\, k^2 e^{-k|z-z'|} J_0(k\rho)\left[J_0(k\rho') - \frac{1}{2} J_2(k\rho') e^{-2i\varphi'} - \frac{1}{2} J_{-2}(k\rho') e^{2i\varphi'}\right]$

$\frac{1}{2\pi}\int d\varphi \int d\varphi'\, G_{xx} = -\pi \int dk\, k^2 e^{-k|z-z'|} J_0(k\rho) J_0(k\rho')$, обобщаем - 27/IV 2023.

Для G_{yy}, $\frac{k_y^2}{k^2} = \sin^2\varphi_k$, $\sin^2\varphi_k = \frac{1}{2}(1 - \cos 2\varphi_k)$, изменится знак недиаг. nn'-терм

$G_{yy}(\vec{r}\vec{r}') = -\frac{1}{2}\int dk\, k^2 e^{-k|z-z'|} \sum_n J_n(k\rho) e^{in(\varphi-\varphi')} \left[J_n(k\rho') + \frac{1}{2} J_{n+2}(k\rho') e^{-2i\varphi'} + \frac{1}{2} J_{n-2}(k\rho') e^{2i\varphi'}\right]$

$G_{xx}(\vec{r}\vec{r}') + G_{yy}(\vec{r}\vec{r}') = -\int dk\, k^2 e^{-k|z-z'|} \sum_n J_n(k\rho) J_n(k\rho') e^{in(\varphi-\varphi')}$

$G_{zz}(\vec{r}\vec{r}') = -\sum_n \int dk \left[-k^2 e^{-k|z-z'|} + 2k\,\delta(z-z')\right] e^{in(\varphi-\varphi')} J_n(k\rho) J_n(k\rho')$

$\mathrm{Tr}(\hat{G}) = G_{xx} + G_{yy} + G_{zz} = -2\sum_n \int dk\, k\, \delta(z-z') e^{in(\varphi-\varphi')} J_n(k\rho) J_n(k\rho') = -4\pi\delta(\vec{r}-\vec{r}')$

$\mathrm{Tr}(\hat{G}) = -4\pi\delta(\vec{r}-\vec{r}')$ верно выражать в декарт. координатах.

G_{xx}, G_{yy} вычисл., т.к. $G_{\alpha\beta}$, $\alpha,\beta = \rho, \varphi$ используя $\frac{\partial}{\partial x} = f_x\left(\frac{\partial}{\partial\rho}, \frac{\partial}{\partial\varphi}\right)$

$\partial/\partial y = f_y\left(\frac{\partial}{\partial\rho}, \partial/\partial\varphi\right)$. Для $\langle G_{xx}\rangle_\varphi$, $\langle G_{yy}\rangle_\varphi$ проще сразу использ.

$G_{\alpha\beta}(\vec{r}\vec{r}') = -\frac{1}{8\pi^3}\int d\vec{k}\, \frac{k_\alpha k_\beta}{k} e^{-k|z-z'|} e^{i\vec{k}(\vec{\rho}-\vec{\rho}')}$ из Gus'JMMM'2011 и усреднить по $\varphi_{\vec{k}}$.

по почта в Киев, пар D. ... ana, 18 kg. 43€. SolSkyMag: принял final Qin переписка ins. Murakami

(11/IV) Л. и Ю. поехали в Santiago, Galicia. написание Strelkov MS

(12/IV) написание Strelkov's MS "Thick & cyl. dots". переписка ICM'2024.

(13/IV) Composed and sent Abstract „BP-DW nanowires" to CMD31, Braga, Portugal

(15/IV) переписка с Orc. (Oviedo, Braga, Bologna, BP-DW). принял AMU travel grant, Poznan.

(16/IV) update SolSkyMag за conf. Program. Meeting „Aramburu". с Ант.

(17/IV) Запис Santander cashback. IVA „Constanto" Feb. 2024.

(18/IV) E-mails Fernandez-Pacheco, Tapia - BP Adv. Func. Mat.

(19/IV) Recom. letter for Ricardo. IVA проверка.

(20-21/IV) IVA проверка. Sent final version MS to Strelkov, Timofeev.

(22/IV) принели „nanoscale DW-BP high ν motion" тъй ст. Recom. letter Ricardo.

(23/IV) IVA проверка. Ариана фактура.

(24/IV) Работа IVA Feb. 2024 (подадена). Туризмо одобр. 3.000€ за SolSkyMag.

(25/IV) Ваня в университета. H-index Cost. H=53. Нови работи Zenkov

(26/IV) С Л. и Ю. в restaurant „Kaskaturi", из града сбогуване с Л. Бимс.

(27/IV) Проверка IVA „Constanto" 2019-2021. Comments IVA - 2022.

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29/V Meeting с Javier „Olazcuh". Probing pintxos. С Л. „Al Campo" Oiartzun.

30/V Form 7-10 Amon. Л. „10. visit to x office, no tax returns ну-да small salary 2023. BPDW-Nonoscale published accepted MS. С Л. в „Carrefour" Oiartzun.

1-15/V Наука. Переписка с Cristina, Maciej, Oxcana (Oviedo trip) BP-DW Nonoscale fluctuations influence on Ref. reports по Artem's papers (in revision) Informe final по министерио PID2019 project (15/V носкан draft Cristina).

16/V Skype с Prof: 1) изпратен Replies to referees draft; 2) contacted Maciej о Maciej's delay 3) изпратен official Braga Conf. invitation letter. Носещи сиб 2×106g + 5×2€ = 52€ Oxcana изпраща draft BP-DW dynamics' manuscript с мегаобяд. Simulations.

17/V Переписка с Oxc. о new simulation Figs. Л. пребедена в Eval Ту. и Nova 300€ Western Union"

18/V С Л. в „Carrefour" „Olaberria и „Carrefour Market".

20/V Oxbo с Г. Волобуев о Fin field tensor (виж знак о Coleman PRB 1977) Запис на MS status „Nature" comments.

21/V Maciej изпраща new Fig. comparison $W_G(R)$ + nanopillar calculated и simulated. $W_G(R)$ - calc. силно забавена. Maciej изпр. MathCad file и свои comments. Нови BP-DW paper sent to Oksana.

22/V Договорено с Аркадий об отмяна Braga Conf. trip и отм. conf. fee 350€ до July 11. Взет Braga invitation letter.

23/V „Localized SW cyl. dots" MS submitted to Ph. Rev. Appl. by Strelkov. Разпр. нар. BPDW manuscript with Oksana.

24/V С Л. „Carrefour" Oiartzun. BP DW manuscript.

25/V С Л. и Ю. в Orio. обяд в Cafeteria.

26/V Изпр. revised BP-DW paper to Oxcana. Разговаряне с paper by Saavedra ACS 2024 „BP dynamics in truncated spheres" - нов gyro-mode azimuthal low-f modes, нехомогенна BP string oscillations.

27/V Santander, San Sebastian (Seguro Hogar, new conditions за смяна на ипотека). Нова нова Seguro Hogar и отмяна by letter с старые стар. Hogar (но говорят на англ., не говорят English). Comments to Cristina - Министерио informe final. Oxcana изпраща GM нанометрес signal. by Tejo.

28/V Записа и submitted министерио PID-2019 grant report. Email to Nature. Tejo GM мерки

29/V Santander, Donostia: отнети 40.000€ на наследник ипотека. Вдовица обезпечена 899€/month out. Внесло 1350€ напълно с June 30, 2024. Запрян credit cards ине и Л. С Л. и Ю. в „Dam" Oiartzuta и „Mercadona".

30/V Подготовка докладов за ICM 2024, Bologna.

31/V С Л. - в „Carrefour". Al Campo и Errotxki. Изпраща отзив ну Nature на нови comments к paper by Zhang et al.

2/VI Наука. Comments to Nature Editor о Zheng paper. Переписка с Пачоли Т. о new project Poznan.

3/VI Отказан Magnebon - MDPI. Нов fusion review by June 22. Нова MCaB review. Отзив Оксана о симулации Tejo-gyromodes short нанометрес.

4/VI Expenses my trip ICM Bologna. Нов fusion review.

5-6/VI Пр. SolSkyMag. Нов fusion-review writing

7/VI Запис на MS for special issue QMT 30 лет A.S. Kvalev Отказан B. Ivanov to submit by Dec. 31, 2024.

8/VI A. Perk canceled participation in SolSkyMag. Заменен ред. на I. Smolyakh.

9-10/VI Нов резервиране Hotel в Bologna (trust $100 in receipts) Нов fusion review. Skype с Tejo, Fri, June 14, 16^00. Отговор на comments by Oxcana на BP-DW manuscript. Turov-AF book sent Ricardo. Запрян travel support за CMD31-Braga у Аркадий (Ох).

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Quarta now apt. in Bologna. Pagamento in conta Danske Bank.

(11/VI) Registered no RDSI - Braga, generated Gaetano e sent it to Elf.

* B. Knapman et al. Space time magnetic hopfions. Comm. Phys. 8, 151 (Accepted) (2025)

Knapman Hopf index $H = -\frac{1}{32\pi^2}\int d^3x\, d^3x'\, \vec{A}\cdot\vec{B}'$

$(\vec{B})_i = \epsilon_{ijk}\,\vec{m}\cdot(\partial_j\vec{m}\times\partial_k\vec{m})$, $\vec{B}=\mathrm{rot}\,\vec{A}$ emergent mag. field e vector potential.

Generalize de toroidal hopfions to m_i $i,j,k=(x,y,t)$ a $m(x,y,t)$.

t - time. $H = n\cdot Q$, n - integer, Q - skyrmion number.

$\vec{A}\cdot\vec{B} = A_0B_0 + A_xB_x + A_yB_y$, $B_0 = \vec{m}\cdot(\partial_x\vec{m}\times\partial_y\vec{m})$

$B_x = \vec{m}\cdot(\partial_y\vec{m}\times\frac{\partial\vec{m}}{\partial t})$, $B_y = \vec{m}\cdot(\frac{\partial\vec{m}}{\partial t}\times\frac{\partial\vec{m}}{\partial x})$.

Aqui B_x, B_y são emergent electric field, não magnetic one.

$\vec{B}=\mathrm{rot}\,\vec{A}$ remove-se emergent magnetic field.

Do emergent electric field $\vec{E} = \nabla A_0 - \frac{\partial\vec{A}}{c\,\partial t}$, $x_0 = ct$.

$E_i = F^e_{i0}$, $F^e_{\mu\nu} = \vec{m}\cdot(\partial_\mu\vec{m}\times\partial_\nu\vec{m})$, $E_i = \vec{m}\cdot(\frac{\partial\vec{m}}{\partial x}\times\frac{\partial\vec{m}}{\partial t})$.

$(B_x \to E_x,\ B_y \to -E_y)$ $E_x = -B_y$, $E_y = B_x$.

Gus' EPL 2016, $B_i = -\frac{1}{8\pi}\epsilon_{ijk}F^e_{jk}$, $E_i = F^e_{i0}$, gives B_i when Bonolomeo

$A^e_\mu = (1-\cos\Theta)\partial_\mu\Phi$, $A^e_0 = (1-\cos\Theta)\frac{\partial\Phi}{\partial t}$, $\vec{A}^e = (1-\cos\Theta)\partial\Phi$, $\mu = x,y,t$. to Knapman '24.

$\int\int dx^2\, A_0B_0 = \int\int dx^2\,(1-\cos\Theta)\frac{\partial\Phi}{\partial t}\,\vec{m}\cdot(\frac{\partial\vec{m}}{\partial x}\times\frac{\partial\vec{m}}{\partial y})$

$= \int\int dx^2\,(1-\cos\Theta)\frac{\partial\Phi}{\partial t}\sin\Theta\,(\partial_x\Theta\,\partial_y\Phi - \partial_x\Phi\,\partial_y\Theta)$, $F^e_{\mu\nu} = \sin\Theta[\partial_\mu\Theta\partial_\nu\Phi - \partial_\mu\Phi\partial_\nu\Theta]$

$\vec{m} = \vec{m}(\Theta,\Phi)$. Helicity $\vec{m}(x,y,t) = \vec{m}$

Quadem, two gauge vector potential $\vec{A}$ in $H = \int \vec{A}\cdot\vec{B}$ obtemos e $\vec{A}^e = (1-\cos\Theta)\nabla\Phi$, $A^e_0 = (1-\cos\Theta)\frac{\partial\Phi}{\partial t}$ emergent vector potential defined Gus' EPL'16.

Dos toroidal hopfion $\vec{m}(x,y,t)$ $Q_H \sim \vec{A}\cdot\vec{B} = \vec{A}\cdot\mathrm{rot}\vec{A}$.

Dos time-hopfion $\vec{m} = \vec{m}(x,y,t)$, $Q_H \sim \vec{A}\cdot\vec{B}$ vemos as expressões p.e. $B_x \sim E_y$, $B_y \sim E_x$ e em el. field $\vec{E} = \nabla A_0 - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$ obtemos e vector potential $\vec{A}$ onde $\vec{B} = \mathrm{rot}\vec{A}$.

* M. Kobayashi, M. Nitta. Winding hopfions in $\mathbb{R}^2\times S^1$. Nucl. Ph. B, 876 625-678 (2013).

New kind of hopfion $\vec{m}(x,y,t)$(!)

Eli envia-nos registo CMD'SI Braga. Confirma receipt.

(12/VI) Observe Referees Ariel's MS. Skype e Teams.

(13/VI) Nature offered to present Comments on Zhang's 2d hopfion paper.

Kamuca Nat. Comm. Phys comments 1ea Knapman - Feuerlein - Sutcliffe Spacetime hopfion paper na Comm. Phys web site e nomeia-nos para ler. Bonfoco de Tobias Nature Editor.

(14/VI) ← A. B Correia e Rebecca. Moche infancia Fig. Gaia(P) gruta P, R, G

(15-16/VI) noite Maia ← A., M. Alex. Restaurante Mejor e Gelateria.

(17/VI) Reunião c Agustín (director dept.) e Ariady ao comeu offre E-mail to Julian: opening the SolSkyMag.

(18/VI) Nuevo Docente Seminar thes for ICM, Bologna. SolSkyMag Padova.

(19/VI) Rebecca Ricardo e multiferroics, topol charges. Amazing papers Kobayashi 2013 e Kobayashi-Nitta, Phys Lett. B 728, 314 (2014). Univ. 2014.

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Kobayashi' 2013 gave Hopfion ansatz in cylindrical coordinates.
Kobayashi' 2014, Kobayashi, arXiv'14 gave Hopf ansatz in spherical coord.

Hopf charge $Q_H = \mathbb{C} = \frac{1}{4\pi^2}\int d^3x\, \varepsilon^{\mu\nu\rho} F_{\mu\nu} A_\rho$ - Kobayashi' 2013.

$R^3(x,y,z) \to S^1(z) \times S^2(x,y)$, $n_i = \varphi^+ \sigma_i \varphi$ K-hopfion

$\varphi^T = (\varphi_1, \varphi_2)$ - spinor, $|\varphi_1|^2 + |\varphi_2|^2 = 1$

Ansatz $\varphi_1 = e^{ip g(z)} \cos(q f(r))$, $\varphi_2 = e^{-in\varphi} \sin(q f(r))$ (Bogan: $q_H \to q$, $q_O \to n$, $\Theta \to \varphi$)

p, q, n - are integers, $\vec{r} = (r, \varphi, z)$ cylindrical coordinates.

$f(r \to 0) = \pi/2$, $f(r) \underset{r\to\infty}{=} 0$ is unknown, no-zero boundary conditions at ∞.

$g(z=0) = 0$, $g(z = L_z) = 2\pi$, L_z - soliton thickness, L_z - размер L_z - Bogan θ_z.

Magnetization $\vec{m}$ components: $m_1 = \cos[pg(z) - n\varphi]\sin(2q f(r))$

$m_2 = -\sin[pg(z) - n\varphi]\sin(2q f(r))$, $m_3 = \cos(2q f(r))$. K-hopfion

Дан $\vec{m}(\Theta, \Phi)$ задает spherical angles: $\Theta(r) = 2q f(r)$, $\Phi = n\varphi - pg(z)$

т.е. Kobayashi hopfion is axially symmetric $\vec{m}(\vec{r}) = \vec{m}(r, z)$. (φ, z)

Дан toroidal hopfion Gus'chaos'2013: $\Theta = \Theta(r,z)$, $\Phi = n\varphi + \chi(r,z)$

$\cos\Theta = m_3 = m_3(\eta)$ задается, $\eta = \eta(\rho, z)$, helicity $\chi(r,z)$ задается.

K-hopfion (Kobayashi hopfion) имеет одно прообраз $\vec{m}(\vec{r})$, toroidal hopfion.

$\Theta = \Theta(r) \to$ $\forall z$ прообраз $m_3(r)$ одинаков, $m_3(0) = \cos(2q\pi) = (-1)^{2q}$

$m_3(r \to \infty) = \cos\Theta(r \to \infty) = 1$. Phase $\Phi(r, z) = n\varphi - pg(z)$ задается Θ и z.

Hopf index для K-hopfion: $\vec{B} = \sin\Theta\, \nabla\Theta \times \nabla\Phi$, $\vec{B} = \mathrm{rot}\vec{A}$. $\vec{A} = 2\,\mathrm{Im}(\varphi^+ \nabla\varphi)$

$\nabla\Theta = 2q \frac{\partial f}{\partial r}\hat{r}$, $\nabla\Phi = \frac{1}{r}\frac{\partial\Phi}{\partial\varphi}\hat{\varphi} + \frac{\partial\Phi}{\partial z}\hat{z} = \frac{n}{r}\hat{\varphi} - p\frac{\partial g}{\partial z}\hat{z}$

$$\nabla\Theta \times \nabla\Phi = \begin{vmatrix} \hat{r} & \hat{\varphi} & \hat{z} \\ 2qf' & 0 & 0 \\ 0 & \frac{n}{r} & -pg' \end{vmatrix} = \frac{n}{r} 2q f' \hat{z} + pg' 2q f' \hat{\varphi}$$

$\vec{A} = 2\,\mathrm{Im}(\varphi^+ \nabla\varphi)$, $\varphi = \begin{pmatrix}\varphi_1 \\ \varphi_2\end{pmatrix}$

$\varphi^+ \nabla\varphi = (\varphi_1^*, \varphi_2^*)\begin{pmatrix}\nabla\varphi_1 \\ \nabla\varphi_2\end{pmatrix} = \varphi_1^* \nabla\varphi_1 + \varphi_2^* \nabla\varphi_2 = e^{-ipg}\cos(q\varphi) \times$

$\times \nabla(e^{ipg}\cos qf) + e^{in\varphi}\sin(qf)\nabla(e^{-in\varphi}\sin qf) = e^{-ipg}\cos qf \times$

$\times \left[e^{ipg}\hat{r}\frac{\partial}{\partial r}\cos qf + \cos qf\, \hat{z}\, e^{ipg} ip\frac{\partial g}{\partial z}\right] + e^{in\varphi}\sin qf\left[\frac{\hat{\varphi}}{r}\frac{\partial}{\partial\varphi}(e^{-in\varphi})\sin qf + \right.$

$\left. + \hat{z}\frac{\partial}{\partial z}(\sin qf)\, e^{-in\varphi}\right] = \cos qf\left[\hat{r}\frac{\partial\cos qf}{\partial r} + ip\cos qf\, \hat{z}\, g'\right] + \sin qf\left[\frac{\hat{\varphi}}{r}(-in)\sin qf + \right.$

$\left. + \hat{z}\frac{\partial\sin qf}{\partial z}\right] = \hat{r}\left[-\cos qf\cdot\sin qf\cdot q f' + \sin qf\cdot\cos qf\cdot q f'\right] + \hat{z}\left[ip\cos^2 qf\cdot g'\right] +$

$+ \hat{\varphi}\left[\frac{(-in)}{r}\sin^2 qf\right] = -\frac{in}{r}\sin^2 qf\,\hat{\varphi} + ip\cos^2 qf\, g'\hat{z}$

$\vec{A} = 2\,\mathrm{Im}\left[-\frac{in}{r}\sin^2 qf\cdot\hat{\varphi} + ip\cos^2 qf\cdot g'\hat{z}\right] = -\frac{2n}{r}\sin^2 qf\,\hat{\varphi} + 2p\cos^2 qf\, g'\hat{z}$

$\vec{B}\cdot\vec{A} = q\sin\Theta\left[2qp g' f'\hat{\varphi} + \frac{2nq}{r} f'\hat{z}\right]\cdot\left[-\frac{2n}{r}\sin^2 qf\,\hat{\varphi} + 2p\cos^2 qf\, g'\hat{z}\right] =$

$= q\sin\Theta\left[-\frac{1}{r}4pqn\sin^2 qf\cdot g' f' + \frac{1}{r}4pnq\cos^2 qf\, f' g'\right] = \sin\Theta\, 4pqn\, g' f'\left[\cos^2 qf - \sin^2 qf\right] =$

$= \frac{1}{2}\sin\Theta\, 4pqn\, g' f'\cos 2qf$. $\vec{B}\cdot\vec{A} = \frac{1}{2}\sin\Theta\cdot 4pqn\cos(2qf)\frac{\partial f}{\partial r}\frac{\partial f}{\partial z} = \frac{4pqn}{2}\sin\Theta\cos\Theta\, f' g'$

$Q_H = \frac{1}{(4\pi)^2}\int d^3x\, \vec{A}\cdot\vec{B} = \frac{1}{(4\pi)^2}\int_0^\infty dr 2\pi r\int_0^{L_z} dz\int_0^{2\pi} d\varphi\, \frac{1}{r}\sin\Theta(z)\, 4pqn\cos(2qf)\frac{\partial f}{\partial r}\frac{\partial g}{\partial z} =$

$= \frac{(2\pi)}{(4\pi)^2}\int_0^\infty dr\int_0^{L_z} dz\,(4pqn)\sin\Theta\cos\Theta\frac{\partial f}{\partial r}\frac{\partial g}{\partial z} = \frac{2\pi\cdot 4pqn}{(4\pi)^2}\int_0^\infty dr\frac{1}{2}\sin(2\Theta)\frac{\partial f}{\partial r}\int_0^{L_z} dz\frac{\partial g}{\partial z}$

$\int_0^{L_z} dz\frac{\partial g}{\partial z} = g(z)\Big|_0^{L_z} = g(L_z) - g(0) = 2\pi - 0 = 2\pi$ by $g(z)$ definition.

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$$\int_0^{\pi} d\theta \sin(2\theta) \frac{\partial \theta}{\partial \theta} = \int_0^{\pi} d\theta \sin(n\theta)\cos(n\theta) = \int_0^{\pi/2} d\theta \sin(n\theta)\cos(n\theta) =$$

$$= \frac{1}{4n}\cos(n\theta)\Big|_0^{\pi/2} = \frac{1}{4n}\left[\cos\left(\frac{n\pi}{2}\right) - 1\right] = \frac{1}{4n}\left[(-1)^{n/2} - 1\right]$$

$Q_n = 0$ (n impar).

y haciendo $n = 2k$ ($k = 1, 2, \ldots$)

$$Q_{2k} = \frac{1}{8k}\left[(-1)^k - 1\right] = -\frac{1}{4k}, \quad k = 1, 2, \ldots$$

$\Theta(x) = \arccos f(x)$, $\Phi(x, \varphi) = \alpha x - \beta g(x)$. Hay función discontinua. $\alpha = (2, 0, 0)$.

α, β, γ constantes

$$\Omega = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & -\beta g' \\ 0 & \beta g' & \alpha \end{pmatrix}, \quad \Omega \times \Omega = \begin{pmatrix} \alpha^2 & 0 & 0 \\ 0 & \alpha^2 - \beta^2 g'^2 & -2\alpha\beta g' \\ 0 & 2\alpha\beta g' & \alpha^2 - \beta^2 g'^2 \end{pmatrix}$$

$$\Omega \times (\Omega \times \Omega) = \begin{pmatrix} \alpha^3 & 0 & 0 \\ 0 & \alpha^3 - 3\alpha\beta^2 g'^2 & -3\alpha^2\beta g' + \beta^3 g'^3 \\ 0 & 3\alpha^2\beta g' - \beta^3 g'^3 & \alpha^3 - 3\alpha\beta^2 g'^2 \end{pmatrix}$$

$$\Omega \cdot \Omega \times (\Omega \times \Omega) = \alpha^4 - 3\alpha^2\beta^2 g'^2 - 3\alpha^2\beta^2 g'^2 + \beta^4 g'^4 = \alpha^4 - 6\alpha^2\beta^2 g'^2 + \beta^4 g'^4$$

(12) Spinor $\chi^\dagger = (\chi_1^*, \chi_2^*)$, $\chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$ normal. $\chi_1 = e^{i\beta g}\cos(\gamma\beta)$, $\chi_2 = e^{-i\alpha x}\sin(\gamma\beta)$.

Magnetización $m_x + i m_y = \chi_1^* \chi_2 = e^{-i\beta g}\cos(\gamma\beta)\, e^{-i\alpha x}\sin(\gamma\beta) =$

$$= \sin(2\gamma\beta)\, e^{-i(\alpha x + \beta g)} = \sin\Theta\, e^{-i(\alpha x + \beta g)}$$

y haciendo $m_x + i m_y = \sin\Theta\left[\cos(\beta g - \alpha x) - i\sin(\beta g - \alpha x)\right] =$

$$= \sin\Theta\, e^{i(\beta g - \alpha x)}$$ se cumplen si βg igualmente βg.

$$m_z = |\chi_1|^2 - |\chi_2|^2 = \cos^2(\gamma\beta) - \sin^2(\gamma\beta) = \cos(2\gamma\beta) = \cos\Theta$$ Se cumple.

$$B_z = 2\gamma\beta'\frac{\sin\Theta}{s}\frac{n}{s} \qquad \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\Theta \frac{n}{s} = \frac{n}{s}\int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\Theta =$$

$$= \frac{n}{s}\int_0^{2\pi} d\varphi \left[-\cos\Theta\right]_0^{\pi} = \frac{n}{s}\int_0^{2\pi} d\varphi\,(1+1) = \frac{2n}{s}\int_0^{2\pi} d\varphi = \frac{4\pi n}{s}$$

$$= \left[\cos(\pi n) - 1\right](-1)\frac{n}{2\pi}\int_0^{2\pi} d\varphi\cos(2\varphi) = \frac{n}{2\pi}\left[(-1)^n - 1\right]\frac{1}{2}\sin(2\varphi)\Big|_0^{2\pi} =$$

$$= \frac{n}{4\pi}\left[(-1)^n - 1\right]\left[\sin(4\pi) - \sin(0)\right] = 0$$

$$\frac{1}{2}\left[m_z(\infty) - m_z(-\infty)\right] = \frac{1}{2}\left[\cos(\infty) - \cos(-\infty)\right] = \frac{1}{2}\left[1 - (-1)\right] = 1$$

$$\text{Nº de bloques } N = \frac{1}{4\pi}\int d\varphi\, B_z = \frac{n}{4\pi}\left[m_z(\infty) - m_z(-\infty)\right] = n$$

Dominio $m_x + i m_y = \sin\Theta e^{i\varphi}$, $m_z = \cos\Theta$.

$$\vec{B} = \frac{\partial\Theta}{\partial s}\sin\Theta\left[\beta g'\hat{\varphi} + \frac{n}{s}\hat{z}\right], \quad B_\varphi = \sin\Theta\,\Theta'\beta g', \quad B_z = \sin\Theta\,\Theta'\frac{n}{s}, \quad B_s = 0$$

$$\text{div}\,\vec{B} = \frac{1}{s}\frac{\partial B_\varphi}{\partial\varphi} + \frac{\partial B_z}{\partial z} = 0. \qquad \vec{A} = \vec{A}(s, z). \text{ No puede } A_s = 0?$$

$$B = \text{rot}\,\vec{A}, \quad B_s = \frac{1}{s}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z} = -\frac{\partial A_\varphi}{\partial z} = 0. \quad A_\varphi = A_\varphi(s).$$

$$B_\varphi = \frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s} = \beta\sin\Theta\,\Theta'g', \quad B_z = \frac{1}{s}\frac{\partial(sA_\varphi)}{\partial s} - \frac{1}{s}\frac{\partial A_s}{\partial\varphi} = \frac{1}{s}\frac{\partial(sA_\varphi)}{\partial s} =$$

$$= \frac{n}{s}\sin\Theta\,\Theta', \quad \frac{\partial}{\partial s}(sA_\varphi) = \sin\Theta\,\Theta' n, \quad A_\varphi(s) = -\cos\Theta + \text{constante}(\Theta)$$

y nuevas formas $A_\varphi = -\frac{2n}{s}\sin^2\frac{\Theta}{2} = -\frac{2n}{s}\frac{1}{2}(1 - \cos\Theta) = \frac{n}{s}(\cos\Theta - 1)$, $A_\varphi = n\cos\Theta$ + constante

- Método gauge Cano-Chacón'23: $A_\varphi = (1 - \cos\Theta)\frac{n}{s}$.

Buscamos Wang'Pkl'19, Cano'2023: $A_\varphi = -(1+\cos\Theta)\frac{\partial\chi}{\partial s}$, $A_\varphi = \frac{n}{s}(1 - \cos\Theta)$, $A_z = -(1+\cos\Theta)\frac{\partial\chi}{\partial z}$

Veamos para cualquier $\Theta = \Theta(s, z)$, $\chi = n\varphi + \gamma(s, z)$ - axial Symmetry,

a ver si vemos las toroidal hopfions,

$\gamma(s, z) \to -\beta g(z)$, $\Theta(s, z) \to \Theta(z)$ para toroidal hopfions. $\frac{\partial A_s}{\partial\varphi} = 0 \to A_s = 0$ ($B_s = 0$)

$\vec{A}$: $A_s = 0$, $A_\varphi = \frac{n}{s}(1 - \cos\Theta)$, $A_z = -(1+\cos\Theta)\frac{\partial\chi}{\partial z}\left(-\beta g'(z)\right)$

$$\vec{A}\cdot\vec{B} = A_s B_s + A_\varphi B_\varphi + A_z B_z = \frac{n}{s}(1 - \cos\Theta)\beta\sin\Theta\,\Theta' g' + \beta(1+\cos\Theta)g'\frac{n}{s}\sin\Theta\,\Theta' =$$

$$= \frac{n}{s}\beta\sin\Theta\,\Theta' g'\left[1 - \cos\Theta + 1 + \cos\Theta\right] = \frac{2\beta n}{s}\sin\Theta\,\Theta' g'$$

$$Q_H = \frac{1}{(4\pi)^2}\int d^3x\, \vec{A}\cdot\vec{B} = \frac{2\beta n}{(4\pi)^2}\int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\Theta \int_0^{\infty} ds\, g' = \frac{\beta n}{2\pi}\int_0^{\pi} d\theta \sin\Theta = \frac{\beta n}{2\pi}\left[-\cos\Theta\right]_0^{\pi} = \frac{\beta n}{\pi}$$

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$= \frac{4\pi P_n}{(4\pi)^2} \int_0^{\pi q} d\theta \sin\theta [g(L_z) - g(0)] = \frac{4\pi P_n \cdot 2\pi}{16\pi^2} (-1) \cos\theta \Big|_0^{\pi q} = \frac{P_n}{2} (-1)[1 - \cos\pi q]$

$Q_H = -\frac{1}{2} P_n [1 - (-1)^q]$, Sk-number $N = Q = -\frac{n}{2}[1-(-1)^q]$, $m_z(0) = (-1)^q$, $m_z(\infty) = 1$

$Q_H = PQ$ в согласии с Kobayashi '2013, если $g(L_z) - g(0) = 2\pi$.

т.е. $Q_H \neq 0$ из-за $\frac{\partial \theta}{\partial z} \neq 0$ (но это 2D soliton) и $\frac{\partial g}{\partial z} \neq 0$ (3D soliton).

$Q_H = \frac{P_n}{4\pi} \int_{\theta(0)}^{\theta(\infty)} d\theta \sin\theta\, g(z) \Big|_0^{L_z} = \frac{P_n}{4\pi} [\cos\theta(0) - \cos\theta(\infty)][g(L_z) - g(0)]$ — K-hopfion

$m_z = \cos\theta$, Sk-number $Q = \frac{1}{4\pi} \int_{z=\text{const}} dx\,dy\, B_z = \frac{1}{2} n [m_z(0) - m_z(\infty)]$ — Hopf

$Q_H = \frac{P}{2\pi} [g(L_z) - g(0)] Q$ — integer если Q - integer и $g(L_z) - g(0) = 2\pi$

Если $g_{max} \neq \infty$, $g_{max} = R$, cylindrical dot, то Q не обязательно integer

$\Phi(\varphi, z) = n\varphi - P g(z)$, $g(L_z) - g(0)$ не обязательно $= 2\pi$. (Q_H - hopfion number)

Note: in thick dots, Skyrmions are Bloch-type при $z = 0$ (m. plane) и Neel-type при $z = \pm L/2$ faces.

При $z \to t$, Karpman '24, $\Phi = n\varphi - \omega t$. Это precession soliton.

$g(t) \to \omega t$

$\Delta\Phi(t) = P\omega t$, за $\Delta t = \frac{2\pi}{\omega P}$, $\Delta\Phi(t)$ меняется от 0 до 2π.

период

Но если $z \to t$, то Q_H не будет $\sim \int dV \vec{A} \cdot \vec{B}$.

* Напомнить Kobayashi, записав вектор $\varphi = (\varphi_1, \varphi_2)$ и $\vec{m}$, так чтоб $\vec{A}$ и $\vec{B}$ совпадали с Eqs (18),(19) Rybakov'23 formula for any axial symmetric $\vec{m}(\vec{r}) = \vec{m}(\rho, z)$ including toroidal и K-hopfions.

* Усилить, что Q_H (K-hopfion) is gauge invariant?

Для toroidal hopfion $Q_H = mn \frac{1}{2}[m_z(0) - m_z(\infty)] = mQ$, m - poloidal vorticity.

У Kobayashi $Q_H = PQ$.

$y = 0$, $(\rho = a, z = 0)$ $m_z = +P$, $y \to \infty$, $\rho = a, z = 0$ $m_z \to -P$.

K-hopfion:

$m_x = \cos\Phi\sin\Theta$, $m_y = \sin\Phi\sin\Theta$, $m_z = \cos\Theta$, $\Theta = 2\beta f(\beta)$, $\Phi = n\varphi - P g(z)$

$B_\beta = 0$, $B_\varphi = \frac{\partial\Theta}{\partial\beta}[P g] \sin\Theta = -\sin\Theta \frac{\partial\Theta}{\partial\beta}\frac{\partial\beta}{\partial z}$, $B_z = \sin\Theta \frac{\partial\Theta}{\partial\beta}\frac{n}{\beta}$. $\gamma(\beta, z) = \gamma(z) = -P g(z)$

Wang'1999: $Q_H' = Q_H + \frac{1}{(4\pi)^2} \oint F \cdot \vec{B} \cdot d\vec{S}$. $Q_H' = Q_H$ при $\oint F \vec{B} \cdot d\vec{S} = 0$, $\vec{A}' = \vec{A} + \nabla F$.

Для цилиндра $\vec{B}\,d\vec{S} = B_\beta dS_\beta + B_z dS_z = |B_\beta = 0| = B_z dS_z$, $dS_z = dx\,dy = \beta\,d\beta\,d\varphi$

$\int F B_z dS_z = \int F B_z \beta\,d\beta\,d\varphi = \int d\beta\,\beta\,d\varphi\, F \sin\Theta \frac{\partial\Theta}{\partial\beta}\frac{n}{\beta} = 2\pi n \int d\beta\, F \sin\Theta \frac{\partial\Theta}{\partial\beta} \neq 0$

Gauge invariance Q_H (K-hopfion) имеет место ∞-цилиндр $B_z(z \to \pm\infty) = 0$

Все время для K-hopfion (winding hopfion) из Kobayashi et al. 2013.

Q1-23/IV) Недоговорка с SolSkyMag. 2024. Покупка и продажа. Забрать чертежи, инструменты. В Ольгин (rented car).

Q4/IV - 27/VI) SolSkyMag 2024 в Ольгин. В программе 60 talks, но реально ~ 5% участников. Conf. dinner в „Каскадии". Catamaran trip. Интересная и довольно conference в San Sebastian (June und Sept. 2025) со Ivan Smalyukh и Shuichi Murakami (Tokyo). Hoffmann said это magnetic Frontiers' Conf. 2025 будет в Czech Republic (topic - electromagnets Cofrend Cash ~ 6000 €. Rent car „Hertz" (бронировать (postcard confirmation etc.) A. Fert on участия отказался. Town Bureau небрежно 3.000 €. Julian на Hamiltonians не был. Возможно (Criterion > 80 чел. - Hotel in Ольгин Зал ~ бесплатно, но very good catering ~ 2.500 €. Sent to Tatara hopf. paper.

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(30/VI - 4/VII) пощувка с П. на ICM Bologna, Italy. Inwita = Sanfilippo by car. Пребиваване с necheuneu apt., Bologna.

S. Bluegel (Co-author Zheng et al. Nature 2023) изпраща ми своето pdf за talk Hopf index и vector potential. Unterschrift talk by A. Kleibert orbital and spin angular momenta of SkW. обсъждане с Оксана manuscripts by Yigo (gyromodes short wires) и BP-DW dynamics nanowires

(8/VII) E-mails to Julian, Murakami (Conf.), Ricardo (Researcher-Site pdf)

(9-10/VII) Оксана gone. g.f. Aranda, ingrice conf. cash и Bank cash. Неучтената Bruegel travel license - Braga. File of - Roma

(11/VII) Пълни: •1) изпращане Braga - travel license (Eli); •2) отиде Bologna; •3) $\omega_G(c)$ - Moalic проверени; •4) отиде apt. Bologna; •5) IVA - Fnb •6) изпращане проверени IVA 2019 и 2022 г. one. in pdf; •7) отиде Ikea chairs; •8) hopfion - review; •9) BP-DW manuscript - Oxc.; •10) неправилно изпратен hotel Braga и резервен hotel Valladolid Aug. 31 •11) club - taxi

Moalic ω_G

неучтената в Оксана в Bologna - travel report.

(12/VII) Проверка много рестарт gyrotropic frequency soft/hard nanopillar $\omega_G(c)$ - Moalic (Nov. 2023) и рестарт $\omega_G(c)$ - Oxc.

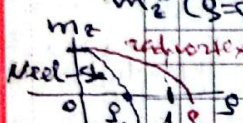
$c = s_c/R$ Soliton reduced radius, берем ид $\min W(c) = W_{ex} + W_m + W_H$

W_H - Zeeman energy of soft layer in stray field of hard layer. Static energy.

Я използвах Bgl-Polyakov ansatz soliton profile $\tan\theta/2 = s/s_c$, $\theta(s)$.

Мой рестарт 4-18 Nov. 2023. BP ansatz $\cos\theta_0(s) = m_z^0(s) = (s_c^2 - s^2)/(s_c^2 + s^2)$.

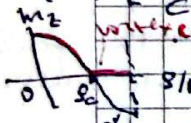
$m_z^0(s=0) = 1$. $s_c < R$, $(c = s_c/R < 1)$ - Neel Skyrmion



$s_c > R$ $(c > 1)$ - radial vortex. Case of Moalic.

Gyrovector $G_s = \frac{2\pi M_s}{\gamma} L\,[m_z(s=R) - m_z(0)] = \frac{2\pi M_s}{\gamma} L\left[\frac{s_c^2 - R^2}{s_c^2 + R^2} - 1\right] =$

$= (\cdot\cdot)\left[\frac{c^2-1}{c^2+1} - 1\right] = -\frac{4\pi M_s}{\gamma} L \frac{1}{1+c^2}$. Реализиране (численото моделиране) equilibrium $c_0 > 1$.



Ако $c<1$, то $W_{vort}(c) < W_{Sk}(c)$, а стандарт $W_{Sk}(c)$ при $c<1$. Реал numerator $W(c)$ for any c. Като $W(c) = \begin{cases} W_{Sk}(c) \text{ или } W_{vort}(c), & c<1 \\ W_{vort}(c), & c>1 \end{cases}$

Ако искаме $c_0 > 1$, то $W_{vort}(c)$ достатъчно.

Вид $W_{Sk}(c)$ не зависи от $c<1, c>1$ и $W_{Sk}(c) = W_{vort}(c)$ при $c>1$.

Вид $W_{vort}(c)$ зависи от $c<1, c>1$. Урок за я surface charges $m_z(R) \neq 0$?

// Magnetostatics of soft layer я (Nov. 2023) е дал in simplified form

$w_m = \frac{1}{V} W_m = 2\pi M_s^2 m_z^2$, т.е. я използвах голямо (face charges $m_z \neq 0$, нужен $\frac{L}{R} \to 0$ of thin soft layer, тогава $m_z(s=R)$ не влиза within $L/R \to 0$ limit.

Проверка: Neel-Sk energy $W_{Sk}(c)$ възпроизвежда ли for any c $(\substack{c>1 \\ c<1})$? и $\omega_G(c)$

Aranda-JMMM'2018:

$$\underline{W(c)} = \frac{1}{t^2}\frac{1}{1+tc^2} + \frac{1}{2}(Q-1)c^2\left[\ln\left(1+\frac{1}{c^2}\right) - \frac{1}{1+tc^2}\right] + \frac{1}{2}$$

in units $4\pi M_s^2 V$. $W = W_{ex} + W_a + W_m$, $w_m = \int ds\, s\, m_z^2(s)$ $p = L/R \to 0$ limit

ограничава ранг Bloch и Neel skyrmions. $t = R/l_{ex}$, $l_{ex}^2 = A/(2\pi M_s^2)$. Bgl-Polyakov ansatz c $m_z(s) = \frac{s_c^2 - s^2}{s_c^2 + s^2}$. $Q = 2K/(4\pi M_s^2)$, $c = s_c/s$ Sk radius $Q=0$.

$\theta(s, s_c) = 2\arctg(s/s_c)$, $\frac{d\theta}{ds} = 2\frac{1}{1+(s/s_c)^2}\frac{1}{s_c} = \frac{2s_c}{(s^2+s_c^2)}$. $\sin\theta = \frac{2ss_c}{(s^2+s_c^2)}$

$W(c) = W[\theta(s)] = \int_0^1 ds\, s\left[\frac{l_{ex}^2}{R^2}\left(\theta'^2 + \frac{1}{s^2}\sin^2\theta\right) + \cos^2\theta\right] = W_{ex} + W_m$ в/ш вече.

Sk-energy $\to \int_0^1 ds(\cdot\cdot)$ $\underline{W(c)}$ is valid for any c $(c>1, c<1)$.

Без hard layer в stray field $W(c)$ има min при $c<1$. При $H_d \neq 0$, $c>1$.

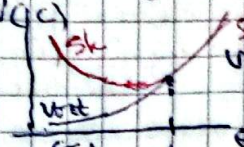
При $c>1$ radial v. $W(c)$ = skyrmion $W(c)$, $s_{max} = 1 < c_0$.

Если $c<1$, то можно ввести vortex $W(c)$ как $\begin{cases} W_s(c), & s<c \\ W_v(c), & s>c, \ c<1, \ \Theta'=0 \end{cases}$ $\Theta(s) = \pi/2$

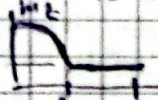
При $c=1$ $W_s(1) = W_v(1)$.

При $c<1$ $W_v(c) < W_s(c)$. При $c>1$

$W_{sk}(c)$ и $W_v(c)$ совпадают by definition.



Это все для stray field hard layer $\vec{H}_d = 0$.



Приняли sk/vortex ansatz с $c_0 > 1$. Roman

Radial vortex ansatz с $b_0 < 1$, и $\cos\theta_0 > 0$ вне core из-за H_z component stray field.

Можно simplified magnetostatics, $W_m = 2\pi M_s^2 t$, когда $L/R \to 0$. Могу учесть finite L/R и W_m (Neel skyrmion с $c_0 > 1$) using Cms – Gareeva JMMM'17 один model parameter, sk/vortex radius $c_0 > 1$.

У Roman 2 parameters in $\vec{m}$-ansatz, θ_0 и b_0, и дает расчет угла меры. Boundaries через volume, face, side surfaces charges and their interaction (это упрощенно и привязано к анизотропии, нужно через ф. Грина) при finite $\frac{L}{R}$.

Skype с Thiaville: принесли Artem's vortex GM paper to Nanoscale Horizons, Moalic's project, ICM-Bologna, CMDSI-Braga.

(13/VII) Booked hotel for Sevilla, Aug. 31. C.N. & "Lidl", "Mercadona", "Hermanos".

(15–18/VII) Переписка с Орл. (ICM reimb., BP-slides). Paper IWA II кв. 2024. Болеет. Proof Artem's paper.

(19/VII) Zoom с Roman и Thiav о WG-moalic project. Модель $\vec{m}_0(\vec{\rho})$ - Roman Обсуждено пробелы с некоторыми приближениями. Решено как model для relative small WG близкие к simulation. Проба – подбор при $L \to 0$. Artem's paper published in Nanoscale Horizons. Зависимость $m_z(\rho)$ by Moalic и сравнить с моей ($c>1$) model и rad. vortex ansatz by Roman. $W_{DM} = 0$.

(21/VII) Переписка с Орл. Report „Publicidad" PID2019 project.

(21–23/VII) Подготовка Audit reports „Publicidad", „Ultra and", etc. PID-2019.

(24/VII) Письмо. Referee reports Strelkov-PRA paper. Comments to Strelkov.

(25/VII) Slides CMDSI talk „BP-DW dynamics"

(26/VII) Нашли, что BP-DW dynamics из the stiffness $\varkappa_1, \varkappa_2$

с $W(\vec{x}) = \frac{1}{2}\varkappa_1(x^2+y^2) + \frac{1}{2}\varkappa_2 z^2$ где BP-DW centered at $\vec{x}=0$ at the middle plane of nanowire.



Видимо при $H_z \neq 0$ DW движется как 1D object и G-gyrovector $\vec{m}(\vec{r})$ internal dynamics DW – не важны, in first approximation.

См. paper $\varkappa_1 \varkappa_2$ rigid DW model Dec. 2022.

(27/VII) File „Bloch_point_stiffness.mcd". Оценили $\varkappa_z^{ex} < 0$. С $\varkappa_z^m$ сложнее

$\varkappa_z^m(vol) = \frac{2}{L^2}\left[W_m^{vol}(L) - W_m^{vol}(0)\right]$ - аналитика бедна $\sim 1-b_0$ ($\mu > 0$)

Сом. 12/IX 2022 $W(z) = \int dz_2 \int d^2\rho \, w\left[\vec{m}(\rho, z_2 - z)\right] = |z_2 - z = z'| = \int_{-\frac{1}{2}-z}^{\frac{1}{2}-z} dz' \, w\left[\vec{m}(\rho, z')\right]$

$\frac{\partial W(z)}{\partial z} = w\left[\vec{m}(-\frac{1}{2}-z)\right] - w\left[\vec{m}(-\frac{1}{2}+z)\right]$ – odd function of z

Расчет W_m^{vol} Сом. 16/II 2022. $div\vec{m} = \frac{\partial m_z}{\partial z}$ при $x=0, y=0$, $\vec{x}(x,y,z)$.

$$W_m^{vol}(\beta, b, \zeta) = \frac{1}{2}\beta \int_{-1/2-\zeta}^{1/2-\zeta} dz_2 \int_{-1/2-\zeta}^{1/2-\zeta} dz' \int_0^\infty dk \, e^{-\beta k |z_2 - z'|} I(\beta, t, k, z_2) I(\beta, t, k, z'), \quad \zeta = z/L.$$

$$I(\beta, t, k, z) = \int_0^1 d\rho\rho \, J_0(k\rho) \frac{\rho^2 t^2}{(\beta^2+t^2)} \frac{1}{(\beta^2 z^2 + \rho^2)^{3/2}}, \quad t = \frac{b}{R}, \ \beta = \frac{L}{R}, \ z$$ in L-units, W_m in $\mu_0 M_s^2$ units.

$\frac{\partial W}{\partial z}$ имеет смысла делать т.к. $W_m \sim \int dz_2 dz'$. $\partial W_{ex}/\partial z$ имеет смысл.

Escaneado con CamScanner

24/11/2022, $\frac{\partial W_m^{vol}}{\partial z}$ и $\chi_m^{vol} = \frac{1}{2}\frac{\partial W_m^{vol}}{\partial z}$ Eigenvalues by Leibniz method

See "Bloch-point_stiffness.nb", Q, P - papers. $\chi_z^{vol} < 0$ и $\approx -10^6 \div 10^5$

$\chi_z^{vol} = \chi_z^{vol}(\beta, \beta)$ и paper equilibrium BP size β, нужен одновременно.

Has also one $W_m^{face-vol}$ ($W_m^{side} = 0$, т.к. $m_s = 0$ для Thiaville ansatz.

$W_m^{f-vol} = (-)\frac{1}{2} m_s^2 \int dV \int_{z=\pm L/2} ds' \frac{div\vec{m} \cdot (\vec{m}'\cdot\vec{n})}{|\vec{r}-\vec{r}'|}$, $div\vec{m} = \frac{\beta^2 s^2}{(\beta^2+s^2)} \frac{1}{(z^2+s^2)^{3/2}}$

$m_z = \frac{z}{\sqrt{z^2+s^2}} \frac{\beta^2}{(\beta^2+s^2)}$ - Thiaville ansatz.



tail-to-tail BP-DW

$\vec{n}$ - external normal

$m_z(z=+L/2)$, $\vec{n} = \vec{z}$, $z = \frac{L}{2}$, $\vec{n} = -\vec{z}$, $z = -\frac{L}{2}$

$z = \frac{L}{2}$, $(\vec{m}\cdot\vec{n}) = m_z \cdot n_z = m_z$

$z = -\frac{L}{2}$, $(\vec{m}\cdot\vec{n}) = m_z \cdot n_z = |n_z = -1| = -m_z$

$W_m^{f-v} = \frac{1}{2} M_s^2 \int dV\, div\vec{m} \left[\int_{z=+\frac{L}{2}} ds' \frac{m_z(\vec{r}')}{|\vec{r}-\vec{r}'|} - \int_{z=-\frac{L}{2}} ds' \frac{m_z(\vec{r}')}{|\vec{r}-\vec{r}'|} \right]$, $dV = \rho d\rho d\varphi dz$, $ds' = \rho' d\rho' d\varphi'$

$W_m^{f-v} \Rightarrow \frac{1}{|\vec{r}-\vec{r}'|} = \frac{1}{\sqrt{\Delta z^2 + (\vec{\rho}-\vec{\rho}')^2}} = \int dk e^{-k|z-z'|} \sum_\mu e^{i\mu(\varphi-\varphi')} J_\mu(k\rho) J_\mu(k\rho')$.

$W_m^{f-v} = \frac{1}{2} M_s^2 \int d\rho d\varphi \rho \int dz\, div\vec{m}(\rho,z) \int d\varphi' \int d\rho' \rho' \left[m_z(\rho', z'=+\frac{L}{2}) \int dk e^{-k|z-z'|} \sum_\mu e^{i\mu(\varphi-\varphi')} J_\mu J_\mu(\cdot) - m_z(\rho', z'=-\frac{L}{2}) \int dk e^{-k|z-z'|} \sum_\mu e^{i\mu(\varphi-\varphi')} J_\mu J_\mu(\cdot) \right]$ *

$\int d\varphi d\varphi' \frac{1}{|\vec{r}-\vec{r}'|} = (2\pi)^2 \int dk e^{-k|z-z'|} J_0(k\rho) J_0(k\rho')$

$W_m^{f-v} = \frac{1}{2} M_s^2 \int d\rho d\rho \rho \int dz\, div\vec{m}(\rho,z) \int d\rho' \rho' \left[m_z(\rho', +\frac{L}{2}) \int dk e^{-k|z-\frac{L}{2}|} J_0(k\rho) J_0(k\rho') - m_z(\rho', z'=-\frac{L}{2}) \int dk e^{-k|z+\frac{L}{2}|} J_0(k\rho) J_0(k\rho') \right] (2\pi)^2 =$

$= \frac{1}{2} M_s^2 (2\pi)^2 \int d\rho d\rho \rho \int dz\, div\vec{m}(\rho z) \int d\rho' \rho' \left[m_z(\rho', +\frac{L}{2}) \int dk e^{-k|z-\frac{L}{2}|} - m_z(\rho', -\frac{L}{2}) \times \right.$

$\left. \times \int dk e^{-k|z+L/2|} \right] J_0(k\rho) J_0(k\rho')$, $m_z(+\frac{L}{2}) - m_z(-\frac{L}{2}) = 2 m_z(\frac{L}{2})$ tail-to-tail DW or head DW

$W_{m(b)}^{f-v} = \frac{1}{2} M_s^2 (2\pi)^2 \int d\rho d\rho \rho \int dz\, div\vec{m}(\rho z) \int d\rho' \rho' m_z(\rho', +\frac{L}{2}) \int_0^\infty dk \left[e^{-k|z-\frac{L}{2}|} + e^{-k|z+\frac{L}{2}|} \right] \times$

$\times J_0(k\rho) J_0(k\rho')$, $\int_{-L/2}^{L/2} dz$ (even func. on z) $\neq 0$. $W_m^{f-v}(b)$ нужна для расчета β^4 BP size.

$m_z(\rho, -\frac{L}{2}) = -m_z(\rho, +L/2)$ только при $z=0$, DW in z=0 middle plane.

$m_z(\rho, z, Z) = \frac{(z-Z)}{\sqrt{(z-Z)^2+\rho^2}} \frac{\beta^2}{(\beta^2+\rho^2)}$ для shifted DW вдоль оси z на Z.

$W_m^{f-v}(b,Z) = 2\pi^2 M_s^2 \int d\rho d\rho \int dz\, div\vec{m}(\rho, z-Z) \int d\rho' \rho' \left[m_z(\rho', \frac{L}{2}) e^{-k|z-\frac{L}{2}|} - m_z(\rho', -\frac{L}{2}) e^{-k|z+\frac{L}{2}|} \right] \times$

$\times \int dk J_0(k\rho) J_0(k\rho')$, $\int d\rho d\rho \int dz\, div\vec{m} \int d\rho' \rho' \int dk = R^2 \int d\rho d\rho L \int dz \frac{1}{R} div\vec{m}\, R^2 \int d\rho' \rho' \frac{1}{R} \int dk =$

$= R^2 L \int d\rho d\rho \int dz\, div\vec{m} \int d\rho' \rho' \int dk$

$W_m^{f-v}(Z) = 2\pi^2 M_s^2 R^2 L \int d\rho d\rho \int dz\, div\vec{m}(\rho, z-\bar{z}) \int d\rho' \rho' \int dk \left[m_z(\rho', \frac{1}{2}) e^{-\beta k|z-\frac{1}{2}|} - m_z(\rho', -\frac{1}{2}) e^{-\beta k|z+1/2|} \right] J_0(k\rho) J_0(k\rho')$, $\bar{z} = z/L$, $\beta = L/R$, $z' = z - \bar{z}$, $\int_{-1/2-\bar{z}}^{1/2-\bar{z}} dz' (\cdots)$

$\rightarrow W_m^{f-v}(\bar{z}) = \frac{1}{2}(4\pi M_s^2 V) \int_0^1 d\rho d\rho \int_{-1/2-\bar{z}}^{1/2-\bar{z}} dz'\, div\vec{m}(\rho, z') \int d\rho' \rho' \int dk \left[m_z(\rho', \frac{1}{2}) e^{-\beta k|z'+\bar{z}-\frac{1}{2}|} - m_z(\rho', -\frac{1}{2}) e^{-\beta k|z'+\bar{z}+\frac{1}{2}|} \right] J_0(k\rho) J_0(k\rho')$, $m_{z(z)} = \frac{\beta z}{\sqrt{\beta^2 z^2 + \rho^2}} \frac{t^2}{(t^2+\rho^2)}$, $div\vec{m}(z) = \frac{t^2\rho^2}{(t^2+\rho^2)} \frac{1}{(\beta^2 z^2+\rho^2)^{3/2}}$

$\rightarrow W_{m(\bar{z})}^{f-v} = \frac{1}{2} \int d\rho d\rho \int_{-1/2}^{1/2} dz \frac{t^2\rho^2}{(t^2+\rho^2)} \frac{1}{(\beta^2(z-\bar{z})^2+\rho^2)^{3/2}} \int d\rho' \rho' \int dk \left[m_z(\rho', \frac{1}{2}) e^{-\beta k|z-\frac{1}{2}|} - m_z(\rho', -\frac{1}{2}) e^{-\beta k|z+1/2|} \right] J_0(k\rho) J_0(k\rho')$. Все через $z = z - \bar{z}$, т.к. $m_z(\rho, \pm\frac{1}{2} - \bar{z})$

$W_m^{f-v}(\bar{z}) = \frac{1}{2} \int d\rho d\rho \int_{-1/2-\bar{z}}^{1/2-\bar{z}} dz' \frac{t^2\rho^2}{(t^2+\rho^2)} \frac{1}{(\beta^2 z'^2+\rho^2)^{3/2}} \int d\rho' \rho' \int dk \left[m_z(\rho', \frac{1}{2}-\bar{z}) e^{-\beta k|z'+\bar{z}-\frac{1}{2}|} - (\cdots) \right] J_0 J_0$ $z = z' + \bar{z}$

Escaneado con CamScanner

Tail-to-tail ΔW. $m_z = \frac{z\,B^2}{r\,(B^2+s^2)^{1/2}}$ $\quad \mathrm{div}\vec{m} = \frac{\partial m_z}{\partial z} = \frac{B^2 s}{(B^2+s^2)}\,\frac{1}{(z^2+s^2)^{3/2}}$, 16/11/2022.

$W_m^{s\text{-vol}} \sim \mathrm{div}\vec{m}\cdot m_z \sim \frac{\partial m_z}{\partial z}\cdot m_z$; a cuenta $m_z \to m_z$ (tail-tail $\to$ head-head)

se aproxima $\sim$ a $W_m^{s\text{-vol}}$.

28/VII Magnetost. potential $\varphi(\vec{r}) = \frac{1}{4\pi}\left[-\int dV \frac{\mathrm{div}\vec{M}(\vec{r}')}{|\vec{r}-\vec{r}'|} + \oint_S dS \frac{\vec{n}\cdot\vec{M}(\vec{r}')}{|\vec{r}-\vec{r}'|} \right]$

Volume charge density $\rho(\vec{r}') = -M_s\,\mathrm{div}\vec{m}(\vec{r}')$; + cross term ρ-b

($b = \vec{m}\cdot\vec{n}$) charge interaction $W_m^{s\text{-vol}}$ luego (-). (A.M. Gelorini.

Nanom de s-vol. abrupt. Vdw function β, b, R, etc.

Antiparallel: use Green function method & abramowitz, ya con hoy

$$W_m = -\frac{1}{2}\int \vec{H}_d \vec{M}\,dV\,, \quad (H_d)_\alpha = \sum_\beta \int d^3r'\, G_{\alpha\beta}(\vec{r},\vec{r}')\, M_\beta(\vec{r}')$$

$W_m = -\frac{1}{2} M_s^2 \int dV\,dV'\, G_{\alpha\beta}(\vec{r},\vec{r}')\, m_\alpha(\vec{r})\, m_\beta(\vec{r}')$, $\vec{m}(\vec{r}) = \vec{m}(s,z)$ a

$m_s = 0$ (vortex-like) que pasara x_2, $\vec{X} = (0,0,Z)$.

$G_{\alpha\beta}(\vec{r}-\vec{r}') = -\frac{\partial^2}{\partial x_\alpha \partial x_\beta'}\,\frac{1}{|\vec{r}-\vec{r}'|}$, $\frac{1}{|\vec{r}-\vec{r}'|} = \int dk\, e^{-k|z-z'|} \sum_\mu e^{i\mu(\varphi-\varphi')} J_\mu J_\mu(\cdot)$

$\frac{\partial}{\partial\varphi}\frac{1}{|\vec{r}-\vec{r}'|} = i\int dk\, e^{-k|z-z'|} \sum_\mu \mu\, e^{i\mu(\varphi-\varphi')} J_\mu(ks) J_\mu(ks')$; hace $G_{\alpha\beta}$ a

$\alpha,\beta = \varphi$, $G_{\varphi\varphi}, G_{\varphi s}, G_{s\varphi}, G_{ss}$ dependencias $\int_0^{2\pi} d\varphi\, d\varphi'(\ldots)$.

$W_m = -\frac{1}{2} M_s^2 \int dV dV' \left[G_{zz}(\vec{r},\vec{r}') \right] m_z(\vec{r})\, m_z(\vec{r}')$, r.r.

$G_{s\varphi}, G_{\varphi s}, G_{\varphi\varphi}, G_{\varphi z}, G_{z\varphi} = 0$ vs $\int d\varphi(\cdot)$, $G_{s\varphi}, G_{sz}, G_{\varphi z}$ no genera energia

vs. pq $m_s = 0$ (mm x_2 paralela, $\vec{X} = (0,0,Z)$).

$G_{zz}(\vec{r},\vec{r}') = -\frac{\partial^2}{\partial z\partial z'}\int dk\, e^{-k|z-z'|} \sum_\mu e^{i\mu(\varphi-\varphi')} J_\mu(ks) J_\mu(ks')$

$\frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_0^{2\pi} d\varphi'\, G_{zz} = -\frac{\partial^2}{\partial z\partial z'}\int dk\, e^{-k|z-z'|} \int d\varphi \int d\varphi' \sum_\mu e^{i\mu(\varphi-\varphi')} J_\mu J_\mu \frac{1}{2\pi} = \left| \int_0^{2\pi} d\varphi\, e^{i\mu\varphi} = 2\pi\delta_{\mu 0} \right|$

$= -\left(\frac{2\pi}{2\pi}\right)^2 \int dk\, e^{-k|z-z'|} \frac{\partial^2}{\partial z\partial z'} J_0 J_0(\cdot) = -(2\pi)^2 \frac{\partial^2}{\partial z\partial z'} \int dk\, e^{-k|z-z'|} J_0(ks) J_0(ks') \frac{1}{2\pi}$

$\frac{\partial^2}{\partial z\partial z'} e^{-k|z-z'|} = -k^2 e^{-k|z-z'|} + 2k\delta(z-z')$ 10/11/2024

$G_{zz}(\vec{r},\vec{r}') = -(2\pi)^2 \frac{1}{(2\pi)^3}\int dk \left[-k^2 e^{-k|z-z'|} + 2k\delta(z-z') \right] J_0(ks) J_0(ks')$

$\int_0^\infty dk\, k\, J_0(ks) J_0(ks') = \frac{1}{s}\delta(s-s')$

$G_{zz}(\vec{r},\vec{r}') = 2\pi \int dk\, k^2 e^{-k|z-z'|} J_0(ks) J_0(ks') - 2\pi \int dk\, 2k\delta(z-z') J_0(ks) J_0(ks') =$

$\langle G_{zz}(\vec{r},\vec{r}') \rangle_\varphi = \frac{1}{2\pi}\int d\varphi \int d\varphi'\, G_{zz}(\vec{r},\vec{r}') = 2\pi \int dk\, k^2 e^{-k|z-z'|} J_0(ks) J_0(ks') - 4\pi\delta(z-z')$

$\frac{1}{2\pi}\int_0^{2\pi} d\varphi\, d\varphi'\, \delta(\varphi-\varphi') = \frac{1}{2\pi}\int d\varphi = 1$. $\quad \delta(\vec{r}-\vec{r}') = \frac{1}{s}\delta(s-s')\,\delta(\varphi-\varphi')\,\delta(z-z')$

$g_{zz}(z,s,z',s') = \langle G_{zz}(\vec{r},\vec{r}') \rangle_\varphi = 2\pi \int dk\, k^2 e^{-k|z-z'|} J_0(ks) J_0(ks') - \frac{4\pi}{s}\delta(s-s')\delta(z-z')$

$W_m[BP] = -\frac{1}{2} M_s^2 \int ds\,dz\,dz' \int_0^{2\pi} d\varphi \int_0^{2\pi} d\varphi' \int_0^{2\pi} d\varphi \int_0^{2\pi} d\varphi' \int_0^{s} s'\,ds' \int_0^{s} s\,ds\, G_{zz}(\vec{r},\vec{r}')\, m_z(z,s)^2 m_z(z',s')^2 =$

$= -\frac{1}{2} M_s^2 \int ds\,dz\,dz' \int_0^{s} s'\,ds' \int_0^{s} s\,ds\, (2\pi)^2 g_{zz}(z,s,z',s')\, m_z(z,s)^2 m_z(z',s')^2 =$

$W_m = -\pi M_s^2 \int_0^s ds \int_0^s s\,ds \int_0^s s'\,ds' \int dz' \left[2\pi \int dk\, k^2 e^{-k|z-z'|} J_0(ks) J_0(ks') - \frac{4\pi}{s}\delta(s-s')\delta(z-z') \right] m_z(z,s)\, m_z(z',s') \doteq -2\pi^2 M_s^2 \int ds\,s \int ds'\,s' \int dz' \int dk\, k^2 e^{-k|z-z'|} J_0(ks) J_0(ks') m_z m_z(\cdot) + 4\pi^2 M_s^2 \int ds\,s \int ds'\,s' \int dz'\, \frac{1}{s}\delta(s-s')\,\delta(z-z')\, m_z m_z(\cdot) =$

$= -2\pi^2 M_s^2 \int ds\,s \int ds'\,s' \int dz' \int dk\, k^2 e^{-k|z-z'|} J_0 J_0(\cdot)\, m_z(s,z)\, m_z(s',z') + 4\pi^2 M_s^2 \int ds\,s \int dz\, m_z^2(s,z)$. Dimensionless $s/R \to s$; $z/L \to z$, $kR \to k$.

$W_m = -2\pi^2 M_S^2 R^4 L^2 \frac{1}{R^3} \int dk k^2 e^{-\beta k|z-z'|} J_0 J_0(\cdot) m_z(\rho z) m_z(\rho' z') +$

$+ 4\pi^2 M_S^2 R^2 L \int d\rho\rho \int dz\, m_z^2 \cdot V = \pi R^2 L \cdot \pi M_S^2 R^4 \frac{L^2}{R^3} = \pi M_S^2 R^2 L \cdot \left(\frac{L}{R}\right) =$

$= \pi M_S^2 V \beta.$

$W_m = 4\pi M_S^2 V \int_0^1 d\rho\rho \int_{-1/2}^{1/2} dz\, m_z^2(\rho z) - 2\pi M_S^2 V \beta \int dk k^2 e^{-\beta k|z-z'|} J_0 J_0(\cdot) m_z m_z(\cdot)$

$\frac{W_m}{4\pi M_S^2 V} = \int_0^1 d\rho\rho \int_{-1/2}^{1/2} dz\, m_z^2(\rho z) - \frac{1}{2}\beta \int dk k^2 e^{-\beta k|z-z'|} \int_0^1 d\rho\rho \int_0^1 d\rho'\rho' \int_{-1/2}^{1/2} dz' J_0(k\rho) J_0(k\rho') \times$

$\times m_z(\rho z) m_z(\rho' z'). \quad I(kz) = \int_0^1 d\rho\rho\, J_0(k\rho) m_z(\rho z)$

$W_m = \int d\rho\rho \int dz\, m_z^2(\rho z) - \frac{1}{2}\beta \int dz \int dz' \int dk k^2 e^{-k|z-z'|} \beta\, I(kz) I(kz')$ – total magnetostatic energy of axially sym. $\vec{m}(\rho, z)$ с $m_\rho = 0$ (BP shifted на Z вдоль Oz and shifted BP-DW вдоль Oz, $z \to z - Z$, rigid BP profile).

$m_z(\rho, z) \to m_z(\rho, z - Z).$

$\int_{-1/2}^{1/2} f(z - Z) = \left|z - Z = z'\right| = \int_{a-Z}^{a-Z} dz' f(z')$

$W_m(Z) = \int d\rho\rho \int_{-a-Z}^{a-Z} dz\, m_z^2(\rho, z) - \frac{1}{2}\beta \int_{-a-Z}^{a-Z} dz \int_{-a-Z}^{a-Z} dz' \int dk k^2 e^{-\beta k|z-z'|} I(kz) I(kz')$ — BP-DW dyn

MathCad Ctrl+Alt $W_m(Z)$ „Bloch-point-stiffness.mcd", но медленно

$W_{tot}(Z) = W_{ex}(Z) + W_m(Z)$ надает с $Z \uparrow$ в большинстве случаев, т.е. $\varkappa_z < 0$.

(29/VIII) Прибыли CONSTANTO за 2023. Переписка с Акс. и Стрелков.

(30/VII) Подготовка доклада CHDSI, Braga.

(31/VII) Переписка с Акс. BP-DW dynamics. Skype с Гроб: Artem's Cover page и Moalic's project.

(1/VIII) E-mails to Caixa Bank.

Magnetostatic energy nanowire in Thiaville model

$W_m = +\frac{1}{2} M_S^2 \int dV \int dV' G_{zz}(\vec{r}, \vec{r}') m_z(\vec{r}) m_z(\vec{r}'), \quad m_z(\vec{r}) = m_z(\rho, z)$

$\int_0^{2\pi} d\varphi d\varphi' G_{zz}(\vec{r}, \vec{r}') = -\left(\frac{k_{max}}{2\pi}\right)^2 \frac{\partial^2}{\partial z \partial z'} \int dk e^{-k|z-z'|} J_0(k\rho) J_0(k\rho')$ $z \to z - Z$ – возможно.

$W_m = -\frac{1}{2} M_S^2 (-1) (R\pi)^2 \int d\rho\rho \int dz \int d\rho'\rho' \int dz' \frac{\partial^2}{\partial z \partial z'} \int dk e^{-k|z-z'|} J_0(k\rho) J_0(k\rho') \times$

$\times m_z(\rho z) m_z(\rho' z'), \quad \int_0^R d\rho\rho \int_0^R d\rho'\rho' (\ldots) = R^4 \int_0^1 dr_z \int_0^1 dr' r' (\ldots), \quad r = \frac{\rho}{R}, \; z = \frac{z}{L}, \; k \to kR$

$W_m = 2\pi^2 M_S^2 R^4 \int_0^1 dr_z \int_0^1 dr' r' \int dz \int dz' \frac{\partial^2}{\partial z \partial z'} \int \frac{dk}{R} e^{\cdot} J_0(kr_z) J_0(kr') m_z(r_z) m_z(r' z')$

$W_m = 2\pi^2 M_S^2 R^3 \int dr_z r_z \int dr' r' \int dz \int dz' \frac{\partial^2}{\partial z \partial z'} \int dk e^{-\beta k|z-z'|} J_0(kr_z) J_0(kr') m_z(r_z) m_z(r' z')$

$\pi M_S^2 R^3 = \pi M_S^2 R^2 L \frac{R}{L} = \frac{1}{\beta} M_S^2 V, \quad V = \pi R^2 L.$

$W_m = 2\pi \frac{1}{\beta} M_S^2 V \int dr_z r_z \int dr' r' \int dz \int dz' \frac{\partial^2}{\partial z \partial z'} \int dk e^{-\beta k|z-z'|} J_0(kr_z) J_0(kr') m_z(r_z) m_z(r' z')$

$I(k, z) = \int_0^1 dr r J_0(kr) m_z(rz)$

$W_m = \frac{1}{2\beta} \int dz \int dz' \frac{\partial^2}{\partial z \partial z'} \int dk e^{-\beta k|z-z'|} I(k, z) I(k, z')$ in $4\pi M_S^2 V$ units

$W_m = \frac{1}{2\beta} \int dz dz' \int dk\, I(kz) I(kz') \frac{\partial^2}{\partial z \partial z'} e^{-\beta k|z-z'|}$ $\partial^2/\partial z \partial z'$ относится только к $e^{-\beta k|z-z'|}$

$W_m = \frac{1}{2\beta} \int dz \int dz' \int dk\, I(kz) I(kz') F(k, z - z'), \quad F(k, z, z') = \frac{\partial^2}{\partial z \partial z'} e^{-\beta k|z-z'|}$

$F(k, z - z')$

$W_m = \frac{1}{2\beta} \int dk \int dz \int dz' I(kz) I(kz') \frac{\partial^2}{\partial z \partial z'} e^{-\beta k|z-z'|}$

$\int dz\, I(kz) \frac{\partial}{\partial z} \int dz' I(kz') \frac{\partial}{\partial z'} e^{-\beta k|z-z'|} = \int_{-a}^{a} dz\, I(kz) \frac{\partial S(z)}{\partial z} =$

$= \left| dz\, \partial S/\partial z = dv, \; u = I(kz) \right| = S(z) I(kz) \Big|_{-a}^{a} - \int S(z) \frac{\partial I(kz)}{\partial z} dz$

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$f(z) = f(k,z) = \int_{-a}^{a} dz' I(kz') \frac{\partial}{\partial z'} e^{-\beta k|z-z'|} = \left| u = I(kz'),\ dv = dz' \frac{\partial e^{\cdot}}{\partial z'} \right| =$

$= I(kz') e^{-\beta k|z-z'|}\Big|_{-a}^{a} - \int e^{-\beta k|z-z'|} \frac{\partial I(kz')}{\partial z'} dz'$, $a = 1/2$ in L units.

$f(k,z) = I(k,a) e^{-\beta k|z-a|} - I(k,-a) e^{-\beta k|z+a|} - \int dz' e^{-\beta k|z-z'|} \frac{\partial I(kz')}{\partial z'}$

$\int dz\, I(kz) \frac{\partial f(kz)}{\partial z} = f(kz) I(kz)\Big|_{-a}^{a} - \int dz\, f(kz) \frac{\partial I(kz)}{\partial z} =$

$= f(k,a) I(ka) - f(k,-a) I(k,-a) - \int dz \frac{\partial I(kz)}{\partial z} \Big[I(k,a) e^{-\beta k|z-a|} - I(k,-a) e^{-\beta k|z+a|}$

$- \int dz' e^{-\beta k|z-z'|} \frac{\partial I(kz')}{\partial z'} \Big] = I(ka) \Big[I(k,a) - I(k,-a) e^{-2\beta ka} -$

$- \int dz' e^{-\beta k|a-z'|} \frac{\partial I(kz')}{\partial z'} \Big] - I(k,-a) \Big[I(ka) e^{-2\beta ka} - I(k,-a) -$

$- \int dz' e^{-\beta k|a+z'|} \frac{\partial I(kz')}{\partial z'} \Big] - \int dz \frac{\partial I(kz)}{\partial z} \Big[I(ka) e^{-\beta k|z-a|} - I(k,-a) e^{-\beta k|z+a|} -$

$- \int dz' e^{-\beta k|z-z'|} \frac{\partial I(kz')}{\partial z'} \Big]$, $I(k,z)$ – четная по z. $I(k,a) = I(k,-a)$

$\int dz\, I(kz) \frac{\partial f(kz)}{\partial z} = -2 I^2(ka) e^{-\beta k 2a} - I(ka) \int dz' e^{-\beta k|a-z'|} \frac{\partial I(kz')}{\partial z'} +$

$+ I(k,-a) \int dz' e^{-\beta k|a+z'|} \frac{\partial I(kz')}{\partial z'} - \int dz \frac{\partial I(kz)}{\partial z} \Big[I(ka) e^{-\beta k|z-a|} - I(ka) e^{-\beta k|z+a|}$

$- \int dz' e^{-\beta k|z-z'|} \frac{\partial I(kz')}{\partial z'} \Big] = -2 I^2(k,a) e^{-2\beta ka} - 2 I(ka) \int dz \frac{\partial I(kz)}{\partial z} e^{-\beta k|z-a|} +$

$+ 2 I(k,a) \int dz \frac{\partial I(kz)}{\partial z} e^{-\beta k|z+a|} + \int dz \int dz' e^{-\beta k|z-z'|} \frac{\partial I(kz)}{\partial z} \frac{\partial I(kz')}{\partial z'} = \Gamma(k)$

$W_m = \frac{1}{2\beta} \int dk \int_{-a}^{a} dz\, I(kz) \frac{\partial f(k,z)}{\partial z} = \frac{1}{2\beta} \int dk\, \Gamma(k)$, если $I(k,a) = I(k,-a)$, что верно благодаря симметрии $z=0$?

$I(k,z) = \int_0^1 dr\, r J_0(kr)\, m_z(r,z)$, $m_z(r,z) = \frac{\beta z}{\sqrt{\beta^2 z^2 + r^2}} \frac{t^2}{(t^2+r^2)}$ – четная по z при $z=0$.

При $z \neq 0$, $m_z(r, z-Z)$, $m_z(z=a) = m_z(a-Z)$, $m_z(z-a) = m_z(-a-Z)$

$I(k,a) = \int_0^1 dr\, r J_0\, m_z(r, a-Z)$, $I(k,-a) = \int_0^1 dr\, r J_0\, m_z(r, -a-Z)$

При $Z=0$, $I(k,a) = -I(k,-a)$.

При $Z>0$ принципиально $I(k,a) \neq 0$ ($\sim m_z(a-Z)$), $I(k,-a) = 0$, т.к. $m_z(r,-a-Z) = 0$ вне нашего отрезка.

$Z>0$ $\int_a dz\, I(kz) \frac{\partial f(kz)}{\partial z} = I^2(k,a) - I(k,a) \int dz' e^{\beta k|a-z'|} \frac{\partial I(kz')}{\partial z'} \ldots$

Нетривиальное $I(k,-a) = 0$ никуда, т.к. $m_z(-a) = \frac{\beta(-a-Z)}{\sqrt{\beta^2(-a-Z)^2 + r^2}} \frac{t^2}{(t^2+r^2)} \neq 0$

$I(k,-a) \sim m_z(-a) \neq 0$.

При $Z \neq 0$, нет простого соотношения $I(k,+a)$ и $I(k,-a)$.

$\int dz\, I(kz) \frac{\partial f(kz)}{\partial z} = I^2(ka) + I^2(k,-a) - 2 I(ka) I(k,-a) e^{-2\beta ka} - I(ka) \int dz' e^{-\beta k|a-z'|} \frac{\partial I(kz')}{\partial z'} +$

$+ I(k,-a) \int dz' e^{-\beta k|a+z'|} \frac{\partial I(kz')}{\partial z'} - \int dz \frac{\partial I(kz)}{\partial z} \Big[I(ka) e^{-\beta k|z-a|} - I(k,-a) e^{-\beta k|z+a|} -$

$- \int dz' e^{-\beta k|z-z'|} \frac{\partial I(kz')}{\partial z'} \Big] = I^2(ka) + I^2(k,-a) - 2 I(ka) I(k,-a) e^{-2\beta ka} -$

$- 2 I(ka) \int dz \frac{\partial I(kz)}{\partial z} e^{-\beta k|z-a|} + 2 I(k,-a) \int dz \frac{\partial I(kz)}{\partial z} e^{-\beta k|z+a|} +$

$+ \int dz \int dz' \frac{\partial I(kz)}{\partial z} \frac{\partial I(kz')}{\partial z'} e^{-\beta k|z-z'|} = \Gamma(k)$ опять. $W_m = \frac{1}{2\beta} \int dk\, \Gamma(k)$.

$I(k,z) = I(k,z,Z) = \int_0^1 dr\, r J_0(kr)\, m_z(r, z-Z)$ при Z displacement $\Rightarrow W \neq 0$.

Энергия $\sim I^2$ всегда. face-face charge energy, $I \frac{\partial I}{\partial z}$ –

– face-volume charge energy, $\frac{\partial I}{\partial z} \frac{\partial I}{\partial z}$ – volume w. energy.

$\frac{\partial I}{\partial z} \sim \frac{\partial m_z}{\partial z} = \operatorname{div} \vec{m}$ при $m_z(z-Z)$ аксиально симметрично.

$I(ka) = \int dr\, r J_0(kr)\, m_z(r, z-Z)$, $m_z(r,z,z) = \frac{\beta(z-Z)}{\sqrt{\beta^2(z-Z)^2 + r^2}} \frac{t^2}{(t^2+r^2)}$ при

$z \to a$ $\beta^2(a-Z)^2 \gg r^2$ ($r \ll 1$), $m_z(r,a,Z) \approx \operatorname{sign}(a-Z) \frac{t^2}{(1+r^2)} \approx \frac{t^2}{(1+r^2)}$ от Z не зависит

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Face - face magn. energy moment interaction $W_m(Z)$ рассчитано

особенно при $\beta \gg 1$, т.к. $\beta^2(a-Z)^2 \approx \beta^2(a)^2 = \frac{1}{4}\beta^2 \gg \pi^2$, $\pi \le 1$.

$$\Gamma(k) = -2I(k,a)\int dz \frac{\partial I(kz)}{\partial z} e^{-\beta k|z-a|} + 2I(k,-a)\int dz \frac{\partial I(kz)}{\partial z} e^{-\beta k|z+a|} +$$

$$+ \int dz \int dz' \frac{\partial I(kz)}{\partial z}\frac{\partial I(kz')}{\partial z'} e^{-\beta k|z-z'|}. \quad \operatorname{div}\vec{m} = \frac{\partial m_z}{\partial z} = \frac{\beta^2 s^2}{(\beta^2+s^2)}\frac{1}{(z^2+s^2)^{3/2}}$$

$$\frac{\partial m_z}{\partial z} = \frac{1}{L}\frac{\partial m_z}{\partial(z/L)} = \frac{1}{L}\frac{\partial m_z}{\partial z} = \frac{1}{L}\frac{\beta^2 s^2}{(\beta^2+s^2)} = \frac{1}{L}\frac{t^2R^2z^2R^2}{R^2(t^2+z^2)}\frac{1}{(L^2z^2+R^2z^2)^{3/2}} =$$

$$= \frac{R^2}{L}\frac{t^2\pi^2}{(t^2+\pi^2)}\frac{1}{R^3}\frac{1}{(\beta^2\xi^2+\pi^2)^{3/2}}. \text{ Точнее } \frac{\partial m_z}{\partial(z/L)} = \frac{L}{1}\frac{\partial m_z}{\partial z} = LR^2\frac{t^2\pi^2}{(t^2+\pi^2)}\frac{1}{R^3}\frac{1}{(\dots)^{3/2}}$$

$$\frac{\partial m_z}{\partial(z/L)} = \frac{\partial m_z}{\partial z} = \beta\frac{t^2\pi^2}{(t^2+\pi^2)}\frac{1}{\left[\beta^2(z-\xi)^2+\pi^2\right]^{3/2}}, \quad \xi = Z/R, \ \beta = L/R, \ \frac{z}{L} \to z.$$

$$\frac{\partial I(kz,Z)}{\partial z} = \int_0^1 d\pi z\, J_0(k\pi z)\frac{\partial m_z}{\partial z} = \int_0^1 d\pi z\, J_0(k\pi z)\frac{t^2\pi^2}{(t^2+\pi^2)}\frac{\beta}{\left[\beta^2(z-\xi)^2+\pi^2\right]^{3/2}}$$

Это выражение от $\Gamma(k)$? $W_m^{vol}(\xi)$ считается также

дано как $W_m^{tot}(Z)$ см 28 / VIII

$$W_m(\xi) = \int_0^1 d\pi z\pi \int_{-a-\xi}^{a-\xi} dz\, m_z^2(\pi,z) - \frac{1}{2}\beta\int dz\int_{-a-\xi}^{a-\xi} dz'\int dk\, k^2 e^{-\beta k|z-z'|} I(kz) I(kz')$$

$$W_m(\xi) = \int_0^1 d\pi z\pi \int_{-a-\xi}^{+a+\xi} dz\, m_z^2(\pi,z) - \frac{1}{2}\beta\int_{-a-\xi}^{a-\xi} dz\, F(z,\xi), \quad F(z,\xi) = \int_{-a-\xi}^{a-\xi} dz' \int dk\, k^2 e^{-\beta k|z-z'|} I(kz) I(kz')$$

$$\frac{\partial}{\partial x}\int_{a(x)}^{b(x)} f(x,t)\,dt = \frac{\partial b}{\partial x} f(x,b) - \frac{\partial a}{\partial x} f(x,a) + \int_a^b dt\,\frac{\partial f(x,t)}{\partial x}$$

$$\frac{\partial W_m}{\partial \xi} = \int d\pi z\pi \left[-m_z^2(\pi, a+\xi) + m_z^2(\pi, -a-\xi)\right] - \frac{1}{2}\beta\left[-F(a-\xi,\xi) + F(-a-\xi,\xi) + \right.$$

$$\left. + \int_{-a-\xi}^{a-\xi} dz\,\frac{\partial F(z,\xi)}{\partial \xi}\right], \quad \frac{\partial^2 W_m}{\partial \xi^2} = \frac{\partial}{\partial \xi}\int d\pi z\pi\left[m_z^2(\pi, a-\xi) - m_z^2(\pi, a-\xi)\right] +$$

$$+ \frac{1}{2}\beta\frac{\partial}{\partial \xi}\left[F(-a-\xi,\xi) - F(a-\xi,\xi) + \int_{-a-\xi}^{a-\xi} dz\,\frac{\partial F(z,\xi)}{\partial \xi}\right], \quad \frac{\partial m_z^2}{\partial \xi} \text{ считается нулем.}$$

$$\frac{\partial}{\partial \xi} F(-a-\xi,\xi) = \frac{\partial}{\partial \xi}\int_{-a-\xi}^{a-\xi} dz' \int dk\, k^2 e^{-\beta k|?|} I(k,z) I(k,z')$$

$$|?| = |z-z'|_{z=-a-\xi} = |-a-\xi-z'| = |a+\xi+z'|$$

$$\frac{\partial F(-a-\xi,\xi)}{\partial \xi} = \frac{\partial}{\partial \xi}\int_{-a+\xi}^{a-\xi} dz' \int dk\, k^2 e^{-\beta k|a+\xi+z'|} I(k,z) I(k,z').$$

(2/VIII) С нов. "Белая банка". Чем отличается. Граница. $\frac{\partial F}{\partial \xi}(z,\xi) = \frac{\partial}{\partial \xi}\int_{-a-\xi}^{a-\xi} dz' \int dk\, k^2 e^{-\beta k|z-z'|} I(kz) I(kz') = \frac{\partial}{\partial \xi}\int_{-a-\xi}^{a-\xi} dz' D(z,z') =$

$$= -D(z, a-\xi) + D(z, -a-\xi) \text{ оператор. } \frac{\partial W_m}{\partial \xi}.$$

$$W_{ex} = \int_{-a+\xi}^{a-\xi} dz\,\omega(z), \quad \omega(z) = \left(\frac{Le}{R}\right)^2 \cdot \int_0^1 d\pi z\pi\left[\frac{1}{(\pi - m_z^2)}(\dots) + \frac{1}{\pi z^2}(\pi - m_z^2)\right]$$

$$\frac{\partial W_{ex}}{\partial \xi} = -\omega(a-\xi) + \omega(-a-\xi) = -\left[\omega(a) - \frac{d\omega}{dz}\Big|_{z=a}\xi\right] + \left[\omega(-a) - \frac{d\omega}{dz}\Big|_{z=-a}\xi\right] =$$

$$= \omega(-a) - \omega(a) + \left[\frac{d\omega}{dz}\Big|_{z=a} - \frac{d\omega}{dz}\Big|_{z=-a}\right]\xi = \frac{\partial}{\partial \xi}\left[W_{ex}(a) + \frac{\partial \chi_z}{\partial z}\xi^2\right] = \chi_z \xi$$

$\chi_z^{ex} = \omega'(a) - \omega'(-a)$, это использовано в Math Cad. $\chi_z^{ex} < 0$.

$\frac{\partial W_m}{\partial \xi} = \chi_z^m \xi$, $\xi = Z/L$ MCad - Block-point-stiffness.mcd - дает $\chi_z^m < 0$

дано $\beta = 20, 30, 50$ и $b/R = 0{,}2, 0{,}3$ $|\chi_z^m(\beta)|$ меньше с ростом β, и $\to \infty$ при $\beta \to \infty$

$\chi_z = \chi_z^{ex} + \chi_z^m < 0$, т.е. $\chi = 0$ (middle of nanowire) – saddle point, т.к. $\chi > 0$

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3-4/VIII Расчет $W_m(2)$ для оценки magnetostatics Thiaville ansatz BP DW. $\chi_z < 0$.

5-9/VIII Ikerbasque promotion report. В/тур посещение Киев - Л.Д. 10kg - 25€

10/VIII Отъезд с Л. в 10, тест. Лора послала WU 250 € Рено и Эд.

11/VIII В родном доме (+39°C). Б.ничг.

12-14/VIII Родной дома. Бынь перепечатка с Евгением и Оксаной.

15-16/VIII Ответы referees Strelkov PRAPPL manuscript.

17-18/VIII Грипп. Навещение корреспонд в доме.

19/VIII Edited, shortened letters arising Zhang'2013 Nature и Submitted as comments после Zhang et al. paper on-line.

20/VIII Рец. MS Chapin STNO. Sci. Rep. 2014.

21/VIII Update citations Web of Sci. в университете.

22-23/VIII Подготовка Ikerbasque prom. application. Travel appeal Porto.

24/VIII Поиск аптеки в Риме. Купил „Ципрофлоксацин" 8 oct./23 oct.

25-28/VIII Подготовка Ikerbasque application. Submitted 28/08.

29-30/VIII Подготовка inv. talk „BP DW dyn.", CMD-31, Braga.

31/VIII - 8/IX Поездка на CMD31, Braga. Дождь сделал 2 inv. talks. „Radial vortex" и „Thin thick dots" с моими со-авторами.

9/IX Travel report Braga, CMD"-31. Письмо прислал Parker - Hopkins.

10/IX Переписка с Maider, Arcady, Oksana. DW-BP dyn. Curvilinear geometry: 1) изогнутая stripe, 2) vortex string in nanocaps.

11/IX Написание BP-DW гирлодин. MS с Оксаной.

$W(\vec{x}) = W(0) + \frac{1}{2}\varkappa(X^2+Y^2) + \frac{1}{2}\varkappa_z Z^2$. $\varkappa_z < 0$ for finite nanowire length.

При $\beta \to \infty$, $L \to \infty$ infinite long nanowire $\varkappa_z \to 0$.

Thiele equations: $-G_S \dot{Z} + D_{\varphi\varphi}\dot{X}_\varphi = \varkappa X_\varphi$, $G_S \dot{X}_\varphi + D_{zz}\dot{Z} = \mu h_z$.

$\Omega^2 = \Gamma^2 + \frac{1}{4}\omega_z^2$. $\Gamma^2 = \varkappa|\varkappa_z|/G_S^2$, $\omega_z = -\varkappa D_{zz}/G_S^2$ при $\varkappa_z \to 0$.

$\Omega G_S^2 = G_S^2\sqrt{\frac{\varkappa|\varkappa_z|}{G_S^2} + \frac{1}{4}\frac{\varkappa^2 D_{zz}^2}{G_S^4}}$. Если $\Gamma \gg \omega_z$, то $\Omega G_S^2 \approx \Gamma G_S^2 = \frac{\sqrt{\varkappa\varkappa_z}}{G_S} G_S^2 = G_S\sqrt{\varkappa\varkappa_z}$. Но при $\varkappa_z \to 0$ так нельзя.

При $\varkappa_z \to 0$ $\Gamma^2 \ll \frac{1}{4}\omega_z^2$ и $\Omega \approx \frac{1}{2}\omega_z$, $\Omega G_S^2 = \frac{1}{2}\omega_z G_S^2 = -\frac{1}{2}\varkappa D_{zz}$.

$Z(t) \sim 1/\Omega G_S^2 + (h_z\text{-term})$.

$\omega_z^2/\Gamma^2 = -\frac{\varkappa D_{zz}}{G_S^2}\frac{G_S^2}{\varkappa|\varkappa_z|} = -\frac{D_{zz}}{|\varkappa_z|}$. Хотя $D_{zz} \sim d_{ZG}$ is small, нельзя $\omega_z^2/\Gamma^2 \ll 1$ при $\varkappa_z \to 0$, long nanowires.

$-G_S\dot{Z} + D_{\varphi\varphi}\dot{X}_\varphi = \varkappa X_\varphi = \varkappa\frac{1}{G_S}[\mu h_z - D_{zz}\dot{Z}]$, $\dot{X}_\varphi = \frac{1}{G_S}[\mu\dot{h}_z - D_{zz}\ddot{Z}]$

$-G_S\ddot{Z} + D_{\varphi\varphi}\frac{1}{G_S}[\mu\dot{h}_z - D_{zz}\dot{Z}] = \frac{\varkappa}{G_S}[\mu h_z - D_{zz}\dot{Z}]$

$-\left[G_S + \frac{D_{\varphi\varphi}D_{zz}}{G_S}\right]\ddot{Z} + D_{zz}\frac{\varkappa}{G_S}\dot{Z} = -\frac{D_{\varphi\varphi}}{G_S}[\mu\dot{h}_z] + \frac{\varkappa\mu}{G_S}h_z$, $D^2/G_S^2 \ll 1$.

$\ddot{Z} - D_{zz}\frac{\varkappa}{G_S^2}\dot{Z} = \frac{D_{\varphi\varphi}}{G_S^2}\mu\dot{h}_z - \frac{\varkappa\mu}{G^2}h_z$. Верно. Можно получить из решения

$Z(t)$ при $\omega_G^2 = 0$ и $\omega_z = -D_{zz}\varkappa/G_S^2$, $\varkappa_z \to 0$, $\beta \to \infty$.

$\ddot{Z} + \omega_z\dot{Z} = F(h_z)$, $F(h_z) = \frac{\mu}{G_S^2}[D_{\varphi\varphi}\dot{h}_z - \varkappa h_z]$. Есть и steady state solution $\dot{Z}(t) = \text{const}$, $\ddot{Z} = 0$. Формально, да.

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Steady solution $\dot{Z} = \frac{1}{\omega_z} \frac{\mu}{G_s} [D_{eff} h_z - \varkappa h_z]$, hyster. $\dot{h}_z = 0$.

$$\dot{Z} = V_z = -\frac{\varkappa \mu}{\omega_z G_s^2} h_z = -\frac{\varkappa \mu h_z}{(-\varkappa D_{zz})} = \frac{\mu h_z}{D_{zz}}$$

Знак зависит от head-to-head и tail-to-tail DW при $h_z > 0$ ($p = \pm 1$).

$\ddot{X}_c + \omega_z \dot{X}_c = \frac{\mu}{G} \dot{h}_z = 0$ при $\dot{h}_z = 0$. Steady $\dot{X}_c = V_c = 0$, т.е. BP движется || Oz, наноwire axis.

$D_{zz} = -2\pi d \frac{M_s}{\gamma} \int_0^R d\rho\, \rho \int_{-H/2}^{H/2} \frac{1}{m_\rho^2} \left(\frac{\partial m_z}{\partial z}\right)^2$ — надо найти. Use Feldkeller ansatz $m_z(\rho, z) = \frac{\rho^2}{\sqrt{\rho^2 + z^2}} \Rightarrow \rho \frac{\rho^2}{(z^2+\rho^2)^{3/2}} = \frac{\partial m_z}{\partial z}$, $\beta \to \infty$.

$m_\rho^2 = 1 - m_z^2 = 1 - \frac{z^4}{(\rho^2+z^2)} = \rho^2/(\rho^2+z^2)$.

$D_{zz} = -(\dots) \int d\rho\, \rho \int dz \frac{(\rho^2+z^2)}{\rho^2} \frac{\rho^4}{(z^2+\rho^2)^3} = -(\dots) \int d\rho\, \rho \int_{-H/2}^{H/2} dz \frac{\rho^2}{(z^2+\rho^2)^2}$.

$\int dz \frac{1}{(z^2+\rho^2)^2} = \frac{z}{2\rho^2(\rho^2+z^2)} + \frac{1}{2\rho^3} \arctg\left(\frac{z}{\rho}\right) \Big|_{-H/2}^{H/2} = \frac{L}{2\rho^2(\rho^2+\frac{L^2}{4})} + \frac{2}{2\rho^3} \arctg\left(\frac{L}{2\rho}\right)$

Long nanowire $H/R \gg 1$, $\rho \le R$. $\arctg(x) \xrightarrow[x\to\infty]{} \frac{\pi}{2} - \frac{1}{x}$.

$\int dz \frac{1}{(z^2+\rho^2)^2} = \frac{L}{2\rho^2 \sqrt{L^2/4}} + \frac{1}{\rho^3} \frac{\pi}{2}$.

$\int d\rho\, \rho^3 \int dz \frac{1}{(z^2+\rho^2)^2} = \int d\rho\, \rho^3 \left[\frac{2}{\rho^2 L} + \frac{\pi}{2} \frac{1}{\rho^3}\right] = \frac{2}{L} \int_0^R d\rho\, \rho + \frac{\pi}{2} \int_0^R d\rho =$

$= \frac{1}{L} R^2 + \frac{\pi}{2} R = \frac{1}{\beta} R + \frac{\pi}{2} R = R\left(\frac{\pi}{2} + \frac{1}{\beta}\right) \approx \frac{\pi}{2} R$ при $\beta \gg 1$.

$D_{zz} = -2\pi d \frac{M_s}{\gamma} \int d\rho\, \rho \int dz \frac{1}{m_\rho^2} \left(\frac{\partial m_z}{\partial z}\right)^2 = -2\pi d \frac{M_s}{\gamma} \frac{\pi}{2} R = -\pi^2 d R \frac{M_s}{\gamma}$.

Итог $D_{eff} = -2\pi^2 d R (M_s/\gamma)$ в пределе $\beta \gg 1$. $D_{zz} = -\pi^2 d R (M_s/\gamma)$

Mobility μ при $\beta \gg R$ (Feldkeller ansatz) $\mu = 8\pi M_s^2 \frac{V}{L}$.

Steady velocity BP-DW $V_z = \frac{\mu}{D_{zz}} h_z = -\frac{8\pi M_s^2 V}{L} \frac{\gamma h_z}{\pi^2 d R M_s} = -\frac{8\pi \cdot \pi R^2 L M_s \gamma h_z}{L \cdot \pi^2 d R} =$

$= -\frac{8R}{d}(\gamma M_s) h_z = -\frac{32}{\pi} \frac{R}{d} \omega_M h_z$, $\omega_M = \mu_0 \gamma M_s$. $V_z = \frac{2R}{\pi d} \omega_M h_z$ Steady BP velocity at $\varkappa h_z \to 0$, $\beta \to \infty$.

$V_z = \frac{4R}{d} \left(\frac{\omega_M}{2\pi}\right) h_z$. Пленка FeNi, $h_z = 10^{-2}$, $d \ge 10^{-3}$, $R = 30$ nm. $V_z = \frac{4 \cdot 30 \cdot 10^{-9}\,m}{10^{-2}} 30\,GHz \cdot 10^{-2} =$

$= 4 \cdot 30 \cdot 30 \frac{m}{s} = 3600 \frac{m}{s} = 3.6 \frac{km}{s}$ (!). Сравнение с Oksana's simulations. Ускорение DW и H_z linear граница до насыщения $t \le 1$ ns. См. Fig. 2, Tejo Nanoscale 2024. $V_z \approx 0{,}8\,\mu m/1\,ns = 0{,}8 \frac{km}{s}$ при $H_z = 50$ mT.

$V_z = 1{,}3 \frac{\mu m}{0{,}2\,ns} = 6{,}5 \frac{km}{s}$ при $H_z = 100$ mT при $M_s = 1700$ G (Re).

$\vec{H}_z$ — BP — tail-to-tail $p=+1$, $H_z \to$ head-to-head BP-DW $p=-1$

Знак V_z при $H_z > 0$, tail DW ($p=+1$) имеет $V_z < 0$, head-DW имеет $V_z > 0$.

$W_H = p \mu h_z Z$. Mobility $\mu \sim \rho$ (уточнить) и steady $\dot{Z} = p \frac{\mu h_z}{D_{zz}} = V_z$

Идея в ночь на 14/I 2023 для $\beta \gg 1$, $\beta \gg 1$ (Feldkeller ansatz).

$D_{eff} = -2\pi^2 d R (M_s/\gamma)$.

Walker velocity: $V_W = \frac{\gamma \Delta}{\alpha} H$, $\Delta = \sqrt{A/K}$.

$V_z = -p \frac{8R}{d} \gamma M_s \frac{H_z}{4\pi M_s} = -p \frac{2R}{\pi} \gamma \frac{H_z}{\alpha}$

BP-DW steady velocity $V_z = \frac{2R}{\pi} \frac{\gamma H_z}{\alpha}$, т.е. effective DW width $\Delta = 2R/\pi$.

12/IV Revised BP-DW artic[illegible]

13/IX A. Bank Cairo" deposit 4060 mE Conf. Soc. на стол CONSTANTO.
В реп. Иван, Ондинеева. N Campo Logarpi c A.. 40.

14/IX Нодгсов cash € Bank: Ивайло ги 8900 € + Кабинет 3,320 €
Бедно: 6,120. €

15/IX Review Hopfions for Magnetism.

16/IX Нагиа чайдъ Maidoris office.

17/IX Напишаните BP DW dyn. MS с Оценки. Информиса с Лекции/Maj ej.

18/IX Нафеждъ в new office. BP DW dyn. MS, E-mails Лена/Maciej.

19/IX Найси hotel B30 airport 7/8 Oct.. Берлин Holiday Inn до 1/10.

• C. Saji et al. Hopfion driven magnonic Hall effect. PRL [31], 166702 (2023)
Разлагане с Ани-Hopfion in spherical coordinates.

$m_x + i m_y = \sin 2\chi\, e^{i(q\varphi - p\theta + \delta)}$, $m_z = \cos 2\chi$, (r, θ, φ) - spherical coordinates.
$\chi = q_1 \pi_2$ - helicity, $|m|^2 = 1$, $\theta(r, \theta) = \tan^{-1}[\tan f(r) \cos\theta]$, $\sin\chi = \sin f(r) \sin\theta$
$f(0) = 0$, $f(\infty) = \pi$, $f(r)$ - is an arbitrary. Это тороидал Hopfion - ?

$m_\mu = \langle Z | \hat{\sigma}_\mu | Z \rangle$, $|Z\rangle$ spinor $= \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$, $z_1 = \cos\chi\, e^{ip\theta}$, $z_2 = i \sin\chi\, e^{iq\varphi}$
$|Z\rangle = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$. Стереографска: $w = \frac{z_2}{z_1} = \frac{i \sin\chi\, e^{iq\varphi}}{\cos\chi\, e^{ip\theta}} = i \operatorname{tg}\chi\, e^{i(q\varphi - p\theta)}$

$m_z = \frac{1 - |w|^2}{1 + |w|^2} = \frac{1 - \operatorname{tg}^2\chi}{1 + \operatorname{tg}^2\chi} = \frac{\cos^2\chi - \sin^2\chi}{\sin^2\chi + \cos^2\chi} = \cos 2\chi$.

$m_x + i m_y = \frac{2w}{1 + |w|^2} = \frac{2i \operatorname{tg}\chi\, \exp[i(q\varphi - p\theta)]}{1 + \operatorname{tg}^2\chi} = i \cdot 2 \sin 2\chi\, e^{i(q\varphi - p\theta)}$

(!) В енлеген Saji'23 г. числа. Тогда $m_x + i m_y = \sin 2\chi\, e^{i(q\varphi - p\theta)}$
Я бран w чрез ст. $S_4 \to S_2$ mapping
$m_x + i m_y = 2 z_2 z_1^* = 2 \cdot i \sin\chi\, e^{iq\varphi} \cdot \cos\chi\, e^{-ip\theta} = i \sin 2\chi\, e^{i(q\varphi - p\theta)}$

Map $w(\vec{r}) = \operatorname{ch}^m\gamma \operatorname{th}^n\gamma\, e^{i(m\beta + n\alpha)}$, (γ, β, α) - toroidal coordinates.
Найти связь тороидал (γ, β, α) и spherical (r, θ, φ) coordinates.
Map $w = \frac{2(x + iy)}{(1 - r^2 + 2iz)}$ sum $|Q_H| = 1$. Sъщ $w = \operatorname{tg}\chi\, e^{i(\varphi - \theta)}$, $p = 1$, $q = 1$.

$w = \frac{2\rho e^{i\varphi}}{(1 - r^2 + 2ir\cos\theta)} = \frac{2r \sin\theta\, e^{i\varphi}}{(1 - r^2 + 2ir\cos\theta)}$ $u = \operatorname{tg}\chi\, e^{i(\varphi - \theta)}$

$\frac{2r\sin\theta}{(1 - r^2 + 2ir\cos\theta)} = \operatorname{tg}\chi\, e^{i\theta}$, $\operatorname{tg}\theta = \operatorname{tg}\beta \cdot \cos\theta$, $\sin\chi = \sin\beta \sin\theta$.

$\frac{2r \sin\chi}{\sin\beta [1 - r^2 + 2ir \cdot \operatorname{ctg}\beta \cdot \operatorname{tg}\theta]} = \operatorname{tg}\chi\, e^{-i\theta}$ - Верно ли?

$\vec{A}^2 \cdot \vec{B} = \frac{2pq}{r^2} \sin^2\beta \delta \cos^2\theta + \frac{p^2 q}{2r^2} \sin^2 2\beta \beta' \sin^2\theta + \frac{2pq^2}{r^2} \sin^4\beta \beta' \sin^2\theta$

$\vec{A}^2 \cdot \vec{B} = \frac{2pq}{r^2} \beta' \left[\sin^2\beta \cos^2\theta + \frac{1}{4} \sin^2 2\beta \sin^2\theta + \sin^4\beta \sin^2\theta \right]$

$\int dr\, r^2 \frac{1}{r^2} \sin^2\beta \beta' = \int dr \beta' \sin^2\beta = \int_0^\pi d\beta \sin^2\beta = \pi/2$.

$\int dr \sin^2 2\beta \beta' = \int_0^\pi d\beta \sin^2 2\beta = |2\beta = x| = \frac{1}{2} \int_0^{2\pi} dx \sin^2 x = \frac{1}{2} \int_0^{2\pi} dx \sin^2 x = \frac{1}{2} \frac{1}{2} 2\pi = \pi/2$

$\int dr \sin^4\beta \beta' = \int_0^\pi d\beta \sin^4\beta = \frac{3}{8} \frac{2\pi}{2} = \frac{3}{8} \pi$

$\int dV \vec{A}^2 \vec{B} = 2pq \int_0^\pi d\theta \sin\theta \cdot 2\pi \left[\frac{\pi}{2} \cos^2\theta + \frac{1}{4} \sin^2\theta \frac{\pi}{2} + \sin^2\theta \frac{3}{8} \pi \right] =$

$= \frac{4\pi^2}{2} pq \int_0^\pi d\theta \sin\theta \left[\cos^2\theta + \sin^2\theta \right] = 2\pi^2 pq \int_0^\pi d\theta \sin\theta = 4\pi^2 pq$.

Escaneado con CamScanner

$Q_H = \frac{1}{4\pi^2}\int dV \vec{A}\cdot\vec{B} = pq$ when $f(0)=0$, $f(\infty)=\pi$, Ranjan's toroidal hopf.

$\Rightarrow$ Eqs. (6), (7) Saji, Nunez 'PRL '23 gives $\vec{A}$, $\vec{B}$ Ranjan, & curl $\vec{A}=\vec{B}$ $Q_H = pq$.

- $\vec{B}$ (Saji) coincides with $\vec{B}$ (Pershoguba'22), & $\vec{A}$ (Saji) is related to $\vec{A}$ (Pershoguba) Pershoguba has $p=1, q=1, Q_H=1$ by Liu-Zang PRL '2020

Liu-Zang began with hopfion by Hietarinta PLB '99
He Hietarinta has m-hopfion with $Q_H = 1$ ($p=q=1$). Normalizing obtained $\vec{m}$-hopfion by Saji '23 via any p,q is analogous (unclear reference).

$\vec{A}$ (Saji) is related to $\vec{A}$ (Pershoguba'22) = $\vec{A}$ (Hietarinta'99) $p=q=1$

Saji: $m_x + i m_y = \sin\gamma\, e^{i(q\varphi - p\theta + \beta)}$, $m_z = \cos\gamma$, $w = \tan\frac{\gamma}{2} e^{i(q\varphi - p\theta + \beta)}$
$\gamma = 0, \pi/2$ - axial, Bloch

(2/6) $f(r)$ monotonic, Bel-P ansatz $\sin f(r) = \frac{2rR_0}{(r^2+R_0^2)}$ (skew range $R_0 > a$).

$\tan\Theta = \tan f \cdot \cos\varphi$, $\Theta(r,\varphi)$ & $\sin\gamma = \sin f \cdot \sin\varphi$. $\Theta \to \Theta$, $\gamma = \gamma_0$ and $\varphi \to \varphi$ is related to $\vec{m}(\Theta,\varphi)$.

Standard form of spherical $\vec{m}(\Theta,\Phi)$ coordinates $\to w = \tan\frac{\Theta}{2} e^{i\Phi}$.

$\tan\frac{\Theta}{2} = \tan f$, $\Phi(r,\varphi) = q\varphi - p\theta_s + \gamma_s$. Axially $\gamma = \Theta/2$, $m_z = \cos\Theta$, $m_x + i m_y = \sin\Theta e^{i\Phi}$.

Gaschler'23: $\Phi(r,\varphi) = q\varphi + \gamma(s,z)$ gives azim. Symmetric toroidal hopfion.

Saji '23: $\Phi(r,\varphi) = q\varphi - p\arctan[\tan f \cdot \cos\varphi] + \gamma_s$, $\gamma_s = \gamma_0 \vee \frac{\pi}{2}$. $\cos\varphi = \frac{z}{r_c} = \frac{z}{\sqrt{\rho^2+z^2}}$

Mee Hietarinta $\gamma(s,z) = -p\arctan[\tan f \cdot \cos\varphi] + \gamma_s = -p\arctan\left[\tan(f(r))\frac{z}{\rho}\right] + \gamma_s$.

$\gamma(s,z) = m\cdot\arctan\left(\frac{2az}{s^2+z^2-a^2}\right)$, Gu. Chaos '23 & my calc. Apr. 2023.

Mee $w = \frac{2(x+iy)}{(1-r^2+2iz)}$, where x,y,z are in "a" hopfion radius units.

$m\cdot\arctan\left(\frac{2az}{r^2-a^2}\right) = -p\arctan\left[\tan f \cdot \frac{z}{\rho}\right] + \gamma_s$, boundaries ok?

Shnir '2018: $\sin f(r) = \frac{2r}{(1+r^2)}$, $\cos f = \frac{1-r^2}{1+r^2}$, $f(0)=0$, $f(\infty)=\pi$ as Sutcliffe '07

$\tan f(r) = \frac{2r}{(1-r^2)} = \frac{2ra}{(a^2-r^2)}$. $\tan f \cdot \frac{z}{\rho} = \frac{2ra}{(a^2-r^2)}\frac{z}{\rho} = \frac{2az}{(a^2-r^2)} = -\tan\beta(s,z)$

$m\cdot\arctan\left(\frac{2az}{r^2-a^2}\right) = -p\arctan\left[\frac{2az}{(a^2-r^2)}\right] + \gamma_s$. Combinations

with $\gamma_s = 0$ (Axial hopfion), $m = p$.

& Liu-Zang PRL '2020 & m-hopfion ($Q_H = m$) analytical (via $\pi, 2^m$ rec), confirmed & Pershoguba '2022.

Ranjan as Saji's $\sin\gamma = \sin f \cdot \sin\Theta = \sin\frac{\Theta}{2}\cdot\frac{z}{\rho}$, $\sin^2\gamma = \sin^2 f \cdot \sin^2\varphi$

$m_z = \cos 2\gamma = \cos^2\gamma - \sin^2\gamma = 1 - 2\sin^2\gamma = 1 - 2\cdot\sin^2 f \cdot \sin^2\varphi = 1 - 2\frac{s^2}{r^2} = 1 - 2\frac{r^2\rho^2}{(a^2+r^2)^2} = 1 - 8\frac{s^2a^2}{(a^2+r^2)^2} = m_z$. Ergo Ranjan gives $Q_H = 1$ [Hietarinta'99]

- Bochgym: 1) Saji '23 eq. $\sin\gamma = \sin f \cdot \sin\varphi$ goes Ranjan $m_z = \cos 2\gamma$ wrong for $Q_H = 1$
2) Saji '23 gives Ranjan hopfion $Q_H = pq$.
3) Saji '23 hopfion analyt $\vec{m}(r,\varphi,\theta)$ unexpressible, Ranjan $\vec{m}(\rho,\varphi,z)$ explicit.
4) Saji '23 vector potential $\vec{A}$ spherical expression for $\vec{A}$ (Pershoguba) gives $Q_H = 1$.
5) Hopfion ansatz Saji '23 - two toroidal hopfions in one space via Ghalandarzadeh-Hietarinta '99 Ranjan-polynomial expressions.

$m_z(\varphi=\pi/2, f=\pi/2) = -1$ when $z=0$, $s=a$ Ranjan Solenoid/toroidal hopf.

Escaneado con CamScanner

Совпадает ли Сэди $\vec{A}^2 = q\sin^2\gamma\,\partial_\mu\varphi - p\cos^2\gamma\,\partial_\mu\theta_s$ с полем Гинзбурга-Чоуза '23 $\vec{A} = -2\nabla\gamma + (1-\cos\theta)\nabla\Phi$? $\mu = 1,2,3$.

$\Phi(\vec{r}) = q\varphi - p\theta_s$. $\vec{A}^2 = q\partial_\mu\varphi - q\cos^2\gamma\,\partial_\mu\varphi - p\cos^2\gamma\,\partial_\mu\theta_s$

$\vec{A}^2 = q\partial_\mu\varphi - \cos^2\gamma\,\partial_\mu[q\varphi - p\theta_s] - 2p\cos^2\gamma\,\partial_\mu\theta_s =$

$= q\partial_\mu\varphi - \cos^2\gamma\,\partial_\mu\Phi - 2p\cos^2\gamma\,\partial_\mu\theta_s = \partial_\mu[q\varphi - p\theta_s) + p\partial_\mu\theta_s -$

$-\cos^2\gamma\,\partial_\mu\Phi - 2p\cos^2\gamma\,\partial_\mu\theta_s = \sin^2\gamma\,\partial_\mu\Phi + p(1-2\cos^2\gamma)\partial_\mu\theta_s$

$\vec{A}^2_\mu = \sin^2\gamma\,\partial_\mu\Phi + p(-1)\cos 2\gamma\,\partial_\mu\theta_s$ - не дано.

Другие усл. $\gamma = -p\theta_s$, $\cos\theta = \cos 2\gamma \to \vec{A} = -2\nabla[-p\theta_s] +$

$+(1-\cos 2\gamma)\nabla\Phi = 2p\nabla\theta_s + (1-\cos 2\gamma)\nabla\Phi = 2\sin^2\gamma\,\nabla\Phi + 2p\nabla\theta_s$

Может ли $\vec{A}^2 = \vec{A}$? $\sin^2\gamma\,\partial_\mu\Phi - p\cos 2\gamma\,\nabla\theta_s = 2\sin^2\gamma\,\nabla\Phi + 2p\nabla\theta_s$

$-p(\cos 2\gamma + 2)\nabla\theta_s = \sin^2\gamma\,\nabla\Phi$, $-p(1+2\cos^2\gamma)\nabla\theta_s = \sin^2\gamma\,\nabla\Phi -$

- это не выполняется.

Вывод: Вектор потенциал Сэди '23 $\vec{A}^2$ - неверен, и не совпадает с $\vec{A}$ (Першагуба '22) для $Q_M = 1$.

Как $\vec{A} = -2\nabla\gamma + (1-\cos\theta)\nabla\Phi$ в spherical coordinates (r, θ, φ)?

$\gamma = \gamma(s,z)$, $\Phi = m\varphi + \gamma(s,z)$. $z = r\cos\theta$, $s = r\sin\theta$

$\gamma(s,z) = m\beta(s,z) = m\,\mathrm{arctg}\left[\frac{2a^2}{r^2-a^2}\right] = m\,\mathrm{arctg}\left[\frac{2r^2}{(r^2-a^2)}\cos\theta\right]$

$\hat{s} = \sin\theta\,\hat{r} + \cos\theta\,\hat{\theta}$, $\hat{\varphi} = \hat{\varphi}$, $\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$

$\vec{A} = A_s\hat{s} + A_\varphi\hat{\varphi} + A_z\hat{z} = A_s(\sin\theta\,\hat{r} + \cos\theta\,\hat{\theta}) + A_\varphi\hat{\varphi} + A_z(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}) =$

$= A_r\hat{r} + A_\varphi\hat{\varphi} + A_\theta\hat{\theta}$, $A_r = A_s\sin\theta + A_z\cos\theta$, $A_\theta = A_s\cos\theta - A_z\sin\theta$

$A_\varphi = (1-m_z)\frac{m}{s}$, A_φ (Першагуба) $= \frac{1}{r}\sin\theta\sin^2\zeta = \frac{1}{s}\sin^2\zeta$ $\quad A_\varphi = A_\varphi$.

$1-m_z = 1-\cos\theta = 2\sin^2\frac{\theta}{2} = \sin^2\zeta$ - точно верно? $(m=1)$

$A_z = -(1+m_z)\frac{\partial\gamma}{\partial s}\sin\theta - (1+m_z)\frac{\partial\gamma}{\partial z}\cos\theta = -(1+m_z)\left[\frac{\partial\gamma}{\partial s}\sin\theta + \frac{\partial\gamma}{\partial z}\cos\theta\right] =$

$= -(1+m_z)(\nabla\gamma\cdot\hat{r})$. A_r (Першаг.) $= 2\cos\theta\sin^2\zeta\,\zeta'$

$(\nabla\gamma\cdot\hat{r}) = \partial\beta/\partial r = m\frac{\partial}{\partial r}\,\mathrm{arctg}\left[\frac{2r^2}{(r^2-a^2)}\cos\theta\right] = m\frac{1}{1+\left(\frac{2r^2}{r^2-a^2}\right)^2\cos^2\theta}\cdot\cos\theta\,\frac{2a(r^2-a^2) - 2r^2\cdot 2r}{(r^2-a^2)^2} =$

$= m\frac{\cos\theta(-2ar^2-2a^3)}{(r^2-a^2)^2 + 4r^4a^2\cos^2\theta}$

$(\nabla\gamma\cdot\hat{z}) = -m\frac{2a\cos\theta(a^2+r^2)}{(r^2-a^2)^2 + 4r^4a^2\cos^2\theta}$, $A_r = m(1+m_z)\frac{2a\cos^2\theta(a^2+r^2)}{(r^2-a^2)^2 + 4r^2a^2\cos^2\theta}$

Совпадение с A_r (Першаг.) не очевидно

$A_\theta \Rightarrow A_\theta = A_s\cos\theta - A_z\sin\theta = -(1+m_z)\frac{\partial\gamma}{\partial s}\cos\theta + (1+m_z)\frac{\partial\gamma}{\partial z}\sin\theta =$

$= -(1+m_z)\left[\frac{\partial\gamma}{\partial s}\cos\theta - \frac{\partial\gamma}{\partial z}\sin\theta\right] = -(1+m_z)(\nabla\gamma\cdot\hat{\theta}) = -(1+m_z)\frac{1}{r}\frac{\partial\gamma}{\partial\theta}$

$\partial\beta(s,z)/\partial\theta = 0 \to A_\theta = 0$ в согласии с A_θ (Першагуба) $= 0$.

A_φ: $1-m_z = \sin^2\zeta$, $m_z = \cos^2\zeta$, $\cos\zeta = \frac{a^2-r^2}{(a^2+r^2)}$, $m_z \neq \cos^2\zeta$.

Першагуба $(\vec{A}\cdot\vec{B}) = 4\cos^2\theta\sin^4\zeta\,\zeta'\frac{1}{r^2} + \frac{1}{r^2}2\sin^2\theta\sin^4\zeta\cdot\zeta'$

$\int dV\,\vec{A}\cdot\vec{B} = 2\pi\int dr\,r^2\int d\theta\sin\theta\,2\sin^4\zeta\frac{\zeta'}{r^2}\left[2\cos^2\theta + \sin^2\theta\right] =$

$= 4\pi\int dr\,\zeta'\sin^4\zeta\int_0^\pi d\theta\sin\theta\left[2\cos^2\theta + \sin^2\theta\right] = 4\pi\cdot\frac{2}{3}\cdot\pi\cdot\frac{4}{3} = 2\pi^2$

Escaneado con CamScanner

Q_H (Pershoguba) $= \frac{1}{4\pi^2}\int d\vec{A}\cdot\vec{B} = 1/2$ (?) изд у них неплотность.

(21/IV) $\vec{B}_\gamma = \frac{1}{4}\vec{m}\cdot(\partial_z\vec{m}\times\partial_\rho\vec{m})$ надо $1/2$ здесь

$B_z = B_\rho\sin\theta + B_z\cos\theta = \frac{n}{\rho}\frac{\partial m_z}{\partial z}\sin\theta - \frac{n}{\rho}\frac{\partial m_z}{\partial\rho}\cos\theta = \frac{n}{\rho}\left[\frac{\partial m_z}{\partial z}\sin\theta - \frac{\partial m_z}{\partial\rho}\cos\theta\right] = \frac{n}{\rho}\left[(\nabla m_z)_z\sin\theta - (\nabla m_z)_\rho\cos\theta\right] = -\frac{n}{\rho}(\nabla m_z\cdot\hat{e}_\theta) = 0$ (?)

Но $m_z = m_z(\rho) = m_z(\rho,z)$, $\nabla m_z = \hat{\rho}\frac{\partial m_z}{\partial\rho} + \hat{z}\frac{\partial m_z}{\partial z}$, $(\nabla m_z\cdot\hat{e}_\theta) = 0$ - но т

$B_\theta = B_\rho\cos\theta - B_z\sin\theta = \frac{n}{\rho}\frac{\partial m_z}{\partial z}\cos\theta + \frac{n}{\rho}\frac{\partial m_z}{\partial\rho}\sin\theta = \frac{n}{\rho}\left[\frac{\partial m_z}{\partial z}\frac{z}{r} + \frac{\partial m_z}{\partial\rho}\frac{\rho}{r}\right]$

$\vec{r} = z\hat{z} + \rho\hat{\rho} + r\varphi\hat{\varphi}$, $r\varphi = 0$, $B_\theta = \frac{n}{\rho r}(\nabla m_z\cdot\vec{r}) = \frac{n}{\rho}(\nabla m_z\cdot\hat{r})$

$B_\theta = \frac{n}{\rho}\frac{\partial m_z}{\partial r}$, $m_z = m_z(\rho,z) = m_z(r\sin\theta, r\cos\theta)$, $\hat{e}_\theta = \hat{\rho}\cos\theta - \hat{z}\sin\theta$

$B_r = -\frac{n}{\rho}(\nabla m_z\cdot\hat{e}_\theta) = -\frac{n}{\rho}\frac{1}{r}\frac{\partial m_z}{\partial\theta}$.

$(\nabla\gamma\cdot\hat{\theta}) = \frac{1}{r}\frac{\partial\gamma}{\partial\theta} \neq 0$, т.к. $\gamma = \gamma(\rho,z) = \gamma(r\sin\theta, r\cos\theta)$, $A_\theta = 0$ не годится.

Note: Бушмо Pershoguba '22 и Sagi '23 нарезали $\mathbb{R}^3$ сферическими координатами $\vec{r}(r,\theta,\varphi)$ из-за Hedgehog ansatz (1999) где $Q_H = 1$, eqs (8.21) Shnir '2018. Hedgehog дает $\vec{m}$-компоненты, $\vec{r} = (x,y,z)$.

Но Sutcliffe (2007) предлагает Hedgehog ansatz через function $f(r)$

$[f(0) = \pi, f(\infty) = 0]$, $f(r) = \arctg\left(\frac{2r}{1-r^2}\right)$, $\sin f(r) = \frac{2r}{(1+r^2)}$, $\cos f(r) = \frac{1-r^2}{(1+r^2)}$,

из-за spinor $(z_1, z_0) = (\sin f\sin\theta e^{i\varphi}, \cos f + i\sin f\cos\theta)$.

Hedgehog-ansatz $\vec{m}(x,y,z)$ Lin '20, Pershoguba $\mathbb{R}^3$ переписали через $f(r)$ (см. 14/IV 2022):

$m_x = \frac{x}{r}\sin 2f + \frac{2yz}{r^2}\sin^2 f$, $m_y = -\frac{y}{r}\sin 2f + \frac{2zx}{r^2}\sin^2 f$, $m_z = \cos 2f + \frac{2z^2}{r^2}\sin^2 f$, $Q_H = 1$

и использовали из-за $f(r)$ - сферич. координаты $\vec{r}(r,\theta,\varphi)$. В этом не было необходимости и Hedgehog '99, Shnir '18 такой группой не пользовались (!).

// Sagi '23 used spherical coordinates, введенные обозначения $\vec{m}, \vec{B}, \vec{A}$ - hedgehog на

Matrix Notice Q_H и без каких ссылок вели $f(r)$ через rotation matrix $\hat{R}_H(\vec{r}) = e^{if(r)\frac{\hat{r}}{r}}$ (!?)

Sagi '23 переправил $\hat{R}_H$ из Terruela PRB 100, 054426 (2019) с ошибкой, надо

$\hat{R}(\vec{r}) = e^{if(r)\hat{e}_r\cdot\hat{L}}$, $[L_\alpha]_{\beta\gamma} = -i\varepsilon_{\alpha\beta\gamma}$, $f(r) = 2\pi\left(1 - \frac{r}{R^*}\right)\Theta(R^* - r)$ $\frac{f(r)}{2\pi}$ $\frac{r}{R^*}$

$m_y(\text{Shnir}) = -m_y(\text{Li})$. Бушмо correct is specific.

Shnir '18 spinor $= \begin{pmatrix} z_1 \\ z_0 \end{pmatrix}$, $z_1 = \sin f\sin\theta e^{i\varphi}$, $z_0 = \cos f + i\sin f\cos\theta$. Не путать "2" Lin '2020 вершины.

У меня Chaos '23: $z = (z_1, z_2)^T$, т.е. $z_2 \to z_0$ (Shnir).

$m_z = |z_1|^2 - |z_2|^2 = \sin^2 f\sin^2\theta - (\cos^2 f + \sin^2 f\cos^2\theta) = \left|\begin{matrix} z = r\cos\theta \\ \rho = r\sin\theta \end{matrix}\right| =$

$= \sin^2 f(1 - \cos^2\theta) - \cos^2 f - \sin^2 f\cos^2\theta = -\cos 2f - \sin^2 f\cdot 2\cos^2\theta = -\cos 2f - 2\frac{z^2}{r^2}\sin^2 f$

- совп. с (Shnir) Lin-corrected с точн. до знака.

$m_x + im_y = 2z_2 z_1^* = 2(\cos f + i\sin f\cos\theta)\sin f\sin\theta e^{i\varphi} = 2\sin f\sin\theta\times$

$\times(\cos f + i\sin f\cos\theta)(\cos\varphi - i\sin\varphi) = 2\sin f\sin\theta[\cos f\cos\varphi + \sin f\cos\theta\sin\varphi +$

$+ i(\sin f\cos\theta\cos\varphi - \cos f\sin\varphi)$. $m_x = 2\sin f\sin\theta\cos f\cos\varphi + \sin^2 f\sin 2\theta\sin\varphi =$

$= \sin 2f\sin\theta\cos\varphi + \sin^2 f\sin 2\theta\sin\varphi = \sin 2f\frac{x}{r} + \sin^2 f\,2\frac{yz}{r^2}\sin\varphi = \frac{x}{r}\sin 2f + 2\frac{yz}{r^2}\sin^2 f$

$m_x(z, z_0)$ совпадает с m_x (Shnir) и corrected Liu '20.

$m_y = 2\sin\beta \sin\varphi \left(\sin\beta\cos\varphi\cos\varphi - \cos\beta\sin\varphi\right) = \sin^2\beta\, 2\sin\varphi\cos\varphi\cos\varphi - \sin 2\beta \sin^2\varphi =$

$= \sin^2\beta\, 2\frac{\rho}{r}\frac{z}{r}\cos\varphi - \sin 2\beta \frac{\rho}{r}\sin\varphi = -\frac{y}{r}\sin 2\beta + 2\frac{xz}{r^2}\sin^2\beta$ как у Shnir '18.

$m_y = -\frac{y}{r}\sin 2\beta + \frac{2xz}{r^2}\sin^2\beta$, $m_x = \frac{x}{r}\sin 2\beta + \frac{2yz}{r^2}\sin^2\beta$

Видно надо $z_1 \rightleftarrows z_0$, тогда смены $m_x \to -m_x$ и $m_y \to -m_y$, давая совпадение с corrected-Liu '20 и перешедшие $z_1, r^2 = \rho^2 + z^2$.

(*) $m_x = \frac{x}{r}\sin 2\beta + \frac{2yz}{r^2}\sin^2\beta$, $m_y = \frac{y}{r}\sin 2\beta - \frac{2xz}{r^2}\sin^2\beta$, $m_z = \cos 2\beta + \frac{2z^2}{r^2}\sin^2\beta$.

$$\begin{cases} m_\rho = m_x\cos\varphi + m_y\sin\varphi = m_x\frac{x}{\rho} + m_y\frac{y}{\rho} = \frac{\rho}{r}\sin 2\beta = m_\rho(\rho, z) \\ m_\varphi = -m_x\sin\varphi + m_y\cos\varphi = -m_x\frac{y}{\rho} + m_y\frac{x}{\rho} = -2\frac{z\rho}{r^2}\sin^2\beta = m_\varphi(\rho, z) \end{cases}$$ см. 14/IV 202.

– components $\vec{m}$ of toroidal hopfion in (ρ, φ, z) coordinates in cylinder. $f(r)$ – profile function

– Sutcliffe (2007) – Liu (2020). $f(r)$ – только меняет, видно $\vec{m}(\vec{r})$ изменена

явно для $P_H = 1$ и ему Q_H из Gao Cheng '2023.

Можно найти $F_{\mu\nu} = \vec{m}\cdot(\partial_\mu\vec{m}\times\partial_\nu\vec{m})$ и em.field $B_\lambda = \frac{1}{2}\varepsilon_{\lambda\mu\nu}F_{\mu\nu}$ $\mu,\nu = \rho, \varphi, z$.

$\vec{m} = \vec{m}(\rho, z)$, $F_{\rho\varphi} = 0$, $F_{\varphi z} = F_{z\rho} = 0$, только $F_{\rho\rho} \neq 0$, $F_{zz} \neq 0$ и $F_{\rho z} = -F_{z\rho}$, тогда $F_{\rho\rho} = 0$, $F_{zz} = 0$

$F_{\rho\rho} = 0$, $F_{zz} = 0$ $F_{\rho z} = \vec{m}\cdot(\partial_\rho\vec{m}\times\partial_z\vec{m}) = -F_{z\rho}$

$\partial_\mu\vec{m} = \vec{\rho}\,\partial_\mu m_\rho + \vec{\varphi}\,\partial_\mu m_\varphi + \vec{z}\,\partial_\mu m_z$. $B_\varphi = \frac{1}{2}\varepsilon_{\varphi\mu\nu}F_{\mu\nu} = \frac{1}{2}\left[\varepsilon_{\varphi\rho z}F_{\rho z} + \varepsilon_{\varphi z\rho}F_{z\rho}\right] = \frac{1}{2}\left[(-1)F_{\rho z} + (+1)F_{z\rho}\right] = F_{z\rho}$, так можно найти B_φ только.

(23/IV) Начало занятий по Murakami и Analytics на церемонии Japanese Conf. в San Sebastian. Hopfion-review.

(24-25/IV) Закончил и submitted „Hopfion-review" в Magnetism.

(26-27/IV) переписка с Oxford и др. о „BP-DW dynamics" MS.

переезд в new office (95 – перевёз, перенес).

(30-29/IV) переслал W_m – BP в new magnetostatics для разных L_x.

(1/V) переписка с Agnesia (Director Dept.) о переезде. W_m-poster.

(2/V) Liverpool meeting, Tabakalera: 1) Перевёл умел не писано в то нес; 2) договорился с Argentina об online Magnonics July 2025, Mallorka и wonder book. JEMS, Frankfurt; 3) к/пузырей Zhou см. Cooke.

(3/V) Miguel Arroyo (Liverpool) написал об „incident" с office change. Объяснил ему, что никакой „инцидент" не было, я согласился переехать в new office в September что и сделал. Завершил переезд в Mansel's office и опробовал internet connection. Skype – Teb, объяснил ему что надо заключить договор подряда с CAM, выписать его этому и получать конверт после работ. Mansel's project давно. Nakagawa дал Teb curvilinear и известил, которые он планирует как для FMR-exp.

(4/V) Получил „договору" неделю из JAEA (Arroyo). → Рос. email 25€ h.D.

Сделал до отъезда в Poznan 7/V: 1) IVA „Enskento" до III кв. 2024;

2) e-mails Murakami, Analytics; 3) $W_m(6)$ расчёт для Oxford; 4) ответил на comments Oxf. / Roberto; 5) Slides Hopfion-talk in Poznan; 6) Rev. Commun. Nanosoft Phys.

7) Strelkov MS. ответил-переслал.

(5-6/V) расчёт IVA III кв. 2024. $W_m(6)$ – BP. Slides „Hopfion" talk Poznan.

(7/V) переезд в hotel-BIO airport. Ресторан с П.

(8/V) приехали в Poznan, taxi, поселились в apt. Gabbary district.

(9/X) Оформил. договора подписал с Maciej (заполнил tax form с home address). Беседа с Sarah, Pietr, Mathieu (ген. мин $F(\omega)$ из-за dynamics of hard layer nanopillar)

(10/X) Slides для "Hopfion" talk

(11/X) Получил 3,200 PLN per-diem. Беседа с Ярослав и Юлия.

(12-22/X) Stay in Poznan - переписка с Оксаной и бухгалтером ICMM. Доклад "Magnetic Hopfions" на Phys. Faculty seminar. UAM. Переписка с "Mirakami" о conference in San Sebastian.

(23/X) Вернулись с Л. в Irun. Принял SW paper-Shelkar в PRApp.

(24/X) Hogan to Maciej: Conf. payments Lufthansa tickets. Hotel in Bilbao и factura/confirmations apartments in Poznan. Min 5,100 PLN to reimburse. Купил 700€ (3,87 PLN/Euro).

→ Корректн и submit paper - ФНТ (Kovalev) по hopfions in cyl. и spherical particle. $Q_H(R)$ для cylinder (легко, стабильность toroidal hopfion доказана), $Q_H(R)$ для sphere (недоказана стабильность, имеет hopfion is stable при $R\to\infty$, проверить). $Q_H(R)$ для BP in spherical dot не проходит, т.к. helicity $\gamma(z,\varphi)$. $\gamma(R,\varphi)-\gamma(0,\varphi)$ не может аналитически при $R\to\infty$, т.к. BP is unstable (does not exist) at $R\to\infty$. неограничена

Gus' chaos' 2023: $Q_H=\frac{n}{4\pi}\int d\rho dz \sin\theta\left[\frac{\partial\gamma}{\partial\rho}\frac{\partial\theta}{\partial z}-\frac{\partial\gamma}{\partial z}\frac{\partial\theta}{\partial\rho}\right]$, $\gamma(\rho,z)=\arctg\beta(\rho,z)$

$\beta(\rho,z)=\arctg\left(\frac{2az}{\rho^2+z^2-a^2}\right)$, $-\sin\theta\frac{\partial\theta}{\partial x_d}=\frac{\partial\cos\theta}{\partial x_d}=\frac{\partial m_z}{\partial x_d}$

→ $Q_H=\frac{mn}{4\pi}\int d\rho dz\left[\frac{\partial\beta}{\partial z}\frac{\partial m_z}{\partial\rho}-\frac{\partial\beta}{\partial\rho}\frac{\partial m_z}{\partial z}\right]$, $m_z=m_z(\gamma)$, (γ,β,φ) - toroidal coordinates

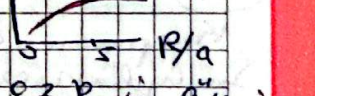
$\frac{\partial m_z}{\partial\rho}=\frac{\partial m_z}{\partial\gamma}\frac{\partial\gamma}{\partial\rho}$, $\frac{\partial m_z}{\partial z}=\frac{\partial m_z}{\partial\gamma}\frac{\partial\gamma}{\partial z}$. $\gamma(\rho,z)=\operatorname{arcth}\left(\frac{2\rho a}{z^2+\rho^2+a^2}\right)$

Сомкано 24.10.2023 toroidal moment $T_z(\vec A)\to\infty$ при $R\to\infty$, критика Nagaosa PRL' 2023.

Для cylindrical particle/dot (L,R) - sizes, $\int_0^R d\rho\int_{-L/2}^{L/2}dz(\dots)=\left|L/R=\beta',\ L=\beta' R\right|=$

$=\int_0^R d\rho\int_{-\beta' R/2}^{+\beta' R/2}dz(\dots)=\left|\rho=az,\ z=a\zeta\right|=$

$=a\int_0^{R/a}dz\,a\cdot\int_{-\beta'(R/a)\frac{1}{2}}^{+\beta' R/2a}d\zeta(\dots)=a^2\int_0^R dz\int_{-\beta' R/2}^{+\beta' R/2}d\zeta(\dots)$, где R в a-units, $\frac{R}{a}\to R$.



$Q_H(R)=\frac{mn}{4\pi}\int_0^R d\rho\int_{-\beta' R/2}^{+\beta' R/2}dz\left[\frac{\partial\beta}{\partial z}\frac{\partial m_z}{\partial\rho}-\frac{\partial\beta}{\partial\rho}\frac{\partial m_z}{\partial z}\right]$, где ρ,z,R в "a" units.

$Q_H(R)=\frac{mn}{4\pi}\int_0^R d\rho\int_{-\beta' R/2}^{+\beta' R/2}dz\,\frac{\partial m_z}{\partial\gamma}\left[\frac{\partial\beta}{\partial z}\frac{\partial\gamma}{\partial\rho}-\frac{\partial\beta}{\partial\rho}\frac{\partial\gamma}{\partial z}\right]$, $\left\{\begin{array}{l}\gamma(\rho,z)=\operatorname{arcth}\left[\frac{2\rho}{z^2+\rho^2+1}\right]\\ \beta(\rho,z)=\arctg\left[\frac{2z}{z^2+\rho^2-1}\right]\end{array}\right\}$

$m_z(\gamma)=\left(1-ch^{2m}\gamma\cdot th^{2n}\gamma\right)/\left(1+ch^{2m}\gamma\cdot th^{2n}\gamma\right)$. $Q_H=Q_H(R,\beta')$, $\beta'=L/R=const$ можно

$\frac{\partial m_z}{\partial\gamma}=m_z'(\gamma)$ или $\gamma=\gamma(\rho,z)$.

(25/X) Spherical particle. Gus' chaos'23: $Q_H=\frac{2mn}{(4\pi)^2}\int dm_z\int d\beta\int d\varphi$

переход от (m_z,β,φ) - toroidal coordinates $(m_z=m_z(\gamma))$ к spherical koords. (r,θ,φ). Jacobian $J(m_z,\beta,\varphi/r,\theta,\varphi)$

Cyl (ρ,φ,z) связь с spherical (r,θ,φ): $\rho=r\sin\theta$, $\varphi=\varphi$, $z=r\cos\theta$

$\int dm_z\int d\beta\int d\varphi=\int dV\cdot J(m_z,\beta,\varphi/r,\theta,\varphi)$.

$J=\begin{pmatrix}\partial m_z/\partial r & \partial m_z/\partial\theta & \partial m_z/\partial\varphi\\ \partial\beta/\partial r & \partial\beta/\partial\theta & \partial\beta/\partial\varphi\\ \partial\varphi/\partial r & \partial\varphi/\partial\theta & \partial\varphi/\partial\varphi\end{pmatrix}=\begin{pmatrix}\partial m_z/\partial r & \partial m_z/\partial\theta & 0\\ \partial\beta/\partial r & \partial\beta/\partial\theta & 0\\ 0 & 0 & 1\end{pmatrix}$

$\det J=\frac{\partial m_z}{\partial r}\frac{\partial\beta}{\partial\theta}-\frac{\partial m_z}{\partial\theta}\frac{\partial\beta}{\partial r}$, $m_z=m_z(\gamma)$, $\beta=\arctg\left[\frac{2r\sin\theta\cos\theta}{r^2-1}\right]$

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$\det J = \frac{\partial m_z}{\partial y}\left[\frac{\partial y}{\partial r}\frac{\partial \beta}{\partial \varphi} - \frac{\partial y}{\partial \varphi}\frac{\partial \beta}{\partial r}\right]$, $y = \operatorname{arcth}\left[\frac{2r\sin\varphi}{r^2+1}\right]$, $\beta = \operatorname{arctg}\left[\frac{2r\cos\varphi}{r^2-1}\right]$, r, φ in φ units

$Q_H = -\frac{2mn}{(4\pi)^2}\int dV_S \det J \Rightarrow$

$= -\frac{2mn}{(4\pi)^2}\int_0^R dr\, r^2 \int_0^{\pi} d\varphi \sin\varphi \int_0^{2\pi} d\varphi \det J = -\frac{mn}{4\pi}\int_0^R dr\, r^2 \int_0^{\pi} d\varphi \sin\varphi \det J(r,\varphi)$

$Q_H = Q_{HS}(R,m,n)$. При $m=1, n=1$ Q_{HS} насочването чрез $Q_H(100) = 2{,}316$ (?)

Бързо Jacobian $\det J$ пресметнат неверно.

Надо в сферата $\det J$ сферата чрез ∇m_z, $\nabla\beta$, $\nabla\varphi$

$\nabla m_z(r,\varphi,\varphi) = \nabla m_z(r,\varphi) = \frac{\partial m_z}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial m_z}{\partial \varphi}\hat{\varphi} + \frac{1}{r\sin\varphi}\frac{\partial m_z}{\partial \varphi}\hat{\varphi}$

$\nabla\beta(r,\varphi) = \frac{\partial \beta}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial \beta}{\partial \varphi}\hat{\varphi}$, $\nabla\varphi = \frac{\partial \varphi}{\partial r}\hat{r} + \frac{\partial \varphi}{\partial \varphi}\frac{1}{r}\hat{\varphi} + \frac{1}{r\sin\varphi}\frac{\partial \varphi}{\partial \varphi}\hat{\varphi}$

$\det J = \det\begin{pmatrix} \partial m_z/\partial r & \frac{1}{r}\frac{\partial m_z}{\partial \varphi} & \frac{1}{r\sin\varphi}\frac{\partial m_z}{\partial \varphi} \\ \partial\beta/\partial r & \frac{1}{r}\partial\beta/\partial\varphi & \frac{1}{r\sin\varphi}\partial\beta/\partial\varphi \\ \partial\varphi/\partial r & \frac{1}{r}\partial\varphi/\partial\varphi & \frac{1}{r\sin\varphi}\frac{\partial\varphi}{\partial\varphi} \end{pmatrix} = \det\begin{pmatrix} \frac{\partial m_z}{\partial r} & \frac{1}{r}\frac{\partial m_z}{\partial \varphi} & 0 \\ \partial\beta/\partial r & \frac{1}{r}\partial\beta/\partial\varphi & 0 \\ 0 & 0 & \frac{1}{r\sin\varphi} \end{pmatrix}$

$= \frac{1}{r^2\sin\varphi}\left[\frac{\partial m_z}{\partial r}\frac{\partial \beta}{\partial \varphi} - \frac{\partial m_z}{\partial \varphi}\frac{\partial \beta}{\partial r}\right]$, где $\frac{1}{r^2\sin\varphi}$ е якобиана.

Sphere $Q_H(R)$ бързо к cylinder $Q_H(R, \beta = \pi)$, $L = 2R$.

Отбелязва Murakami, nogaдо IVA „Constants", „V. string GM wires (тел)

(26/X) Написване $Q_H(R)$ - dot MS и построяване Figs. „Hopf index – cylinder med" Започване IVA - форма Ba III ісв. 2024

(27/X) Написване $Q_H(R)$ - dot MS. Писане MS draft done.

(28/X) Поездка в UPV/EHU. Проверяване своя new office. Мой ключ (карта) позволява и к old office. Скоро Maider de-activate карта на new office и improve equipment? (Телефонен по моем old office)

(30/X) Нерешителна с Ленчо Т. по $Q_H(R)$ paper дано ФНТ. Update paper

(31/X) Среща с Ленчо. Нерешителна с Орландо по Knapman_arXiv_2024 „Simulations of the Hopf index". Hellowish gone.

(1/XI) С Л.и М. За публикуване. „Нейно препринт Knapman – 2024 „Hopfions". Knapman дава $\vec{m}(\vec{r})$ ansatz $\vec{m}(\theta,\varphi)$, справянии, уб. $Q_H = 1$.

In cylindrical $\vec{r}(\rho,\varphi,z)$ coordinates. $\varphi = \operatorname{arctg}(y/x)$

$\Phi(\vec{r}) = \varphi - \psi$, $\psi = \operatorname{arctg}\left[\frac{z}{x\cos\varphi + y\sin\varphi - R}\right] = \psi(\vec{r})$.

$\rho'^2 = z^2 + (x\cos\varphi + y\sin\varphi - R)^2$. По $x = \rho\cos\varphi$, $y = \rho\sin\varphi$ с $\vec{r}(\rho,\varphi,z)$.

$\psi(\rho,z) = \operatorname{arctg}\left[\frac{z}{\rho - R}\right]$. $\Theta(\vec{r}) = \begin{cases} \pi\exp\left[1 - \frac{1}{1-(\rho'/z)^2}\right] & \text{ако } \rho' < z \\ 0 & \rho' > z \end{cases}$

$\rho'^2 = z^2 + (\rho - R)^2$

Тор $(z-R)$ Hopfion, artificial construction. Вероятно $Q_H = 1$ дава ли Hopfion?

(2/XI) Подреждане Figs. дано Hopfion $Q_H(R)$ paper.

(3/XI) Изпращане to Murakami с San Sebastian Conf. Sept. 2025 Написване $Q_H(R)$ paper дано ФНТ.

(4/XI) Писане $Q(R)$ manuscript to M. Krenczyk. Proof Strelkov PRA Ленчо О. Volkov et al. Chirality Coupling in Nat. Com. 14 91 (2023) disk with Gauss bump. Coupling of volume and V. string сингулярни magn. charges. $W_m = pC$ – of vortex. of vortex core string. Нанесени забележки в ръкописа

(5/XI) Отказ Referees of Hopfion review „Magnetism.

(6/XI) Review MS Barman. Vortex. JPhCM

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(7/XI) Replies to referees Hopfion-review, Magnetism. Корректные Figs. Nikitarina 'PL'1999 дает аномал $m_{x,y,z}(x,y,z)$ для $Q_H=1$ для Hopfions. Различие Bloch hopfions (Nikitarina) и Neel hopfions (Wang PRL'1999) в phase angle φ_0, $\varphi \to (\varphi+\varphi_0)$. $\varphi_0 = 0, \pi$ Bloch hopfions, $\varphi_0 = \pm\pi/2$ для Neel hopfions.

Вышло в Nanomat'2024 и Ursan stability $W(Q)$ Bloch hopfions.

Winding hopfions [Kobayashi Nucl. Phys. 2013], см. 19.20/VI 2024.

$m_x = \cos(n\varphi - P g(z)) \sin\Theta(\rho)$, $m_y = \sin(n\varphi - P g(z)) \sin\Theta(\rho)$, $m_z = \cos\Theta(\rho)$.

Kobayashi дает $\Theta(\rho) = 2\arctg f(\rho)$, $f(\rho=0) = \pi/2$, $f(\rho\to\infty) = 0$, $\vec{r}(\rho,\varphi,z)$

$\Phi(\rho,z) = n\varphi - P g(z)$, $g(z=0) = 0$, $g(L) = 2\pi$. $Q_H = \frac{P}{2\pi}[g(L) - g(0)]Q = PQ$ Hopfs

is integer, если $g(L) - g(0) = 2\pi$, и Pontryagin number $Q = \frac{1}{4\pi}\int dx\,dy\, B_z$ is integer.

Infinite film: $\cos\Theta(\rho) = \frac{\rho^2 - \rho_c^2}{\rho_c^2 + \rho^2}$, $\cos\Theta(0) = -1$, $\Theta(0) = \pi$, $\Theta(0) = 2\arctg f(0) = 2\pi$,

берем $q = 1$. $\cos\Theta(\rho\to\infty) = 1$, $\Theta(\infty) = 0$.

$\Phi(\rho,z) = n\varphi - P g(z)$, $g(z) = \frac{2\pi}{L} z$, $g(0) = 0$, $g(L) = 2\pi$ - берем Bug. $z \in [0, L]$.

$\cos\Theta(\rho) = m_z(\rho) = \left(\frac{\rho^2 - \rho_c^2}{\rho^2 + \rho_c^2}\right) = \frac{\rho^2 - 1}{\rho^2 + 1}$, $\Phi(\rho,z) = \varphi - P\frac{2\pi}{L} z$.

$Q = \frac{n}{2}[1 - (-1)^q] = |q=1| = n$, берем $n=1$, $P=1$, $Q_H = PQ = nP = 1$.

$m_z(\rho) = \frac{(\rho^2-1)'}{(\rho^2+1)}$, $m_x = \cos\Phi \sin\Theta$, $m_y = \sin\Phi \sin\Theta$, $\Phi(\rho,z) = \varphi - 2\pi\left(\frac{z}{L}\right)$

$\sin\Theta(\rho) = \pm\frac{2\rho}{(1+\rho^2)}$. В xz, $\varphi=0$ плане $\rho = sign(x)\cdot x$, $\sin\Theta(x) = \frac{2x}{(1+x^2)}$.

$\sin\Theta(x) = \frac{2|x|\,sign(x)}{(1+x^2)}$ в $\varphi=0$ плане. $x = sign(x)|x| = \rho\, sign(x)$.

(8-11/XI) Нумерованные new figures для Hopfion-review Magnetism - $\vec{m}(\vec{r})$ images toroidal and winding (Kobayashi) hopfions. Text revisions. Re-submitted the manuscript to Magnetism.

(12/XI) Among inv. speakers Sol-Skyrmag-2025. Нужен список invitations.

(13/XI) M. Azhar (Duisburg-Essen) прислал две ArXiv papers по hopfions: analytics ++. Invitations Sol-Skyrmag.

(14/XI) Preprint книги V. Kravchuk "Problems in micromagnetism" 2024 Немагнит internet в office UPS/EPFL по общей компьютерах. Q_H $A(\vec{r})$

(15/XI) Kravchuk'ин вспомогательное без ссылки eq. Kunda'1982 для Hopf index Q_H через $\vec{B}(\vec{r})$, $\vec{B}(\vec{r})$ инвариант. Требуется это $\operatorname{div}\vec{A} = 0$ не вычисляется для toroidal hopfions. $\operatorname{div}\vec{A} = 0$ gauge.

Проблема чего $\operatorname{div}\vec{A} = 0$ для $\vec{A}(\vec{r}) = -\frac{1}{4\pi}\int d^3\vec{r}' \cdot \frac{\vec{R}\times\vec{B}(\vec{r}')}{R^3}$, $\vec{R} = \vec{r} - \vec{r}'$ (*)

Верно ли $\vec{A}(\vec{r})$ (*) для finite domain? Надо ли $\vec{B}(\vec{r}) \sim 1/r^n \to 0$ при $r = |\vec{r}| \to \infty$?

См. MFE 2020 по Q_H Bloch point, вообще $\vec{A}(\vec{r})$ и Helmholtz theorem.

См. 20/VIII 2020. Helmholtz theorem.

// При вводе $\vec{A}(\vec{r}) = -\frac{1}{4\pi}\int d^3\vec{r}' \cdot \frac{\vec{R}\times\vec{B}(\vec{r}')}{R^3}$ Aug. 20, 2023 использовано только $\vec{B} = \operatorname{rot}\vec{A}$ и $\operatorname{div}\vec{B} = 0$. Ограничено (finite cross-domain). $\operatorname{div}\vec{A} = 0$ не надо.

Gyrocoupling density $\vec{g} = \frac{1}{8\pi}\vec{e}_i \varepsilon_{ijk}\vec{m}[\partial_j\vec{m}\times\partial_k\vec{m}]$, $\vec{g} = \frac{1}{4\pi}\cdot\vec{B}$ - energy of mag. field $\vec{B} = \operatorname{rot}\vec{A}$, $\vec{A} = \vec{e}_i(\vec{a}_1\cdot\partial_i\vec{a}_2)$ spin connection, $(\vec{a}_1, \vec{a}_2, \vec{m})$ is orthonormal basis, $\vec{a}_1\times\vec{a}_2 = \vec{m}(\vec{r})$ - local magnetization, Kravchuk сомневается Mermin-Ho, PRL'1976 где нет $\vec{A} = \vec{e}_i(\vec{a}_1\cdot\partial_i\vec{a}_2)$ equation.

Gus' EPL'16: $\vec{A}_\mu = 2\partial_\mu\vec{n}\times\vec{n}$, $\vec{A}_\mu^e = (\vec{A}_\mu\cdot\vec{m})\vec{e}$, $\vec{B} = \operatorname{rot}\vec{A}^e$, $\vec{n} = \frac{\vec{z}+\vec{m}}{|\vec{z}+\vec{m}|}$ rotation axis

Gus' chaos 23: $\vec{B} = \frac{4a}{(a^2+r^2)}$ $\vec{A}(\vec{r}) = f(r)\vec{A}(\vec{r})$ give hopfion $Q_H = 1$.

$0 = \mathrm{div}\vec{B} = \mathrm{div}[f\vec{A}] = f\,\mathrm{div}\vec{A} + \nabla f \cdot \vec{A} = f\cdot\mathrm{div}\vec{A} + \frac{\partial f}{\partial r}(\hat{r}\cdot\vec{A})$

$(\hat{r}\cdot\vec{A}) = (\sin\theta\,\hat{\rho} + \cos\theta\,\hat{z})\vec{A} = A_\rho \sin\theta + A_z\cos\theta = A_z \neq 0$ $A_z \neq 0$ - Pershoguba PRB'21

$\mathrm{div}\vec{A} = 0$ дает $\nabla f \cdot \vec{A} = \frac{\partial f}{\partial r} A_z = 0$, но $\frac{\partial f}{\partial r} \neq 0$ и $A_z \neq 0$.

Gauge $\mathrm{div}\vec{A} = 0$ неверно, что можно сделать и в cyl. coordinates $\vec{A} = (A_\rho, A_\theta, A_z)$.

Точнее $\mathrm{div}\vec{A}' = 0$ за счет $\vec{A}' = \vec{A} + \nabla f$, но зачем это делать?

Надо дать natural gauged $\vec{A}'(\vec{r}) = -\frac{1}{4\pi}\int d^3\vec{r}'\,\frac{\vec{R}\times\vec{B}(\vec{r}')}{R^3}$ какого gauge $\vec{A}$, вычислив $\mathrm{div}\vec{A} = 0$, предполагается не надо [Krawczyk · 2024 preprint

Намекал comment to U. Krawczyk. приглашение invited Speakers Sol. Sky mag, 2025.

(18-17/XI) Работа над $Q_H(t)$ manuscript дает Low Temp. Phys.

(18/XI) Hopfion-review" accepted by magnetism. Переписка с Magnetism.

(19/XI) " Вычела proofs Hopfion-review.

(20/XI) Hopfion-review is published. E-mails to M. Ahrzer (hopfions). Krawczyk, Лена поздравление Peter и Лена.

(21/XI) Invited Yamamoto и Donnelly. E-mail to Tatara о DW-resonance in bended stripe [Saitoh et al. Nature 2004]. DW bended stripe

(22/XI) Tatara eqn. Видимо, добавь DW res. frequency ω_0 и y через [Saitoh'2004] Кивера. Надо различать $(x,y,z)\,\vec{r}$ и $\vec{X}(X,Y,Z)$ - current coordinates ∈ stripe и DW-center position $\vec{X}$

Tatara: $0z \perp$ plane, $0y$ - transverse dir., $0x$ along stripe length



Tatara $U_m(y) = -Q_M H_y y$, $y = z - \frac{X^2}{2r}$ → X^2-term in Thiele eq.

Надо $\vec{M}(x,y,z,X,Y,Z)$. For thin stripes 2D-model $\vec{M}(x,y,X,Y)$. Видимо y (current coord.) → Y (DW center coordinate)

$U_m(Y) \sim Y$. Если так, что $y = z - \frac{X^2}{2r}$, $\frac{X}{r} \ll 1$, не вычисляется.

$Q_M = 2\mu_0 M_S S$, S-cross-section area (в xy plane).

Видимо $U_m = -\frac{1}{2} H_y^d M_y$, $M_y \sim H_y y$, y - DW displacement $\parallel H_y$. center

(23/XI) С п. в "Life" Кернами. Бинц.



Момент рас $E(X)$ of DW in bended stripe [Saitoh '2004] сведен с расчетом Zeeman energy in transverse field H_y,

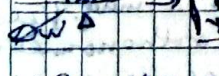
$E_z = -H_y\int dV M_y = -\int dV \vec{H}\cdot\vec{M}$, $\vec{H}\cdot\vec{M} \simeq H_y M_S \cos\theta$,

где θ угол поворота DW-plane, поэтому, если DW $0y$ go new displaced DW position. Displacement будем $\hat{\varphi}$-direction in $\vec{s}(R,\varphi)$ polar coordinate system ⇒ x.

$\cos\theta = 1 - \frac{1}{2}\theta^2 = 1 - \frac{1}{2}\frac{X^2}{r^2}$, $\sin\theta \approx \theta = \frac{X}{r}$

$E_z(X) = -\int dV H_y M_S \cos\theta = -\int dV H_y M_S\left[1 - \frac{1}{2}\theta^2\right] =$

$= \mathrm{const} + \frac{1}{2} H_y M_S \int dV\,\theta^2 = \frac{1}{2} H_y M_S V_D \frac{X^2}{r^2}$, $V_D = \Delta\cdot L\cdot w$ L - stripe thickness, w - stripe width, Δ - DW width. V_D - volume где $M_y \neq 0$



$E_z(X) = +\frac{1}{2} H_y M_S (\Delta L w)\frac{X^2}{r^2} = \frac{1}{2}\varkappa X^2$, $\varkappa = H_y M_S \frac{\Delta L w}{r^2}$

ЗД: $M_{DW}\ddot{X} + \varkappa X = F(t)$ дает $\omega_0^2 = \frac{\varkappa}{M_{DW}} \sim H_y$,

$\omega_0^2 = \frac{H_y M_S L w \Delta}{M_D r^2}$, M_D - Döring DW mass. DW mass density [g/cm²]

$m_{DW} = \frac{1}{2\pi r^2 \Delta}$, Δ - DW width. Для 1D DW model

Сомаков Hubert - Schäfer '1998: $m_{DW} = \frac{1}{2}\mu_0 \cdot 4\sqrt{AK}\,\frac{1}{A\gamma^2} = 2\mu_0 \frac{1}{\gamma^2\Delta}$,
где $\Delta^2 = A/K$. $M_{DW} = S\, m_{DW} = \frac{S}{2\pi\gamma^2\Delta}$, S - DW cross section area

$\chi = H_y M_s \frac{\Delta \cdot S}{2}$, $S = wL$ - за stripe. $\omega_D^2 = \frac{\chi}{M_{DW}} = \frac{1}{S m_{DW}} H_y M_s \frac{S\Delta}{2} = \frac{H_y M_s \Delta}{m_{DW}\, 2}$. Ecau $m_{DW} = \frac{1}{2\pi\gamma^2\Delta}$ Döring:

то $\omega_D^2 = \frac{H_y M_s \Delta}{\left(\frac{1}{2\pi\gamma^2\Delta}\right) 2} = 2\pi\gamma^2 H_y M_s \left(\frac{\Delta}{2}\right)^2$ - DW resonance frequency in bended strip, z-Bending radius.

Saitoh' team gave bending radius $r = 50\,\mu m$, $\Delta \approx 50$ nm.

$\left(\frac{\Delta}{2}\right)^2 \approx \left(\frac{50\text{ nm}}{50\,\mu m}\right)^2 = (10^{-3})^2 = 10^{-6}$. $2\pi\gamma M_s = \frac{1}{2}\omega_M$.

$\omega_D^2 = \gamma H_y \frac{1}{2}\omega_M \left(\frac{\Delta}{2}\right)^2 = \frac{1}{2}\frac{\gamma H_y}{\omega_M}\omega_M^2\left(\frac{\Delta}{2}\right)^2$, $\frac{1}{2}\frac{\gamma H_y}{\omega_M} = \frac{\gamma H_y}{4\pi\gamma M_s}\frac{1}{2} = \frac{1}{2}\left(\frac{H_y}{4\pi M_s}\right)$

$\omega_D^2 = \frac{1}{2}\left(H_y/4\pi M_s\right)^2 \omega_M^2 \left(\frac{\Delta}{2}\right)^2$, $\left(\omega_D/\omega_M\right) = \frac{1}{\sqrt{2}}\sqrt{\frac{H}{4\pi M_s}}\cdot\left(\frac{\Delta}{2}\right)$.

$\frac{\omega_D}{\omega_M} \approx \frac{1}{\sqrt{2}}\sqrt{\frac{H}{4\pi M_s}}\cdot 10^{-3}$. Дано $H_y = 100$ Oe, $\frac{H}{4\pi M_s} = \frac{100\text{ Oe}}{10\,100\text{ Oe}} = 0{,}01$

$\frac{\omega_D}{\omega_M} \approx \frac{1}{1.4}\sqrt{0.01}\cdot 10^{-3} = \frac{1}{1.4}10^{-4}$, $\omega_M/2\pi\,(FeNi) = 30$ GHz

$f_D = \frac{\omega_D}{2\pi} = \frac{1}{1.4}10^{-4}\left(\frac{\omega_M}{2\pi}\right) = \frac{1}{1.4}10^{-4}\cdot 30\text{ GHz} = \frac{10^{-4}}{1.4}30\cdot 10^3\text{ MHz} =$

$= \frac{3}{1.4}\text{ MHz} \approx 2$ MHz, $f_D(100\text{ Oe}) \approx 2$ MHz. Експериментално Saitoh' team дава $f_D^{exp}(100\text{ Oe}) \approx 20$ MHz. Calc - f_D занижена на порядък, защото DW mass - calculated завишена (M_{DW} завишена).

Сомаков 14/XI 2014 г.: $\frac{1}{\gamma r} = 2{,}95\cdot 10^{-15}\left(\frac{g}{cm}\right)$ при $\gamma/2\pi = 2.93$ (FeNi).

Döring mass for stripe Saitoh с transverse DW:

$M_{DW} = \frac{S}{2\pi\gamma^2\Delta} = \frac{1}{\gamma^2}\frac{wL}{2\pi\Delta} = \frac{1}{\gamma^2}\frac{(70\text{ nm})(45\text{ nm})}{2\pi\,50\text{ nm}} = \frac{10.03}{\gamma^2}\text{ nm} =$

$= 2{,}95\cdot 10^{-15}\frac{g}{cm}\cdot 10{,}03\text{ nm} \approx 30\cdot 10^{-15}\,g\cdot\frac{nm}{cm} = 30\cdot 10^{-15}\,g\cdot\frac{nm}{10^7\,nm} = 3\cdot 10^{-21}\,g$

Дава съвпадение с exp. Saitoh' team ако вземем $M'_{DW} = 0{,}01\,M_{DW}$ (Döring)

тогава $\omega_D' = \sqrt{\frac{\chi}{M'_{DW}}} = 10\sqrt{\frac{\chi}{M_{DW}}} = 10\,\omega_D$ и при $H_y = 100$ Oe получаваме

$\omega_D'/2\pi = 10\cdot 2\text{ MHz} \approx 20$ MHz. $M'_{DW} = 10^{-2} M_{DW}(Döring) \approx 10^{-2}\cdot 3\cdot 10^{-21}\,g = 3\cdot 10^{-23}\,g$.

Saitoh' team дава оценка DW mass $M_{DW} = 6.5\cdot 10^{-20}\,g$ на основа на Eq.(1).

→ Вибод: ако вземем $E(X)$ bended stripe и изчислим stiffness χ от $E_{Zeem}(X) = \frac{1}{2}\chi X^2$ от DW pure Zeeman energy, то при M_{DW}(Döring)

Calc. $\omega_D/2\pi$ занижена $\approx$ в 10 пъти. За нормално изпълнение, използвайки M_{DW}(Döring)

(24/XI) Написал о выводе зав. frequency ω_D to G.Tatara [Saitoh Nat 2.]
Татара отвечал: будет 3 ч. через 6 ч. обратно в Saitoh

(25/XI) Reminders to invited speakers SolSkyMag 25. нет ответа от ICMM account dept. в Декември. Обратен път Simulations of bended stripes to Maciej - Нева. UAM January 5050 PLN. compensation trip to Poznan Oct. 2024

(26/XI) Набран 23 invited speakers SolSkyMag '2025: 15 agreed и 8 усилено agreed, waiting for confirmation.

- H.K. Moffatt. J. Fluid Mech. 35, 117-129 (1969). The degree of knottedness of tangled vortex lines
Vortex filaments in fluid mechanics

$I = \int dV\, \vec{u}\cdot\vec{\omega} = \alpha \chi_1 \chi_2 = const$, $\alpha = 0, \pm 1$, χ_i - strength of vortex filaments.

$\vec{u} = \vec{u}(x,t)$ – velocity field of inviscid fluid, $\vec{\omega} = \nabla \times \vec{u}$ – vorticity.

$\vec{u}$ analog of gauge vector potential $\vec{A}$, $\vec{\omega} = \vec{B} = rot\,\vec{A}$ en. mag. fld.

I – Hopf index. $I = \sum_{ij} \alpha_{ij} \chi_i \chi_j$, $\alpha_{ij} = \frac{1}{4\pi} \oint_{C_i} \oint_{C_j} \frac{\vec{R}\cdot d\vec{l}_i \times d\vec{l}_j}{R^3}$.

– Gauss linking numbers, $\vec{R} = \vec{x}_i - \vec{x}_j$.

$\vec{u}(\vec{r}) = -\frac{1}{4\pi} dV' \frac{\vec{R}\times\vec{\omega}(\vec{r}')}{R^3}$, аналог $\vec{A}(\vec{r}) = -\frac{1}{4\pi}\int dV' \frac{\vec{R}\times\vec{B}(\vec{r}')}{R^3}$

Hopf index $I = \frac{1}{4\pi}\int dV dV' \frac{\vec{R}\cdot[\vec{\omega}(\vec{r})\times\vec{\omega}(\vec{r}')]}{R^3}$ даст $I = \sum_{ij} \alpha_{ij} \chi_i \chi_j$

• L. Woltjer. Proc. Natl. Acad. Sci. USA 44, 489-491, 833-841 (1958).
Magnetohydrodynamic invariants. $\vec{B} = rot\,\vec{A}$, $\int dV \vec{A}\cdot\vec{B} = const$.

If we minimize the magnetic energy $E_m = \frac{1}{8\pi}\int dV\, |rot\vec{A}|^2$ при условии $I = \int dV \vec{A}\cdot\vec{B} = const$, $\vec{B} = rot\,\vec{A}$, то $\vec{B}$ is force-free field $rot\vec{B} = \alpha\vec{B}$ (!)

• E.A. Kuznetsov, A.V. Mikhailov. Phys. Lett. A 77, 37-38 (1980).
Hopf invariant $\Gamma = \int dV\, \vec{V}\cdot rot\vec{V}$ недавно Moffatt 1969.

• Wiki (Hydrodynamical helicity) $H = \int dV\, \vec{u}\cdot\vec{\omega}$, $\vec{\omega} = rot\,\vec{u}$, $\vec{u}$ - velocity.
For two linked unknotted vortex tubes $H = 2n\chi_1\chi_2$, where
n is Gauss linking number, $link(\gamma_1,\gamma_2) = \frac{1}{4\pi}\oint_{\gamma_1}\oint_{\gamma_2} \frac{(\vec{r}_2-\vec{r}_1)\cdot d\vec{r}_2 \times d\vec{r}_1}{|\vec{r}_2-\vec{r}_1|^3}$ of curves γ_1, γ_2
χ is circulation, $\chi = \oint d\vec{l}\cdot\vec{V} = \int d\vec{S}\cdot\vec{\omega}$, т.е. $\chi = \int_S d\vec{S}\cdot\vec{B}$ flux, $\vec{V}\sim\vec{A}$
Недалеко как Gauss S интегрирует?
Conditions for $\Gamma, H = const$: 1) fluid is inviscid, 2) the flow is incompressible ($div\,\vec{u} = 0$, $div\,\vec{A} = 0$).
Inviscid is a fluid with zero viscosity.

// $div\,\vec{A} = 0$ дает нормировку для векторного потенциала, но $div\vec{A} = 0$ неприемлемо. Предлагаем „natural" gauge $\vec{A} = -2\nabla\chi + (1-\cos\theta)\nabla\varphi$

Поток жидкости vorticity $\vec{\omega} = rot\,\vec{u}$ аналогичен эм. field $\vec{B}$, а fluid velocity $\vec{u}$ аналогичен эм. vector potential $\vec{A}$, в fluid theory нет выражения $\vec{\omega}\sim\vec{B}$ через $\partial m/\partial x_2$. Нет такого Hopf mapping $R^3 \to S_2(\vec{m})$ → Em. field tensor $F_{\mu\nu} = \vec{m}\cdot(\partial_\mu\vec{m}\times\partial_\nu\vec{m})$ не выражается в fluid theory, sphere $\vec{m}^2 = 1$.
для Hopf index eq. $Q_H = \frac{1}{(4\pi\chi)^2}\int dV \vec{B}(\vec{r})\cdot\vec{A}(\vec{r})$, $\vec{A}(\vec{r}) = -\frac{1}{4\pi}\int dV' \frac{\vec{R}\times\vec{B}(\vec{r}')}{R^3}$, $\vec{R} = \vec{r}-\vec{r}'$
соответствие. Также Q_H не удается as интегрирование $\vec{B}(\vec{r}) \sim \vec{\omega}(\vec{r})$ в вихрях
области.

(27/IV) Domcan + vocuan to B. Ivanov MS "Hopf index in dots" для Low T Phys
(28/IV) Search new papers on Hopfions в университет. office. Посылки в Киев.
1. D. (3,5 kg), Tablet (1,0 kg): 2,1 + 10 × 2 + 5 € × 2 = 52 €
(29/IV) E-mails to Maciej & visit Poznań. Oral + project compensation + DW bended stripe.
→ $\vec{m}(\vec{r}) = \hat{z}\cos 2\chi + \hat{R}(\hat{R}\cdot\hat{z})(1-\cos 2\chi) + (\hat{R}\times\hat{z})\sin 2\chi$, $\chi(r,\theta)$ в сферич. координатах $\vec{r}(r,\theta,\varphi)$. $\chi = \chi(r,\theta)$, $\chi(0,\theta) = \pi$, $\chi(\infty,\theta) = 0$.
В Dzyaloshinskii et al. JETP Lett. 29, 54 (1979) и Kosevich et al. Phys. 3D
Доказано, что $\vec{m}(\vec{r})$ это Hopfion с $Q_H = \pm 1$ Это также $\frac{1}{R}\frac{1}{R^2}\frac{1}{R^3} = \delta(\vec{r})$ также.
Shivamoggi - Leach, Angular Momentum in Quantum Physics. 1981.

Escaneado con CamScanner

Вращение (active) вектора $\vec{r}$ на угол φ вокруг оси $\vec{n}$ $\hat{R}(\varphi, \vec{n})$ $(|\vec{n}|=1)$:

$\hat{R}(\varphi,\vec{n}) \cdot \vec{r} = \vec{r}'$, $\vec{r}' = R(\varphi,\vec{n})\vec{r} = \vec{r}\cos\varphi + \vec{n}(\vec{n}\cdot\vec{r})(1-\cos\varphi) + (\vec{n}\times\vec{r})\sin\varphi$

$\vec{r}'$ имеет new components $\vec{r}'(x',y',z') = (r_x', r_y', r_z')$ in the fixed Coordinate system (x,y,z), $\hat{x},\hat{y},\hat{z}$ – basis. Vectors $\vec{r}', \vec{r}$ are arbitrary.

Varshalovich 1988: passive rotation, rotation of coordinate system $S(x,y,z) \to S'(x',y',z')$

radius vector $\vec{r}' = \vec{r}\cos\omega + \vec{n}(\vec{n}\cdot\vec{r})(1-\cos\omega) + (\vec{n}\times\vec{r})\sin\omega$, ω – rotation angle вокруг оси $\vec{n} = \vec{n}(\theta;\Phi)$. Rotation operator $\hat{U}(\omega,\vec{n}) = e^{-i\omega\vec{n}\cdot\vec{J}} = \hat{D}(\alpha,\beta,\gamma)$.

For vector $\vec{A}$: $(\vec{A}')_i = \sum_k a_{ki} A_i$, $i,k = x,y,z$, $\hat{e}_i' = \sum_k a_{ki}\hat{e}_k$, $a_{ik} = \hat{e}_i'\cdot\hat{e}_k$

– rotation matrix (3×3).

$a_{ik} = \cos\omega\,\delta_{ik} + (1-\cos\omega)\,n_i n_k - \sin\omega\,\varepsilon_{ikl}n_l$.

For active rotation (Biedenharn): rotation matrix $R_{ij} = \hat{e}_i\cdot\hat{R}\hat{e}_j = \vec{e}_i\cdot\vec{e}_j'$

$x_i' = \sum_j R_{ij}x_j$, в passive rotation $A_i' = \sum_k a_{ki}A_i$ (Varshalovich).

$\vec{m} = \hat{z}\cos 2\chi + \hat{R}(\hat{R}\cdot\hat{z})(1-\cos 2\chi) + (\hat{R}\times\hat{z})\sin 2\chi$ – Biquino active rotation (вращение вектор $\hat{z}$ at fixed coordinate system) согласно [Volovik, Mineev 1977]

Volovik, Mineev 1977: Hopf index $N = \frac{1}{(2\pi)^2}\int dV\, \varepsilon_{\alpha\beta\gamma\delta}\varepsilon_{ijk} n_\alpha \frac{\partial n_\beta}{\partial x_i}\frac{\partial n_\gamma}{\partial x_j}\frac{\partial n_\delta}{\partial x_k}$,

$\alpha,\beta,\gamma,\delta = 1,\dots,4$, $i,k = 1,\dots,3$. $n_\alpha = (\vec{n}, n_4)$, $\sum_\alpha n_\alpha^2 = \vec{n}^2 + n_4^2 = S_3$ sphere in 4-space

4-vector $n_\alpha(\vec{n}, n_4) = (\hat{r}\sin\chi, \cos\chi)$ $\hat{r} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ unit 3-vector

$\hat{r}(r,\theta,\varphi)$ – Spherical Coordinates. $\chi = \chi(r)$ – undefined, $\chi = \chi(r)$ можно.

$V_i(\vec{r}) = R_{ik}[n_\alpha(\vec{r})]\hat{z}_k$, $\vec{V}$ is order parameter, $\vec{V}^2 = 1$, $0 \le \chi \le \pi$, $N=1$

$\vec{V} = \vec{m}$ for 3D magnetization unit vector $\vec{m}(\vec{r})$.

$R_{ik}[\vec{n}, n_4] = \delta_{ik} + 2(n_i n_k - \vec{n}^2\delta_{ik}) - 2\varepsilon_{ikl}n_l n_4$, $R_{ik}[n_\alpha]$.

$m_i = R_{ik}[n_\alpha(\vec{r})]\hat{z}_k$, rotation matrix is parameterized by $n_\alpha(\vec{n}, n_4)$ 4-vector

Если $\hat{z}$ is unit vector of Cartesian coordinate system, то $\hat{z} = \begin{pmatrix}0\\0\\1\end{pmatrix}$,

$\hat{z}_k = \delta_{kz}$, $i,k = 1,2,3$ $(\vec{m})_i = \sum_k R_{ik}\hat{z}_k = \sum_k R_{ik}\delta_{kz} = R_{iz}$, $m_i = R_{iz}$.

$R_{iz} = \delta_{iz} + 2(n_i n_z - \vec{n}^2\delta_{iz}) - 2\varepsilon_{izl}n_l n_4$, $(x,y,z) \to (1,2,3)$.

$m_1 = R_{13} = 2n_1n_3 - 2\varepsilon_{13l}n_l n_4$, $\sum_{l=1,2,3}\varepsilon_{13l}n_l = -n_2$, $3 \to z$.

$m_1 = 2n_1n_3 - 2n_4(-n_2) = 2(n_1n_3 + n_2n_4)$.

$m_2 = R_{23} = 2n_2n_3 - 2n_4\sum_l \varepsilon_{23l}n_l = 2n_2n_3 - 2n_4n_1 = 2(n_2n_3 - n_1n_4)$

$m_3 = R_{33} = 1 + 2(n_3^2 - \vec{n}^2) = 1 - 2(n_1^2+n_2^2)$, так как $\sum_1 n_\alpha^2 = \vec{n}^2 + n_4^2 = 1$

$n_3^2 - \vec{n}^2 = n_3^2 - (1-n_4^2) = n_3^2 + n_4^2 - 1$. $m_3 = 1 + 2[n_3^2 + n_4^2 - 1] = 2(n_3^2+n_4^2) - 1$

$m_3 = m_2 = 2(n_3^2+n_4^2) - 1 = \left| n_i = \vec{n} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta),\ n_4 = \cos\chi \right| =$

$= 2(\cos^2\theta\sin^2\chi + \cos^2\chi) - 1 = 2\cos^2\theta\sin^2\chi + \cos 2\chi$

$m_1 = 2(n_1n_3 + n_2n_4)$, $m_2 = 2(n_2n_3 - n_1n_4)$.

Согласно Gus: chaos in n_3: $m_2 = m_3 = |z_1|^2 - |z_2|^2 = |X_4 + iX_3|^2 - |X_1 + iX_2|^2 =$

$= X_4^2 + X_3^2 - X_1^2 - X_2^2$ $(X_1^2 + X_2^2) + (X_3^2 + X_4^2) = 1$, $m_2 = 2(X_3^2 + X_4^2) - 1$, Biquino

$m_1 = 2\mathrm{Re}(z_1 z_2^*) = 2\mathrm{Re}[(X_1 + iX_2)(X_4 - iX_3)] = 2(X_1X_4 + X_2X_3)$

$m_2 = 2\mathrm{Im}(z_2 z_1^*) = 2(X_2X_4 - X_1X_3)$. Вероятно ось n_4 переобозначена $3 \to 4$, $4 \to 3$

Escaneado con CamScanner

Спинор $Z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$ $z_1 = x_1 + ix_2$, $z_2 = x_4 + ix_3$ $Z_2 \to z_2' = x_3 + ix_4$ если $x^3 \leftrightarrow x^4$.

в Volovik'e m_i совпадает с Gus'chaos m_i через $(\vec{n}, n_4)$ $(\vec{X}, X_4)$ вектор компонент

$X_4 = (\vec{X}, X_4)$, $X_i = \frac{2x_i}{(1+r^2)}$, $\vec{X} = \frac{2\vec{r}}{(1+r^2)}$, $r^2 = \sum_{i=1,3} x_i^2$ у меня Chaos'a

Volovik 1977: $n_4 = (\vec{n}, n_4)$, $\vec{n} = \hat{r} \sin\chi$, $\vec{X} = \hat{r} \frac{2r}{(1+r^2)}$, совместимо

если $\sin\chi(r) = \frac{2r}{(1+r^2)}$, $\chi(0) = 0$ или π, $\chi(\infty) = 0$ или π.

$m_3 = ?$ $X_4 = \left(\frac{1-r^2}{1+r^2}\right)$, $n_4 = \cos\chi$, $\cos^2\chi = 1 - \sin^2\chi = 1 - \frac{4r^2}{(1+r^2)^2} = \left(\frac{1-r^2}{1+r^2}\right)^2$.

$m_3 = 2(\cos^2\theta \sin^2\chi + \cos^2\chi) - 1 = \left|\cos\theta = \frac{z}{r}\right|$ θ-polar angle $\cos^2\theta \sin^2\chi + \cos^2\chi =$

$= \cos^2\theta \sin^2\chi + (1 - \sin^2\chi) = (\cos^2\theta - 1)\sin^2\chi + 1 = -\sin^2\theta \sin^2\chi + 1$

$m_3 = 2(-\sin^2\theta \sin^2\chi + 1) - 1 = 1 - 2\sin^2\theta \sin^2\chi$. $\sin^2\theta = 1 - \frac{z^2}{r^2} =$

$= s^2/r^2$. $\sin^2\chi = \frac{4r^2}{(1+r^2)^2}$, $m_3 = 1 - 2\frac{s^2}{r^2}\frac{4r^2}{(1+r^2)^2} = 1 - 8\frac{s^2}{(1+r^2)^2}$.

Nicarinta 1999: $m_3 = 1 - \frac{8s^2a^2}{(a^2+s^2+z^2)^2}$. Эти m_3 и m_2 совпадают если s, z, r are inunits of hopfion radius a^4.

$m_1 + im_2 = 2(n_1n_3 + n_2n_4) + 2i(n_2n_3 - n_1n_4) = 2n_3(n_1 + in_2) + 2n_4(n_2 - in_1) =$

$= 2n_3(n_1 + in_2) - in_4 2(n_1 + in_2) = 2(n_1 + in_2)(n_3 - in_4) \to$

$m_4 = 2(X_1 + iX_2)(X_3 - iX_4) = 2z_2z_1^*$, $m_x + im_y = 2z_2z_1^*$ как в Gus'chaos'23

Вывод: $\vec{m}$ - components $m_i = R_{ik}[n_d(r)]\hat{z}_k$ - Volovik'77 однократно даёт $\vec{m}(\vec{r})$ - hopfion из Gus'chaos'23 если $Q_H = 1$, если $\sin\chi(r) = \frac{2r}{(1+r^2)}$.

// Но Degeltshinski'79 и Kosevich'81 использовал другую параметризацию R_{ik} - rotation matrix, через угол поворота ω и ось поворота $\hat{P}$ (?) Обозначим ось поворота как $\vec{P}$ (а не $\vec{R}$), чтоб не путать с Rik matrix.

$\vec{m}^* = \hat{z}\cos 2\chi + \vec{P}(\vec{P}\cdot\hat{z})(1 - \cos 2\chi) + [\vec{P}\times\hat{z}]\sin 2\chi$, видимо $\omega = 2\chi$ (?).

Varshalovich rot. matrix $(\omega, \vec{n})$: $a_{ik} = \cos\omega\,\delta_{ik} + (1 - \cos\omega)n_in_k - \sin\omega\,\varepsilon_{ikl}n_l$

$\vec{n} \to \vec{P}$ $a_{ik}(\omega, \vec{P}) = \cos\omega\,\delta_{ik} + (1 - \cos\omega)P_iP_k - \sin\omega\,\varepsilon_{ikl}P_l$

Volovik'77: $R_{ik}[n_d] = \delta_{ik} + 2(n_in_k - \vec{n}^2\delta_{ik}) - 2n_4\varepsilon_{ikl}n_l$, $n_d = (\vec{n}, n_4)$.

$m_i = \sum_k R_{ik}\hat{z}_k = R_{i3}[n_d]$. $m_1 = R_{13}$, $m_2 = m_{23}$, $m_3 = R_{33}$.

$m_1 = 2(n_1n_3 + n_2n_4)$, $m_2 = 2(n_2n_3 - n_1n_4)$, $m_3 = 2(n_3^2 + n_4^2) - 1$

$R_{ik}[n_d] = (1 - 2\vec{n}^2)\delta_{ik} + 2n_in_k - 2n_4\varepsilon_{ikl}n_l$, $\vec{n}^2 + n_4^2 = 1$.

$1 - 2\vec{n}^2 = 1 - 2(1 - n_4^2) = -1 + 2n_4^2 = -1 + 2\cos^2\chi = \cos 2\chi$, $n_4 = \cos\chi$.

$R_{ik}[n_d] = \cos 2\chi\,\delta_{ik} + 2n_in_k - 2\cos\chi\,\varepsilon_{ikl}n_l$. $\hat{z}_k = \delta_{k3}$

$R_{i3}[n_d] = \cos 2\chi\,\delta_{i3} + 2n_in_3 - 2\cos\chi\,\varepsilon_{i3l}n_l$. $n_3 = \cos\theta\sin\chi$

$m_i = (\vec{m}^*\cdot\hat{z}) = \cos 2\chi + (\vec{P}\cdot\hat{z})^2(1 - \cos 2\chi)$, $(\vec{m}^*\cdot\vec{P}) = (\vec{P}\cdot\hat{z})\cos 2\chi + (\vec{P}\cdot\hat{z})(1 - \cos 2\chi) =$

$\vec{m}$ - component || оси вращения совпадающая $= (\vec{P}\cdot\hat{z}) = m_z$ т.к. R_0, т.к.

Хотим найти $\vec{P}$ - ось вращения где $(\vec{m}^*\cdot\vec{P}) = (\vec{P}\cdot\hat{z})$. Будет $\vec{m} = \hat{z}$ до вращения.

$m_3 = 1 - 2\sin^2\theta\sin^2\chi = 1 - \sin^2\theta(1 - \cos 2\chi)$ и $m_3 = m_z = \cos 2\chi + (\vec{P}\cdot\hat{z})^2(1 - \cos 2\chi)$

$m_1 + im_2 = 2(n_1 + in_2)(n_3 - in_4) = 2(\sin\theta e^{i\varphi}\sin\chi)(\cos\theta\sin\chi - i\cos\chi) =$

$= 2\sin\theta\sin\chi e^{i\varphi}(\cos\theta\sin\chi - i\cos\chi)$

$m_x^* + im_y^* \Rightarrow$ $m_x^* = (\hat{x}\cdot\vec{m}^*) = (\hat{x}\cdot\vec{P})(\hat{z}\cdot\vec{P})(1 - \cos 2\chi) + \hat{x}\cdot(\vec{P}\times\hat{z})\sin 2\chi$

$m_y^* = (\hat{y}\cdot\vec{m}^*) = (\hat{y}\cdot\vec{P})(\hat{z}\cdot\vec{P})(1 - \cos 2\chi) + \hat{y}\cdot(\vec{P}\times\hat{z})\sin 2\chi$

$m_x^* + im_y^* = (\hat{x} + i\hat{y})\cdot\vec{P}(\hat{z}\cdot\vec{P})(1 - \cos 2\chi) + (\hat{x} + i\hat{y})(\vec{P}\times\hat{z})\sin 2\chi$ $\hat{x} + i\hat{y}$

$m_x^* = P_xP_z(1 - \cos 2\chi) + (\vec{P}\times\hat{z})_x\sin 2\chi$, $m_y^* = P_yP_z(1 - \cos 2\chi) + (\vec{P}\times\hat{z})_y\sin 2\chi$

Escaneado con CamScanner

Dual active rotation ($\omega \to -\omega$), $a_{ik} = \cos\omega\,\delta_{ik} + (1-\cos\omega)\,p_i p_k + \sin\omega\,\varepsilon_{ikl}\,p_l$

$m_i^* = \sum_k a_{ik} z_k = \sum_k a_{ik}\delta_{k3} = a_{i3}$, $a_{i3} = \cos\omega\,\delta_{i3} + (1-\cos\omega)\,p_i p_3 + \sin\omega\,\varepsilon_{i3l}\,p_l$

$m_z^* = a_{33} = \cos\omega + (1-\cos\omega)\,p_z^2$ и $m_3 = 1 - \sin^2\varphi\,(1-\cos 2\chi)$

$m_z^* = m_3$ даёт $\cos 2\chi + (1-\cos 2\chi)\,p_z^2 = 1 - \sin^2\varphi\,(1-\cos 2\chi)$ при $\omega = 2\chi$,

что даёт $p_z = \cos\varphi$ (?). $\vec{p}(\theta,\Phi)$ и $p_z = \cos\theta$ by definition.

(30/II) Kamchatnov comments to V.B. Ivanov about Eq. (1) [Dzyaloshinski '79]

Hopfion [Dzyal. '79]: $\vec{m}^* = \hat{z}\cos 2\chi + \vec{p}(\vec{p}\cdot\hat{z})(1-\cos 2\chi) + (\vec{p}\times\hat{z})\sin 2\chi$.

$\vec{p}$ (больше $\hat{p}$) is unit vector along rotation axis. $\vec{p} = a(\vec{m}^*\times\hat{z})$ ⊥ плоскости вращения $\vec{m}^* - \hat{z}$. $p_z = \vec{p}\cdot\hat{z} = a(\vec{m}^*\times\hat{z})\cdot\hat{z} = 0$.

$m_z^* = (\vec{m}^*\cdot\hat{z}) = \cos 2\chi + (\vec{p}\cdot\hat{z})^2(1-\cos 2\chi) = \cos 2\chi$ (это!?)

$m_x^* = (\vec{p}\times\hat{z})_x \sin 2\chi$, $m_y^* = (\vec{p}\times\hat{z})_y \sin 2\chi$, Def $\vec{p}\times\hat{z} = \vec{\mu}$, $\mu^2 = 1$, $\mu_z = 0$

$m_x^* = \mu_x \sin 2\chi$, $m_y^* = \mu_y \sin 2\chi$, $(\vec{m}^*)^2 = \cos^2 2\chi + (\mu_x^2 + \mu_y^2)\sin^2 2\chi = 1$.

Вывод: если $\vec{p}$ ($\vec{R}$) - rotation axis of $\hat{z}$ к $\vec{m}^*$ (hopfion), как усвоено в [Dzyal. '79], [Kosevich '81] то $m_z^* = \cos 2\chi = 1 - 2\sin^2\chi$, $\sin\chi = \frac{z}{(1+r^2)}$, это явно неверно. Хотя формально $m_z^*(r=0) = 1$, $m_z^*(r=\infty) = 1$, что верно для hopfion $m_z(r)$. Может $\sin\chi \neq z/(1+r^2)$?

Тогда m_z (hopfion, $Q_H = 1$): $m_z = 1 - \frac{2\varrho^2 a^2}{(a^2+r^2)^2}$, $\vec{r} = (r, \varrho, \varphi)$, $\varrho = r\sin\varrho$, $z = r\cos\varrho$

$m_z^* = 1 - 2\sin^2\chi$, $\sin^2\chi = \frac{4\varrho^2 a^2}{(a^2+r^2)^2}$, $\sin\chi(r,\varrho) = \frac{2ra\sin\varrho}{(a^2+r^2)}$.

$\sin\chi(r=0,\varrho) = 0$, $\sin\chi(\infty,\varrho) = 0$, $\chi(0,\varrho) = 0,\pi$, и $\chi(\infty,\varrho) = 0,\pi$.

Как найти rotation axis $\vec{p}$? $\vec{p} = (p_x, p_y, 0)$. $\vec{m}^*$ in-plane $\vec{m}^* = (\vec{p}\times\hat{z})\sin 2\chi$.

Gin'equation 2011: $m_x + i m_y = \frac{a\varrho\, e^{i(\varphi+\varphi_0)}\,[2za + i(r^2 - a^2)]}{(a^2+r^2)^2}$.

$m_x + i m_y = 2\sin\chi\, e^{i(\varphi+\varphi_0)}\,\frac{[2za + i(r^2-a^2)]}{(a^2+r^2)^2}(a^2+r^2) = \frac{2\sin\chi}{(a^2+r^2)}\, e^{i(\varphi+\varphi_0)}[2za + i(r^2-a^2)]$

$e^{i\varphi'}[2za + i(r^2-a^2)] = (\cos\varphi' + i\sin\varphi')[2za + i(r^2-a^2)] = 2za\cos\varphi' - \sin\varphi'(r^2-a^2) +$

$+ i[2\sin\varphi' za + \cos\varphi'(r^2-a^2)]$, $\varphi' = \varphi + \varphi_0$

$m_x + i m_y = \frac{2\sin\chi}{(a^2+r^2)}\{2za\cos\varphi' - \sin\varphi'(r^2-a^2) + i[2za\sin\varphi' + \cos\varphi'(r^2-a^2)]\}$

$m_x^* = \mu_x \sin 2\chi$, $m_y^* = \mu_y \sin 2\chi$, $\vec{\mu} = \vec{p}\times\hat{z} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ p_x & p_y & p_z \\ 0 & 0 & 1 \end{vmatrix} = \hat{x}p_y - p_x\hat{y}$

$m_x^* = p_y \sin 2\chi$, $m_y^* = -p_x \sin 2\chi$.

Уравнения на p_x, p_y: $\begin{cases} p_y \sin 2\chi = \frac{2\sin\chi}{(a^2+r^2)}[2za\cos\varphi' - \sin\varphi'(r^2-a^2)] \\ -p_x\sin 2\chi = \frac{2\sin\chi}{(a^2+r^2)}[2za\sin\varphi' + \cos\varphi'(r^2-a^2)] \end{cases}$ Bloch, Neel $\varphi_0 = 0,\pi$.

Надо $\vec{p}(\chi,\beta,\varphi)$ $\varphi' = \varphi + \varphi_0$

→ Может $\vec{p}(r,\varrho,\varphi) = \hat{R}[\text{Dzyal. '79}] = (p_x, p_y, 0)$ и $\chi(r,\varrho)$ явно. Тогда $\vec{r}(r,\varrho,\varphi)$ spherical coord.

только? $\vec{m}^* = \hat{z}\cos 2\chi + \hat{R}(\hat{R}\cdot\hat{z})(1-\cos 2\chi) + (\hat{R}\times\hat{z})\sin 2\chi$ — неясно, из-за $\hat{R}(\vec{r})$ произвольно.

[Dzyal. '79]

$(\vec{p}\cdot\hat{z}) = p_z = (\vec{R}\cdot\hat{z}) = 0$, т.к. $\hat{R}\perp\hat{z}$ - $\vec{m}^*$ plane as rotation axis; $\vec{m}^* = \hat{z}\cos 2\chi + (\hat{R}\times\hat{z})\sin 2\chi$,

$\vec{m}^* = \hat{z}\cos 2\chi + \hat{x}\hat{R}_y \sin 2\chi - \hat{y}\hat{R}_x \sin 2\chi$ - убедились, что с произвольного $\hat{R}(\vec{r})$.

→ Вывод: hopfion $\vec{m}^*(\vec{r})$ [Dzyaloshinski '79], и [Kosevich '81] не имеют смысла; rotation не нужно.

+ т.к. $p_z = p_z = 0$, то $\vec{R}(R_x, R_y, 0)$ и вообще $\vec{R}$ в плоскости xOy - плоскость, в которой "живут" $\vec{m}^*$ на unit sphere, т.е. magnetization space $\vec{m}^2 = 1$.

Rotation Rik Eq. (2.3) - Volovik-Mineev '77 имеет смысл и берет в $\vec{m}(\vec{r})$ hopfion $Q_H = 1$

Escaneado con CamScanner

(1/III) G.E. Volovik, V.P. Mineev, Sov. Phys. JETP 46, 401-404 (1977).

Rotation of $\vec{m}' = m_z \hat{z} = \hat{z}$ $\times$ $\vec{m}(\vec{r})$ hopfion, $R_{ik}(\vec{n})$ $n_4 = (\vec{n}, n_4)$ - 4-vector

Volov: $R_{ik} = \delta_{ik} + 2(n_i n_k - \vec{n}^2 \delta_{ik}) - 2\varepsilon_{ikl} n_l n_4$, $\vec{n} = \hat{r} \sin\chi$, $n_4 = \cos\chi$.

$\hat{r} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ in spherical coordinates $\vec{r}(r, \theta, \varphi)$.

Rodrigues: $R[\varphi, \hat{n}] = \hat{I} + \hat{N}\sin\varphi + \hat{N}^2(1 - \cos\varphi)$, $\hat{n}$ - ось, φ - угол вращения

$\hat{N}_{ij} = -\varepsilon_{ijk} n_k$. $R_{ik}(\varphi, \hat{n}) = \delta_{ij} + N_{ij}\sin\varphi + (\hat{N}^2)_{ij}(1 - \cos\varphi)$

$R_{ik}(\varphi, \hat{n}) = \cos\varphi\, \delta_{ik} + n_i n_k (1 - \cos\varphi) - \sin\varphi\, \varepsilon_{ikj} n_j$. $\vec{n}$ - ось вращения

у Volovik $\vec{n}$ не ось вращения, а radius vector $\vec{n} = \hat{r}\sin\chi$

$\vec{n}^2 = (\hat{r}\sin\chi)^2 = \sin^2\chi \quad 1 - 2\sin^2\chi = \cos 2\chi = 1 - 2\vec{n}^2$.

R_{ik}[Volovik] $= \delta_{ik}\cos 2\chi + 2n_i n_k - 2\cos\chi\, \varepsilon_{ikl} n_l$ имеет похожий на $R_{ik}[\varphi, \hat{n}]$

Видно R_{ik}[Volovik] обвязана с quaternion parametrization for rotations.

Quaternion $\hat{q} = \begin{pmatrix} q_x \\ q_y \\ q_z \\ q_r \end{pmatrix} = q_x\hat{x} + q_y\hat{y} + q_z\hat{z} + q_r = \vec{q} + q_r$

помимо $\vec{q} = \vec{n}\sin\frac{\theta}{2}$ $\quad \vec{q}^2 + q_r^2 = 1$ подходит для S_3 sphere.

$q_r = \cos\frac{\theta}{2}$, где $(\vec{n}, \theta)$ - ось и угол вращения.

Rotation matrix $\hat{R} = (q_r^2 - \vec{q}^T\vec{q})\hat{I}_3 + 2\vec{q}\vec{q}^T + 2q_r\hat{O}$, $\hat{O} = \begin{pmatrix} 0 & -q_z & q_y \\ q_z & 0 & -q_x \\ -q_y & q_x & 0 \end{pmatrix}$

(Wiki) $\vec{q} = (q_x, q_y, q_z)$. $\vec{q}^T\vec{q} = |\vec{q}|^2$. $|\vec{q}|^2 + q_r^2 = 1$.

$\hat{R} = (1 - 2|\vec{q}|^2)\hat{I}_3 + 2\vec{q}\vec{q}^T + 2q_r\hat{O}$. $O_{ij} = -\varepsilon_{ijk} q_k$

$R_{ij} = (1 - 2|\vec{q}|^2)\delta_{ik} + 2q_i q_j - 2q_r \varepsilon_{ijk} q_k$ in quaternion representation

R_{ik}[Volovik] $= \delta_{ik}\cos 2\chi + 2n_i n_k - 2\cos\chi\, \varepsilon_{ikl} n_l$.

Чтобы $q_r = \cos\chi$, тогда $1 - 2|\vec{q}|^2 = \cos 2\chi$, или $|\vec{q}|^2 = \sin^2\chi$

R_{ik}[quaternion] $= \cos 2\chi\, \delta_{ik} + 2q_i q_k - 2\cos\chi\, \varepsilon_{ikl} q_l = R_{ik}$[Volovik] или

$\vec{q} = \vec{n}\sin\chi$, $q_r = \cos\chi$. у Volovik $\vec{n} = \hat{r}\sin\chi$ $\quad (\vec{n}, n_4)n_4 = \hat{q} = (\vec{q}, q_r)$.

$\hat{q} = (\vec{q}, q_r) = (\vec{n}\sin\chi, \cos\chi)$, т.е. ось вращения $\vec{n} = \hat{r}$ - radius vector

Babog, (1/1) Rotation matrix R_{ik}[Volovik'77] это quaternion representation of rotation вокруг оси $\vec{n} = \hat{r}$, на угол χ. $\hat{q} = (\vec{q}, q_r)$

Volovik'77: R_{ik} rotation matrix in 3D space вокруг оси $\hat{r} = \frac{\vec{r}}{|\vec{r}|}$ на угол χ. quaternion

Если rotation matrix R явно задана то можно [Gos' EPL' 2016]

Найти gauge matrix $\hat{A}_\mu = \partial_\mu R \cdot R^{-1} = (\vec{A}_\mu \cdot \vec{G})$, $\vec{A}_\mu$ - emergent field

gauge potential $A^e_\mu = (\vec{A}_\mu \cdot \vec{m})$, $A^e_\mu = (A^e_0, \vec{A})$, $\vec{A}$ is the em. field potential

v em. fields $\vec{B} = \mathrm{rot}\vec{A}$.

Rotation operator $R(\vec{n}, \omega) = e^{i\omega\vec{n}\cdot\vec{J}}$ $\quad \omega = \omega(\vec{r})$, $\vec{n} = \vec{n}(\vec{r})$ rotation

$\partial_\mu R = \partial_\mu e^{i\omega\vec{n}\cdot\vec{J}} = i\partial_\mu(\omega\vec{n}\cdot\vec{J})\, e^{i\omega\vec{n}\vec{J}}$, angle and axis

$\hat{A}_\mu = \partial_\mu R \cdot R^{-1} = i[\partial_\mu\omega\vec{n}\cdot\vec{J} + \omega\partial_\mu\vec{n}\cdot\vec{J}]\, e^{+i\omega\vec{n}\vec{J}}\, e^{-i\omega\vec{n}\vec{J}}$

$\hat{A}_\mu = i(\partial_\mu\omega\cdot\vec{n}\vec{J} + \omega\partial_\mu\vec{n}\cdot\vec{J}) = |\vec{G} = i\vec{J}| = (\partial_\mu\omega\vec{n} + \omega\partial_\mu\vec{n})\cdot\vec{G} = \vec{A}_\mu\cdot\vec{G}$

$\vec{A}_\mu = \partial_\mu\omega\vec{n} + \omega\partial_\mu\vec{n}$ - получается так просто? $A^e_\mu = (\vec{m}\cdot\vec{A}_\mu) = \vec{m}\cdot\vec{n}\,\partial_\mu\omega +$

Gauge vector potential of emergent field $\vec{A} = \vec{m}\cdot\vec{n}\,\nabla\omega + \omega(\vec{m}\cdot\nabla\vec{n})$ $+\omega\vec{m}\cdot\partial_\mu\vec{n}$

$\vec{m}\cdot\partial_\mu\vec{n} = m_x\partial_\mu n_x + m_y\partial_\mu n_y + m_z\partial_\mu n_z = m_x\nabla n_x + m_y\nabla n_y + m_z\nabla n_z$

$\vec{A} = (\vec{n}\cdot\vec{m})\nabla\omega + \omega\sum m_k\nabla n_k$. $\omega = 2\chi(\vec{r})$. $\vec{n} = \hat{r}$, $\vec{n} = (1, \theta, \varphi)$ in (r, θ, φ) spher. coord

$\nabla n_b = \nabla(\sin\theta\cos\varphi) = \frac{1}{r}\hat{\theta}\frac{\partial}{\partial\theta}(\sin\theta\cos\varphi) = \hat{\theta}\frac{1}{r}\cos\theta\cos\varphi + \frac{1}{r\sin\theta}\hat{\varphi}\frac{\partial}{\partial\varphi}$

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$= \frac{1}{r}\hat{e}_\theta \cos\theta\cos\varphi + \frac{\hat{e}_\varphi}{r\sin\theta}(\sin\theta(-1)\sin\varphi) = \frac{1}{r}\hat{e}_\theta\cos\theta\cos\varphi - \frac{1}{r}\hat{e}_\varphi(\sin\varphi)$

$\nabla n_y = \nabla(\sin\theta\sin\varphi) = \frac{1}{r}\hat{e}_\theta\cos\theta\sin\varphi + \frac{1}{r}\hat{e}_\varphi\cos\varphi \quad \nabla n_z = \nabla(\cos\theta) = \frac{-\hat{e}_\theta}{r}(\sin\theta)$

$\sum_i m_i \nabla n_i = \sin\theta\cos\varphi\left[\frac{1}{r}\hat{e}_\theta\cos\theta\cos\varphi - \frac{\hat{e}_\varphi}{r}\sin\varphi\right] + \sin\theta\sin\varphi\left[\frac{\hat{e}_\theta}{r}\cos\theta\sin\varphi + \frac{1}{r}\hat{e}_\varphi\cos\varphi\right] + \cos\theta(-1)\frac{\hat{e}_\theta}{r}\sin\theta = \frac{\hat{e}_\theta}{r}\left[\sin\theta\cos\theta\cos\varphi - \sin\theta\cos\theta\right] + \frac{\hat{e}_\varphi}{r}\sin\theta\left[-\cos\varphi\sin\varphi + \sin\varphi\cos\varphi\right] = 0$

Bryngar $\vec{A} = 2(\vec{m}\cdot\vec{n})\nabla f$, no $f(r) = f(-r)$, $\sin f(r) = \frac{r^2}{(1+r^2)}$

$\vec{A} = 2(\vec{m}\cdot\vec{n})\frac{\partial f}{\partial r}\hat{r}$, $\vec{A} = (A_r, 0, 0)$ in spherical coordinates? Bryngar, nebogno.

Hagga $\vec{m} = (m_x, m_y, m_z) = (\sin\Theta\cos\Phi, \sin\Theta\sin\Phi, \cos\Theta)$, a ne $(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$

Θ, Φ - spherical angles of $\vec{m}$, $\sum_i m_i \nabla n_i \neq 0$.

$\sum_i m_i \nabla n_i = \sin\Theta\cos\Phi\left[\frac{1}{r}\hat{e}_\theta\cos\theta\cos\varphi - \frac{\hat{e}_\varphi}{r}\sin\varphi\right] + \sin\Theta\sin\Phi\left[\frac{\hat{e}_\theta}{r}\cos\theta\sin\varphi + \frac{1}{r}\hat{e}_\varphi\cos\varphi\right] + \cos\Theta(-1)\frac{\hat{e}_\theta}{r}\sin\theta = \frac{1}{r}\hat{e}_\theta\left[\sin\Theta\cos\theta\cos\varphi\cos(\Phi-\varphi) - \cos\Theta\sin\theta\right] + \frac{\hat{e}_\varphi}{r}\left[-\sin\Theta\cos\Phi\sin\varphi + \sin\Theta\sin\Phi\cos\varphi\right] = \frac{1}{r}\hat{e}_\theta[\ldots] + \frac{\hat{e}_\varphi}{r}\sin\Theta\sin(\Phi-\varphi)$

Ein field

$\vec{A} = 2(\vec{m}\cdot\vec{n})\frac{\partial f}{\partial r}\hat{r} + 2f(r)\frac{1}{r}\left[\hat{e}_\theta(\sin\Theta\cos\theta\cos(\Phi-\varphi) - \cos\Theta\sin\theta) + \hat{e}_\varphi\sin\Theta\sin(\Phi-\varphi)\right]$

Φ,φ bxogeo kak Φ−φ, ⇒ gaer $\vec{A}(r,\theta) = \vec{A}(\beta,z)$ gne $Q_H = 1$ hopfion.

$\vec{m}\cdot\vec{n} = (\vec{m}\cdot\hat{r}) = \sin\Theta\cos\Phi\sin\theta\cos\varphi + \sin\Theta\sin\Phi\sin\theta\sin\varphi + \cos\Theta\cos\theta =$

$= \sin\Theta\sin\theta\cos(\Phi-\varphi) + \cos\Theta\cos\theta$. Em field vector potential $\vec{A} = \vec{A}(r,\theta) = \vec{A}(\beta,z)$

$\vec{B}(\vec{r}) = \frac{4a}{(a^2+r^2)}\vec{A}(\vec{r})$ gne $Q_H = 1$, hopfion magnetization $\vec{A}\cdot\vec{B}$ ugbocno u Q_H [dW $\vec{A}\vec{B}$ ugbocno

Cpabnuo noe $\vec{A}$ u $\vec{B}$ c $\vec{A},\vec{B}$ Pershoguba'2021 u $\vec{A},\vec{B}$ - Sagi '2023. Bryngo, cno

$\vec{B} = \frac{4a}{(a^2+z^2)}\vec{A}$ ne bonoxnaerca uu gne [Pershoguba'21], uu gne [Sagi '2023].

(2/III) Dober to Maciej kak umsara DW resonance in bended stripe (Sinc.)

Revision hopfion-dot MS gne LTP (9+11+17). no chen ero rone.

- Kosevich Phys. Rep. 1990: rotation matrix $O_{ij} = \delta_{ij} + 2(n_i n_j - \vec{n}^2\delta_{ij}) - 2n_4\epsilon_{ijk}n_k$,

$n_d = (\vec{n}, n_4)$. O_{ik} cobnagaet c R_k [Volovik '77]. $\sum_d n_d^2 = 1$, $d = 1..4$.

- Hopfion magnetization $\vec{m} = \hat{e}_z(1-2\vec{n}^2) + 2\vec{n}n_3 + 2n_4(\vec{n}\times\hat{e}_z)$. Cno

paccuraeno ocnobnaerca ot $\vec{m}$ b [Ozyalakh '79] u [Kosevich '81]

Volovik '77: $\vec{n} = \hat{r}\sin\chi$, $n_4 = \cos\chi$. $\hat{r}$ unit radius vector. $\hat{r} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$

$\vec{r}(x,y,z)$ in spherical coordinates $\vec{r}(r,\theta,\varphi)$.

Kosevich '90: $n_4 = \cos\chi$, $n_3 = \sin\chi\cdot\cos\psi$, $n_1 + in_2 = \sin\chi\sin\psi\cdot e^{i\Phi}$, $\cos\theta$

cnu $\chi = \chi$: $\chi = f$ [Volovik '77], $\Phi = \varphi$, $\psi = \theta$.

29/III: $m_3 = 2(n_3^2 + n_4^2) - 1 = 2\cos^2\theta\sin^2\chi + \cos^2\chi = 1 - 2\sin^2\chi\sin^2\theta$ (beno χ Kosevich)

$m_1 = 2(n_1n_3 + n_2n_4) = 2[\sin\chi\sin\theta\cos\varphi\cos\theta + \sin\chi\cos\chi\sin\theta\sin\theta] =$

$= 2\sin\chi\sin\theta[\sin\chi\cos\varphi\cos\theta + \cos\chi\sin\varphi]$

$m_2 = 2(n_2n_3 - n_1n_4) = 2[\sin\chi\sin\theta\sin\varphi\cos\theta - \sin\chi\cos\chi\sin\theta\cos\varphi] =$

$= 2\sin\chi\sin\theta[\sin\chi\sin\varphi\cos\theta - \cos\chi\cos\varphi]$

- [Kosevich '90] $m_x + im_y = 2\sin\chi\sin\theta(\cos\theta\sin\chi - \cos\chi)(\cos\varphi + i\sin\varphi) =$

$= 2\sin\chi\sin\theta[\cos\theta\sin\chi\cos\varphi + \cos\chi\sin\varphi + i(-\cos\chi\cos\varphi + \cos\theta\sin\chi\sin\varphi)]$

m_1, m_2 - Kosevich cobnagaet c n_1, n_2 - Volovik '77, nuu $\chi = f$, $\Phi = \varphi$, $\psi = \theta$

Ho bce gne $|Q_H| = 1$. Hopfion gne $|Q_H| > 1$ nyx' nepexogu

$\vec{n} = \hat{r}\sin\chi$, $n_4 = \cos\chi$ Eonee cnoxnbie, cnoct θ beco azimuthal, nuu poloidal. nuu Mathieu?

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• Saji '2023: $SO(3) \to SU(2)$ rotation operator $\hat{R} = R_0 + i\sum_\mu R_\mu \sigma_\mu$, $\mu = 1,2,3$

$\hat{R}_0^2 + \sum_\mu \hat{R}_\mu^2 = 1$ unit sphere S_3. $\vec{n} = \hat{R}^{-1}\sigma_z \hat{R} = \sum_\mu n_\mu \sigma_\mu$

Hopfion $\vec{m}$ is constructed by $R_H(\vec{r}) = \exp[i f(r)\hat{r}]$ rotation angle $f(0)=0$, $f(\infty) = \pi$. Осе вращения $-\hat{r}$, угол вращения $f(r)$.

Hopfion $m_z = \cos 2\chi(r,\theta)$, $m_x + i m_y = \sin 2\chi\, e^{i(q\varphi - p\theta + \varphi_0)}$,

$\sin f(r,\theta) = \sin[f(r)]\sin\theta = \sin f(r)\cdot\sin\theta$, $\operatorname{tg}\theta(r,\theta) = \operatorname{tg} f(r)\cdot\cos\theta$ (!!)

q - azimuthal, p - poloidal vorticity. Где брали $\vec{m}(\vec{r})$ - hopfion с такой параметризацией. Нет ссылок. $\vec{r}(r,\theta,\varphi)$ - spherical coordinates.

(3/XII) Note: $\vec{m}$ (Saji '23) совпадает с $\vec{m}^* = \hat{R}\cos 2\chi + (\hat{R}\times\hat{z})\sin 2\chi$, $(\hat{R})_z = 0$, что дало и нам ось вращения [Dzyal. '79] это $\hat{R}$, т.е. χ, θ [Saji '23] описывают ось вращения $\hat{R}$ из Ref. [Ozgal. '79] (!)

• C. Saji et al. Phys. Rev. Lett. 131, 166702 (2023) + arXiv Feb. 2023 (I saw on arXiv).

Написал со-автору Saji et al. 2023, Vagson Carvalho вопросы о Bulge hopfion $\hat{R}_H(\vec{r})$, $f(r,\theta)$, $\operatorname{tg}\theta(r,\theta)$. Откуда они это взяли? Самих hopfion [Saji '23] не содержит ссылок. $R_H(\vec{r})$ видимо из [Volovik-Mineev 1977]

E-mails to Javier/Ruben, Oksana (GM pillars, DW resonance, hopfion simul.), Boris Murakami (San Seb. conference?), Vagson (hopfions).

(4/XII) Получил Ikerbasque promotion на 2025-27 с оценкой "very good".

Оксана написала что ее PhD student занимается simulations of bent stripe DW resonance в духе [Saji, Nature 2004] and will conduct an experiment. Оксана прислала the paper [Bitencourt – Chubykalo-Fesenko, Nanotechn. 35 (2023) 065709] по analytical calcs of DW resonance of bent stripe (!)

DW res. Slonczewski's eqs. for DW position $q(t)$ и azimuthal angle $\varphi(\theta)$ in DW center.

θ - direction along bent stripe, $\hat{r}$ - normal to stripe in xOy plane, φ as out-of-plane or out-of-plane axis Oz, $+\varphi$ is polar angle of spherical coordinate system $\hat{x}\hat{y}\hat{z}$, $\hat{x},\hat{y} \to \hat{r}\hat{\theta}$. $\varphi = \pi/2 - \vec{m}$ is xOy plane.

Slonc. equations linearized $\dot{q} = -\frac{\gamma}{2M_s}F_\varphi$, $\dot{\varphi} = \frac{\gamma}{2M_s}F_q$, $F_q = -\frac{\partial E}{\partial q} = -K_1 q$, $F_\varphi = -\frac{\partial E}{\partial \varphi} = -K_2(\varphi - \frac{\pi}{2})$

$\dot{q} = -\frac{\gamma}{2M_s}(-K_2)(\varphi - \frac{\pi}{2})$, $\dot{\varphi} = \frac{\gamma}{2M_s}(-K_1 q)$, $\begin{cases} \dot{q} = \frac{\gamma K_2}{2M_s}\varphi \\ \dot{\varphi} = -\frac{\gamma}{2M_s}qK_1 \end{cases}$ даст $\ddot{q} = \frac{\gamma}{2M_s}\dot{\varphi} = \frac{\gamma}{2M_s}\left(-\frac{\gamma}{2M_s}K_1 q\right)K_2$

DW res.

$\ddot{q} = -\left(\frac{\gamma}{2M_s}\right)^2 K_1 K_2 q = -\omega_0^2 q$, ω_0 DW resonance frequency $\omega_0^2 = \left(\frac{\gamma}{2M_s}\right)^2 K_1 K_2$.

При $\chi\lambda \ll 1$, $\chi = \frac{1}{R}$ curvature of bent stripe, $\lambda \approx \sqrt{\frac{A}{2\pi K_s M_s^2}} \approx \sqrt{\frac{A}{2\pi M_s^2}} = l_e \approx$ $\times 5$nm

$K_1 \approx 2\pi M_s H$ (Hy), $K_2 = 2\pi A\chi + 8\pi\lambda M_s^2 N$, $N = N_z - N_x$ демаг. factors, $N > 0$

for that stripe ($L \gg w$, L infinity for long stripe). L, w stripe thickness, width. $K_2 \approx 8\pi\lambda M_s^2 N$, если $R \gg l_e$.

$\omega_0^2 = \frac{\gamma^2}{(2M_s)^2} 2\pi M_s H \cdot 2\pi\lambda M_s^2 N = \frac{\gamma^2}{4M_s^2} 2\pi M_s H \cdot 8\pi\lambda M_s^2 N = 4\pi\gamma^2 \pi M_s H N\lambda \sim \frac{H_y}{R}$

$\omega_0^2 = 4\pi\gamma^2 \pi M_s H N\lambda$. Как обычно DW mass m? $m_s = \frac{m}{S}$, $S = lw$

Сравнение: 200 m DW момент инерции момент ω_0 (slone) $= \omega_0$ (oscil)

ω_0^2 (oscil) $= K_1/M_s$ 1D-harmonic oscillator model.

$4\pi\gamma^2(\pi\lambda)$; $\omega_0^2 = \left(\frac{\gamma}{2M_s}\right)^2 K_1 K_2$ slone; $\frac{K^2}{(2M_s)^2} K_1 K_2 = \frac{K_1}{M_s}$. m_s DW mass density

$m_s = \left(\frac{2M_s}{\gamma}\right)^2 \frac{1}{K_2} = \frac{4M_s^2}{\gamma^2} \frac{1}{8\pi\lambda M_s^2 N} = \frac{1}{\pi^2} \frac{\lambda}{2\pi\lambda N}$, $\left(m_s = \frac{1}{\pi^2}\frac{1}{2\pi N}\right)$

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1D - Döring mass density $m = \frac{1}{\delta} \frac{1}{\gamma^2} \frac{1}{2\pi\Delta}$, Δ - DW wall width $\Delta^2 = \frac{A}{K_d}$.

В [Rama-Eiros 'PRB 2020] получено m_s не зависит от $E_{DW}(v) = E_0 \gamma(v)$ при $v^2 \ll c^2$, $E_{DW} = E_0 + \frac{1}{2} m_s v^2$, see [Schlomann 'APL '1971] при Walker approximation $\varphi(x) = \varphi(x - vt) = \varphi_0 = const$.

m_s [Bitter count '23] $= \frac{1}{\gamma^2} \frac{1}{2\pi\lambda N}$ похожа на m_s [Döring] $= \frac{1}{\gamma^2 2\pi\Delta}$.

то $\lambda \propto l_e$, а $\Delta = \sqrt{A/K_u}$ зависит от анизотропии K_u. $l_e^2 = A/2\pi m_s^2$.

Bitter: $\omega_0^2 = \frac{\gamma^2}{4\pi M_s^2} K_1 K_2 = \frac{\gamma^2}{4\pi M_s^2} K_1 \frac{4\pi M_s^2}{\gamma^2} \frac{1}{m_s} = \frac{K_1}{m_s} = \frac{2\pi M_s H}{m_s}$, $f_0^2 = \left(\frac{\omega_0}{2\pi}\right)^2 = \frac{\pi M_s H}{2\pi^2 m_s}$

Saitoh '2004: $f_0^2 = \frac{Q_M H \pi}{4\pi^2 m} = \frac{2 \cdot 4\pi M_s S \cdot H\pi}{4\pi^2 m} = \frac{2}{\pi} \frac{M_s H \pi}{m_s}$, $\pi = 1/R$.

т.е. $\frac{f_0^2 [\text{Bitter}]}{f_0^2 [\text{Saitoh}]} = \frac{1}{2\pi^2} \frac{1}{(2/\pi)} = \frac{1}{4\pi}$, $f_0 [\text{Bitter}] = \frac{1}{\sqrt{4\pi}} f_0 [\text{Saitoh}]$

$\frac{1}{\sqrt{4\pi}} = 0{,}282$. Чтобы получить $f_0 = f_0(\exp)$ Saitoh должен занизить DW mass density m_s. $m = m_s \cdot S$ (Saitoh'ом) $= 6.6 \cdot 10^{-20}$ g.

Если Оливер f_0 (Bitter '23) верной, то m не может быть $9 \cdot 10^{-20}$ g $\approx$ $\approx m$ (Saitoh). m [Bitter '23] завышена, надо взять в $\sqrt{4\pi} \approx 3{,}54$ меньшую m.

m (Bitter, corrected) $= \frac{1 \cdot m_B}{\sqrt{4\pi}} = 0{,}282\, m$ (Bitter '2023)

K.V. Yershov et al. Curvature induced DW pinning. PRB 92, 104412 (2015)

Рассм. fast nanostripe contact с exchang and anisotropy. Sloncz. equations lead to finite (1,6 GHz) DW frequency.

(5/XII) переделан PHT manuscript и послан to B. Ivanov. Ранчесм с А. небыл слушно на кафедру, условился ёлку.

(6/XII) Совпадение на UAM - travel grant application (2 weeks).

Analog Rotations of Saji et al. PRL '2023 Saji '23 Hopfion

Будем rotation operator Saji '23 $\hat{R}_H(\vec{\pi}) = e^{i\frac{f}{2}\vec{n}\cdot\vec{J}} = e^{i\frac{f}{2}\hat{\pi}\cdot\vec{\sigma}}$, $\vec{J} = \vec{\sigma}/2$.

$f = f(\vec{\pi})$ - rotation angles, Rodrigues representation дает сигма $SU(2)$.

$\hat{\pi}$ - unit vector $\frac{\vec{\pi}}{|\vec{\pi}|}$. $\hat{\pi}(\theta, \Phi) = \hat{\pi}(\theta, \varphi)$ $\vec{\pi}(\pi, \theta, \varphi)$ spherical Coord. System.

Для $\vec{J} = V_2 \cdot \vec{\sigma} = \frac{1}{2}\vec{\sigma}$ $\hat{R}_H = e^{i\frac{f}{2}\vec{n}\cdot\vec{J}} = e^{i\frac{f}{2}\hat{\pi}\cdot\vec{J}} = \hat{I}\cos\frac{f}{2} + i(\hat{\pi}\cdot\vec{\sigma})\sin\frac{f}{2}$.

и $\vec{n} = \hat{\pi}$ $\hat{R}_H = R_0 I_2 + i\sum_j R_j \hat{\sigma}_j$, $R_0^2 + \sum_j R_j^2 = 1$.

Quaternion components $R_0 = \cos\frac{f}{2}$ $R_j = (\hat{\pi})_j \sin\frac{f}{2}$. $\hat{\pi}\cdot\vec{\sigma} = \hat{x}\sigma_x + \hat{y}\sigma_y + \hat{z}\sigma_z$.

$\vec{R} = \hat{\pi}\sin\frac{f}{2}$. Это $SU(2)$ представление $\hat{R}_H$. $\hat{\pi}\cdot\vec{\sigma} = \sum_j (\hat{\pi})_j \hat{\sigma}_j$

у Volovik '1977 $SO(3)$ представление $\hat{R}_H$, R_{ij} и $(\vec{m})_i = \sum_j R_{ij} (\vec{e})_j$.

Hopfion $\vec{m} = \hat{R}^{-1} \hat{z} \hat{R}$ или $(\vec{m})_i = \sum_j R_{ij} (\hat{z})_j$ в $SO(3)$, (3×3) представление.

В $SO(3)$, $\hat{R} = e^{i\chi \vec{\pi}\cdot\vec{J}}$, $J = 1$,

из Eq. (1) Saji '23 $m_x + i m_y = \sin 2\chi\, e^{i(q\varphi - p\theta + \varphi_0)}$ и $m_x + i m_y = \sqrt{1 - m_z^2}\, e^{i(m\varphi + m\beta)}$

будем $m_z = \cos 2\chi$ и $\Theta(\pi, \varphi) = \arctg[\tg f(\pi) \cdot \cos\varphi]$ будем, что $\Theta(\pi, \varphi) = \beta(\delta, \pi)$ из Gaididei '23

$\beta(\delta, \pi) = \arctg\left[\frac{2a\pi}{(\pi^2 - a^2)}\right]$, $\cos\varphi = \frac{x}{\pi}$, $\tg f \cdot \frac{x}{\pi} = \frac{2a\pi}{(\pi^2 - a^2)}$,

$\tg f = \frac{2a\pi}{(\pi^2 - a^2)}$. $m_z = \cos 2\chi = 1 - 2\sin^2\chi$. по Gaididei $f = \chi$, по $f(\pi)$

$\chi(\pi, \varphi) = \arcsin[\sin f \cdot \sin\varphi]$. $\sin^2 f = \frac{\tg^2 f}{1 + \tg^2 f} = \frac{4a^2\pi^2}{(\pi^2 - a^2)^2} \Big/ \left(1 + \frac{4a^2\pi^2}{(\pi^2 - a^2)^2}\right) =$

$= \frac{4a^2\pi^2}{(\pi^2 + a^2)^2}$, $\sin f(\pi) = \frac{2a\pi}{(a^2 + \pi^2)}$ - верно для $\chi(\pi)$ - Walker

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Volovik $\sin\chi(r) = \frac{2ar}{(r^2+a^2)}$ и $m_z = 1 - 2\sin^2\theta \sin^2\chi$. У Suji $\chi(r,\theta)$ функция, χ_s

Suji – $\sin\chi_s = \sin\chi \cdot \sin\theta$, $m_z = \cos 2\chi_s = 1 - 2\sin^2\chi_s = 1 - 2\sin^2\chi \cdot \sin^2\theta$ – верно.

Окружа $\zeta(r) = \chi(r)$ – Volovik. $\Theta(r,\theta) = \beta(r,\theta) = \arctg\left[\frac{2ar\cos\theta}{(r^2-a^2)}\right]$

$e^{i(q\varphi + p\Theta + \varphi_0)}$ верно, и $\sim e^{i(n\theta + m\beta)}$ Sus' Chaos'23.

Но $m_z = \cos 2\chi_s = 1 - 2\sin^2\chi \sin^2\theta$, $\sin\chi = \frac{2ar}{(r^2+a^2)}$, – верно. Имаме знак $\deg(Q_H) = \pm 1$.

Вывод: Видяно Suji '23 построение $\vec{m}(\vec{r})$ – hopfion для $|Q_H| = 1$, $p=1, q=1$, а после навеждане хедгехог винтинг (p,q) произволно.

Eq.(1) Suji'23 для $\vec{m}$-hopfion директно съвпада с $\hat{n}$ [Dzyal.'89][Kosevich'81].

$m_z^* = \cos 2\chi = 1 - 2\sin^2\chi$, $\sin\chi\ (z_0/\vec{r}) = \frac{2ra}{(a^2+r^2)}\sin\theta = \sin\chi_s(r,\theta)$.

$m_x^* + i m_y^* = +p_y \sin 2\chi - i p_x \sin 2\chi = -i(p_x + i p_y)\sin 2\chi$, где $\vec{p} = \vec{R}$ (Dzyal.'89) ос z_0 вращение.

$p_x + i p_y = \cos\varphi_p + i\sin\varphi_p = e^{i\varphi_p}$, $\operatorname{tg}\varphi_p = p_y/p_x$. $(-i)(p_x+ip_y) = e^{-i\pi/2} e^{i\varphi_p}$

$m_x^* + i m_y^* = e^{i(\varphi_p - \pi/2)}\sin 2\chi$ и Suji'23: $m_x + i m_y = \sin 2\chi\, e^{i(q\varphi - p\Theta + \varphi_0)}$

У Suji'23 и Dzyal.'89 $\sin 2\chi$ одинаково, $\sin\chi = \sin\chi_s = \frac{2ra}{(a^2+r^2)}\sin\theta$.

А вот φ_p трябва да зависи от φ (Azimuthal angle).

$\varphi_p(r,\theta,\varphi) \overset{p=1}{=} \frac{\pi}{2} = \varphi - \Theta + \varphi_0$ при $p=1, q=1$. p_x, p_y нагледно $z_0/\vec{r}$.

$\operatorname{tg}\varphi_p = \frac{p_y}{p_x} = -\frac{[2ra\cos\theta' - \sin\theta'(r^2-a^2)]}{[2ra\sin\theta' + \cos\theta'(r^2-a^2)]}$, $\theta' = \theta + \theta_0$.

$\varphi_p \overset{(r,\theta,\varphi)}{=} \varphi + \beta(r,\theta) + \text{const}$ при $Q_H = \pm 1$ hopfion, $\beta(r,\theta) = \arctg\left[\frac{2ar}{(r^2-a^2)}\right]$

$m_z^* = \cos 2\chi_s$, $m_x^* + i m_y^* = \sin 2\chi_s e^{i(\varphi_p - \pi/2)}$, $\sin\chi_s = \frac{2ar}{(a^2+r^2)}\sin\theta$.

$\varphi_p(r,\theta,\varphi) = \varphi + \beta(r,\theta) + \frac{\pi}{2} + \varphi_0$, $\varphi_0 = 0, \pi$ – Bloch hopfion, $\varphi_0 = \pm\pi/2$ – Neel hopfion как в Sus' magn. '24.

Вывод: rotation axis $\vec{p} = \hat{R}$ в [Dzyal.'89][Kosevich'81], $\vec{p} = (p_x, p_y, 0)$ явно експресна чрез ъгъл φ_p, $p_x + i p_y = e^{i\varphi_p}$, $\varphi_p = \varphi_p(r,\theta,\varphi) = \varphi + \beta(r,\theta) + \text{const}$, β – poloidal ang.

// Eq.(1) Suji'23, $m_z = \cos\chi_s$, $m_x + i m_y = \sin\chi_s e^{i(q\varphi + p\beta + \varphi_0)} = \sin\chi_s e^{i(n\theta + m\beta)}$

можем сделано верояно, ако $\cos 2\chi_s = m_z$ определено из $m_z(\gamma)$ – Sus'chaos'23, за $\forall m,n(p,q)$ $m_z(\gamma) = 1 - 2\operatorname{th}^2\gamma$ получено явно за $Q_H = \pm 1$, $m_z^{mn}(\gamma)$.

Любой (m,n) toroidal hopfion, $Q = mn$, можем представить как вращение uniform magnetization $\vec{m}' = \hat{z}$ вокруг оси $\vec{p}(p_x, p_y, 0)$ на ъгъл $(2\chi_s)$.

$\vec{m}(\vec{r}) = \hat{R}(2\chi_s \vec{p})\vec{m}' = \hat{R}(2\chi_s, \vec{p})\hat{z} = e^{i(2\chi_s \vec{p}\cdot\vec{J})}\hat{z}$. $\cos 2\chi_s = m_z(\gamma)$

$\varphi_p = n\varphi + m\beta(r,\theta) + \text{const} = \varphi_p(r,\theta,\varphi)$.

Това обобщава [Dzyal.'89][Kosevich'81] for any Q_H и направлява $\vec{m}$ Suji'2023.

Щом $\chi_s(\gamma)$ имаме явно, $\varphi_p(p_x, p_y)$ – тривиално, но просто в toroidal coordinates.

Alternative by Volovik-Ц (вероятно) $\sin\chi = \frac{2ar}{(r^2+a^2)}$ вокруг оси $\hat{z}$ или $Q_H = \pm 1$.

Для $Q_H = 1$ възможно и $\hat{n}$-rotation [Volovik'77] и $\vec{p}$-rotation [Dzyal.'89].

4/XII С Л. в и Самедани Olaberria.

3/XII Тел. разговор с Ruben Jaque, докато уточняваме детайлите.

Също и уточняваме Ваше с Supervisor Javier и уточняваме ему участие: chapters of Ph.D thesis, writing comments. Формално нужно Ваше/Ваш co-supervisor и с Editor A. Zhukov от UPV/EHU.

Тел. разговор с Оксана о Javier thesis, DW resonance, hopfions и GM-short wires.

Escaneado con CamScanner

(10/XII) Предложили себя to Trieste conf., Sept. 1-5, Bari, Italy как invited Speaker по magnetic hopfions. Org. committee согласился.

Papers Radu '2008, Protogenov '2006 по hopfions.

Radu'08: $w = \frac{m_1 + im_2}{1 + m_3}$, $Z_2 = \sin f \sin\theta e^{i\varphi}$, $Z_1 = \cos f + i m_3 \sin f$ с $|Z_1|^2 + |Z_2|^2 = 1$ дает S^3 для hopfion $\vec{m}$ если $w = \frac{Z_2^n}{Z_1^m}$ как в Gus' chaos'23. $\vec{r}(r, \varphi, \theta)$ - spherical coordinates, $f = f(r)$, $f(0) = \pi$, $f(\infty) = 0$, n, m - integer winding numbers.

$m_1 + i m_2 = e^{i(n\varphi - m\psi)} \sin\theta$, $m_3 = \cos\theta$, $\theta = \theta(\vartheta, \tau)$, $\psi = \psi(\vartheta, \tau)$ ref. to Kundu 1982.

у Kundu'82 нет $m_1 + i m_2$ только $\psi(\vartheta, \tau)$.

Kundu'82: em. magnetic field $\vec{B} = \frac{2}{m_3} \nabla m_1 \times \nabla m_2$ ($\vec{B}$(Kundu) $= 2\vec{B}$(Ran))

$m_3 = \cos\Theta = w(r, \theta)$, $\Phi = \operatorname{arctg}\left(\frac{m_2}{m_1}\right) = n\varphi + \psi(r, \varphi)$, $\vec{r}(\vec{r}, \varphi, \theta)$ spherical coordinates

$\vec{B} = -2(\nabla m_3 \times \nabla\Phi)$, $\vec{B} = -\frac{2n}{r^2 \sin\theta}\left[\hat{e}_r \frac{\partial w}{\partial\theta} - \hat{e}_\theta \frac{\partial w}{\partial r} r\right] - \frac{2}{r}\hat{e}_\varphi\left[\frac{\partial w}{\partial r}\frac{\partial \psi}{\partial\theta} - \frac{\partial w}{\partial\theta}\frac{\partial\psi}{\partial r}\right]$

Очевидно $\psi(r, \varphi) = n\beta(r, \varphi) = \gamma(r, \varphi)$ - Guschasi'23, где $m \rightleftarrows n$, $\gamma(\vec{r})$ - helicity по $\vec{B}(\vec{r})$ можно найти vector potential $\vec{A}$ в $\vec{r}(r, \theta, \varphi)$ coord. β - poloidal angle. и сравнить с Bershoguba'21 и Saji 2023.

$\vec{B}$(Saji'23) $= \frac{2Q}{r^2}\sin^2 f \cos\theta\, \hat{r} - \frac{2}{r}\sin 2f f' \sin\theta \hat{e}_\theta + \frac{2P}{r}\sin^2 f f' \sin\theta \hat{e}_\varphi$, $f = f(\vec{r}) = \gamma(\vec{r})$ Volovik где $Q_H = 1$.

$\vec{B}$(Saji) нефизично, т.к. $\hat{r}, \hat{e}_\theta$ - компоненты не содержат $m_3 = w$, $\hat{e}_\varphi$ - компонент не содержит $\nu = \gamma$ - helicity, $\Phi = n\varphi + \gamma(r, \varphi)$.

(11/XII) Переписка с WPI-SKCM2 (Japan), on-line seminar. Запросил Оксана to nominate me as invited speaker JEMS-2025. Запрос Смолуха о конф. San Seb 2025

(12/XII) Ответ Murakami о Singular "Chimera" conf. San Sebastian 2025. hopfion rotation MS.

(13/XII) С Л. в Centre Fourier Olaberria в Tolosa. Hopfion rotation MS. Техосмотр в Leganes 3 часа.

(14/XII) Если $n_\mu = (\vec{n}, n_4)$ - 4-vector [Volovik'77], то [Kosevich'90], hopfion $\vec{m}$

$\vec{m} = \hat{z}(1 - 2\vec{n}^2) + 2\vec{n}n_z + 2n_4(\vec{n} \times \hat{z})$, rotation matrix $R_{ik} = (1 - 2\vec{n}^2)\delta_{ik} + 2n_i n_k - 2\varepsilon_{ikl} n_l n_4$ [Volov.4]

$(\vec{m})_i = R_{ik}(\hat{z})_k$, $\hat{z} = (0, 0, 1)$, $n_x^2 + n_y^2 + n_z^2 + n_4^2 = 1$.

Верно ли $\vec{m}(\vec{n})$ для any Q_H или только для $Q_H = 1$? Хотелось бы parametrization [Volovik], $\vec{n} = \hat{n}\sin\chi$, $n_4 = \cos\chi$. Хорошо было бы для $Q_H = 1$?

$m_3 = m_z = (1 - 2\vec{n}^2) + 2n_z^2 = 1 - 2(n_x^2 + n_y^2)$, ~~Нет см. 29/12~~

$m_1 = m_x = (\vec{m}\cdot\hat{x}) = 2n_x n_z + 2n_4 \hat{x}\cdot(\vec{n}\times\hat{z})$, $\hat{x}\cdot(\vec{n}\times\hat{z}) = \vec{n}\cdot(\hat{z}\times\hat{x}) = \vec{n}\cdot\hat{y} = n_y$

$m_2 = m_y = (\vec{m}\cdot\hat{y}) = 2n_y n_z + 2n_4 \hat{y}\cdot(\vec{n}\times\hat{z}) = 2n_y n_z + 2n_4\vec{n}\cdot(\hat{z}\times\hat{y}) = 2n_y n_z - 2n_4 n_x$

$m_x = 2n_x n_z + 2n_4 n_y$, $m_y = 2n_y n_z - 2n_4 n_x$. $\hat{z}\times\hat{y} = -\hat{x}$

$m_x + i m_y = 2n_x n_z + 2n_4 n_y + (2n_y n_z - 2n_4 n_x)i = 2n_z(n_x + i n_y) + 2i n_4(n_x + i n_y)(-1) =$

$= 2(n_z - i n_4)(n_x + i n_y)$. $\vec{m}^2 = m_z^2 + |m_x + i m_y|^2 = [1 - 2(n_x^2 + n_y^2)]^2 +$

$+ 4|(n_z - i n_4)(n_x + i n_y)|^2 = [1 - 2(n_x^2 + n_y^2)]^2 + 4|(n_z - i n_4)|^2 \cdot |(n_x + i n_y)|^2 =$

$= [1 - 2(n_x^2 + n_y^2)]^2 + 4(n_z^2 + n_4^2)(n_x^2 + n_y^2) = [\ldots]^2 + 4(1 - n_x^2 - n_y^2)(n_x^2 + n_y^2) =$

$= 1 - 4(n_x^2 + n_y^2) + 4(n_x^2 + n_y^2)^2 + 4(1 - n_x^2 - n_y^2)(n_x^2 + n_y^2) = 1$, $\vec{m}^2 = 1$ (Верно)

$R_{ik}(\vec{n})$ - Volovik form, quaternion representation [XII] of $SO(3)$ rotation это вращение на угол (2χ), $n_4 = \cos\chi$, вокруг оси $\vec{n}$ (n_x, n_y, n_z).

$\begin{cases} R_{ik}(\text{Volovik}) = (1 - 2\vec{n}^2)\delta_{ik} + 2n_i n_k - 2\varepsilon_{ikl} n_l n_4 \\ R_{ik}(\text{quaternion}) = (1 - 2|q|^2)\delta_{ik} + 2q_i q_k - 2q_4 \varepsilon_{ikl} q_l \end{cases}$, $q = (\vec{q}; q_4)$ quaternion, $\vec{q}^2 + q_4^2 = 1$

Escaneado con CamScanner

Берем $\vec{q} = \vec{n}$, $q_4 = n_4$. Можно $\vec{q} = \vec{n}' \sin\frac{\theta}{2}$, $q_4 = \cos\frac{\theta}{2}$, тогда $R_{ik}(q)$ описывает вращение на угол θ вокруг оси $\vec{n}'$. Пусть $\theta = 2\gamma$, $q_4 = \cos\gamma$.

$\vec{q} = \vec{n} = \vec{n}' \sin\gamma$, $q_4 = n_4 = \cos\gamma$. $R_{ik}(n_4)$ вращение на угол 2γ вокруг оси $\vec{n}'$, $|\vec{n}'|^2 = 1$.

$1 - 2\vec{n}^2 = 1 - 2(\vec{n}')^2 \sin^2\gamma = 1 - 2\sin^2\gamma = \cos 2\gamma$.

$\vec{m} = \hat{z}\cos 2\gamma + 2\vec{n} n_z + 2\cos\gamma\,(\vec{n} \times \hat{z})$ [Kosevich 1990]. γ и $\vec{n}'$ undefined

Hopfion

$\vec{m}(\cos 2\gamma)$ даст $m_z = 1 - 2(n_1^2 + n_2^2)$, $m_x = 2n_1 n_3 + 2n_4 n_2$, $m_y = 2n_2 n_3 - 2n_4 n_1$.

Верно ли $\vec{m}(\cos 2\gamma)$ for any Hopf index Q_H? И если только для $Q_H = 1$?

$(x, y, z) \to (1, 2, 3)$, $m_z = 1 - 2(n_1^2 + n_2^2)$, $m_x = 2n_1 n_3 + 2n_4 n_2$, $m_y = 2n_2 n_3 - 2n_4 n_1$

Проверим m_z: $m_z^2 = 1 - 2\vec{n}^2 + 2n_z^2$, $\vec{n}^2 + n_4^2 = 1 \Rightarrow m_z = 2(n_3^2 + n_4^2) - 1$.

$n_1^2 + n_2^2 + n_3^2 + n_4^2 = 1$, $\vec{n}^2 = n_x^2 + n_y^2 + n_z^2$. $m_z^2 = 1 - 2(n_1^2 + n_2^2)$ верно.

Если $\vec{m} = Z^+ \vec{\sigma} Z$, $Z = \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}$ спинор, то $m_z = |Z_1|^2 - |Z_2|^2$, $m_x + i m_y = 2 Z_2 Z_1^*$ (Gus-chas)

$m_x + i m_y = 2(n_3 - i n_4)(n_1 + i n_2)$, $w = \frac{Z_2}{Z_1}$ или $w = \frac{Z_2}{Z_1^n}$

$m_x + i m_y = \frac{2w}{1 + |w|^2}$, $m_z = \frac{1 - |w|^2}{1 + |w|^2}$. $Z_1 = |Z_1| e^{i\varphi_1}$, $Z_2 = |Z_2| e^{i\varphi_2}$

$w = \frac{Z_2}{Z_1} = \frac{|Z_2| e^{i\varphi_2}}{|Z_1| e^{i\varphi_1}} = \frac{|Z_2|}{|Z_1|} e^{i(\varphi_2 - \varphi_1)} \sim e^{i(n\varphi + m\beta)}$, $\vec{r}(\gamma, \beta, \varphi)$ - toroidal coord.

Берем $Z_2 = (n_1 + i n_2)$, $Z_1 = n_3 + i n_4$, $m_x + i m_y = 2(n_1 + i n_2)(n_3 - i n_4) = 2 Z_2 Z_1^*$.

$m_z = |Z_1|^2 - |Z_2|^2 = 1 - 2|Z_2|^2 = 1 - 2(n_1^2 + n_2^2)$.

Если $Q_H \neq 1$, $m, n \neq 1$, то $Z_2 = (n_1^0 + i n_2^0) e^{in\varphi} = Z_2^0 e^{in\varphi} = (Z_2^0)^n$

$Z_1 = (n_3^0 + i n_4^0) e^{im\beta} = (Z_1^0)^m$, где $Z_2^0 = (n_1^0 + i n_2^0) e^{i\varphi}$, $Z_1^0 = (n_3^0 + i n_4^0) \times e^{i\beta}$.

$w = \frac{Z_2}{Z_1}$, то $Z_2 \to (Z_2^0)^n$, $Z_1 \to (Z_1^0)^m$, и $w = \frac{(Z_2^0)^n}{(Z_1^0)^m}$.

$n_i^0 = f_i(\gamma)$ только. $\sum_i (n_i^0)^2 = 1$.

Тогда $m_z(\gamma) = 1 - 2|Z_2|^2 = 1 - 2[(n_1^0)^2 + (n_2^0)^2]$,

$m_x + i m_y = 2 Z_2 Z_1^* = (n_1^0 + i n_2^0) e^{in\varphi} \cdot (n_3^0 - i n_4^0) e^{-im\beta} = (n_1^0 + i n_2^0)(n_3^0 - i n_4^0) e^{i(n\varphi + m\beta)}$

$Z_2^0 = \text{th}\gamma \cdot e^{i\varphi} = (n_1^0 + i n_2^0) e^{i\varphi}$, $n_2^0 = 0$, $n_1^0 = \text{th}\gamma$ (?)

$Z_1^0 = (n_3^0 + i n_4^0) e^{-i\beta} = -\frac{1}{\text{ch}\gamma} e^{-i\beta}$, $n_4^0 = 0$, $n_3^0 = -\frac{1}{\text{ch}\gamma}$ (?)

так много $n_4^0 = \cos\gamma = 0$, где $\gamma = \pi/2$, $m_z(\gamma) = 1 - (n_1^0)^2 \cdot 2 = 1 - 2\text{th}^2\gamma$,

Это верно только для $Q_H = 1$. Так ли так?

Wolfram 2023: $w = \frac{2(x + iy)}{r^2 + 1}$, $w = \frac{1 + r^2 + 2iz}{r^2 + 1}$, $w = \frac{w^n}{w^m} = \frac{2^n (x + iy)^n}{(r^2 + 1)^n} \frac{(r^2 + 1)^m}{(1 - r^2 + 2iz)^m}$

Будем считать, что $e^{i\varphi}$ входило в $Z_2^0 = n_1^0 + i n_2^0 = |Z_2^0| e^{i\varphi}$

$Z_2^0 = n_1^0 + i n_2^0 = \text{th}\gamma\, e^{i\varphi} = |Z_2^0| e^{i\varphi}$, $|Z_2^0| = |n_1^0 + i n_2^0| = \text{th}\gamma$.

$(n_1^0)^2 + (n_2^0)^2 = \text{th}^2\gamma$. $e^{-i\beta}$ входило в $Z_1^0 = n_3^0 + i n_4^0 = |Z_1^0| e^{-i\beta}$

$Z_1^0 = |Z_1^0| e^{-i\beta} = e^{-i\beta} \frac{1}{\text{ch}\gamma}$; (n) $= e^{i\beta}$ умножим через него, $|Z_1^0|^2 = \frac{1}{\text{ch}^2\gamma}$

$(n_3^0)^2 + (n_4^0)^2 = \frac{1}{\text{ch}^2\gamma}$, $\sum_i (n_i^0)^2 = \text{th}^2\gamma + \frac{1}{\text{ch}^2\gamma} = 1$.

Но $m_z = 1 - 2[(n_1^0)^2 + (n_2^0)^2] = 1 - 2\text{th}^2\gamma$ верно лишь для $m = 1$, $n = 1$ vortexities.

Если $w = \frac{(Z_2^0)^n}{(Z_1^0)^m}$, $|w|^2 = w\bar{w} = \frac{(Z_2^0)^n (\overline{Z_2^0})^n}{(Z_1^0)^m (\overline{Z_1^0})^m} = \frac{|Z_2^0|^{2n}}{|Z_1^0|^{2m}}$, $|Z_2^0|^2 = \text{th}^2\gamma$, $|Z_1^0|^2 = 1/\text{ch}^2\gamma$

$w = \frac{1 - |w|^2}{1 + |w|^2} = \frac{1 - |Z_2^0|^{2n} / |Z_1^0|^{2m}}{1 + |Z_2^0|^2 / |Z_1^0|^{2m}} = \frac{\text{ch}^{2m}\gamma - \text{th}^{2n}\gamma}{\text{ch}^{2m}\gamma + \text{th}^{2n}\gamma} = \frac{1 - \text{ch}^{2m}\gamma \cdot \text{th}^{2n}\gamma}{1 + \text{ch}^{2m}\gamma \cdot \text{th}^{2n}\gamma}$ Если $m, n \neq 1$, надо

[Gus'chas'23] $Z_1 = \frac{1}{\sqrt{N}} \frac{(\text{th}\gamma)^m e^{-im\beta}}{\text{ch}^m\gamma}$, $Z_2 = \frac{1}{\sqrt{N}} \text{th}^n\gamma\, e^{in\varphi}$, $N = \text{ch}^{2m}\gamma + \text{th}^{2n}\gamma$.

Escaneado con CamScanner

$n_4 = |\tilde z_1|^2 - |\tilde z_2|^2 = 1 - 2|\tilde z_2|^2$ — вернемся. $w = \frac{\tilde z_2}{\tilde z_1}$ дает вернемся $w_2(y)$.

$\tilde z_2 = n_1 + i n_2 = \frac{1}{\sqrt{\lambda}} th^m y\, e^{im\varphi}$, $\tilde z_1 = n_3 + i n_4 = \frac{1}{\sqrt{\lambda}} \frac{(-1)^m e^{-im\beta}}{ch^m y}$

$n_1 = \frac{th^m y}{\sqrt{\lambda}} \cos m\varphi$, $n_2 = \frac{th^m y}{\sqrt{\lambda}} \sin m\varphi$, $n_3 = \frac{(-1)^m}{\sqrt{\lambda}\, ch^m y} \cos m\beta$,

$n_4 = \frac{(-1)^m}{\sqrt{\lambda}\, ch^m y} (-1) \sin m\beta$. $\sum n_i^2 = \frac{th^{2m} y}{\lambda}(\cos^2 m\varphi + \sin^2 m\varphi) + \frac{1}{\lambda ch^{2m} y}(\cos^2 m\beta + \sin^2 m\beta) =$

$= \frac{1}{\lambda}\left[th^{2m} y + \frac{1}{ch^{2m} y}\right] = 1$, $\lambda = th^{2m} y + \frac{1}{ch^{2m} y}$.

$n_1 = f(y)\cos m\varphi$, $n_2 = f(y)\sin m\varphi$, $n_3 = g(y)\cos m\beta$, $n_4 = -g(y)\sin m\beta$,

$f(y) = \frac{1}{\sqrt{\lambda}} th^m y$, $g(y) = \frac{(-1)^m}{\sqrt{\lambda}\, ch^m y}$, $f^2(y) + g^2(y) = 1$.

Вектор $n_d = (\vec n, n_4)$ однородный. Можно $f(y) = \sin\delta$, $g(y) = \cos\chi$ (Hopf parametrization)

Note: $(n_i, n_4) = (\vec n, n_4)$ parametrization отличается от parametrization [Volovik '77], где можно $n_d = (\vec n, n_4) = (\hat z \sin\chi, \cos\chi)$, $Q_4 = 1$,

а этот от parametrization [Kosevich 'go] $n_4 = \cos\chi$, $n_3 = \sin\chi\cos\psi$, $n_1 + i n_2 = \sin\chi e^{i\varphi}$

[Kosevich 'go] при $\varphi = \varphi$, $\psi = \theta$, $\vec n(r, \theta, \varphi)$ spherical coord. Вернемся к [Volovik '77]

$n_d = (\vec n, n_4)$ parametrization дана в toroidal (χ, β, φ) coordinates.

Можно $n_4 = \cos\chi$, 2χ угол вращения вокруг оси $\vec n'$, $\vec n = \vec n' \sin\chi$.

$\vec n = (\sin\delta\cos m\varphi, \sin\delta\sin m\varphi, \cos\delta\cos m\beta)$, $n_x + i n_y = \sin\delta e^{im\varphi}$, $n_z = \cos\delta\cos m\beta$.

[Volovik '77] взял $\vec n' = \hat z$, $(\vec n')^2 = 1$, $\vec n'$ - ось вращения.

$\vec n' = \frac{1}{\sin\chi}(\sin\delta\cos m\varphi, \sin\delta\sin m\varphi, \cos\delta\cos m\beta)$, и $\cos\chi = n_4 = -\sin\delta\sin m\beta$.

$tg\delta = f(y)/g(y) = (-1)^m th^m y\, ch^m y$, $tg^2\delta = th^{2m} y\, ch^{2m} y = |w|^2$, $tg\delta = (-1)^m |w|$.

Угол вращения (2χ) и ось вращения $\vec n'$ определяются, $(\vec m)_i = R_{ik}(n_d)(\hat z)_k$, (исходный $\hat z = \vec m$) для any $Q_4 = m.n.$

Hopfion $\vec m(\cos^2 g_0) = \hat z(1 - 2\vec n^2) + 2\vec n n_z + 2 n_4 (\vec n \times \hat z)$, и имеет $(\vec m \cdot \vec n) = n_z (1 - 2\vec n^2) +$

$+ 2\vec n^2 n_z = n_z$ компонент вдоль оси вращения $\vec n' = \frac{\vec n}{\sin\chi}$. $(\vec m \cdot \vec n) = n_z$

Если требовать $\vec m \cdot \vec n = 0$ ($\vec n \perp$ плоскости $\hat z - \vec m$), то $n_z = 0$. Но $n_z = g\cos m\beta = 0$

невозможно. Что не так?! $\vec n' = \vec n / \sin\chi$ не ось вращения?

[Volovik '77] принял ось вращения $\vec n' = \hat z$, $(\hat z)_z = \cos\delta\delta = (\vec n')_z \neq 0$

и получил вернуто $\vec m$ (Hopfion) для $Q_4 = 1$.

(15/XII) Wiki: Rotation angle δ, $\cos\delta = \frac{1}{2}(R_{11} + R_{22} + R_{33} - 1)$, $\hat R \to R_{ik}$ rotation matrix

Rotation axis $\vec n$ из $2\sin\delta\, \vec n' = (R_{32} - R_{23}, R_{13} - R_{31}, R_{21} - R_{12})$

$R_{ik}[n_d] = (1 - 2\vec n^2)\delta_{ik} + 2 n_i n_k - 2 n_4 \varepsilon_{ike} n_e$, $n_d = (\vec n, n_4)$ - 4-вектор,

$tr(R_{ik}) = \sum_{i=1,2,3} R_{ii} = \sum_i \left[(1 - 2\vec n^2) + 2 n_i^2\right] = 3(1 - 2\vec n^2) + 2\sum_i n_i^2$, $\vec n^2 = \sum n_i^2$ quaternion

$tr(R_{ik}) = 3(1 - 2\vec n^2) + 2\vec n^2 = 3 - 4\vec n^2$, $\vec n^2 + n_4^2 = 1$

$\cos\delta = \frac{1}{2}[tr(R_{ik}) - 1] = \frac{1}{2}[2 - 4\vec n^2] = 1 - 2\vec n^2$

$2\sin\delta(\vec n')_x = R_{32} - R_{23} = -2 n_4[\varepsilon_{32e} n_e - \varepsilon_{23e} n_e] = -2 n_4[-n_1 - n_1] \Rightarrow$

$2\sin\delta(\vec n')_y = R_{13} - R_{31} = -2 n_4[\varepsilon_{13e} n_e - \varepsilon_{31e} n_e] = -2 n_4[-n_2 - n_2] = 4 n_4 n_2$

$2\sin\delta(\vec n')_z = R_{21} - R_{12} = -2 n_4[\varepsilon_{21e} n_e - \varepsilon_{12e} n_e] = -2 n_4[-n_3 - n_3] = 4 n_4 n_3$

$\sin\delta\, \vec n' = 2 n_4 \vec n$, ось вращения $\vec n'$. $\cos\delta = 1 - 2\vec n^2 = (1) - 2(1 - n_4^2) = 2 n_4^2 - 1$

Def. $n_4 = \cos\chi$ $\cos\delta = 2\cos^2\chi - 1 = \cos 2\chi$, $\vec n' = \frac{2 n_4}{\sin\delta}\vec n = \frac{2\cos\chi}{\sin 2\chi}\vec n = \frac{1}{\sin\chi}\vec n$ (!) - rot. axis

$\delta = 2\chi$ - rotation angle

Cayley-Klein SU(2) rotation matrix $U = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}$, $|a|^2 + |b|^2 = 1$.

Angle (ω), $\vec{n}(\theta,\varphi)$ axis parametrization $a = \cos\frac{\omega}{2} - i\sin\frac{\omega}{2}\cos\theta$,

$b = -i\sin\frac{\omega}{2}\sin\theta e^{i\varphi}$, θ,φ spherical angles of rotation axis $\vec{n}$ (θ,φ).

[Vershalovich] SU(2) rotation operator $\hat{R} = \begin{pmatrix} z_1 & z_2 \\ -z_2^* & z_1^* \end{pmatrix}$, spinor $|z\rangle = \begin{pmatrix} \cos\gamma_s\, e^{i\varphi_s} \\ i\sin\gamma_s\, e^{i\varphi} \end{pmatrix}$,

γ_s, φ_s angles defined by Eq. (4) SU(2).

How do we spinors $\begin{pmatrix} \cos\cdot \\ \sin\cdot\, e^{i\cdot} \end{pmatrix}$, $z_2 = b = -i\sin\frac{\omega}{2}\sin\theta e^{i\varphi}$, $z_1 = \cos\frac{\omega}{2} - i\sin\frac{\omega}{2}\cos\theta$? $z_1 = a$.

$m_z = |z_1|^2 - |z_2|^2 = 1 - 2|z_2|^2 = 1 - 2\sin^2\frac{\omega}{2}\sin^2\theta$

$m_x + i m_y = 2 z_2 z_1^* = -2i\sin\frac{\omega}{2}\sin\theta e^{i\varphi}\left[\cos\frac{\omega}{2} + i\sin\frac{\omega}{2}\cos\theta\right] =$

$= 2\sin\frac{\omega}{2}\sin\theta[-i e^{i\varphi}]\left[\cos\varphi + i\sin\varphi\right]\left[\sin\frac{\omega}{2}\cos\theta - i\cos\frac{\omega}{2}\right]$

$m_x = 2\sin\frac{\omega}{2}\sin\theta\left[\cos\varphi\sin\frac{\omega}{2}\cos\theta + \sin\varphi\cos\frac{\omega}{2}\right]$ — 3-ro gaec parametrization

[Kosevich'go] when $\chi = \frac{\omega}{2}$, $\psi = \theta$, two obhacnaec Kosevich'go angles (χ,ψ,φ).

[Kosevich'go]: $n_4 + i n_3 = \cos\chi + i\sin\chi\cos\psi \rightarrow a = z_1 = \cos\frac{\omega}{2} - i\sin\frac{\omega}{2}\cos\theta$

$n_2 + i n_1 = \sin\chi\sin\psi e^{i\varphi} \xrightarrow{\text{Cortes}} b = z_2 = -i\sin\frac{\omega}{2}\sin\theta e^{i\varphi}$

$m_i = \sum_j R_{ij}(\vec{z})_j = \sum_j R_{ij}\delta_{j3} = R_{i3}$, $R_{ij} = (1-2\vec{n}^2)\delta_{ij} + 2n_i n_j - 2\varepsilon_{ije} n_e n_4$

$m_i = R_{i3} = (1-2\vec{n}^2)\delta_{i3} + 2n_i n_3 - 2\varepsilon_{i3e} n_e n_4$.

$\mu_i = \varepsilon_{i3e} n_e$, $\mu_1 = \varepsilon_{132} n_2 = -n_2$, $\mu_2 = \varepsilon_{232} n_2 = n_1$, $\mu_3 = \varepsilon_{332} n_2 = 0$

$\vec{u} = (\vec{n}\times\hat{z}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ n_x & n_y & n_z \\ 0 & 0 & 1 \end{vmatrix} = \hat{x} n_y - \hat{y} n_x$, $\mu_x = -n_y$, $\mu_y = n_x$

$\vec{u} = -(\vec{n}\times\hat{z})$, $u_i = -(\vec{n}\times\hat{z})_i = \varepsilon_{i3e} n_e$.

$\vec{m} = (\lambda - 2\vec{n}^2)\hat{z} + 2\vec{n} n_z + 2 n_4 (\vec{n}\times\hat{z})$ — совпадает с Eq. $\vec{m}$ (Kosevich'go).

Вектор $\vec{m}$ — вектор, лежащий в плоскости $\hat{z}(A)$, $\vec{m}(B)$ и O, образующей с проведенным $\omega/2$ плоскость AOB, плоскость вращения $\hat{z}$-$\vec{m}$. Rotation axis $\vec{n}$: $\vec{n} \perp (\hat{z}-\vec{m})$ plane, т.е. $(\vec{m}\cdot\vec{n}) = 0 \rightarrow (\vec{n}\cdot\hat{z}) = n_z = 0$. Вторая с Вершалович

(6/VII) Vershalovich: volume element of 3D rotation group $(\omega, \vec{n}(\theta,\varphi))$ интеграл.

$dU = 4\sin^2\frac{\omega}{2}\, d\omega \sin\theta\, d\theta\, d\varphi$ → the total volume $\int dU = 4\int d\omega \sin^2\frac{\omega}{2} \times$

$\times \int_0^{\pi}\sin\theta\, d\theta \int_0^{2\pi} d\varphi = 8\pi^2$.

[Kosevich'go]: $Q_H = \frac{1}{2\pi^2}\int d\chi\, d\psi\, d\varphi \sin^2\chi\sin\psi = \left|\chi = \frac{\omega}{2},\ \psi = \theta,\ \varphi = \varphi\right| =$

$= \frac{1}{2\pi^2}\int_0^{\pi}\frac{d\omega}{2}\sin^2\frac{\omega}{2}\int_0^{\pi} d\theta\sin\theta\int_0^{2\pi} d\varphi = \frac{1}{4\pi^2}\, 8\pi^2 = 2$ (!?) Должно $= 1$.

Вершалович и [Kosevich'go] надо $Q_H = \frac{1}{4\pi^2}\int d\chi\, d\psi\, d\Theta(-)$, чтобы $Q_H = 1$, ед.

Wiki: Hypersphere S_3 $x^2+y^2+z^2+w^2 = 1$, each point on S_3 (x,y,z,w) corresponds to rotation regular 4, $w = \cos\theta/2$, Вектор $\vec{n}(x,y,z)$, т.е. $SO(3)$-element

S_3 Surface (of radius R) $= 2\pi^2 R^3$ или $2\pi^2$ для unit hypersphere S_3

• Hypersphere coordinates $x_4 = (x_0, \vec{x})$, $x_0 = r\cos\psi$, $\vec{x} = r\sin\psi\cdot\hat{r} = (x_1, x_2, x_3)$

(ψ, θ, φ) angles $\hat{r} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$, $\hat{r}(\theta,\varphi)$. $\sum x_i^2 = r^2$

When S_3 radius $r = 1$ $x_4 = (x_0, \vec{x})$ дает Вершалович параметризацию

$x_4 = (n_4, n_4)$, $\vec{n} = \hat{r}\sin\chi$, $n_4 = \cos\chi$, $\chi = \psi = \pi/2$, α - rotation angle.

• Hopf coordinates on S_3 $z_1 = (x_0 + i x_1) = e^{i\varphi_1}\sin\xi$, $z_2 = x_2 + i x_3 = e^{i\varphi_2}\cos\xi$

$|z_1|^2 + |z_2|^2 = 1$ - S_3 sphere, $\xi \in [0, \pi/2]$

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$w = \frac{z_1}{z_2} = th\zeta\, e^{i(\varphi_1-\varphi_2)}$, $m_2 = \frac{1-|w|^2}{1+|w|^2} = \frac{1-th^2\zeta}{1+th^2\zeta} = \frac{1/ch^2\zeta}{1+th^2\zeta} = \frac{1}{(2ch^2\zeta-1)}$

Иначе $w = \frac{z_1}{z_2} = tg\frac{\zeta}{2} e^{i(\varphi_1-\varphi_2)}$, $|w| = tg\frac{\zeta}{2}$, $m_z = \frac{1-tg^2\frac{\zeta}{2}}{1+tg^2\frac{\zeta}{2}} = \cos 2\zeta$.

$m_x + i m_y = \frac{2w}{1+|w|^2} = \frac{2tg\frac{\zeta}{2} e^{i(\varphi_1-\varphi_2)}}{1+tg^2\frac{\zeta}{2}} = \sin 2\zeta\, e^{i(\varphi_1-\varphi_2)}$, $\vec{m}^2 = 1$.

$\varphi_1 = n\varphi$, $\varphi_2 = -m\beta$ & toroidal coordinates (ζ, β, φ). $m_z \in [-1, +1]$.

Параметры $\varphi_1, \varphi_2, \zeta$ задают mapping S_3 (x_0, x_1, x_2, x_3) на S_2 $(\vec{m}^2=1)$, $\vec{m} = \vec{m}(\varphi_1, \varphi_2, \zeta)$

Спиноры $z_1 = x_0 + i x_1$ и $z_2 = x_2 + i x_3$. Hopf coordinates ≡ spinor mapping.

[Volovik '77] Hopf index $Q_H = \frac{1}{16\pi^2}\int dV\, \varepsilon_{\alpha\beta\gamma\delta}\varepsilon_{ijk} n_\alpha \frac{\partial n_\beta}{\partial x_i}\frac{\partial n_\gamma}{\partial x_j}\frac{\partial n_\delta}{\partial x_k}$, Q_H — это the area of unit sphere S_3 $(\sum_\alpha n_\alpha^2 = 1)$ multiplied by the number of times the vector $n_\alpha = (\vec{n}, n_4)$ traces this $S_3(n_\alpha)$ sphere as the vector $\vec{r}$ runs through entire coordinate space (R^3). Q_H degree of R^3 to S_3 mapping. $S_3(\vec{r})$ to $S_3(n_\alpha)$.

$m_z(\eta) = \cos 2\zeta$ for given vortices (m, n) дает mapping $S_3(x_0, \vec{x})$ на $S_2(\vec{n})$. Хопф нашел $m_z^{mn}(\eta)$ надо задать обычный вид w. [Gus'Chaos'23].

[Volovik '77], $\vec{n} = \hat{i}\sin\chi$, $n_4 = \cos\chi$, $n_\alpha = (\vec{n}, n_4) \in S_3$, $\chi = \chi(r)$ undefined.

Surface area of S_3 - sphere $r=1$ есть $\frac{2\pi^{n/2}}{\Gamma(n/2)}\Big|_{n=4} = \frac{2\pi^2}{\Gamma(2)} = 2\pi^2$

$Q_H = \frac{1}{2\pi^2}\int dV \frac{1}{8}\varepsilon_{\alpha\beta\gamma\delta}(\dots)$, $2\pi^2$ - surface of S_3, $1/8$ относится к $\varepsilon_{\alpha\beta\gamma\delta}, \varepsilon_{ijk}$.

$\vec{r} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$, $\vec{r}(r, \theta, \varphi)$ spher. coordinates mapping means the vector n_α traces S_3 one time, или $Q_H = 1$. Для $Q_H = m\cdot n$ надо $\varphi \to n\varphi$, $\theta \to m\theta$ (или $\beta \to m\beta$ in toroidal coordinates ζ, β, φ).

Хотя $\vec{m}\cdot\vec{n} = n_z$ формально $=0$, как $\vec{m}, \vec{n}$ components $\frac{1}{2}(\vec{m}-\vec{z})$ plane $n_z = 0$, т.к. $n_\alpha(\vec{n}, n_4)$ должно проходить всю sphere $S_3(n_\alpha)$ [B. Ivanov] на $\hat{R} = \vec{p}$ vector $SO(3)$ rotation axis [Ozyal. '79] таких определений нет и можно $p_z = 0$ (Зо/Хп).

$S_3(n_\alpha)$, [Volovik] $SO3$ [Ozyal.], Spinor [Gus'Chaos'23], $SU(2)$ представление.

Tatara et al. Phys. Reports 2008: $SU(2)$-matrix U, $U^+(\vec{m}\cdot\vec{\sigma})U = \sigma_z = m_z\sigma_z$ - инвариантно к оси z-заряда $\vec{m} = (0, 0, 1)$, $U = \vec{n}\cdot\vec{\sigma}$, $\vec{n} = (\sin\frac{1}{2}\theta\cos\Phi, \sin\frac{\theta}{2}\sin\Phi, \cos\frac{\theta}{2})$ $\vec{m} = (\sin\theta\cos\Phi, \sin\theta\sin\Phi, \cos\theta)$ in $\vec{m}(\theta, \Phi)$ spherical angles - representation.

Gaikwad LTP '2024: $U = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}$, $|a|^2 + |b|^2 = 1$, $a = b_0 + ib_3$, $\beta = b = b_2 + ib_1$, $\sum_i b_i^2 = 1$.

$b_i = (\vec{b}, b_0) \Rightarrow U = b_0\hat{I} - i\vec{b}\cdot\vec{\sigma}$ - rotation matrix, $b_i = (\hat{n}\sin\frac{\theta}{2}, \cos\frac{\theta}{2})$.

θ - угол, $\hat{n}$ - ось вращения. $b_i \in S_3$.

E-mail to Maciej: имел travel grant и OPUS project report 2024.

(17/XII) Напомнил "Hopfion-rotation" paper. E-mail Лене Т о OPUS-research.

(18/XII) Напомнил "Hopfion rotation" paper. Surface of S_3 hypersphere

$Surf_3 = \int_0^\pi d\varphi_1 \sin^2\varphi_1 \int_0^\pi d\varphi_2 \sin\varphi_2 \int_0^{2\pi} d\varphi_3 = \frac{\pi}{2}(-1)(-1-1)\cdot 2\pi = 2\pi^2$

$Surf(\theta) = \int_0^\pi d\chi \sin^2\chi \int_0^\pi d\theta \sin\theta \int_0^{2\pi} d\varphi = 2\pi^2$, при $n_4 = \cos\chi$, $\vec{n} = \hat{i}\sin\chi$ [Volovik]

$Q_H = -\frac{2mn}{16\pi^2}\int_{-1}^{1} dm_z \int_{-\pi}^{\pi} d\beta \int_0^{2\pi} d\varphi = -\frac{2mn}{16\pi^2}\int_{y=0}^{y=\infty} dm_z \cdot 4\pi^2 = -\frac{2mn}{4}[-p - p] = mnp$

$Q_H = |m_z| = \cos 2\zeta$, $\zeta \in [0, \frac{\pi}{2}]$, $dm_z = -2\sin 2\zeta\, d\zeta$ $= -\frac{2mn}{16\pi^2}(-2)\int_0^{\frac{\pi}{2}} d\zeta \sin 2\zeta \int d\beta \int d\varphi =$

$= -\frac{2mn}{4}[\cos 2\zeta(y=\infty) - \cos 2\zeta(y=0)] = -\frac{1}{2}mn[-p-p] = mnp$.

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19/XII Начать Hopfion papers в HPV/EHU. Отправили посылки в Киев

Hopfion paper writing.

Hopf parametrization: $m_z = \cos 2\xi$, $m_x + i m_y = \sin 2\xi \cdot e^{i(n\varphi + m\beta)}$ toroidal coord.

Dzyal. '79 parametrization

при $p_z = (\hat{p})_z = 0$: $\vec{m} = \hat{z}\cos 2\psi + (\hat{p}\times\hat{z})\sin 2\psi$, $m_z = \cos 2\psi$

$\vec{p}\times\hat{z} = \hat{x}p_y - p_x\hat{y}$, $m_x = (\vec{p}\times\hat{z})_x \sin 2\psi = p_y \sin 2\psi$, $m_y = -p_x \sin 2\psi$, $\operatorname{tg}\varphi_p = \frac{p_y}{p_x}$

$m_x + i m_y = \sin 2\psi (p_y - i p_x) = -i(p_x + i p_y)\sin 2\psi = e^{i(\varphi_p - \pi/2)}\sin 2\psi$ [Dzyal.]

[Hopf param.] $m_x + i m_y = \sin 2\xi\, e^{i(n\varphi + m\beta)}$, откуда $\psi = \xi(y)$,

$\varphi_p = \frac{\pi}{2} + n\varphi + m\beta$, $p_x + i p_y = e^{i\varphi_p}$, $p_x = \cos\varphi_p$, $p_y = \sin\varphi_p$.

$\varphi_p(\vec{r}) = \varphi_p(\beta, \varphi) = n\varphi + m\beta + \frac{\pi}{2}$ in toroidal coordinates $\vec{r}(y, \beta, \varphi)$.

Boiling: SO(3) rotation [Dzyal '79] $\vec{m}^* = \hat{z}\cos 2\psi + \hat{p}(\hat{p}\cdot\hat{z})(1-\cos 2\psi) + (\hat{p}\times\hat{z})\sin 2\psi$, $\psi(y)$, $\varphi_p = \varphi_p(\beta, \varphi)$ — never parameterized in toroidal coordinates for any Hopf index $Q_H = mnp$.

20/XII N. и W. неделя в Cuadonga, Asturias.

Дан $J = 1/2$ rotation operator $U^{1/2}(\omega, \vec{n}) = e^{i\omega\vec{n}\vec{S}} = \hat{I}\cos\frac{\omega}{2} - 2i(\vec{n}\cdot\vec{S})\sin\frac{\omega}{2}$, $\vec{S} = \frac{1}{2}\vec{\sigma}$, $U^{1/2}(\omega, \vec{n}) = e^{i\frac{\omega}{2}\vec{n}\vec{\sigma}} = \hat{I}\cos\frac{\omega}{2} - i(\vec{n}\vec{\sigma})\sin\frac{\omega}{2}$.

$u \to +u$, $R_\mu = e^{if(\hat{r}\cdot\vec{\sigma})} = \hat{I}\cos f + i(\hat{r}\cdot\vec{\sigma})\sin f$. $f = f(r) = \omega/2$, $\vec{n} = \hat{r}$ rotation axis's

$R = R_0\hat{I} + i\sum_{\mu=1,2,3} R_\mu \sigma_\mu$, $R_0 = \cos f$, $R_\mu = (\hat{r})_\mu \sin f$ — Saji'23 parametriz.

Универсальный rotation operator [Saji'23] is $R_H = e^{if(\eta)(\hat{r}\cdot\vec{\sigma})}$, где $f(\eta) = f(z)$ - Volovik '77. Но это означает только что Hopf index $Q_H = 1$.

(half-rotation angle вокруг $\hat{z}$-axis)

Hopfion $\vec{m}(\vec{r})$: $m_z(y) = \cos 2\xi(y)$, $m_x + i m_y = \sin 2\xi(y) e^{i(n\varphi + m\beta)}$

Особые точки $\vec{r} = 0$, $|\vec{r}| = \infty$ и $(z = 0, \rho = a)$ - hopfion radius.

a) $\vec{r} \to 0$ $m_z(y \to 0) = p \cdot \frac{1 - \operatorname{ch}^{2m} y \operatorname{th}^{2n} y}{1 + \operatorname{ch}^{2m} y \operatorname{th}^{2n} y} \approx p\,\frac{1 - y^{2n}}{1 + y^{2n}} = p + O(y^{2n})$

$\beta(\vec{r} \to 0) = \operatorname{arctg}\left[\frac{2az}{(\rho^2 + z^2 - a^2)}\right] \approx \operatorname{arctg}\left[-2\frac{z}{a}\right] \to 0$

$\sin^2 2\xi = 1 - \cos^2 2\xi = 1 - p^2(1 - 2y^{2n})^2 \approx 1 - [1 - 4y^{2n}] = 4y^{2n}$.

$\sin 2\xi(y) = 2y^n$, $m_x + i m_y = 2y^n e^{i(n\varphi + m\beta)} \approx 2y^n e^{in\varphi}$

b) $|\vec{r}| \to \infty$ $y(\rho, z) = \operatorname{arcth}\left(\frac{2a\rho}{\rho^2 + z^2 + a^2}\right) = \begin{cases} \operatorname{arcth}\left(\frac{2a}{\rho}\right), & \rho > z \\ \operatorname{arcth}\left(2a\frac{\rho}{z^2}\right), & z > \rho \end{cases}$

$\rho, z \gg a$

$y(\rho, z) \approx \frac{2a}{\rho} \to 0$ или $y(\rho, z) = 2a\frac{\rho}{z^2} \to 0$.

$m_z(y \to 0) = p\left(\frac{1 - y^{2n}}{1 + y^{2n}}\right) = p(1 - 2y^{2n})$, $\beta(\vec{r} \to \infty) \approx \operatorname{arctg}\left[\frac{2az}{\rho^2 + z^2}\right] \to 0$

$|\vec{r}|$ сбоится к $(\vec{r} \to 0)$ случаю $m_z = p + O(y^{2n})$, $m_x + i m_y = 2y^n e^{in\varphi}$.

c) hopfion ring, $z = 0$, $\rho = a$, $y(\rho, z = 0) = \operatorname{arcth}\left(\frac{2a\rho}{\rho^2 + a^2}\right) \underset{\rho \to a}{=} \operatorname{arcth} x \underset{x \to 1}{=}$

$= \frac{1}{2}\ln\left[\frac{1 + 2a\rho/(\rho^2 + a^2)}{1 - 2a\rho/(\rho^2 + a^2)}\right] = \frac{1}{2}\ln\frac{(\rho + a)^2}{(\rho - a)^2} = \ln\frac{(\rho + a)}{|\rho - a|} = \ln\left[\frac{2a}{|\rho - a|}\right]_{\rho \to a} \to \infty$.

$y(\rho \to a, z = 0) = \ln\frac{2a}{|\rho - a|} \to \infty$.

$\operatorname{ch} y \approx \frac{1}{2}e^y$, $\operatorname{th} y \approx 1 - 2e^{-2y}$, $y \to \infty$ $\operatorname{ch}^{2n} y \approx \left[\frac{1}{2}e^y(1 + e^{-2y})\right]^{2m}$

$\operatorname{ch}^{2m} y \operatorname{th}^{2n} y \approx \frac{1}{2^{2m}} e^{2my}(1 + e^{-2y})^{2m}(1 - 2e^{-2y})^{2n} \approx \frac{1}{2^{2m}} e^{2my}[1 + 2m e^{-2y}][1 - 4n e^{-2y}] =$

$\approx \frac{1}{2^{2m}} e^{2my}[1 + 2(m - 2n)e^{-2y}] \approx \frac{1}{2^{2m}} e^{2my}[1 + \varepsilon]$, $\varepsilon = 2(m - 2n)e^{-2y} \ll 1$, $y \to \infty$.

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$m_2 = p\,\frac{1-ch^{2m}y\,th^{2m}y}{1+ch^{2m}y\,th^{2m}y} = p\,\frac{1-\frac{1}{2^{2m}}e^{2my}(1+\varepsilon)}{1+\frac{1}{2^{2m}}e^{2my}(1+\varepsilon)} = p\,\frac{2^{2m}e^{-2my}-(1+\varepsilon)}{2^{2m}e^{-2my}+(1+\varepsilon)} =$

$= p\,\frac{-1-\varepsilon+2^{2m}e^{-2my}}{1+\varepsilon+2^{2m}e^{-2my}}$, $\lambda_\pm = 2^{2m}e^{-2my} \pm \varepsilon \ll 1$.

$m_2(y\to\infty) = p\,\frac{-1+\lambda_-}{1+\lambda_+} = (-p)\,\frac{1-\lambda_-}{1+\lambda_+} = (-p)(1-\lambda_-)(1+\lambda_+) = (-p)[1-\lambda_- -\lambda_+]$

$\lambda_- + \lambda_+ = 2^{2m}e^{-2my}(1+\varepsilon) - \varepsilon + 2^{2m}e^{-2my} + \varepsilon = 2^{2m+1}e^{-2my}$

$m_2(y\to\infty) = (-p)\left[1-2^{2m+1}e^{-2my}\right]$, $\sin^2 2\gamma = 1-m_2^2 = 1-\left[1-2^{2m+1}e^{-2my}\right]^2 =$

$= 1-\left[1-2\cdot 2^{2m+1}e^{-2my}\right] = 4\cdot 2^{2m}e^{-2my}$, $\sin 2\gamma = 2\cdot 2^m e^{-my}$

~~$\beta(\rho\to a, z=0) = \operatorname{arctg}\left(\frac{2az}{\rho^2-a^2}\right) = 0$~~ при $z=0$ (ядро вдоль оси z).

Если $|z|<a$, но z - finite, то $\beta(\rho,z) = \operatorname{arctg}\left(\frac{2az}{z^2+(\rho^2-a^2)}\right) = \begin{cases}\operatorname{arctg}\left(\frac{2a}{z}\right), \rho=a, \frac{\pi}{2}\operatorname{sign}(z)\\ \operatorname{arctg}\left(\frac{2az}{a^2(x^2-1)}\right), \rho\neq a\end{cases}$

$\rho = x\cdot a$, $\rho^2-a^2 = a^2(x^2-1)$, $\beta(\rho,z) = \operatorname{arctg}\left[\frac{z}{a}\frac{1}{(x^2-1)}\right] = \frac{z}{a}\frac{1}{(x^2-1)} \to 0$ при $x^2\neq 1, \rho^2\neq a^2$.



ρ (ядро вдоль оси $\rho, z=0$) ядро вдоль z, $\rho\neq a$. $\beta(\rho,z)=0$ на кружке $\rho=a$ (боковая кружка) Вблизи $\rho=a$, вдали $\beta(\rho z) = -\frac{\pi}{2}\operatorname{sign}(z)$

$m_x+im_y = 2^{m+1}e^{-my}e^{i(mz+\beta\cdot m)}$

$\beta(\rho,z)$ ~(e-mp gyroscope) надо брать при $z=0$, $\rho=a$, $\operatorname{sign}(z)=0$ при $z=0$.

вдали $m_x+im_y = 2^{m+1}e^{-my}e^{imz}$, $y\to\infty$, $\rho=a$ - circle.

$m_2(y\to\infty) = (-p)[\dots] = (-p)\left[1-2^{2m+1}e^{-2my}\right]$ $z=0$

(21/III)

$S=\frac{1}{2}$. $SU(2)$ rotation matrix $U^{1/2}(\omega,\vec{n}) = e^{i\frac{\omega}{2}\vec{n}\cdot\vec{\sigma}} = \hat{I}\cos\frac{\omega}{2} + i(\vec{n}\cdot\vec{\sigma})\sin\frac{\omega}{2}$ -

- rotation на угол ω вокруг оси $\vec{n}$.

Shniz: 2018 $SU(2)$ group space is isomorphic to the sphere S_3. $SU(2)\approx S_3$

- (2+1) dimensions: $O(3)$ nonlinear σ-model and baby-Skyrme model (planar model)

triplet of real scalar fields $\varphi^a = (\varphi_1, \varphi_2, \varphi_3)$ на S_2-sphere $\varphi_a\varphi_a = 1$

Skyrmion number (2D) $Q = \frac{1}{4\pi}\int d^2x\,\vec{\varphi}\cdot\left(\frac{\partial\vec{\varphi}}{\partial x}\times\frac{\partial\vec{\varphi}}{\partial y}\right) = \frac{1}{8\pi}\int d^2x\,\varepsilon_{abc}\varepsilon_{ij}\,\varphi^a\partial_i\varphi^b\partial_j\varphi^c$

Q - number of times that $\vec{\varphi}$ covers the S_2 ($\varphi^a\varphi^a=1$) space as (x,y) ranges over the spatial domain space (R_2) once.

Topological current $O(3)$ σ-model $j_\mu = \varepsilon_{\mu\nu\lambda}\varepsilon_{abc}\varphi^a\partial_\nu\varphi^b\partial_\lambda\varphi^c$, $Q = j_0$

Q - temporal component of the topol. current, $Q = j_0$ ($\mu=0$). $\mu = 0,1,2$ (t,x,y).

тогда $Q = \int d^2x\, j_0$ (baby Skyrme model).

- Skyrme model (3+1) space.

the field of (3+1) Skyrme model is $U(\vec{r},t) = SU(2)$ matrix

$U = \hat{\pi}_0 + i\pi^a\tau^a = \pi_0\hat{1} + i\pi^a\tau^a$, quartet of scalar fields $\hat{\pi} = (\pi_0, \vec{\pi})$

$\vec{\pi} = (\pi_1, \pi_2, \pi_3)$, $\hat{\pi}\in S_3$ sphere, $\pi^\mu\pi^\mu = 1$, $\mu = 0,1,2,3$.

U is map from coordinate space S_3 (R_3) to $SU(2)$-space, $SU(2) = S_3$.

т.е. $\hat{\pi}^a$ описывает mapping from $R_3(\vec{r})$ to S_3, $\pi^\mu\pi^\mu = 1$.

chiral current (Shniz'2018), $R_i = \partial_i U\cdot U^+$,

(3+1) topological charge $Q = -\frac{1}{24\pi^2}\int d^3x\,\varepsilon_{ijk}\,\mathrm{tr}(R_iR_jR_k)$

Quartet of scalar fields $\hat{\varphi}(\varphi_0,\vec{\varphi})$ linear $\hat{\pi}(\pi_0,\vec{\pi})$; пропорционально to Jacobian

$U = \hat{I}\varphi_0 + i\vec{\varphi}\cdot\vec{\sigma}$ $\Rightarrow Q = -\frac{1}{12\pi^2}\int d^3x\,\varepsilon_{abcd}\varepsilon^{ijk}\,\varphi^a\partial_i\varphi^b\partial_j\varphi^c\partial_k\varphi^d$; $\left(\begin{matrix}a=0,1,2,3\\ i=1,2,3\end{matrix}\right)$

Q (Shniz, Skyrme model) - топ. инвариант отображения сферы - с) Hopf index

Левая $\hat{n}^d$ ($n_d, \vec{n}$) d-вектор компоненты of R_{ik} (n_d) rotation matrix.

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$Q=1$ Skyrmion [Shnir]: $U(\vec{r}) = e^{i f(r)\,\hat{r}\cdot\vec{\tau}} = \cos f(r) + i \sin f(r)\,\hat{r}\cdot\vec{\tau}$ – hedgehog ansatz

– famous $SU(2)$ matrix used [Volovik '77]: $n_4 = \cos f$, $\vec{n} = \hat{r}\sin f$.

Any Q Skyrmion [Shnir]: $U(\vec{r}) = I\cos f(r,\theta\varphi) + i(\vec{\pi}\cdot\vec{\tau})\sin f(r,\theta\varphi)$

$\hat{\pi} \to (\pi_0, \vec{\pi})$, $\pi_0 = \cos f$, $\pi^1 = \sin g \cdot \cos n\varphi$, $\pi^2 = \sin g \cdot \sin n\varphi$, $\pi^3 = \cos g$

$g(r,\theta\varphi)$, $f(r,\theta\varphi)$ are undefined, except bound. conditions at $r=0,\infty$.

$Q = n$ (azimuthal vorticity), A how grows "m" (poloidal vorticity)? m_p

$f(0,\theta\varphi)=\pi$, $f(\infty,\theta\varphi)=0$, $g(\infty,\theta\varphi)=\pi$, $g(0,\theta\varphi)=0$ – [Shnir]

Note: $\hat{\pi} \to \hat{\pi}'$ components in $U(\hat{\pi})$ are the same as $\hat{\pi}$ in $U = \pi_0 + i(\vec{\pi}\cdot\vec{\tau})$.

$\pi_0' = \pi_0 = \cos f$, $\vec{\pi}' = \vec{\pi}\sin f$, $(\pi')^2 = 1$.

[S. Krusch, P. Sutcliffe '2004] J. Phys. A37, 9037 (2004). Sphalerons in the Skyrme model.

$SU(2)$ – $U = I\pi_0 + i\vec{\pi}\cdot\vec{\tau}$, $\pi_0^2 + \vec{\pi}^2 = 1$, $\pi_0 = \cos f$, $\pi_1 = \frac{g}{2}\sin f \cdot \cos n\varphi$, $\pi_3 = \frac{g}{2}\sin f$

(g,φ) – polar coordinates, (g,z,φ) cylindrical coords. $\pi_2 = \frac{g}{2}\sin f \cdot \sin n\varphi \Rightarrow Q = n$.

$f(0)=\pi$, $f(\infty)=0$.

[Shnir '18]. Hopfions $Q_H = \frac{1}{4\pi^2}\int F\wedge A$, $F = dA$, $F_{ij} = \frac{1}{2}\varepsilon_{abc}\varphi^a \partial_i\varphi^b \partial_j\varphi^c$

$F_{ij} = \frac{1}{2}\vec{\varphi}\cdot(\partial_i\vec{\varphi}\times\partial_j\vec{\varphi})$. $dA = n\cos^2\frac{F}{2}\,dG + n\sin^2\frac{F}{2}\,d\varphi$

$F\wedge A = n^2 \cos^2\frac{F}{2}\sin F\, dG\wedge dF\wedge d\varphi$ $\left\{\begin{array}{l}\varphi_1 + i\varphi_2 = \sin F\, e^{i(n\varphi - mG)} \\ \varphi_3 = \cos F\end{array}\right.$ [Gladikowski '97]

G – poloidal angle β, $\varphi_3 = m_3(\gamma)$.

$Q_H = \frac{1}{4\pi^2}\int (mn)\cos^2\frac{F}{2}\sin F\, dG\, dF\, d\varphi$, $Q_H = mn$, т.к. $\int_0^\pi dF \cos^2\frac{F}{2}\sin F = 1$

Geometrical Gladikowski's hopfions are axially symmetric.

Hopfions as minima of Faddeev-Skyrme lagrangian are axially symmetric torons and have Q_H ($Q_H = 1, 2$) Hopf charges.

(22/XII) Eq. $\vec{m} = \hat{z}\cos 2\psi + (\vec{p}\times\hat{z})\sin 2\psi$, $\vec{p} = (p_x, p_y, 0)$, $2\psi = \Theta$, $\vec{m}(\Theta,\Phi)$

describes a 2D skyrmion (Bloch or Neel). – ver, $\vec{m}_{Sk}(\infty) = -\vec{p}$, $\vec{m}(0) = \vec{p}$.

Bloch skyrmion $\vec{p} = \hat{\vec{s}}$, Neel skyrmion $\vec{p} = \hat{\varphi}$, $\vec{r} \to \vec{s}(\rho,\varphi)$ 2D plane.

Polar $\rho = 0, z = 0$ give ordinary 2D skyrmion.

$\hat{\vec{s}} = \hat{x}\cos\varphi + \hat{y}\sin\varphi$, $\hat{\varphi} = -\hat{x}\sin\varphi + \hat{y}\cos\varphi$.

Bloch skyrmion $p_x + ip_y = \hat{s}_x + i\hat{s}_y = \cos\varphi + i\sin\varphi = e^{i\varphi}$

Neel Sk: $p_x + ip_y = -\sin\varphi + i\cos\varphi = i(\cos\varphi + i\sin\varphi) = ie^{i\varphi} = e^{i(\varphi+\pi/2)}$

Вывод: $\vec{m}(\vec{r},\vec{p})$ [Dzyal. 79] we suggest that 2D skyrmions.

(23/XII) R.A. Battye, P.M. Sutcliffe, arXiv:1998. "Solitons, Links and Knots" Proceed. Royal. Soc. A 455, 4305–4331 (1999).

$\vec{m} = \xi^+\vec{\sigma}\xi$, $\xi = (z_1, z_2)^+$, $U = \begin{pmatrix} z_1 & -\bar{z}_2 \\ z_2 & \bar{z}_1 \end{pmatrix}$, $U = e^{if\vec{v}\cdot\vec{\tau}}$, $\vec{v} = (v_1, v_2, v_3)$

Hopfion $\vec{m}$: $\left\{\begin{array}{l} m_1 = 2(v_3 v_1 \sin f - v_2(v_3^2-1)\sin f \\ m_3 = 1 - 2(1 - v_3^2)\sin^2 f \end{array}\right.$, $f = f(r)$, $\vec{v}(r,\theta,\varphi)$, $\vec{v} = v(r\theta,\varphi)$, $f(0)=\pi$

$\vec{m}(m_1, m_2, m_3) \to m_i\sigma_j = m_1\begin{pmatrix}0&1\\1&0\end{pmatrix} + m_2\begin{pmatrix}0&-i\\i&0\end{pmatrix} + m_3\begin{pmatrix}1&0\\0&-1\end{pmatrix} = \begin{pmatrix} m_3 & m_1 - im_2 \\ m_1 + im_2 & -m_3\end{pmatrix}$ matrix, $f(\infty)=0$

$\hat{m} = \begin{pmatrix} m_3 & m_1 - im_2 \\ m_1 + im_2 & -m_3\end{pmatrix} = U\hat{m}U^{-1}$, $U^{-1} = U^+$, $\hat{m} = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$, $\vec{m}$

$\hat{m} = \begin{pmatrix} z_1 & -\bar{z}_2 \\ z_2 & \bar{z}_1\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}\bar{z}_1 & \bar{z}_2 \\ -z_2 & z_1\end{pmatrix} = \begin{pmatrix} z_1 & -\bar{z}_2 \\ z_2 & \bar{z}_1\end{pmatrix}\begin{pmatrix}\bar{z}_1 & \bar{z}_2 \\ -z_2 & z_1\end{pmatrix} = \begin{pmatrix} |z_1|^2 - |z_2|^2 & 2\bar{z}_1 z_2 \\ z_1\bar{z}_2 + \bar{z}_1 z_2 & |z_2|^2 - |z_1|^2\end{pmatrix}$

Escaneado con CamScanner

$m_z = |z_1|^2 - |z_2|^2$ $m_1 + i m_2 = 2 z_1 \bar{z}_2$ $m_1 - i m_2 = 2 \bar{z}_1 z_2$.

Gus' chaos $z = (z_1, z_2)^T$, $m_z = |z_1|^2 - |z_2|^2$, $m_x + i m_y = 2 z_2 \bar{z}_1$, $(\cdot)^*$ conjugation

$z_1 = n_1 + i n_2$, $z_2 = n_3 + i n_4$, $n_4 = (\vec{n}, n_4)$, $m_z = n_1^2 + n_2^2 - n_3^2 - n_4^2$, $\sum_\alpha n_\alpha^2 = 1$.

$n_4 = (\vec{u} \sin\frac{\omega}{2}, \cos\frac{\omega}{2})$, $\omega = 2\chi$ Darwin [Drosophila] [Roserich '1990] $\vec{n}^2 + n_4^2 = 1$

$m_z = \vec{m} \cdot \hat{z} = (1 - 2\vec{n}^2) + 2 n_z^2 = 1 - 2 n_x^2 - 2 n_y^2 + 2 n_z^2 = 1 - 2(n_x^2 + n_y^2) =$

$= \vec{n}^2 + n_4^2 - (n_x^2 + n_y^2) - (n_x^2 + n_y^2) = n_z^2 + n_4^2 - n_x^2 - n_y^2 = n_3^2 + n_4^2 - n_1^2 - n_2^2$, отличие от $m_z = |z_1|^2 - |z_2|^2$.

Hopfion $\vec{m} = (1 - 2\vec{n}^2)\hat{z} + 2\hat{n} n_z + 2 n_4 (\vec{n} \times \hat{z})$, $m_z = (1 - 2\vec{n}^2) + 2 n_z^2$

(no Pik matrix) $m_x = \vec{m} \cdot \hat{x} = 2 n_x n_z + 2 n_4 \hat{x} \cdot (\vec{n} \times \hat{z})$, $\hat{x} \cdot (\vec{n} \times \hat{z}) = \vec{n} (\hat{z} \times \hat{x}) = \vec{n} \cdot \hat{y}$

$m_x = 2 n_x n_z + 2 n_4 n_y$, $m_y = 2 n_y n_z + 2 n_4 \hat{y} \cdot (\vec{n} \times \hat{z})$, $\hat{y} \cdot (\vec{n} \times \hat{z}) = \vec{n} \cdot (\hat{z} \times \hat{y}) = -\vec{n} \cdot \hat{x}$

$m_y = 2 n_y n_z - 2 n_4 n_x$. $m_x + i m_y = 2 n_z (n_x + i n_y) + 2 n_4 (n_y - i n_x) = 2 n_3 (n_1 + i n_2) - 2 i n_4 (\cdot) =$

$= 2 (n_1 + i n_2)(n_3 - i n_4) = 2 z_1 \bar{z}_2$, совпадает с $m_1 + i m_2 = 2 z_1 \bar{z}_2$ из $\hat{U}$ rotation.

Связь $z_1 = n_1 + i n_2 \sim \sin\frac{\chi}{2}$, $z_2 = n_3 + i n_4 \sim \cos(\frac{\chi}{2})$, $\frac{\chi}{2} = \Theta/2$.

$m_z = \cos\Theta = \cos^2\frac{\chi}{2} = |z_2|^2 - |z_1|^2 \Rightarrow$ верное определение, т.е.

m_z (Kosterlitz '1990) $= (1 - 2\vec{n}^2)^2 + 2 n_z^2 = n_3^2 + n_4^2 - n_1^2 - n_2^2 = |z_2|^2 - |z_1|^2$

$m_x + i m_y$ (Kosterlitz '1990) $= 2 (n_1 + i n_2)(n_3 - i n_4) = 2 z_1 \bar{z}_2$. $w = \frac{z_1}{z_2} = \operatorname{tg}\left(\frac{\Theta}{2}\right) e^{i(n\varphi + m\beta)}$

$2 z_1 \bar{z}_2 = 2 e^{i n \varphi} \sin\frac{\chi}{2} \cdot e^{i m \beta} \cos\frac{\chi}{2} = \sin(\chi) e^{i(n\varphi + m\beta)} = \frac{2w}{1 + |w|^2} = m_x + i m_y$

т.е. $m_x + i m_y$ (new) $= 2 z_1 \bar{z}_2 = m_x + i m_y$ (Kosterlitz '1990).

m_z (new) $= |z_1|^2 - |z_2|^2$, m_z (Kosterlitz '1990) $= |z_2|^2 - |z_1|^2 \Rightarrow$ надо $z_1 \rightleftarrows z_2$ в $\hat{U}$ (new).

Надо $U = \begin{pmatrix} z_2 & -\bar{z}_1 \\ z_1 & \bar{z}_2 \end{pmatrix}$. $\hat{R}(n_4) = \hat{I} n_4 + i(\vec{n} \cdot \vec{\sigma}) = \hat{I} n_4 + i n_1 \sigma_1 + i n_2 \sigma_2 + i n_3 \sigma_3 =$

$= \begin{pmatrix} n_4 & 0 \\ 0 & n_4 \end{pmatrix} + i n_1 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + i n_2 \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + i n_3 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} n_4 + i n_3 & n_2 + i n_1 \\ -n_2 + i n_1 & n_4 - i n_3 \end{pmatrix}$

$\hat{R}(\omega, \vec{u}) = \hat{I} \cos\frac{\omega}{2} + i(\vec{u} \cdot \vec{\sigma}) \sin\frac{\omega}{2} = \left|\frac{\omega}{2} = \chi\right| = \hat{I} \cos\chi + i(\vec{n} \cdot \vec{\sigma}) = \hat{I} n_4 + i(\vec{n} \cdot \vec{\sigma})$

$\vec{n} = \vec{u} \sin\chi$, $n_4 = \cos\chi$.

New definition $z_2 = n_4 + i n_3$, $z_1 = n_2 + i n_1$ и U (new) $= \begin{pmatrix} z_2 & z_1 \\ -\bar{z}_1 & \bar{z}_2 \end{pmatrix}$

$m_z = |z_2|^2 - |z_1|^2$, $m_x + i m_y = \frac{2w}{1 + |w|^2} = \sin\Theta e^{i\varphi} = 2 \sin\frac{\chi}{2} \cos\frac{\chi}{2} e^{i n \varphi} e^{i m \beta}$

New def. $z_2 = n_4 + i n_3$, $z_1 = n_2 + i n_1$, $m_x + i m_y = 2 z_1 \bar{z}_2$

U (new) $= \begin{pmatrix} z_2 & z_1 \\ -\bar{z}_1 & \bar{z}_2 \end{pmatrix}$, U^+ (new) $= \begin{pmatrix} \bar{z}_2 & -z_1 \\ \bar{z}_1 & z_2 \end{pmatrix}$, $U^+ = U^{-1}$.

$\hat{m} = \begin{pmatrix} m_z & m_1 - i m_2 \\ m_1 + i m_2 & -m_z \end{pmatrix} = U \hat{m}' U^{-1} = \begin{pmatrix} z_2 & z_1 \\ -\bar{z}_1 & \bar{z}_2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \bar{z}_2 & -z_1 \\ \bar{z}_1 & z_2 \end{pmatrix} = \begin{pmatrix} z_2 & z_1 \\ -\bar{z}_1 & \bar{z}_2 \end{pmatrix} \begin{pmatrix} \bar{z}_2 & -z_1 \\ -\bar{z}_1 & -z_2 \end{pmatrix} =$

$= \begin{pmatrix} |z_2|^2 - |z_1|^2 & -2 z_1 z_2 \\ -2 \bar{z}_1 \bar{z}_2 & |z_1|^2 - |z_2|^2 \end{pmatrix}$, $m_z = |z_2|^2 - |z_1|^2$ верно, $m_1 + i m_2 = -2 \bar{z}_1 \bar{z}_2$ неверно.

$U = \hat{R} = \hat{I} n_4 + i(\vec{n} \cdot \vec{\sigma})$, $U^+ = \hat{I} n_4 - i(\vec{n} \cdot \vec{\sigma}^T) = \hat{I} n_4 - i(\vec{n} \cdot \vec{\sigma})$, $\sigma_i = \sigma_i^+$ hermitian.

$\hat{m} = U \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} U^+$, $(\vec{a} \cdot \vec{\sigma})(\vec{b} \cdot \vec{\sigma}) = (\vec{a} \cdot \vec{b}) \hat{I} + i(\vec{a} \times \vec{b}) \vec{\sigma}$

$SU(2)$ matrix $\begin{pmatrix} a + bi & c + di \\ -c + di & a - bi \end{pmatrix}$ can be mapped to quaternion $a\hat{1} + b\hat{i} + c\hat{j} + d\hat{k}$ [Wiki]

$Q = n_4 I + n_1 \hat{i} + n_2 \hat{j} + n_3 \hat{k}$ $i = (n_4, \vec{n})$ $U = \begin{pmatrix} n_4 + i n_1 & n_2 + i n_3 \\ -n_2 + i n_3 & n_4 - i n_1 \end{pmatrix}$

Чтобы выполнялось $\hat{R}(n_4) = \hat{I} n_4 + i(\vec{n} \cdot \vec{\sigma}) = \begin{pmatrix} n_4 + i n_3 & n_2 + i n_1 \\ -n_2 + i n_3 & n_4 - i n_3 \end{pmatrix}$

(24) XII $\hat{m} = Z^+ \hat{\sigma}^* Z$, $Z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$, $Z^+ = (z_1^*, z_2^*)$, $m_x + i m_y = Z^+ (\sigma_x + i \sigma_y) Z$

$\sigma_x + i \sigma_y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + i \begin{pmatrix} 0 & +i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$, $m_x + i m_y = 2 (z_1^*, z_2^*) \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = 2 (z_1^*, z_2^*) \begin{pmatrix} z_2 \\ 0 \end{pmatrix}$

Escaneado con CamScanner

$m_x + i m_y = 2 z_1^* z_2$. $m_z = (z_1^*, z_2^*)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}z_1\\ z_2\end{pmatrix} = (z_1^* z_2)\begin{pmatrix}z_1\\ -z_2\end{pmatrix} = |z_1|^2 - |z_2|^2$

Battye-Sutcliffe '1999 берут $U = \begin{pmatrix} z_1 & -\bar{z}_2 \\ z_2 & \bar{z}_1 \end{pmatrix}$ или $Z = (z_1, z_2)$

$U = e^{i\vec{f}\cdot\vec{\sigma}} = \hat{I}\cos f + i(\vec{n}\cdot\vec{\sigma})\sin f$. $U = \hat{I} n_4 + i(\vec{n}\cdot\vec{\sigma})$, $n_4 = \cos f$, $\vec{n} = \vec{r}\sin f$.

Def. $U = \hat{I} n_4 + i(\vec{n}\cdot\vec{\sigma}) = \begin{pmatrix} n_4 + i n_3 & n_2 + i n_1 \\ -n_2 + i n_1 & n_4 - i n_3 \end{pmatrix}$, $n_4 = (\vec{n}, n_4)$ S_4 spheres coord.

тогда z_1 (Bat) $= n_4 + i n_3$, z_2 (Bat) $= -n_2 + i n_1$.

$m_x + i m_y = 2 z_1^* z_2 = 2(n_4 - i n_3)(-n_2 + i n_1) = 2\left[-n_4 n_2 + n_1 n_3 + i(n_2 n_3 + n_1 n_4)\right]$

$m_x = 2(n_1 n_3 - n_2 n_4)$, $m_y = 2(n_2 n_3 + n_1 n_4)$ – Battye'99

Из $R_{ik}(n_i)$ следует $m_x = 2(n_1 n_3 + n_2 n_4)$, $m_y = 2(n_2 n_3 - n_1 n_4)$, согласно [Kosevich]

m_x (Bat), m_y (Bat) неверно. Использовали $\vec{n}' = (1 - 2n^2)\hat{z} + 2\vec{n} n_z + 2n_4(\vec{n}\times\hat{z})$ [Kosevich]

m_z (Bat) $= |z_1|^2 - |z_2|^2 = n_3^2 + n_4^2 - n_1^2 - n_2^2$, m_z (cos) $= 1 - 2n^2 + 2n_3^2$ совп. с m_z (Bat).

Либо у Battye'99 надо $n_4 \to -n_4$, либо $\vec{n} \to -\vec{n}$. $n_4 = \cos\chi$, нужно $\vec{n} \to (-1)\vec{n}$.

Corrected z_1 (Bat) $= n_4 - i n_3$, z_2 (Bat) $= n_2 - i n_1$.

$$\begin{pmatrix} m_z & m_x - i m_y \\ m_x + i m_y & -m_z \end{pmatrix} = U\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}U^+ = \begin{pmatrix} z_1 & -\bar{z}_2 \\ z_2 & \bar{z}_1 \end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix} \bar{z}_1 & \bar{z}_2 \\ -z_2 & z_1 \end{pmatrix} =$$

$U^+ = \begin{pmatrix} \bar{z}_1 & \bar{z}_2 \\ -z_2 & z_1 \end{pmatrix}$

$$= \begin{pmatrix} z_1 & -\bar{z}_2 \\ z_2 & \bar{z}_1 \end{pmatrix}\begin{pmatrix} \bar{z}_1 & \bar{z}_2 \\ z_2 & -z_1 \end{pmatrix} = \begin{pmatrix} |z_1|^2 - |z_2|^2 & 2z_1\bar{z}_2 \\ 2\bar{z}_1 z_2 & |z_2|^2 - |z_1|^2 \end{pmatrix}.$$

$\Rightarrow m_z = |z_1|^2 - |z_2|^2$, $m_x + i m_y = 2\bar{z}_1 z_2$ – верно с $Z = (z_1, z_2)$

$\sim z_1 = n_4 - i n_3$, $z_2 = -n_2 - i n_1$. // Eq. $U = \begin{pmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{pmatrix}$ с $\alpha = n_4 + i n_1$, $\beta = n_2 + i n_3$ неверно.

Верное SU(2)-represent. $U = \begin{pmatrix} z_1 & -\bar{z}_2 \\ z_2 & \bar{z}_1 \end{pmatrix}$ $z_1 = n_4 - i n_3 = e^{-i\varphi}\cos\frac{\theta}{2}$, $z_2 = n_2 - i n_1 = e^{i\varphi}\sin\frac{\theta}{2}$.

Надо $w = \frac{z_2}{z_1}$ как [Gusichaos]. $Z(z_1, z_2)$

$$w = \frac{z_2}{z_1} = \frac{e^{i\varphi}\sin\frac{\theta}{2}}{e^{-i\varphi}\cos\frac{\theta}{2}} = \operatorname{tg}\frac{\theta}{2} e^{i(\varphi + i\varphi)} = \operatorname{tg}\frac{\theta}{2} e^{i\Phi}.$$

Hopf coordinates (wiki): $z_1 = e^{i\varphi}\sin\eta = n_4 + i n_1$, $z_2 = e^{i\xi}\cos\eta = n_2 + i n_3$. не дают верные z_1, z_2.

В [Gus'chaos'23]: $Z = (z_1, z_2)$, $w = \frac{z_2}{z_1}$, $z_1 = x_4 + i x_3$, $z_2 = x_1 + i x_2$, отличается от $z_1 = n_4 - i n_3$, $z_2 = n_2 - i n_1$, комп. сопряжением $+\Phi$ вместо $-\Phi$.

$\Rightarrow$ Бэкер-Хаусдорф-Пауль: $R_{ij} = \frac{1}{2}\operatorname{Tr}\left[\sigma_i U \sigma_j U^+\right]$, проверим это.

$\sigma_i U = \sigma_i\left[\hat{I} n_4 + i(\vec{n}\cdot\vec{\sigma})\right] = \sigma_i n_4 + i\sigma_i\cdot(\vec{n}\cdot\vec{\sigma}) = \sigma_i n_4 + i\left[n_i\hat{I} + i(\hat{x}_i\times\vec{n})\vec{\sigma}\right]$

$\sigma_j U^+ = \sigma_j\left[\hat{I} n_4 - i(\vec{n}\cdot\vec{\sigma})\right] = \sigma_j n_4 - i\left[n_j\hat{I} + i(\hat{x}_j\times\vec{n})\cdot\vec{\sigma}\right]$

$\sigma_i U \sigma_j U^+ = \left[\sigma_i n_4 - (\hat{x}_i\times\vec{n})\vec{\sigma} + i n_i\hat{I}\right]\left[\sigma_j n_4 + (\hat{x}_j\times\vec{n})\vec{\sigma} - i n_j\hat{I}\right] =$

$= \sigma_i\sigma_j n_4^2 + \sigma_i n_4(\hat{x}_j\times\vec{n})\vec{\sigma} - i n_j\sigma_i n_4 - (\hat{x}_i\times\vec{n})\vec{\sigma}\sigma_j n_4 - (\hat{x}_i\times\vec{n})\vec{\sigma}(\hat{x}_j\times\vec{n})\vec{\sigma} +$

$+ (\hat{x}_i\times\vec{n})\vec{\sigma} n_j + i n_4 n_i\sigma_j + i n_i(\hat{x}_j\times\vec{n})\vec{\sigma} + n_i n_j\hat{I}$

$\operatorname{Tr}(\vec{\sigma}) = 0$ $\operatorname{Tr}(\hat{S}_i\hat{S}_k) = \operatorname{Tr}\left(\frac{1}{2}\sigma_i\frac{1}{2}\sigma_k\right) = \frac{1}{4}\operatorname{Tr}(\sigma_i\sigma_k) = \frac{(1)^0}{4}\cdot\frac{1}{2}\delta_{ik}$ $\operatorname{Tr}(\sigma_i\sigma_k) = 2\delta_{ik}$

$\operatorname{Tr}\left[\sigma_i U \sigma_j U^+\right] = n_4^2 2\delta_{ij} + n_4\operatorname{Tr}\left[\sigma_i(\hat{x}_j\times\vec{n})_k\sigma_k\right] -$

$- n_4\operatorname{Tr}\left[(\hat{x}_i\times\vec{n})_k\sigma_k\sigma_j\right] - \operatorname{Tr}\left[(\hat{x}_i\times\vec{n})_k\sigma_k(\hat{x}_j\times\vec{n})_k\sigma_k\right] = 2n_4^2\delta_{ij} +$

$+ n_4 2(\hat{x}_j\times\vec{n})_i - 2n_4(\hat{x}_i\times\vec{n})_j - 2\operatorname{Tr}\left[(\hat{x}_i\times\vec{n})_k(\hat{x}_j\times\vec{n})_k\right]$

$\operatorname{Tr}(\sigma_i U\sigma_j U^+) = 2n_4^2\delta_{ij} + 2n_4(\hat{x}_j\times\vec{n})_i - 2n_4(\hat{x}_i\times\vec{n})_j - 2(\hat{x}_i\times\vec{n})\cdot(\hat{x}_j\times\vec{n})$

$(\hat{x}_i\times\vec{n})(\hat{x}_j\times\vec{n}) = (\hat{x}_i\hat{x}_j)\vec{n}^2 - n_j n_i$, $\operatorname{Tr}[\Gamma_{ij}] = 2n_4^2\delta_{ij} - 2\delta_{ij}\vec{n}^2 + 2n_i n_j +$

$+ 2n_4\left[(\hat{x}_j\times\vec{n})_i - (\hat{x}_i\times\vec{n})_j\right]$. $R_{ij} = n_4^2\delta_{ij} - \vec{n}^2\delta_{ij} + n_i n_j + 2n_4[\dots]$

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$R_{ij} = (1-2\vec{n}^2)\delta_{ij} + n_i n_j + 2n_4 [(\vec{x}_j \times \vec{n})_i - (\vec{x}_i \times \vec{n})_j]$ ↙ 2 чернушенка, $[\dots] = 2\varepsilon_{ijk} n_k$ (-1)

$R_{ij} = (1-2\vec{n}^2)\delta_{ij} + 2n_i n_j - 2\varepsilon_{ijk} n_k n_4$. $R_{ij} = \frac{1}{2} Tr[\sigma_i U \sigma_j U^+]$ - сверка

$SO(3)$ rotation matrix и $SU(2)$ $U = e^{i\frac{\omega}{2}\vec{n}\vec{\sigma}} = \hat{I}\cos\frac{\omega}{2} + i(\vec{n}\vec{\sigma})\sin\frac{\omega}{2}$

$U = U^{1/2}(\omega, \vec{n}) = \hat{I} n_4 + i\vec{n}\cdot\vec{\sigma}$ - rotation matrix, $\frac{\omega}{2} = \chi$, $n_4 = \cos\chi$, $\vec{n} = \vec{u}\sin\chi$

(5/VII) Battye 1999, $U = e^{i\chi \vec{v}\cdot\vec{\sigma}}$, $\chi \to \gamma$, $\vec{v} \to \vec{u}$, $U = e^{i\gamma \vec{u}\cdot\vec{\sigma}}$, $\vec{n} = \vec{u}\sin\gamma$

$\vec{v}\sin\chi = \vec{n}$ в $\hat{R}[n_4]$ параметризации.

Battye: $m_1 = 2(n_1 n_3 - n_2 n_4)$, $m_2 = 2(n_2 n_3 + n_1 n_4)$, $m_3 = 1 -$
(Hopfion $\vec{m}$) $-2(1-v_3^2)\sin^2\chi = 1 - 2(\sin^2\chi - n_3^2)$, $\sin^2\chi = 1 - \cos^2\chi = 1 - n_4^2$

$m_3 = 1 - 2(1 - n_4^2 - n_3^2) = 1 - 2(n_1^2 + n_2^2)$, $\sum n_i^2 = 1$.

Hago y Battye '99 $\vec{n} \to -\vec{n} \Rightarrow$ $m_1 = 2(n_1 n_3 + n_2 n_4)$, $m_2 = 2(n_2 n_3 - n_1 n_4)$, $m_3 = 1 - 2(n_1^2 + n_2^2)$

Тогда $\vec{m}$ (Battye '99) совпадает с $\vec{m}$ (Kosevich '1990), причем только из $\hat{R}(n_4, \vec{n})$.

Вывод: $\vec{m}$ (Battye) $\to$ $\vec{m}$ (Kosevich) с учетом $\vec{n} \to -\vec{n}$.
(Hopfion)

Сначала Battye '99 rotation axis $\vec{v} = \vec{u}$ in spherical $(\vec{r}, \theta, \varphi)$ coord.

$\vec{u}(\theta, \varphi)$, $\vec{u}^2 = 1$, $\vec{u} = \frac{1}{1+|P|^2}(P + \bar{P}, i(\bar{P} - P), |P|^2 - 1)$, $P = P(z)$

$z = e^{i\varphi} tg\frac{\theta}{2}$. Taking $P(z) = z^Q$ - rational function, получим тот

Hopf index $Q_H = Q$. $\vec{u}(P)$ аналогично $\vec{m}(W)$, $W = tg\frac{\Theta}{2} e^{i\Phi}$, $\vec{m}(\Theta, \Phi)$.

$\hat{R}(n_4)$, $n_4 = (\vec{n}, n_4)$, $n_4 = \cos\chi$, $\vec{n} = \vec{u}\sin\chi$, $\vec{u}(\theta, \varphi)$ через $P(z)$, $z = e^{i\varphi} tg\frac{\theta}{2}$.

$P(z)$, например $P(z) = z^Q$, определяет $(\vec{n}, n_4) = n_\alpha$ с индексом Q.

Для (m,n) вихрей $Q_H = m \cdot n$, $e^{i\varphi} \to e^{im\varphi}$ это θ_k, но $\theta \to (m\theta)$ не

инвариант. Инвариант $z^Q = (e^{im\varphi} tg\frac{\theta}{2})^Q = e^{iQm\varphi} tg^Q\frac{\theta}{2}$, $\varphi \to Q\varphi$, $tg\frac{\theta}{2} \to$

Вместо $\varphi \to m\varphi$, $\theta \to m\theta$, Battye '99 offers $\varphi \to Q\varphi$, $tg\frac{\theta}{2} \to (tg\theta/2)^Q$.

(26/VII - 27/VII) Назвали "Hopfion rotation" ms. Introduction + Results.

(28/VII) Sugic et al, Nat. Commun. 2021. Hopf index via $S_3(n_\alpha)$

Suppl. S. Три 4-х векторов $(\vec{a}\times\vec{b}\times\vec{c})\cdot\vec{e}_\alpha = \det(\vec{a},\vec{b},\vec{c},\vec{e}_\alpha) = \varepsilon_{\alpha\beta\gamma\delta} a_\beta b_\gamma c_\delta$

$(\vec{a}\times\vec{b}\times\vec{c})\cdot\vec{n} = (\vec{a}\times\vec{b}\times\vec{c})\sum_\alpha n_\alpha \vec{e}_\alpha = \sum_\alpha \varepsilon_{\alpha\beta\gamma\delta} n_\alpha a_\beta b_\gamma c_\delta$

$\vec{a} \to \frac{\partial\vec{n}}{\partial x_i}$, $\vec{b} \to \frac{\partial\vec{n}}{\partial x_j}$, $\vec{c} \to \frac{\partial\vec{n}}{\partial x_k}$: $\left(\frac{\partial\vec{n}}{\partial x_i}\times\frac{\partial\vec{n}}{\partial x_j}\times\frac{\partial\vec{n}}{\partial x_k}\right)\cdot\vec{n} = \varepsilon_{\alpha\beta\gamma\delta} n_\alpha \frac{\partial n_\beta}{\partial x_i}\frac{\partial n_\gamma}{\partial x_j}\frac{\partial n_\delta}{\partial x_k} \sim q$

$Q_H = \int dV q$, q - Hopf density. Также q дает Q_H [Volovik '1977]

q should be a Jacobian for change from S_3 sphere $n_\alpha = (n_4, \vec{n})$ coordinates to cartesian (or R_3) coordinates.

$\varepsilon_{\alpha\beta\gamma\delta}\frac{\partial n_\beta}{\partial x_i}\frac{\partial n_\gamma}{\partial x_j}\frac{\partial n_\delta}{\partial x_k} n_\alpha = \det\left(\frac{\partial\vec{n}}{\partial x_i}, \frac{\partial\vec{n}}{\partial x_j}, \frac{\partial\vec{n}}{\partial x_k}, \vec{n}\right)$ как называется это Jacobian?

Точнее $J = \varepsilon_{ijk}\det\left(\frac{\partial\vec{n}}{\partial x_i}, \frac{\partial\vec{n}}{\partial x_j}, \frac{\partial\vec{n}}{\partial x_k}, \vec{n}\right)$ есть Jacobian of n_α to $\vec{r}(x_1 x_2 x_3)$ coordinate change?

$Q_H \sim \int dS_3(\dots) = \int d^4\vec{n}(\dots)\,\delta(|n_\alpha| - 1)$ интеграл по гиперсфере S_3 hypersphere.

J is not standard Jacobian, т.к. это $S_3(n_\alpha = 1)$ sphere to $\vec{r}(x_1 x_2 x_3)$ change.

У Sugic '2021 не говорится, что J is Jacobian. Ref. P.R. Jennings, PhD thesis

Jennings: $(\vec{a}\times\vec{b}\times\vec{c})\cdot\vec{d} = \det(\vec{a},\vec{b},\vec{c},\vec{d})$ for 4-vectors $(\vec{a},\vec{b},\vec{c},\vec{d})$. [arXiv 2022]

"Hyperspherical Trigonometry"

J. Ruostekoski, Ph. Rev. A 2004, Bose-Einstein Condensates, $\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix} = \begin{pmatrix}\cos\lambda & -i e^{-i\gamma}\sin\lambda\cos\beta\\ -i e^{i\gamma}\sin\lambda\cos\beta & \cos\lambda\end{pmatrix}\begin{pmatrix}z_1\\ z_2\end{pmatrix}$

$n_\alpha = (n_4, \vec{n})$, $n_4 = \cos\lambda$, $\vec{n} = \sin\lambda(\sin\beta\cos\gamma, \sin\beta\sin\gamma, \cos\beta)$ — S_3

Winding number (Hopf index) $= W = \frac{1}{2\pi^2}\int d^3x \sin^2\lambda\sin\beta\det\left(\frac{\partial\lambda_i}{\partial x_j}\right)$, $\lambda = (\lambda, \beta, \gamma)$

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J. Ruostekoski, JRAnglin PRL'2001, BEC. S_3 sphere $\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$, $|\psi_1|^2+|\psi_2|^2=1$

S_3 (α,β,γ). Вихрь $\gamma(\vec r)=\varphi$, $\beta(\vec r)=\varphi$, $\alpha(\vec r)=\lambda(r)$, $\lambda(0)=0$, $\lambda(\infty)=\pi$

$\vec r(r,\theta,\varphi)$ spherical coord. in R_3. $W=\frac{1}{2\pi^2}\int_{R_3}\sin^2\alpha\sin\beta\det\left(\frac{\partial\alpha_i}{\partial x_j}\right)$

$\det(\dots)$ – Jacobian $x(r,\theta,\varphi)\to\alpha_i(\alpha,\beta,\gamma)$. W – volume of the unit S_3 sphere, regardless the exact form $\alpha_i(x_j)$. Аналог χ – [Volovik], вихрь по $f(r,\theta)$. $(\alpha,\beta,\gamma)\to(\chi,\theta_u,\varphi_u)$

29/XI С п.зр. на Онтологията „Хаос"

$SU(2)$ matrix $\hat R\{n_\mu\}=I n_4+i(\vec n\cdot\vec\sigma)$, $n_\mu(\vec n,n_4)$ quaternion.

$n_4=\cos\omega/2$, $\vec n=\vec u\sin\omega/2$, $\omega=2\chi$ – rotation angle, $\vec u$ – rotation axis, $\vec u^2=1$.

Toroidal representation $z_1=n_4-in_3=e^{-i\eta\beta}\cos(\xi)$, $z_2=e^{i\nu\varphi}\sin(\xi)=n_2-in_1$.

$\vec m(\theta,\varphi)$ $\Theta(\xi)=\Theta(\chi)=2\xi(\chi)$, $\Phi=\nu\varphi+\eta\beta\to w=\operatorname{tg}\frac{\Theta}{2}e^{i\Phi}$, $m_z=\cos\Theta$.

$n_4=\cos(\eta\beta)\cos\xi$, $n_3=\sin(\eta\beta)\cos\xi$, $n_2=\cos(\nu\varphi)\sin\xi$, $n_1=-\sin(\nu\varphi)\sin\xi$

Standard $n_4=\cos\chi$, $\vec n=\sin\chi\,(\sin\theta_u\cos\varphi_u,\sin\theta_u\sin\varphi_u,\cos\theta_u)=\hat\chi\cdot\vec u$.

$\vec u(\theta_u,\varphi_u)$ in 3D space R_3. θ_u,φ_u – spherical angles of vector $\vec u$.

$n_4=\cos\chi=\cos(\eta\beta)\cdot\cos\xi$, $\xi(\chi)$, $\beta=\beta(\theta,\varphi)=\beta(r,\theta)$, $\cos\chi\to\chi=\chi(r,\theta)$.

$n_3=\sin\chi\cos\theta_u=\sin(\eta\beta)\cos\xi=f(r,\theta)$, $\chi=\chi(\theta,\varphi)=\chi(r,\theta)$

$n_1=\sin\chi\sin\varphi_u\cos\theta_u=-\sin(\nu\varphi)\sin\xi$.

$n_4=n_4(\beta,\chi)$, $n_3=n_3(\beta,\chi)$, $n_2=n_2(\varphi,\chi)$, $n_1=n_1(\varphi,\chi)$. Components $n_\mu(n_4,\vec n)$

J. Jäykkä, J. Hietarinta, P. Salo, PRB 77, 094509 (2008)

$\vec m=\langle z|\vec\sigma|z\rangle=z^+\vec\sigma z$. $z=\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$ – spinor. Babaev'2002 $\psi_\mu\sim\chi_\mu$ two 2-component BEC

$\psi_1=\rho_1+i\rho_2=g(\chi)e^{i\beta\eta}$, $\psi_2=\rho_3+i\rho_4=\sqrt{1-g^2}\,e^{i\nu\varphi}$, (χ,β,φ) toroidal coord.

$m_1+im_2=2g\sqrt{1-g^2}\,e^{i(\beta\eta+\nu\varphi)}$, $m_3=2g^2(\chi)-1$.

$m_3=\cos\Theta=2g^2-1$, $g^2(\chi)=\frac12(1+\cos\Theta)=\cos^2\frac{\Theta}{2}$, $g(\chi)=\cos\frac{\Theta}{2}$, $\Theta(\chi)=2\xi(\chi)$.

$\vec\rho=(\rho_1,\rho_2,\rho_3,\rho_4)=\left(g\cos\beta\eta,\ g\sin\beta\eta,\ \sqrt{1-g^2}\cos\nu\varphi,\ \sqrt{1-g^2}\sin\nu\varphi\right)$, $\sqrt{1-g^2}=\sin\frac{\Theta}{2}$.

$n_\mu=(n_4,\vec n)=\left(\cos\eta\beta\cdot\cos\xi,\ \sin\eta\beta\cos\xi,\ \cos\nu\varphi\sin\xi,\ -\sin\nu\varphi\sin\xi\right)$ $g=\cos\frac{\Theta}{2}$.

n_4 n_3 n_2 n_1

→ Т.е. $\rho_\mu(\rho_1,\rho_2,\rho_3,\rho_4)$ – Jäykkä'2008 съвпада с нашия $n_\mu(n_4,\vec n)$, с точ. до знака

Ruostekoski angles $\alpha_i(\alpha,\beta,\gamma)$, (λ,β,γ) → $(\chi,\theta_u,\varphi_u)$ и Hopf index $Q_H=\frac{1}{2\pi^2}\int_{R_3}\sin^2\chi\sin\theta_u\det\left(\frac{\partial\alpha_i}{\partial x_j}\right)$ и чрез отношението $\frac{\partial\chi}{\partial x_i}$, $\frac{\partial\theta_u}{\partial x_i}$, $\frac{\partial\varphi_u}{\partial x_i}$ в (x_1,x_2,x_3) coord. system, toroidal нам/мисир. $\vec u(\theta_u,\varphi_u)$ и $\vec n=\vec u\sin\chi$.

Ако $Q_H=1$, $\vec u=\hat r$ и $\theta_u=\theta$, $\varphi_u=\varphi$ [Volovik 1977].

[Kosevich 1990], $Q_H=\frac{1}{2\pi^2}\int d\chi\,d\Psi\,d\Phi\sin^2\chi\sin\Psi$, $\Psi=\theta_u$, $\Phi=Q_H\varphi_u$ (?!), $Q_H=1$.

$Q_H=1$

Аналогична $g_{\mu\nu}$ с Q_H-равенство: отново $\frac{\partial n_\mu}{\partial x_i}$, чрез които в toroidal coordinates.

30/XI С п. в „Еловски" Wbie „Lidl"

Jacobian $J(\chi,\theta_u,\varphi_u/\chi,\beta,\varphi)=\det\left(\frac{\partial\alpha_i}{\partial x_j}\right)=\begin{pmatrix}\partial\chi/\partial\chi & \partial\chi/\partial\beta & \partial\chi/\partial\varphi\\ \partial\theta_u/\partial\chi & \partial\theta_u/\partial\beta & \partial\theta_u/\partial\varphi\\ \partial\varphi_u/\partial\chi & \partial\varphi_u/\partial\beta & \partial\varphi_u/\partial\varphi\end{pmatrix}$

$\chi=\chi(\chi,\beta)$, $\partial\chi/\partial\varphi=0$, $\theta_u=\theta_u(\chi,\beta)$, $\frac{\partial\theta_u}{\partial\varphi}=0$

$J=\begin{pmatrix}\times&\times&0\\ \times&\times&0\\ \times&\times&\times\end{pmatrix}=\frac{\partial\varphi_u}{\partial\varphi}\det\begin{pmatrix}\partial\chi/\partial\chi & \partial\chi/\partial\beta\\ \partial\theta_u/\partial\chi & \partial\theta_u/\partial\beta\end{pmatrix}=\frac{\partial\varphi_u}{\partial\varphi}\left[\frac{\partial\chi}{\partial\chi}\frac{\partial\theta_u}{\partial\beta}-\frac{\partial\chi}{\partial\beta}\frac{\partial\theta_u}{\partial\chi}\right]$

$\cos\chi=\cos(\eta\beta)\cos\xi(\chi)$ $\frac{\partial}{\partial\beta}[\sin\chi\cos\theta_u]=\cos\chi\frac{\partial\chi}{\partial\beta}\cos\theta_u+\sin\chi(-1)\sin\theta_u\frac{\partial\theta_u}{\partial\beta}=\cos(\eta\beta)(-1)\sin\xi\frac{\partial\xi}{\partial\chi}\cdot\frac{\partial\chi}{\partial\beta}(-\sin\xi)=\eta\sin(\eta\beta)\cos\xi$

$\frac{\partial\chi}{\partial\beta}=\eta\cos(\eta\beta)\cos\xi=\eta\cos\chi$

$[\dots] = \frac{\cos(m\beta)\sin\xi}{\sin\chi}\frac{\partial\xi}{\partial\chi}\frac{\partial u_4}{\partial\beta} - m\frac{\sin(m\beta)\cos\xi}{\sin\chi}\frac{\partial u_4}{\partial\chi}$

$\frac{\partial}{\partial\chi}[\sin\chi\cos\xi u_4] = \cos\chi\frac{\partial\chi}{\partial\chi}\cos\xi u_4 - \sin\chi\sin\xi u_4\frac{\partial\xi u_4}{\partial\chi} = \sin(m\beta)(-1)\sin\xi\frac{\partial\xi}{\partial\chi}$

$\frac{\partial\xi u_4}{\partial\beta}(-1)\sin\chi\sin\xi u_4 = m\cos\chi - \cos\chi\cos\xi u_4\frac{\partial\chi}{\partial\beta}$, $\sin\chi\sin\xi u_4\frac{\partial\xi u_4}{\partial\beta} =$

$= \sin(m\beta)\sin\xi\frac{\partial\xi}{\partial\chi} + \cos\chi\cos\xi u_4\frac{\partial\chi}{\partial\chi}$

$[\dots] = \frac{\cos(m\beta)\sin\xi}{\sin\chi}\frac{\partial\xi}{\partial\chi}\frac{1}{\sin\chi\sin\xi u_4}\left[-m\cos\chi + \cos\chi\cos\xi u_4\frac{\partial\chi}{\partial\beta}\right] - m\frac{\sin(m\beta)\cos\xi}{\sin\chi}\times$

$\times\frac{1}{\sin\chi\sin\xi u_4}\left[\sin(m\beta)\sin\xi\frac{\partial\xi}{\partial\chi} + \cos\chi\cos\xi u_4\frac{\partial\chi}{\partial\chi}\right] = \frac{1}{\sin^2\chi\sin\xi u_4}\left[\cos(m\beta)\sin\xi\frac{\partial\xi}{\partial\chi}\times\right.$

$\left.\times\left\{-m\cos\chi + \cos\chi\cos\xi u_4\frac{\partial\chi}{\partial\beta}\right\} - m\sin(m\beta)\cos\xi\left\{\sin(m\beta)\sin\xi\frac{\partial\xi}{\partial\chi} + \cos\chi\cos\xi u_4\frac{\partial\chi}{\partial\chi}\right\}\right]$

$Q_H = \frac{1}{(2\pi)^2}\int d^3x\,\sin^2\chi\sin\xi u_4\frac{\partial\varphi u}{\partial\varphi}\frac{1}{\sin^2\chi\sin\xi u_4}[\dots]$, $d^3x = dV = \frac{a^3}{\tau^3}\sinh\eta\,d\eta\,d\beta\,d\varphi$, $\tau = \cosh\eta - \cos\beta$.

→ множител. $\frac{1}{\sin\chi\sin\xi u_4}$ – берем, $[\dots]$ ограничено.

можно $\int_0^{2\pi}d\varphi\frac{\partial\varphi u_4}{\partial\varphi} = \varphi u_4(\eta,\beta,\varphi)\Big|_{\varphi=0}^{\varphi=2\pi}$ ≠ 0, будет, $\sim(2\pi n)$, n – azimuthal vorticity

$\varphi u_4(\eta,\beta,\varphi) = n\varphi + f(\eta,\beta)$ — как это ≠ 0?

В (r,ξ,φ) spherical coord. $n_4 = n_4(r,\xi)$, $n_3 = (r,\xi)$, $n_2 = n_2(r,\xi,\varphi)$, $n_1 = n_1(r,\xi,\varphi)$.

В toroidal coord. (η,β,φ) $n_4 = n_4(\beta,\eta)$, $n_3(\beta,\eta)$, $n_2 = n_2(\varphi,\eta)$, $n_1 = n_1(\varphi,\eta)$.

$\frac{\partial n_4}{\partial\varphi} = 0$, $\frac{\partial n_3}{\partial\varphi} = 0$, $\frac{\partial n_2}{\partial\beta} = 0$, $\frac{\partial n_1}{\partial\beta} = 0$.

[Gus'chaos'23]: $Q_H = -\frac{2}{(4\pi)^2}\int d\beta\int d\theta\sin\theta\int d\varphi$, $\gamma = m\beta$, $d\beta = \frac{\partial\beta}{\partial\varphi}d\varphi$, (η,β,φ) toroid. coord.

$\partial\beta/\partial\varphi = n$. $Q_H = (+)\frac{2}{(4\pi)^2}m\int d\beta(-1)\int d(\cos\theta)\int d\varphi\frac{\partial\beta}{\partial\varphi} = +\frac{2m}{(4\pi)^2}\int_{-\pi}^{\pi}d\beta(-1)\int_0^{\eta=\infty}d\eta\,n_3\int d\varphi\,n =$

$= +\frac{2\cdot mn}{(4\pi)^2}\int_{-\pi}^{\pi}d\beta(-1)\int_0^{\infty}d\eta\frac{\partial n_3}{\partial\eta}\int_0^{2\pi}d\varphi = -\frac{2mn}{(4\pi)^2}\int_0^{\infty}d\eta\int_0^{2\pi}d\beta\int_0^{2\pi}d\varphi\frac{\partial n_3}{\partial\eta}^{\eta=0} = mn\rho$.

$\rho = n_3(\eta=0)$.

Написание „Hopfion rotation" article.

(31/XII) Приготовления к встрече New Year.

(1/I) **2025** Написание „Hopfion rotation" article.

(2/I) Отказ to S. Murakami „Chimera" conf. San Sebastian 2025 Sept. Получил link-Overleaf to J. Vélez PhD thesis's к ознак. to comment/edit the thesis's

(3/I) Подсчет IUA „Confiendo" VIrch. 2024 и приобрел 2024. удалил 42€.

(4/I) Оплата „Aranburu" за 2024: 32,50 × 12 = 390,40 €.

(5/I) Hopfion vorticities $m<0$, $n<0$, имеют ли они cusps?

NewMS: $Z_1 = e^{-im\beta}\cos\xi$, $Z_2 = e^{in\varphi}\sin\xi$, $w = \frac{Z_2}{Z_1} = \operatorname{tg}\xi\cdot e^{i(n\varphi+m\beta)}$

$\xi = \xi(\eta)$, $\xi = \Theta/2$, $\vec{m}(\theta,\varphi)$, $\varphi = n\varphi + m\beta$. $m_z = \cos\Theta(\eta) = \frac{1-\operatorname{ch}^{2m}\eta\,\operatorname{th}^{2n}\eta}{1+\operatorname{ch}^{2m}\eta\,\operatorname{th}^{2n}\eta}$ f.

$m_z(\eta, m, n)$, дефекты $m>0$, $n>0$ и неудобно $m\to-|m|<0$, $n\to-|n|<0$.

[Gus'chaos'23]: $Z_1 = -e^{i\beta}\frac{1}{\operatorname{ch}\eta}$, $Z_2 = e^{i\varphi}\operatorname{th}\eta$, $w = \frac{(Z_2)^n}{(Z_1)^m} = \operatorname{ch}^m\eta\,\operatorname{th}^n\eta\,e^{i(m\beta+n\varphi)}$

$Q_H = mn\rho$. $m=0, n=0$ не имеют cusps, т.к. $Q_H = 0$.

a) $m\to-|m|<0$, $n>0$, $w = Z_2^n\frac{1}{Z_1^{-|m|}} = Z_2^n Z_1^{|m|}$, $w = e^{in\varphi}\operatorname{th}^n\eta(-1)^m e^{-im\beta}\frac{1}{\operatorname{ch}^{|m|}\eta}$

$m_z = \frac{1-|w|^2}{1+|w|^2} = \left[1-\operatorname{th}^{2n}\eta\cdot\frac{1}{\operatorname{ch}^{2|m|}\eta}\right]/\left[1+\operatorname{th}^{2n}\eta\frac{1}{\operatorname{ch}^{2|m|}\eta}\right]$, $m_z(\eta\to\infty) = 1$

b) $m>0$, $n\to-|n|<0$, $w = \operatorname{ch}^m\eta\frac{1}{\operatorname{th}^{|n|}\eta}e^{i\varphi}$, $|w|^2 = \operatorname{ch}^{2m}\eta/\operatorname{th}^{2|n|}\eta$, $m_z(\eta\to0) = -1$

это не Hopfion, $m_z(\eta\to\infty) = -1$

c) $m\to-|m|<-M<0$, $n\to-|n|<-D<0$, $w = \frac{1}{\operatorname{ch}^{|m|}\eta\,\operatorname{th}^{|n|}\eta}e^{i(\gamma\beta+\nu\varphi)}$

$w = 1/w'$, $w' = \operatorname{ch}^{\mu}\eta\,\operatorname{th}^{\nu}\eta\,e^{i(\mu\beta+\nu\varphi)}$. $m_x + im_y = 2w/[1+|w|^2]$

Escaneado con CamScanner

Escaneado con CamScanner

(6) $m_x + i m_y = \frac{2(1/w')}{1+|1/w'|^2} = \frac{2|w'|^2 \frac{1}{w'}}{1+|w'|^2} = \frac{2\bar{w}'}{1+|w'|^2}$, $m_z = \frac{1-|1/w'|^2}{1+|1/w'|^2} = \frac{|w'|^2-1}{|w'|^2+1}$

т.е. $w \to 1/w'$ меняет знак m_z $(x>0, y>0)$ и $(m_x+im_y) \to (m_x+im_y) = \frac{2\bar{w}'}{1+|w'|^2}$.

$w' = ch^m y\, th^n y\, e^{i(m\beta+n\varphi)}$, $m_x+im_y \sim \bar{w}' \sim e^{-i(m\beta+n\varphi)} \sim e^{i(m\beta+n\varphi)}$

$m_x+im_y = \frac{2\bar{w}'}{1+|w'|^2} = \frac{2|w'|e^{-i(m\beta+n\varphi)}}{1+|w'|^2} = \frac{2|w'|}{1+|w'|^2} e^{i(m\beta+n\varphi)}$, $\begin{matrix} m=-\mu<0 \\ n=-\nu<0 \end{matrix}$

$|w'| = ch^m y\, th^n y$, $\mu>0, \nu>0$, $\mu=|m|$, $\nu=|n|$.

Вывод: $m=-|m|=-\mu<0$ и $n=-|n|=-\nu<0$ меняет знак m_z $\left(\begin{matrix} m=-\mu \\ n=-\nu \end{matrix}\right) = -m_z \left(\begin{matrix} m=\mu>0 \\ n=\nu>0 \end{matrix}\right)$

$m_x+im_y \left(\begin{matrix} m<0 \\ n<0 \end{matrix}\right) = \frac{2|w'|}{1+|w'|^2} e^{i(m\beta+n\varphi)}$, где $|w'| = ch^{|m|}_y th^{|n|}_y = |w(m>0, n>0)|$

Смена m_z - знак — эквивалентно $p \to -p$, т.к. $m_z = p\frac{1-|w|^2}{1+|w|^2}$

$m_x+im_y \sim e^{i(m\beta+n\varphi)}$, $m<0, n<0$ — это anti-hopfion с $Q_H = mnp \to (-|m|)(-|n|)\times(-p) = |m||n|(-p)$

$\to Q_H(\text{anti-hopfion}) = -|m||n|p$. $Q_H(\text{hopfion}) = mnp$ $(m>0, n>0)$.

$m_x+im_y (m<0, n<0) = \frac{2|w'|}{1+|w'|^2} e^{-i(|m|\beta+|n|\varphi)} = \frac{}{(m_x+im_y)(m>0, n>0)}$, т.е. $\cos\alpha \to (-i)$

$m_x+im_y (m>0, n>0)$. $i \to (-i)$ сопряжение. Смена $m_y \to -m_y$.

MathCad „Hopf_index_cylinder, med" зеркало скирмион antihopfion $\vec{m}(\vec{r})$ в xy plane при $m=-1, n=-1$. Bloch anti-hopf поворот на anti-vortex near $\vec{r}=0$, Neel - anti hopf $(\varphi_0 = \pi/2)$ это Bloch - anti hopf с rotation на $\pi/4$ вокруг оси oz. $(\varphi_0=0)$

(7/2) Убедимся корректно Bloch-anti hopf = Neel anti hopf + $\pi/4$ rotation.

[Gus' magn. 24]: $m_x+im_y = \frac{4as\, e^{i(\varphi+\varphi_0)}}{(a^2+s^2+z^2)^2}\left[2za + i(s^2+z^2-a^2)\right]$

Bloch $m_x+im_y \sim (x+iy)e^{i\varphi_0}\left[2z + i(s^2+z^2-1)\right] = e^{i\varphi_0}\left[2zx - y(s^2+z^2-1) + i2yz + ix(s^2+z^2-1)\right]$, Neel $\varphi_0 = \pm\pi/2$, $e^{\pm i\pi/2} = i(-1)$

Neel $m_x+im_y = 2yz + x(s^2+z^2-1) - 2izx + iy(s^2+z^2-1)$

$z=0$ plane. Bloch hopfion $m_x+im_y = -y(s^2-1) + ix(s^2-1) \sim (-y+ix)$ vortex like.

Neel hopfion $m_x+im_y = x(s^2-1) + iy(s^2-1) \sim (x+iy)$. Antihopfion $m_x \to -m_x$, $m_y \to -m_y$, $i \to (-i)$

Bloch antihopfion $m_x+im_y \sim (-y-ix)$, Neel antihopf $m_x+im_y \sim (x-iy)$

$m_x+im_y \sim (-i)(x-iy)$

Получили Bloch/Neel antihopfions в $\pi/2$ $(-i) = e^{-i\pi/2}$, $\pi/2$ angle rotation.

MathCad Neel - antihopf. строит новелла? Или Hedgehog '99 знак (m_x+im_y) меняется $\to -m$?

Bloch hopfion near $(\vec{r}=0)$ и $z=0$ plane - это vortex

Vortex $m_x = \cos\varphi \cos(\varphi+\pi/2) = -\sin\varphi \sim (-y)$, $m_y = \sin(\varphi+\pi/2) = \cos\varphi \sim x$

CCW $m_x+im_y = (-y+ix)$. По eq. (1) Hedgehog дает CW. vortex

тот Hedgehog '99 дает $m_x+im_y \sim (-y+ix)$ CCW vortex, по MathCad строит как CW.

В [Gus' Magnetism 24] на Eq. 1,2 надо $m_x \to +m_x$, $m_y \to -m_y$ для CW. совпадет с вершин Eq. (2). По это не согласуется новые Bloch/Neel antihopfions в MathCad на $\varphi_0 = \pi/4$.

Hopfion for $Q_H>1$, $m>1, n>1$.

$m_x+im_y = \sin\Theta\, e^{i(n\varphi+m\beta)}$, $\beta = arctg\left[\frac{2az}{s^2+z^2-a^2}\right]$, Центр кривой $z=0$

в плоскости $z=0$ дает hopfion images. т.к. $s=a$ дает 0/0 неопределенность.

$arctg(x+iy) = \frac{i}{2}\ln\left[\frac{1+y-ix}{1-y+ix}\right]$, при $y=0$ $arctg(x) = \frac{i}{2}\ln\left[\frac{1-ix}{1+ix}\right]$

$\exp(im\beta) = \exp\left\{-\frac{m}{2}\ln\left(\frac{1-ix}{1+ix}\right)\right\} = \exp\left[\ln\left(\frac{1-ix}{1+ix}\right)^{-m/2}\right] = \left(\frac{1-ix}{1+ix}\right)^{-m/2}$

$x = \frac{2az}{(s^2+(z^2-a^2)}$ $W\beta(s,z) = \frac{1}{2i}\ln\left[\frac{z+i(s+a)}{z+i(s-a)}\right]$ $m_x+i\beta = \ln\left[\frac{z+i(s+a)}{z+i(s-a)}\right]$

$i m\varphi = m \ln\left[\frac{z+i(\xi+a)}{z+i(\xi-a)}\right] - my$, $e^{im\varphi} = \left[\frac{z+i(\xi+a)}{z+i(\xi-a)}\right]^m e^{-my}$, μεβάλλει

μιγαδικό $e^{im\varphi}$ κατά $e^{i(n\psi+m\varphi)}$. $w_x + i w_y = \sin\theta\, e^{i n\psi}\left[\frac{z+i(\xi+a)}{z+i(\xi-a)}\right]^m e^{-my}$.

$y = y(\xi,z) = \operatorname{arcth}\left[\frac{2a\xi}{\xi^2+z^2+a^2}\right]$, $\operatorname{arcth} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$

$y(\xi,z) = \frac{1}{2}\ln\left[\frac{\xi^2+z^2+a^2+2a\xi}{\xi^2+z^2+a^2-2a\xi}\right] = \frac{1}{2}\ln\left[\frac{z^2+(\xi+a)^2}{z^2+(\xi-a)^2}\right]$. $\theta = \theta(y)$.

$w_x + i w_y \sim e^{in\psi}\left[\frac{z+i(\xi+a)}{z+i(\xi-a)}\right]^m$. Όταν $m=n=1$, $w_x+iw_y \sim e^{i\psi}\frac{z+i(\xi+a)}{z+i(\xi-a)} =$

$= e^{i\psi}\frac{[z+i(\xi+a)][z-i(\xi-a)]}{z^2+(\xi-a)^2} = e^{i\psi}\frac{[z^2+\xi^2-a^2+i(\xi+a)z - i(\xi-a)z]}{z^2+(\xi-a)^2} =$

$= e^{i\psi}\frac{[z^2+\xi^2-a^2+2iaz]}{z^2+(\xi-a)^2}$, $e^{-my} = \left(\frac{1+x}{1-x}\right)^{-m/2} \frac{(z^2+(\xi-a)^2)}{e^{i\psi}[2az+i(\xi^2+z^2-a^2)]}$

Εναλλαγή στην Εξ. (*) για w_x+iw_y της [δικής μας μορφής], παίρνω +.

→ $w_x+iw_y \sim e^{im\varphi}\left[\frac{z+i(\xi+a)}{z+i(\xi-a)}\right]^m$ άρα w_x+iw_y for any m,n ομοπολικούς.

Για δοσμένο κομμάτι τροχιάς μιγαδικού $\beta(t=0)=0$ το xOy μένει, $t=0$.

$\beta(t=a)=0$ παίρνουμε πάλι $\xi \times a$.

(3/1) Πλαν: •1) Theoretical update άρθρο; •2) Συνεργ. Β Ακαν, hotel Ακαν
•3) Murakami conf.; •4) Συναγωνιστ. conference; •5) Volos PhD edit;
•6) RAL-hopfion NLS; •7) OPUS 1st report

(4/1) Ανέφικτα με Macis-OPUS πάρτε + Γραικά χρήματα ΕΜΠ.
Τελειώσαμε RAL-hopfion NLS, κάνετε συνεργ BIO ⇒ Ακαν Αρχ. 1-16.

(10/1) Στερεοτυπία για $Q_H \geq 1$. $\beta = \arctg\left(\frac{2z}{\xi^2+z^2-1}\right)$ & εξ. β'': $\arctg(x) = \frac{i}{2}\ln\left(\frac{1-ix}{x+ix}\right)$

$e^{i\alpha\beta} = \left(\frac{1-ix}{x+ix}\right)^{-m/2}$, $x = \frac{2z}{(\xi^2+z^2-1)}$. $\frac{1+ix}{1-ix} = \frac{(1+ix)^2}{(1+x^2)}$.

$e^{i\alpha\beta} = \left(\frac{1+ix}{1-ix}\right)^{m/2} = \left[\frac{(1+ix)^2}{1+x^2}\right]^{m/2} = \frac{(1+ix)^m}{(1+x^2)^{m/2}}$. Όταν $m=1$ $e^{i\beta} = \frac{1+ix}{1+x^2}$

$e^{i\beta} = \frac{1+i\frac{2z}{(\xi^2+z^2-1)}}{1+\frac{-z^2}{(\xi^2+z^2-1)^2}} = (\xi^2+z^2-1)\frac{(\xi^2+z^2-1)+i2z}{(\xi^2+z^2-1)^2+4z^2}$

$(\xi^2+z^2-1)^2+4z^2 = \xi^4+z^4+1+2\xi^2z^2-2z^2-2\xi^2+4z^2 =$

$= (\xi^2+z^2+1)^2 - 4\xi^2$, Συμπεραίνουμε ευκολονοήτως όταν $z=0$, $\xi=1$. $\sin\theta$ μένει,

$w = \operatorname{ch}^m y \cdot \operatorname{th}^n y\, e^{i(m\varphi+n\alpha)}$, $w_x+iw_y = \frac{2w}{1+|w|^2} = \frac{2\operatorname{ch}^m y \operatorname{th}^n y}{1+\operatorname{ch}^{2m}y \operatorname{th}^{2n}y} e^{i(n\psi+m\varphi)} = \sin\theta e^{i\varphi}$

$\sin\theta = \frac{2\operatorname{ch}^m y \cdot \operatorname{th}^n y}{1+\operatorname{ch}^{2m}y \cdot \operatorname{th}^{2n}y}$. Όταν $m=1, n=1$ $\sin\theta = \frac{2\operatorname{sh} y}{1+\operatorname{sh}^2 y} = \frac{2\operatorname{sh} y}{\operatorname{ch} y} = \frac{1}{\operatorname{ch} y}\operatorname{th} y$

$\operatorname{th} y = \frac{2\xi}{(\xi^2+z^2+1)}$. $\operatorname{th}^2 y + \frac{1}{\operatorname{ch}^2 y} = 1$, $\frac{1}{\operatorname{ch}^2 y} = 1-\operatorname{th}^2 y = 1-\frac{(2\xi)^2}{(\xi^2+z^2+1)^2} = \frac{(\xi^2+z^2+1)^2-4\xi^2}{(\xi^2+z^2+1)^2} =$

$= \frac{(\xi^2+z^2-1)^2+4z^2}{(\xi^2+z^2+1)^2}$. $w_x+iw_y = \frac{4\xi}{(\xi^2+z^2+1)}\frac{\sqrt{(\xi^2+z^2-1)^2+4z^2}}{(\xi^2+z^2+1)} e^{i\varphi}$ $\frac{(\xi^2+z^2-1)\left[(\xi^2+z^2-1)+i2z\right]}{(\xi^2+z^2-1)^2+4z^2}$

w_x+iw_y $(m=1,n=1)$ $= \frac{4\xi e^{i\varphi}}{(\xi^2+z^2+1)^2}\frac{(\xi^2+z^2-1)}{\sqrt{(\xi^2+z^2-1)^2+4z^2}}\left[(\xi^2+z^2-1)+i2z\right]$ απλοποίηση σε [Nietzsche '99]: $w_x+iw_y = \frac{4\xi e^{i\varphi}}{(\xi^2+z^2+1)^2}\left[(\xi^2+z^2-1)+i2z\right]$

Όταν $m=1, n=1$, $w = \frac{2(x+iy)}{[1-z^2+2iz]} = \frac{2\xi e^{i\varphi}}{(1-z^2+2iz)} = 2\xi e^{i\varphi}\frac{(1-z^2-2iz)}{(1-z^2)^2+4z^2}$

$w_x+iw_y = \frac{2w}{1+|w|^2} = 4\xi e^{i\varphi}\frac{(1-z^2-2iz)}{[(1-z^2)^2+4z^2]}\frac{1}{1+\frac{4\xi^2}{(1-z^2)^2+4z^2}} = \frac{4\xi e^{i\varphi}(1-z^2-2iz)}{(1-z^2)^2+4z^2+4\xi^2}$

$= \frac{4\xi e^{i\varphi}(1-z^2-2iz)}{(1+\xi^2+z^2)^2} = (-i)\frac{4\xi e^{i\varphi}\left[2z+i(z^2-1)\right]}{(1+\xi^2+z^2)^2}$, $(-i)e^{i\varphi} = e^{i(\varphi-\pi/2)}$ = σύνθετο [Nietzsche '99]

знаком $-i(r^2-1)$, у нетарных $+i(r^2-1)$, и доп. множитель $e^{-i\pi/2}$.

$e^{-i\pi/2} e^{i(\varphi+\varphi_0)} = e^{i(\varphi+\varphi_0-\pi/2)}$ даст Bloch hopfions при $\varphi_0 = 0, \pi$ Néel при $\varphi_0 = \pm\pi/2$

но $-i(r^2-1)$ разное. Что не так с $e^{im\varphi}$ - разное?

(**) $m_x + im_y = 4\rho e^{i(\varphi+\varphi_0-\pi/2)} \cdot \frac{[2z - i\cdot(r^2-1)]}{(1+r^2)^2}$, $r^2 = \rho^2 + z^2$.

при $\rho=0$, $m_x + im_y = 4\rho e^{i(\varphi+\varphi_0)} \frac{(1-r^2)}{(1+r^2)^2}$, Bloch hopfion CCW ($\varphi_0 = \pi/2$, $\varphi_0 = -\pi/2$)

(*) осталось знаком $-i(r^2-1)$ от [Gus' Magn.] и дает вторую Bloch hopfions ($\varphi_0 = \pm\pi/2$) и Néel hopfions ($\varphi_0 = 0, \pi$). Néel hopfion $\varphi_0 = 0, \pi$ ($m_z > 0, \varphi_0 = 0$; $m_z < 0, \varphi_0 = \pi$; $r^2 < 1$)

$m_x + im_y = 4\rho e^{i(\varphi+\varphi_0)} \cdot \frac{[(a^2-r^2) - 2iz a]}{(a^2+r^2)^2}$, $m=n=1$ hopfion.

[Gus' Chaos'23]. $w = Z_2^n / Z_1^m$, $Z_2 = X_1 + iX_2 = \frac{2}{(1+r^2)}(x+iy)$, $Z_1 = X_4 + iX_3 = \frac{(1-r^2+2iz)}{(1+r^2)}$

$m_x + im_y = \frac{2w}{1+|w|^2}$ for any m,n.

11 - 12/I Написание hopfion solution MS.

13/I 1st report "ORUS" project. Переписка с Murakami, Chimera conf.

14-16/I Написание hopfion solution MS, Figs. Submitted to RRL 01/06/25.

17/I Invited C. Donnelly declined. Answer to Ricardo и предложение to simulate mag. hopfions in cylind. & spherical dots. Comments on Vlad's simulations

18/I

19/I

20 - 21/I Переписка с Владиславом по DW resonance bent stripes.

in bent stripe. He simulated vortex DW resonance.

Reminder Murakami о "Chimera" conf. San Sebastian, Sept. 2025

22/I Переписка с Олегом по DW resonance in bent stripes.

23/I Обзор статьи на мои frustrion и hopfion papers.

24/I Ст. в "Eroski" Wirbel и "Mercadona" Adocin. Skyrmions.

25/I Разбор отзыва Vagson [Bittencourt'25] моей критики on переписки DW mass mixing Slonczewski (q-φ) и 1D Döring (m, q) models.

E-mails Олегу о DW resonance bent stripes и DW single-vortex wires by Tejo

Разбор paper Y. Nakatani JMMM 290-291, 750 (2005) DW stripes.

Both models (q(t)-Döring) и (q-φ Slonczewski) are 1D models, но DW mass определяется только в Döring model из $E_{DW}(0) = \frac{1}{2} m V^2$, где $V \ll V_W$.

A. Thiaville Y. Nakatani DW dynamics in nanowires

Spin Dyn. III - Top. Appl. Phys. 101, 161-205 (2006).

Vortex DW in rectangular stripe: Vortex + two Antivortices at the edges. AV appear local $m_y \neq 0$ и interact with mag. field H_y. Does stripe bending?

$\Lambda = \sqrt{2A/\mu_0 M_s^2}$, $\Lambda = \sqrt{A/2\pi M_s^2} \approx 5$ nm for P_y.

M. Michael 1957, line separating TW и V-DW is $w/L = 60\Lambda^2$ in rect. stripe (width w, thick. L).

M. Kleui et al. Head-to-head DW in mag. rings. TW и V-DW finite curvature

APL 85, 5637 (2004), of the ring $2R = 1.65$ μm characteristic area of

stability of TW. Wide area of TW - metastability as TW-V-DW transition

is of 1st-order

[Thiaville-2006] q-φ equations Slonczewski.

$F_a(\theta,\varphi) = K_0 \sin^2\theta + K_1 \sin^2\theta \sin^2(\varphi)$, θ - polar angle $\vec{m}$ $\angle Oz = 0x$ - length

Hard $F_a = K_0 \sin^2\theta + K_1 \sin^2\theta \sin^2\varphi$ φ in yz - plane transverse to Ox

$E_a = K_0(1-m_x^2) + K_1(m_y^2+m_z^2)\sin^2\varphi = K_0(1-m_x^2) + K_1 m_z^2$ - two axial ani-

(rhombic anisotropy). $\frac{\partial E_a}{\partial \varphi} = \frac{\partial}{\partial \varphi} K_1 \sin^2\theta \sin^2\varphi = K_1 \sin^2\theta \sin 2\varphi$.

Escaneado con CamScanner

1D q-φ model ansatz: $\theta=\theta(x,t)=2\arctg e^{(x-q(t))/\Delta}$, $\varphi(x,t)=\Phi(t)$

Torque $\frac{\alpha}{\Delta}\dot q+\dot\Phi=\gamma H$, $\frac{1}{\Delta}\dot q-\alpha\dot\Phi=\gamma H_K\frac12\sin2\Phi$, $\Delta(\Phi)=\sqrt{\frac{A}{K_0+K_1\sin^2\Phi}}$

$H_K=2K/M_S$, $K=K_1$ - transverse anistropy, $K_1>0$, xOy - easy plane

$\varkappa=K/K_0$, $\Delta(\Phi)=\frac{\Delta_0}{\sqrt{1+\varkappa\sin^2\Phi}}$, $\Delta_0=\sqrt{A/K_0}$; $K_1<0$, easy Oz axis.

$\dot\Phi=-\frac{\alpha}{\Delta}\dot q+\gamma H$; H applied $\parallel Ox$.

$=\gamma H-\alpha\left[\alpha\dot\Phi+\frac12\gamma H_K\sin2\Phi\right]=\gamma H-\alpha^2\dot\Phi-\frac12\alpha\gamma H_K\sin2\Phi$

$\dot\Phi=\frac{\gamma}{1+\alpha^2}\left[H-\frac12\alpha H_K\sin2\Phi\right]$, $H_w=\frac12\alpha H_K$, $\dot\Phi=\frac{\gamma}{1+\alpha^2}\left[H-H_w\sin2\Phi\right]$

При $H<H_w$ есть stationary solution $\Phi(t)=\Phi^*$, $\sin2\Phi^*=H/H_w$

Stationary (steady) DW velocity $\dot q=\left[\frac{\gamma H}{\Delta(\Phi^*)}\right]=\frac{\gamma H}{\alpha}\Delta(\Phi^*)$, $\dot\Phi=0$.

можно ли исходные $\Phi(t)$ из q-φ system eqs of motion?

$\frac{\alpha}{\Delta(\Phi)}\dot q+\dot\Phi=\gamma H$, $\frac{1}{\Delta(\Phi)}\dot q-\alpha\dot\Phi=\frac12\gamma H_K\sin2\Phi$. $\Phi(t)$ исключить. Можно.

При $\Phi\ll1$, $\Delta(\Phi)=\Delta_0+O(\Phi^2)$, $\frac{\alpha}{\Delta_0}\dot q+\dot\Phi=\gamma H$, $\frac{1}{\Delta_0}\dot q-\alpha\dot\Phi=\gamma H_K\Phi$

$\frac{1}{\Delta_0}\dot q-\alpha\left[\gamma H-\frac{\alpha}{\Delta_0}\dot q\right]=\gamma H_K\Phi$. $\frac{d}{dt}(\dots)$ $\frac{1}{\Delta_0}(1+\alpha^2)\ddot q=\gamma H_K\dot\Phi=$

$=\gamma H_K\left[\gamma H-\frac{\alpha}{\Delta_0}\dot q\right]$, $\frac{(1+\alpha^2)}{\Delta_0\gamma^2H_K}\ddot q=H-\frac{\alpha}{\gamma\Delta_0}\dot q$, $m_D=\frac{(1+\alpha^2)2\mu_0M_S}{\gamma^2\Delta_0H_K}$

Döring mass. $m_D\ddot q+\frac{\alpha}{\gamma\Delta_0}\dot q=H$ - oscillator like equation.

т.е. Döring mass m_D можно ввести в 1D q-φ model, если $\Phi(t)\ll1$.

[Rama-Eiroa' 2020] : Döring mass $m_{DW}=\frac{1}{2\pi\gamma^2\Delta_0}$, $\Delta_0^2=\frac{A}{K_2}$, $E_a(\vec m)=K_xm_x^2-K_zm_z^2$

Oz, Oy are transverse к DW вдоль Ox.

[Hubert' 1988], $\gamma_w=\gamma_w^0+\frac12mv^2$, $m(180^\circ DW)=\frac12\frac{\mu_0\gamma_w^0}{A\gamma^2}$, $\gamma_w^0=4\sqrt{AK_u}$

$m(\text{Hubert})=\frac{\mu_0}{2}\frac{4\sqrt{AK_u}}{A\gamma^2}=2\mu_0\frac{1}{\gamma^2\sqrt{A/K_u}}$, $E_a=K_u\sin^2\theta$

K_u uniaxial anis, $K_u>0$ ПЛН xOy

у Thiaville 2006 в m_D входит $\Delta_0^2=\frac{A}{K_0}$ и $H_K=2K/M_S$.

$K_0>0$ easy axis anisotropy $\parallel Ox$, DW вдоль ox. K - transverse anisotropy

$E_a=K_0(1-m_x^2)+Km_z^2$. $[m_{DW}]=[M_{DW}/S]=\left[\frac{1}{\gamma^2\Delta}\right]=\frac{g}{cm}\left[\frac{1}{\Delta}\right]=\frac{g}{cm^2}$

у Thiaville'2006 $m_D=\frac{(1+\alpha^2)2\mu_0M_S}{\gamma^2\Delta_0H_K}$ - вероятно размерность, $[m_D]=\frac{g}{cm^2}\frac{M_0M_S}{[H_K]}$

$\rightarrow m_D(\text{Thiaville})=\frac{2\mu_0}{\gamma^2}\frac{M_S}{\sqrt{A/K_0}H_K}$ отличается от $m(\text{Hub})=\frac{2\mu_0}{\gamma^2}\frac{1}{\sqrt{A/K_u}}$

Hubert "m" из $E_{DW}=E_{DW}(\gamma)=E_0+\frac12mv^2$.

Т.е. m_D(Thiaville) из $\Phi(t)\ll1$ ($\Phi=\Phi(t)$) q-φ model отличается от m_{DW} из $\Phi(t)=\Phi_0=\text{const}$. Неверно. DW mass

(27/I) A. Stankiewicz, JE) TMag. 30, 878 (1994) : Döring mass $m_0=\frac{1}{2\pi\gamma^2\sqrt{A/F}}$

$E=\frac12d(\Phi\vec m)^2-W$, $d=2A/M_S^2$. $w=\frac12\beta\sin^2\theta\left(1+\frac{1}{q}\sin^2\varphi\right)$, $W_m=2\pi(\vec m\cdot\vec n)^2\frac{M_S^2}{N}$ $\vec n\parallel Ox$

W. Jantz JMMM 28, 285 (1982), Döring mass $m_D=\frac{1}{2\pi\gamma^2}\frac{1}{\sqrt{A/K}}$.

J. Miltat, P. Wigen JAP 52, 2344 (1981) : Döring mass $m_0=\frac{1}{2\pi\gamma^2\Delta}$

$\Delta=\sqrt{A/K}$, $W_a=K\sin^2\theta$, θ от оси z = ось

L. Thomas ... S.S.P. Parkin, Nature 443, 197 (2006) DW resonance, TW, V-DW confinement potential due to stripe bending ($R_c=3\mu m$) и artificial pinning potential $E_p(q)=Vq_0(q/q_0)^2\sim q^2$. Track navigation TW-DW resonance ($f_0\approx2$ GHz) и V-DW resonance ($f_0\approx0.5$ GHz)! By current exat

a) No bending, $\psi(t) = \Phi(t) \ll 1$, res. frequency of DW $\omega^2 = \frac{\gamma^2 H_k \Delta}{M_s q_0}$

Oscillator model $\omega^2 = \varkappa / m_D$. $E_p = V q_0 (q/q_0)^2 = \frac{1}{2}\varkappa q^2$.

$\varkappa = 2V/q_0$. $\omega^2 = \frac{\varkappa}{m_D} = \gamma^2 \frac{V H_k \Delta}{M_s q_0} = \gamma^2 \frac{H_k \Delta}{M_s} \frac{\varkappa}{2}$, откуда

$\frac{1}{m_D} = \gamma^2 \frac{H_k \Delta}{2 M_s}$, $m_D = \frac{1}{\gamma^2} \frac{2M_s}{\Delta \cdot H_k}$ Сравнил с m_D (Thiav) $= \frac{(1+\alpha^2)}{\gamma^2} \frac{2\mu_0 M_s}{\Delta_0 H_k}$

и совпадает с m_D (Thiav '06) при $\alpha \ll 1$, $M_s \to \mu_0 M_s$. Thiaville also assumed $\Phi(t) \ll 1$ to get „Döring" DW mass.

b) No pinning, $V = 0$, $\frac{\partial E}{\partial q} = -2 H_y M_s \sin(q/R_c)$, small q · $q/R_c \ll 1$.

$\frac{\partial E}{\partial q} = -2 H_y M_s \frac{q}{R_c}$. Система ур-ий для $q(t)$, $\psi(t) = \Phi(t)$ при $\alpha \ll 1$, $\psi \ll 1$, $j = 0$

$\dot{q} = \frac{1}{2}\gamma \Delta H_k \cdot 2\psi$, $\psi = -\frac{\gamma}{2M_s}\frac{\partial E}{\partial q} = -\frac{\gamma}{2M_s}\left[-2H_y M_s \frac{q}{R_c}\right] = \gamma H_y q \frac{1}{R_c}$.

$\ddot{q} = \gamma \Delta H_k \dot{\psi} = \gamma \Delta H_k \gamma H_y q \frac{1}{R_c} = (\text{знак}_y\text{-}") = -\gamma^2 \Delta H_k H_y \frac{1}{R_c} q = -\omega_0^2 q$

DW resonance при $\omega_0^2 = \gamma^2 \frac{\Delta H_k H_y}{R_c}$, $f_0^2 = \left(\frac{\omega_0}{2\pi}\right)^2 = \gamma^2 \frac{\Delta H_k H_y}{4\pi^2 R_c}$ –

nanowire Saitoh'04 res. frequency $f_e^2 = \frac{Q_M H}{4\pi^2 m_0 R}$, $Q_M = 2\mu_0 M_s$, отсюда $m_D = 2\mu_0 M_s / \gamma^2 \Delta H_k$

H_k (Thomas) - shape anisotropy field. $H_k \simeq 1800$ Oe для Py. H_k (Thiav) $= 2K_1/M_s$

H_k (Thiaville) is transverse anisotropy field in bulk sample $E_a = K_0(1-m_x^2) + K_1 m_z^2$, $Oz \perp$ plane, Ox - DW axis

[Shryer '1974]: $E_a = 2\pi M_x^2 - \frac{K}{M_0^2} M_z^2$, Ox-DW axis, $m_z = \pm 1$ - vacuum (domains). (1D model)

[Rana-Ebrahim '20]: $E_a = 2\pi M_s^2 m_x^2 - K_2 m_z^2$ как у Shryer'74, the same xyz system

E_m (thin stripe) $= 2\pi M_s^2 m_z^2 = K_1 m_z^2$ как в 1D model Thiaville'06

$E_a = K_0(1-m_x^2) + K_1 m_z^2$, a K_0 (OH $\parallel Ox$ DW axis) неизвестна Thiaville 2006.

(stripe magnetized)

→ Вывод: из Thomas model получаем осциллятор model с Döring mass (no bending) и Saitoh model с $f_0^2 \sim H_y/R$ (no pinning, только bending).

28/I

m_D (Thiaville) $= \frac{1}{\gamma^2} \frac{2M_s}{\Delta_0 H_k}$ и m_D (Thomas) $= \frac{1}{\gamma^2} \frac{2M_s}{\Delta H_k}$ получаем в пределе $\mu_0 = 1$, $\Delta_0^2 = A/K_0$

предела $\psi = \Phi(t) \ll 1$ из q-φ Slonc. equations. m_D (Thiav) $\gg m$ (Döring) $= \frac{1}{2\pi\gamma^2\Delta}$

→ Для Thiaville m_D из q-φ model назван „Döring mass".

Schlömann APL'1974: $E_a = \int dx \left[2\pi M_s^2 m_x^2 + K(m_x^2 + m_y^2)\right] \to \varepsilon_a = 2\pi M_s^2 m_x^2 - K m_z^2$,

по совпадает с [Shryer'74] и [Rana, Ebrahim'2020]. ОЛН $K = K_z$ - vacuum $m_z = \pm 1$,

из разложения $E_{DW}(v)$ - Schlomann'74 ($K_x = 2\pi M_s^2$, $K_x = K_z$) $Ox \perp$ DW plane,

получим $E_{DW}(v) = E_0 + \frac{1}{2} m_{DW} v^2$, (Döring mass $m_{DW} = \frac{1}{2\pi\gamma^2\Delta_0}$, $\Delta_0^2 = \frac{A}{K} = \frac{A}{K_e}$.

См. $E_{DW}(v)$ 22/I 2019.

→ предел $\Phi(t) \ll 1$ из q-φ model и $E_{DW}(v)$ дают разные m_{DW}.

m_{DW} ($\Phi \ll 1$) $\gg m_{DW}$ (Döring) $= \frac{1}{2\pi\gamma^2\Delta_0}$.

В соотношениях [Rana-Ebrahim'20] $E_{DW} = E_0 + \frac{1}{2} m_{DW} v^2$, $m_{DW} = \frac{E_0}{c^2} = \frac{1}{\gamma^2 \Delta_0} \frac{M_s^2}{K_x}$ Rana-Ebrahim'20

$E_0 = 4\sqrt{A K_z}$, $c^2 = \left(\frac{2\gamma}{M_s}\right)^2 A K_x$, $E_a = K_x m_x^2 - K_z m_z^2$, где $K_x = 2\pi M_s^2$, $m_{DW} = \frac{1}{2\pi\gamma^2\Delta_0}$

$\Delta_0^2 = A/K_z$. m_D (Thiav) (при $\mu_0 \to 1$) $= \frac{(1+\alpha^2)}{\gamma^2 \Delta_0} \frac{2M_s}{H_k}$, $K = \frac{1}{2} M_s H_k$ дает

m_D (Thiav) $= \frac{(1+\alpha^2)}{\gamma^2 \Delta_0} \frac{M_s^2}{K_1}$, $\Delta_0 = A/K_0$, $E_a = K_0(1-m_x^2) + K_1 m_z^2$. Формально

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m_D (Thiav'06) совпадает с m_{DW} (Rama-Eiras), но Δ_0^2 (Rama) $= \frac{A}{K_2}$, Δ_0^{12} (Thiav) $= \frac{A}{K_0}$

m_D (Thiav) $\sim \frac{1}{K_1}$, m_{DW} (Rama) $\sim 1/K_x$. Anisotropy constants в версии

m_{DW} (Rama) - Döring mass по Ослеб. Anisotropy constants (Thiaville'06)

E_a (Rama) $= K_x m_x^2 + K_2 m_z^2$, E_a (Thiaville'06) $= K_0 (1 - m_x^2) + K_1 m_z^2$.

$Ox \perp$ DW plane. K_0 ($OA \parallel Ox$), но K_2 ($OA \parallel Oz$); K_1 (easy plane xOy), но K_x (easy plane yOz) = DW plane. Thiaville с анизотропу E_a (m) написал и $\mu_0 = 4\pi$ для

Ослеб, что m_D (Thiav) >> m_{DW} (Rama) - Döring.

Döring DW mass получается в 1D model при $\Phi(x,t) = \text{const}$ [Schlömann'71, Schlyer2'74] as $E_{DW} = E_0 + \frac{1}{2} m_D v^2$, где в 1D q-φ model при $\Phi(x,t) = \Phi(t) \ll 1$, используя Φ из Φ, q-ур 1D и получая $m_D \dot{q} = (..)$ eq. как у [Thiaville'06] и [Thomas'06]?

Применимость 1D q-φ model: TW-wire. BP DW wire

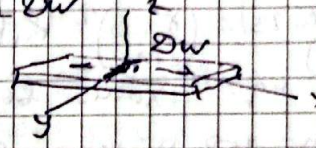
TW-stripe (yes in small fields), V-DW-stripe (no).

Bulk (wire) $E_a = K_x m_x^2 - K_2 m_z^2$ $Ox \perp DW$

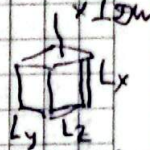
Thin nanostripe $E_a = K_2 m_z^2 - K_x m_x^2$

$K_x > 0$, $K_2 > 0$

E-минимум DW-stripe Онсаге, Внадисчав.



$K_2 \simeq 2\pi M_s^2$



wire $L_y \approx L_z \times 10$ nm.

(29/I) Домашен обзор DW dyn. Thiaville'2006.

(30/I) Запознаване с характеристики на IVA, Constanto" 2024.

(31/I) С Л. „Aldi", „Mercadona" Андрей.

Cash

Cash остатък: ужкар 2800€ + кафинет (1000 + 550 + 1500) 3050€ = 5.850€

приход: 700€ Роспен (23/Σ 24).

Расчет. остатък: 6220 (окт. 14/IX 24) + 700€ (23/Σ) − 550 − 500 (сотки) = 5870€

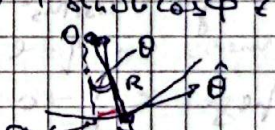
(1/II) Проверка DW res. frequency TW nanostripe [Bittencourt'23].

q-φ model 1D. $\vec{m} = \sin\Theta \sin\Phi \hat{i} + \cos\Theta \hat{\theta} + \sin\Theta \cos\Phi \hat{z}$

$\hat{\theta} = \hat{z}$, polar angle (unit vector)

$\hat{i}$ $\hat{z}$ - stripe normal, $\hat{r}$ - in plane radius vector





Ansatz $\Theta(\theta) = 2\arctg \exp\left[\frac{s-q}{\lambda}\right]$, $s = R\theta$, $\varkappa = \frac{1}{R}$

$\dot{q} = \frac{\gamma}{2M_s} \frac{\partial E}{\partial \Phi} + \alpha \dot{\Phi} \lambda$, $\dot{\Phi} = -\frac{\gamma}{2M_s} \frac{\partial E}{\partial q} - \frac{\alpha}{\lambda} \dot{q}$ $E = E(q, \Phi)$ energy density $\left(\frac{erg}{cm^2}\right)$

λ - DW width

В центре DW $s = q$, $\Theta(s=q) = \pi/2$ и $\vec{m}(s=q) = \sin\Phi \hat{i} + \cos\Phi \hat{z}$, где TW

видно что TW $m_z = \langle m_y \rangle_V$ должно < 1. Недостаток 1D q-φ model.

Считаем $\varkappa\lambda \ll 1$, $\lambda/R \ll 1$, тогда [Bittencourt'23] удерживая const(q, Φ) имеем:

$E_{ex} = -2\pi \varkappa A \sin\Phi - A\varkappa^2 \lambda (1 + \cos^2\Phi) \approx -2\pi\varkappa A \sin\Phi$

$E_d = 4\pi\lambda M_s^2 (N_x \sin^2\Phi + N_z \cos^2\Phi)$, хотя мы знаем уже N_θ, т.к. $N_x \ll 1$

$E_Z = M_s H [-\pi\lambda \sin\Phi + \varkappa q^2]$. дает линейную ступ.

$E(q,\Phi) = E_{ex} + E_d + E_Z = -2\pi\varkappa A \sin\Phi + 4\pi\lambda M_s^2 (N_x \sin^2\Phi + N_z \cos^2\Phi) + M_s H [-\pi\lambda \sin\Phi + \varkappa q^2]$

$\partial E/\partial\Phi = -2\pi\varkappa A \cos\Phi + 4\pi\lambda M_s^2 (-N) \sin\Phi \cos\Phi - \pi\lambda M_s H \cos\Phi$, $N = (N_z - N_x)$

$\partial E/\partial q = 2 M_s H q \varkappa$. Хотим УД при $q \ll 1$, $(\Phi - \pi/2) = -\varepsilon \ll 1$, $\varepsilon = \pi/2 - \Phi$

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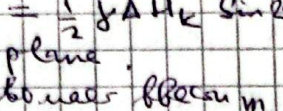
Пока $d=0$. $\dot{q} = \frac{\gamma}{2M_s}\frac{\partial E}{\partial \Phi} = \frac{\gamma}{2M_s}\left[-2\pi\varkappa A\cos\Phi - 4\pi\lambda N M_s^2 \sin 2\Phi - \pi\lambda M_s H\cos\Phi\right]$

$\dot{\Phi} = -\frac{\gamma}{2M_s}\frac{\partial E}{\partial q} = -\frac{\gamma}{2M_s}\cdot 2M_s H q\varkappa$. $\cos\Phi = \cos\left(\frac{\pi}{2}-\varepsilon\right) = \sin\varepsilon \approx \varepsilon \ll 1$.

$\sin 2\Phi = 2\sin\Phi\cos\Phi = 2\varepsilon\cdot\sin\left(\frac{\pi}{2}-\varepsilon\right) = 2\varepsilon$.

[Thiaville] Eq. (25): $\dot{\Phi} = \gamma H$, $\dot{q} = \frac{1}{2}\gamma\Delta H_k \sin 2\Phi$

Φ - magnetization angle in xOy plane.



У Thiaville'ов $\Phi(x,t) = \Phi(t) \ll 1$ подобно нашему ввести m_{DW}.

[Bittencourt'23]: $\dot{\Phi} = -\gamma H q\varkappa$, $\dot{q} = -\frac{\gamma}{2M_s}\left[2\pi\varkappa A\varepsilon + 4\pi\lambda N M_s^2\cdot 2\varepsilon + \pi\lambda M_s H\varepsilon\right]$

$\varepsilon = \frac{\pi}{2} - \Phi$. $\dot{\varepsilon} = \gamma H q\varkappa$, $\dot{q} = -\frac{\gamma}{2M_s}\left[2\pi\varkappa A + 8\pi\lambda N M_s^2 + \pi\lambda M_s H\right]\varepsilon$

$\left[2\pi\varkappa A + 8\pi\lambda N M_s^2 + \pi\lambda M_s H\right] = 2\pi M_s^2\left[2\pi\varkappa\frac{A}{2\pi M_s^2} + 4\pi\lambda N + \frac{\pi\lambda M_s H}{2\pi M_s^2}\right] =$

$= \left| A/(2\pi M_s^2) = l_e^2,\ \frac{\pi\lambda M_s H}{2\pi M_s^2} = \frac{\lambda}{2}\left(\frac{M_s}{H}\right)^{-1} = \frac{\lambda}{2}\frac{H}{M_s} = \frac{1}{2}\lambda h,\ H/M_s = h \right| =$

$= 2\pi M_s^2\left[2\pi\varkappa l_e^2 + 4\pi\lambda N + \frac{1}{2}\lambda h\right]$. $\dot{q} = -\frac{K_\varepsilon}{2M_s}2\pi M_s^2\left[2\pi\varkappa l_e^2 + 4\lambda N + \frac{1}{2}\lambda h\right] =$

$= -\gamma\pi M_s\left[2\pi\varkappa l_e^2 + 4\lambda N + \frac{1}{2}\lambda h\right]\varepsilon$, $\dot{\varepsilon} = \gamma M_s h q\varkappa$.

$\ddot{q} = -\pi\gamma M_s\left[2\pi\varkappa l_e^2 + 4\lambda N + \frac{1}{2}\lambda h\right]\dot{\varepsilon} = -\pi\gamma M_s\left[2\pi\varkappa l_e^2 + 4\lambda N + \frac{1}{2}\lambda h\right]\gamma M_s h\varkappa q$

$\ddot{q} = -\omega_0^2 q$, $\omega_0^2 = \pi(\gamma M_s)^2 h\varkappa\left[2\pi\varkappa l_e^2 + 4\lambda N + \frac{1}{2}\lambda h\right]$ - res. frequency.

ω_0^2 (Bittencourt'23) $= \left(\frac{\gamma}{2M_s}\right)^2 K_1 K_2 = \left(\frac{\gamma}{2M_s}\right)^2 (2M_s H\varkappa)\left[2\pi A\varkappa + 8\pi\lambda M_s^2 + \pi\lambda M_s H\right] -$

$(\varkappa h\lambda \ll 1)$

- верное eq. Хотим ввести m_D. $m_D\ddot{q} = -\omega_0^2 m_D q$. $H = H_y$ здесь.

Формально это можно, если принять $m_D\ddot{q} = F_q = -K_1 q$, $\left(\omega_0^1\right)^2_{\text{Döring}} = \frac{K_1}{m_D}$

и Ошибка $\omega_0(q-e) = \omega_0^i$ (Döring). На уД $m_D\ddot{q} = -K_1 q$ не существует, т.к. работает $q-\varphi$ model и m_D(Döring) определяется не этим уД, а из $E_{DW}(v) = E_{DW}(0) + \frac{1}{2}m_D v^2$. $E_{DW}(v)$ из Schöbinger'ом.

Формально можно $m_D = \frac{1}{K_2}\left(\frac{2M_s}{\gamma}\right)^2$ и $\omega_0^2 = \frac{K_1}{m_D}$. Но тогда разве $\omega_0^2 = \frac{K_1}{m_D}$?

$$m_D = \left(\frac{2M_s}{\gamma}\right)^2\frac{1}{K_2} = \left(\frac{2M_s}{\gamma}\right)^2\frac{1}{\left[2\pi A\varkappa + 8\pi\lambda M_s^2 N + \pi M_s\lambda H\right]}$$

$$m_D = \frac{4M_s^2}{\gamma^2}\frac{1}{2\pi M_s^2}\frac{1}{\left[2\pi\varkappa l_e^2 + 4\lambda N + \frac{1}{2}\lambda h\right]} = \frac{2}{\pi\gamma^2}\frac{1}{\left[2\pi\varkappa l_e^2 + 4\lambda N + \frac{1}{2}\lambda h\right]}$$

- DW mass in $q-\varphi$ Bittencourt'23 model.

Если предел $\varkappa = \frac{1}{R} \to 0$ straight stripe и $h \to 0$, то $m_D = \frac{1}{2\pi\gamma^2}\frac{1}{\lambda N}$, $N = N_z - N_x$.

$m_D = \frac{1}{2\pi\gamma^2}\frac{1}{\lambda N}$ похожа на Döring mass [Rama-Eiras'20] $m_{DW} = \frac{1}{\gamma^2\Delta_0}\frac{M_s^2}{K_x}$, $\Delta_0^2 = \frac{A}{K}$

где $E_a = K_x m_x^2 - K_z m_z^2$, $Ox \perp$ DW, вакуум $m_z(\pm\infty) = \pm 1$ вдоль оси Oz.

m_{DW} (Döring) можно вычислить из $E_{DW}(v)$ полагая $\vec{m}(x) \to \vec{m}(x-vt)$ где v - DW in stripe. Travel wave ansatz $\vec{m}(x-vt)$ описывает отношение к DW resonance на имеет.

(2.11) Anyway, N_z [Bittencourt'23] надо заменить на $N_\theta = N_y$, т.к. $N_z \to 0$ for long stripe. Хотя $N_z \ll 1$, но задает vacuum state $\vec{m}(x \to \pm\infty) = \pm\hat{x}$, $m_x(\infty) = \pm 1$.

$m_\theta = \cos\Omega$ и $E_d(\theta) = 2\pi M_s^2 (m_\theta)^2 = 2\pi M_s^2\cos^2\Omega$ (dOR $E_d(\theta)$ от Φ, q не зависит для ansatz $\Omega(\theta) = 2\arctg\exp\left[\frac{s-q}{\lambda}\right]$. (dOR $\sin^2\Omega = 2\lambda$

No не даст вклад в $\dot{q}, \dot{\Phi}$ уД.



M. Pulton, K.D. Leaver, Phys Lett. 5, 15 (1963) xOy - DW plane

$\to H_x$ | Dyn. M_z demag. field, Dwe-rigid DW motion

$H_e = -NM_z$. $\omega_L = \gamma H_e$. $\partial\Phi/\partial t = -\frac{1}{v}\frac{\partial\Phi}{\partial x}$, $E_d = -\frac{1}{2}\int dV\vec{M}\cdot\vec{H_e} =$

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$= \frac{1}{2N}\int dW H_e^2 = \frac{1}{2N}\frac{\pi^2}{\gamma^2}\int_{-\infty}^{\infty}\left(\frac{\partial P}{\partial x}\right)^2$, если $H_e = \frac{1}{\gamma}\frac{\partial P}{\partial t}$ $E_k = \frac{1}{2}m_b v^2 = \frac{1}{2}m_b \frac{1}{N\gamma^2}\int \dot{P}^2$

Для $P(t) = 2\arctg e^{t/\Delta}$, отсюда $m_b = \frac{2}{N\gamma^2\Delta}$, Δ - DW width.

При $N = N_0 = 4\pi$, $m_b = \frac{1}{2\pi\gamma^2\Delta}$ Döring mass.

• N. Smith IEEE TM 27, 1229 (1991) $m_b = \frac{F_{DW}}{2\pi\gamma^2\Delta}$, $F_{DW} \propto 2E_{ex}$.

Основание: для thin film/stripe $\Theta \propto \pi/2$, $m_z = 0$ (0) для $\Theta \perp$ film plane

т.е. $P(t)$ dynamics при $\Theta(t) \approx \pi/2$, как у Thiaville'06.

• S.W. Yuan, H.N. Bertram PRB 44, 12395 (1991)

Ref. [Döring'48] $m_D = \frac{1}{\gamma^2}\frac{\sigma_{ex}}{NA}$, σ_{ex} - exch. energy density. For bulk

of thin film $m_b = \frac{1}{4\pi\gamma^2}\frac{\sigma_w}{2A}$, $\sigma_{ex} = \frac{1}{2}\sigma_w$, $N = 4\pi$, $\sigma_w = 4\sqrt{AK}$

m_b [Yuan'91] $= \frac{1}{2\pi\gamma^2 A}$, $\Delta = \sqrt{A/K}$ DW width, $m_b = \frac{1}{2\pi\gamma^2\Delta}$ - Döring 1D mass.

→ Вывод: если $\vec{m}(\Theta, \Phi)$ dynamics is reduced к one-angle dynamics

$\Theta \propto \pi/2$, $\Phi(t)$ у [Walker'63] [Schryer'74] [Thiaville'06] $\Phi(t) \ll 1$, $\Theta(t)$], то

можно дать эта ур-ия (или $q(t)$) вблизи DW mass - Döring mass.

Döring mass из $E_{DW}(v) = E_{DW}(0) + \frac{1}{2}m_b v^2$ ($x = 0$).

У [Bittencourt'23] $\Phi \propto \pi/2$ и dynamics $R(t)$ in stripe plane - можно m_b

вывести, хотя и не доказано что $\varkappa = 1/R \neq 0$, bent stripe.

DW resonance

[Bittencourt'23] дает DW width $\lambda(\Phi)$, Eq.(9)

Но DW dynamics при $\Phi \propto \pi/2$. $\lambda(\pi/2) = \sqrt{\frac{A}{2\pi M_s^2 N_z - \frac{\pi}{2}H_y M_s}}$.

$N_z \sim \frac{L_z}{L_0}$, $\frac{L_z}{L_0}$, L_0 - stripe length, L_z = w stripe width, L_z stripe thickness. $N_z \ll 1$.

Поэтому $\lambda(\pi/2) \to \infty$ даже при $H_y = 0$ и в 1D q-e model нет

тв DW. Solitons $K_1(\lambda) \to \infty$, $K_2(\lambda) \to -\infty$ при $\Phi \to \pi/2$.

DW resonance frequency $\omega_0 = \frac{1}{2M_s}\sqrt{K_1 K_2} \sim \lambda^n \to \infty$ при $\Phi = \pi/2$.

Нашли DW width in soft stripes in 2D-model.

(3/II) Проверим Zeeman energy в поле H_y - by [Bittencourt'23], Eq.(3)

поле $\Phi = \pi/2$, DW rotation in $x'oy'$ - $\hat{r}\hat{\Theta}$ plane - stripe plane.

$\vec{m} = \sin\Theta\sin\Phi\,\hat{r} + \cos\Theta\,\hat{\Theta} + \sin\Theta\cos\Phi\,\hat{z} \to \vec{m}(\Phi = \frac{\pi}{2}) = \sin\Theta\,\hat{r} + \cos\Theta\,\hat{\Theta}$

$\vec{H} = H_y\hat{y} = H_y(\sin\varphi\,\hat{\varphi} + \cos\varphi\,\hat{e})$, $(\rho, \varphi, z) \to (r, \Theta, z)$. $\vec{H} = H_y\hat{y} = H_y(\hat{r}\sin\Theta + \hat{\Theta}\cos\Theta)$

$\vec{M}\cdot\vec{H} = (\sin\Theta\,\hat{r} + \cos\Theta\,\hat{\Theta})M_s H_y(\hat{r}\sin\Theta + \hat{\Theta}\cos\Theta) = M_s H_y(\sin\Theta\sin\Theta + \cos\Theta\cos\Theta)$

Cyl. (ρ, φ, z)

Ansatz $\Theta(\theta) = \arctg\exp\left[\frac{\theta - \theta_0}{\varkappa\lambda}\right]$, $\varkappa = \frac{1}{R}$ - curvature.

$E_z = -\int dW \vec{M}\vec{H}$, $dW = \rho\,d\rho\,d\varphi\,dz \to r\,dr\,d\theta\,dz$, 1D model $r = R$

E_z $\hat{r}/\hat{\theta}$ Element giving DW (1D) string $dL = R\,d\theta$

$E_z = -\int R\,d\theta\,\vec{M}\vec{H} = -M_s H_y R\int d\theta\,[\sin\Theta\sin\theta + \cos\Theta\cos\theta]$, θ counts от $-\hat{y}$

$= -\int_{-\psi/2}^{\psi/2} d\theta\,[\sin\Theta(\theta)\sin\theta + \cos\Theta(\theta)\cos\theta]$, $\theta_1 = -\psi/2$, $\theta_2 = \psi/2$

$\tg\frac{\Theta}{2} = e^{\pm(\theta - \theta_0)/\varkappa\lambda}$

- domain wall $m_\theta(\theta) = m_{y'}(x')$. Но $\left(\frac{\theta - \theta_0}{\varkappa\lambda}\right) \to +\infty$ при $\theta \to \infty$

в области $\left(\frac{x - x_0}{\Delta}\right) \to \infty$ даже infinite stripe с 1D-тв DW, где $\int dW(-\infty \to \infty)$

$\max\left(\frac{\theta - \theta_0}{\varkappa\lambda}\right) = \frac{\psi/2 - \theta_0}{\varkappa\lambda} = \frac{\pi/2 - \theta_0}{\varkappa\lambda}$ при half-ring $\psi = \pi$

$\min\left(\frac{\theta - \theta_0}{\varkappa\lambda}\right) = -\frac{\psi/2 - \theta_0}{\varkappa\lambda} = -\frac{(\psi/2 + \theta_0)}{\varkappa\lambda} = -\frac{(\pi/2 + \theta_0)}{\varkappa\lambda}$

Escaneado con CamScanner

$\sin\Omega = \frac{2\,\mathrm{tg}\,\Omega/2}{(1+\mathrm{tg}^2\Omega/2)}$ $\cos\Omega = \frac{1-\mathrm{tg}^2\Omega/2}{(1+\mathrm{tg}^2\Omega/2)} = m_\theta(\theta)$

Note: в эксперименте работа $w_0^2 + w$-DW 23/XI 2024 а не интегрируется по θ, о взял $\theta = \frac{x}{R} \ll 1$ и получил $E_2(q) \sim x^2 \sim q^2$ potent. well.

Длина string-дуги $L = R\psi \to \pi R$, $\max\left(\frac{\theta-\theta_0}{\varkappa\lambda}\right) = \frac{\pi}{2}\frac{R}{\lambda}$

Если принять $\frac{1}{\varkappa\lambda} \gg 1$, $\varkappa\lambda = \frac{\lambda}{R} \ll 1$, то все Ok. Но можно ли?

$\lambda(\pi/2) = \sqrt{\frac{A/2\pi M_s^2}{N_2}} = \frac{l_e}{\sqrt{N_2}}$, $l_e^2 = A/2\pi M_s^2 \sim 1$ nm.

$\varkappa\lambda = \frac{1}{R}\frac{l_e}{\sqrt{N_2}}$ малое. Найти размер. фактор rectang. stripe $\parallel$ границе.

Gus JMMM '2011: $N_{yy} = -\frac{4P}{w}\ln\frac{P}{2} = -\frac{4}{4}\frac{L_z}{L}\ln\left(\frac{L_z}{w}\right)$ при $\frac{L_z}{w} \ll 1$, $L \sim w$, L - stripe length

$(\varkappa\lambda)^2 = \frac{l_e^2}{R^2}\frac{1}{N_2} = \frac{l_e^2}{R^2}\left(-\frac{1}{4}\right)\frac{L}{L_z}\ln^{-1}\left(\frac{L_z}{w}\right) = \left(-\frac{1}{4}\right)l_e^2\frac{\pi}{RL_z}\cdot\frac{1}{\ln\left(\frac{L_z}{w}\right)}$.

Если stripe thickness $L_z \ll w$ (stable +w DW).

$(\varkappa\lambda)^2 = \frac{\pi}{4}\frac{l_e^2}{RL_z}\frac{1}{\ln\left(\frac{w}{L_z}\right)}$, $\frac{w}{L_z} \sim 10$, $(\varkappa\lambda)^2 = \frac{\pi}{4\ln 10}\frac{l_e^2}{RL_z} \approx \frac{\pi}{8}\left(\frac{l_e}{R}\right)$ при $L_z \approx l_e$

$(\varkappa\lambda) \sim \sqrt{\frac{l_e}{R}}$, вполне $(\varkappa\lambda) \ll 1$. при $R \approx 10\,\mu$m, $l_e \approx 5$ nm.

Проверить N_{yy} (Aharoni при $w \ll L$), может $(\varkappa\lambda)$ тогда иначе?

[Aharoni JAP' 1998]: N_{xx}, N_{yy}, N_{zz} rectangular dot for arbitrary a, b, c.

$\mathrm{tg}\frac{\Omega}{2} = e^x$, $x = \left(\frac{\theta-\theta_0}{\varkappa\lambda}\right)$. $\sin\Omega = \frac{2e^x}{1+e^{2x}} = \frac{1}{\mathrm{ch}\,x}$, $\cos\Omega = \frac{1-e^{2x}}{1+e^{2x}} = \mathrm{th}\,x$ (± 1)

$J(\psi) = \int_{-\psi/2}^{\psi/2} d\theta\,[\sin\Omega(\theta)\sin\theta + \cos\Omega(\theta)\cos\theta] = J_1 + J_2$.

$J_1 = \int d\theta \sin\theta \underbrace{\sin\Omega(\theta)}$, $J_2 = \int d\theta \cos\theta \underbrace{\cos\Omega(\theta)}$

$J_1 = \int d\theta \sin\theta \frac{1}{\mathrm{ch}\left[\frac{\theta-\theta_0}{\varkappa\lambda}\right]}$, $J_2 = \int d\theta \cos\theta\,\mathrm{th}\left[\frac{\theta-\theta_0}{\varkappa\lambda}\right]$, $J_n = J_n(\psi, \varkappa\lambda, \theta_0)$

$\theta_0 = q/R \ll 1$

при $\psi = \pi$, $\varkappa\lambda = \mu$, $\psi \to \pi$. $\theta_0 = \varkappa$

$J_1(\mu,\theta_0) = \int_{-\pi/2}^{\pi/2} d\theta \frac{\sin\theta}{\mathrm{ch}\left[\frac{\theta-\theta_0}{\mu}\right]}$, $J_2(\mu,\theta_0) = \int_{-\pi/2}^{\pi/2} d\theta \cos\theta\,\mathrm{th}\left[\frac{\theta-\theta_0}{\mu}\right]$

$J(\mu,\theta_0) = J_1(\mu,\theta_0) + J_2(\mu,\theta_0)$

File zeeman_energy_stripe.mcd. $J(\mu,\theta_0) = F(\theta_0) = F\left(\frac{q}{R}\right)$ is linear function of $\theta_0 = q/R$. A rge potential well, $\sim q^2$ - term?

$F(\theta_0)$ be ще range close, rem $\theta_0 = q/R$, при $q/R \geq 0{,}07$.

$J(\mu, q/R)$ - range close rem (q/R) при $\mu < 1$, $\mu = 0{,}005;\ 0{,}01$.

$J(q/R)$ похожа на $a_1(q/R) + a_3(q/R)^3$ - cubical, при $\mu \ll 1$.

Предел $\mu \to 0$ $\frac{1}{\mathrm{ch}\left(\frac{\theta-\theta_0}{\mu}\right)}$ резкий пик при $\theta = \theta_0$.

$J_1(\mu\to 0, \theta_0) = \int d\theta \sin\theta \frac{1}{\mathrm{ch}\left[\frac{\theta-\theta_0}{\mu}\right]} \approx \int_{-\pi/2}^{\pi/2} \frac{d\theta}{\mathrm{ch}\left[\frac{\theta-\theta_0}{\mu}\right]} \sin\theta_0 =$

$= \left|\frac{\theta-\theta_0}{\mu} = x,\ dx = \frac{1}{\mu}d\theta\right| = \sin\theta_0 \int_{(-\frac{\pi}{2}-\theta_0)/\mu}^{(\pi/2-\theta_0)/\mu} \frac{\mu\,dx}{\mathrm{ch}\,x} =$

$= \mu\sin\theta_0 \int_{x_1}^{x_2} \frac{dx}{\mathrm{ch}\,x} = \mu\sin\theta_0\left[2\,\mathrm{arctg}\,e^x + C\right]_{x_1}^{x_2} = \mu\sin\theta_0 2\left[\mathrm{arctg}\,e^{x_2} - \mathrm{arctg}\,e^{x_1}\right] =$

$= \left|x_2 = \frac{1}{\mu}(\pi/2 - \theta_0) \to \infty,\ x_1 = -\frac{1}{\mu}(\pi/2 + \theta_0)\right| = 2\mu\sin\theta_0\left[\mathrm{arctg}\,e^{x_2} - \mathrm{arctg}\,e^{x_1}\right]$

$\mathrm{arctg}(e^{x_2}) = \frac{\pi}{2} - e^{-x_2}$, $\mathrm{arctg}\,e^{x_1} = \mathrm{arctg}\,e^{-\frac{1}{\mu}(\pi/2+\theta_0)} \approx e^{-\frac{1}{\mu}(\pi/2+\theta_0)} \underset{\mu\to 0}{\to} 0$

$J_1(\mu\to 0, \theta_0) = 2\mu\sin\theta_0\left[\frac{\pi}{2} + e^{-x_2} - e^{x_1}\right] \approx \pi\mu\sin\theta_0$, $\mu \to 0$.

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$J_2(\mu\to 0,\theta_0) = \int_{\theta_1}^{\theta_2} d\theta \cos\theta \, th\left(\frac{\theta-\theta_0}{\mu}\right) = \left| th\left(\frac{\theta-\theta_0}{\mu}\right) = \begin{cases} +1, & \theta>\theta_0 \\ 0, & \theta<\theta_0 \end{cases} \right| =$

$= \int_{\theta_0}^{\theta_2} d\theta \cos\theta = -\sin\theta \Big|_{\theta_0}^{\theta_2} = \sin\theta_0 - \sin\theta_2 = \sin\theta_0 - \sin\frac{\pi}{2} = \sin\theta_0 - 1$

$J(\mu\to 0,\theta_0) = J_1 + J_2 = \pi\mu\sin\theta_0 + \sin\theta_0 - 1, \quad \sin\theta_0 = \sin\frac{\varrho}{R} \simeq \frac{\varrho}{R} = x\varrho$

$J = \pi\mu x\varrho + x\varrho - 1 = \left|\mu = x\lambda\right| = \pi x^2\lambda\varrho + x\varrho - 1$

$E_z(\mu\to 0) = -M_s H_y R\, J(\mu\to 0) = -M_s H_y \left[\pi x\lambda\varrho + \varrho - R\right], \quad \Big|_{\Phi=\pi/2}^{\mu\to 0}$

[Bittencourt '23] даёт $E_z = M_s H_y\left[-\pi\lambda + \left(1+\frac{\pi}{2}x\lambda\right)x\varrho^2\right]$, $\mu\to 0, \Phi=\pi/2$

мое $\mu\to 0$ интегрирование недопустимо.

Надо найти $J(\mu,\theta_0)$ even function of $\theta_0 = \varrho/R$ (?). Нет.

(4/II) $J_1(\mu,\theta_0) = \int_{\theta_1}^{\theta_2} \frac{d\theta\sin\theta}{ch\left[\mu^{-1}(\theta-\theta_0)\right]}$, $J_1(-\theta_0) = \int_{\theta_1}^{\theta_2} \frac{d\theta\sin\theta}{ch\mu^{-1}(\theta+\theta_0)} = \left|\theta = -\varphi\right| =$

$= \int_{-\theta_1}^{-\theta_2} \frac{d\varphi\sin\varphi}{ch\mu^{-1}(-\varphi+\theta_0)} = \int_{-\theta_1}^{-\theta_2} d\varphi \frac{\sin\varphi}{ch\mu^{-1}(\varphi-\theta_0)} = \left|\begin{matrix}\theta_1 = -\pi/2 \\ \theta_2 = \pi/2\end{matrix}\right| =$

$= \int_{\pi/2}^{-\pi/2} \frac{d\varphi\sin\varphi}{ch\mu^{-1}(\varphi-\theta_0)} = -\int_{-\pi/2}^{\pi/2} \frac{d\varphi\sin\varphi}{ch\mu^{-1}(\varphi-\theta_0)}$, $J_1(\theta_0) = \int_{-\frac{\pi}{2}}^{\pi/2} \frac{d\theta\sin\theta}{ch\mu^{-1}(\theta-\theta_0)}$

$J_1(-\theta_0) = -J_1(\theta_0)$.

$J_2(\theta_0) = \int_{\theta_1}^{\theta_2} \cos\theta\, th\mu^{-1}(\theta-\theta_0)\, d\theta$; $J_2(-\theta_0) = \int_{\theta_1}^{\theta_2} d\theta\cos\theta\, th\mu^{-1}(\theta+\theta_0) =$

$= \left|\theta = -\varphi\right| = -\int d\varphi \cos\varphi\, th\mu^{-1}(-\varphi+\theta_0) = +\int_{-\theta_1}^{-\theta_2} d\varphi\cos\varphi\, th\mu^{-1}(\varphi-\theta_0) =$

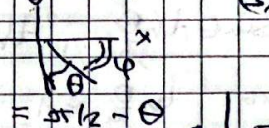
$= \int_{\pi/2}^{-\pi/2} d\varphi\cos\varphi\, th\mu^{-1}(\varphi-\theta_0) = -\int_{-\pi/2}^{\pi/2} d(\dots)^{-\theta_1}_{-\theta_2} = -J_2(\theta_0)$.

$J_1(\theta_0=0) = \int_{-\pi/2}^{\pi/2} d\theta \frac{\sin\theta}{ch\mu^{-1}\theta} = 0^{-\pi/2}$, $J_2(\theta_0=0) = \int_{-\pi/2}^{\pi/2} d\theta \frac{\cos\theta}{th\mu^{-1}\theta} = 0$.

$E_z(\theta_0) = -M_s H_y R\, J(\theta_0)$, $J(\theta_0) = J_1(\theta_0) + J_2(\theta_0)$, $E_z(\theta_0=0) \sim J(\theta_0=0) = 0$?

$\Omega(\theta) = 2\,\mathrm{arctg}\exp\left[\mu^{-1}(\theta-\theta_0)\right]$. $\vec{H} = H_y\hat{y} = H_y\left[\hat{r}\sin\theta + \hat{\theta}\cos\theta\right]$ неверно, (3/II)

т.к. θ отсчитывается от оси $-\hat{y}$.

$\theta + \varphi = \pi/2$  где cylindr. coord

$\vec{H} = H_y\hat{y}' = H_y\left[\sin\varphi\hat{\rho} + \cos\varphi\hat{\varphi}\right] = \left|\begin{matrix}\varphi = \pi/2 - \theta \\ \hat{\rho}\to\hat{r},\ \hat{\varphi}\to\hat{\theta}\end{matrix}\right| = H_y\left[\sin\left(\frac{\pi}{2}-\theta\right)\hat{r} + \right.$

$\left. + \cos\left(\pi/2-\theta\right)\hat{\theta}\right] = H_y\left[\cos\theta\hat{r} + \sin\theta\hat{\theta}\right]$, $\vec{H}(\theta=0) = H_y\hat{r}$, $\vec{H}(\theta=\frac{\pi}{2}) = H_y\hat{\theta}$

– это верно при $\mu, H_y > 0$.  $\theta=0$, $\hat{r} = -\hat{y}$, $\vec{H}(\theta=0) = H\hat{r} = -H\hat{y}$, $H_y = -H$

$\theta = \frac{\pi}{2}$. $\hat{\theta} = +\hat{y}$, $\vec{H}(\theta=\frac{\pi}{2}) = H\hat{\theta} = H\hat{y}$

Надо $\vec{H}(\theta=\pi/2) = -H\hat{\theta} = -H\hat{y}$, $H>0$.

$\vec{H} = H\left[\cos\theta\hat{r} - \sin\theta\hat{\theta}\right]$, $H>0$ как у [Bittencourt '23] $\vec{H} = (0, -H, 0)$, $H_y = -H$

$\vec{M}\cdot\vec{H} = M_s\left(\sin\Omega\sin\Phi\hat{r} + \cos\Omega\hat{\theta} + \sin\Omega\cos\Phi\hat{z}\right)\cdot H\left(\cos\theta\hat{r} - \sin\theta\hat{\theta}\right) =$ (θ отсчит. от $-\hat{y}$)

$= M_s H\left(\sin\Omega\sin\Phi\cos\theta - \cos\Omega\sin\theta\right)$

$E_z = -R\int d\theta\, \vec{M}\cdot\vec{H} = -M_s HR\int_{\theta_1}^{\theta_2} d\theta\left[\sin\Omega\cos\theta\sin\Phi - \cos\Omega\sin\theta\right]$

Def. $J_1(\theta_0) = \int_{\theta_1}^{\theta_2} d\theta\sin\Omega\cos\theta$, $J_2(\theta_0) = \int_{\theta_1}^{\theta_2} d\theta\cos\Omega\sin\theta$.

$E_z(\theta_0) = -M_s HR\left[\sin\Phi\, J_1(\theta_0) - J_2(\theta_0)\right]_{\theta_1}^{\theta_2}$, $J_1(\theta_0) = \int_{\theta_1}^{\theta_2} d\theta \frac{\cos\theta}{ch\mu^{-1}(\theta-\theta_0)}$

$J_2(\theta_0) = \int_{\theta_1}^{\theta_2} d\theta\sin\theta\, th\mu^{-1}(\theta-\theta_0)$. Evident, $J_1(\theta_0), J_2(\theta_0)$ are even functions of θ_0.

Зеемана ан. [Bittencourt '23] $E_z = M_s HR\pi\left[\lambda(\mu\mu^2 - \pi)\sin\Phi + \left(1+\frac{\pi}{2}\mu\sin\Phi\right)x\varrho^2\right]$, $\theta_0 = \varrho/R$

$E_z(Bit) = M_s HR\left[\mu(\mu\mu^2 - \pi)\sin\Phi + \left(1+\pi/2\cdot\mu\sin\Phi\right)\theta_0^2\right]$, $\theta_0 = x\varrho$, $\mu = x\lambda$

Escaneado con CamScanner

$\mu/\mu_0 = 0$ $\Phi = \pi/2$ $\delta\Phi$ $\vec{m}$ $\quad m_z = \sin\Omega = \frac{1}{ch\mu^{-1}\Theta}$ (Θ) $\quad m_z > 0$ $\quad m_z \| (-\hat{y})$ $\quad m_z \| \vec{H}$, $\vec{H} = -H\hat{y}$

$m_\Theta = \cos\Omega = th\mu^{-1}\Theta$ $\quad m_\Theta(\Theta = \pi/2) = th\frac{\pi}{2\mu} <$

$E_z(\theta_0) = -\int R d\theta \vec{M}\cdot\vec{H} = -R\int d\theta M_s m_z(-\hat{y})\cdot H(-\hat{y}) = -M_s HR\int_{\theta_1}^{\theta_2} d\theta \frac{1}{ch\frac{\theta}{\mu}} =$

$= \left| \frac{\theta}{\mu} = x,\ x_{1,2} = \frac{\theta_{1,2}}{\mu} = \pm\frac{\pi}{2\mu} \right| = -M_s HR\mu\int_{x_1}^{x_2} dx \frac{1}{ch x} = -M_s HR\mu\left[2\,\mathrm{arctg}\, e^x\Big|_{x_1}^{x_2}\right] =$

$= -M_s HR\mu\cdot 2\left[\mathrm{arctg}\, e^{\pi/2\mu} - \mathrm{arctg}\, e^{-\pi/2\mu}\right] = -M_s HR\cdot 2\mu\left[\frac{\pi}{2} - e^{-\pi/2\mu} - e^{-\pi/2\mu}\right]$

$\propto \left|\mu << 1\right| \simeq -M_s HR\cdot\mu\pi = \left|\mu = \pi\lambda = \frac{\lambda}{R}\right| = -M_s H\lambda\pi$ Совпадает с Bitter

$E_z = -M_s H\lambda\pi$.

$J_1(\theta_0 \to 0) = \int_{\theta_1}^{\theta_2} d\theta \frac{\cos\theta}{ch\frac{\theta}{\mu}} = \left|\frac{\theta}{\mu} = x\right| = \mu\int_{x_1}^{x_2} dx \frac{\cos(\mu x)}{ch x}$, $\mu x_2 = \frac{\pi}{2}$, $\mu x_1 = -\pi/2$

$J_2(\theta_0 \to 0) = \int_{\theta_1}^{\theta_2} d\theta \sin\theta\, th\frac{\theta}{\mu} = \mu\int_{x_1}^{x_2} dx \sin(\mu x)\, th x$, но (μx) нельзя разлагать

$x_2 = \pi/2\mu \gg 1$, $x_1 = -\pi/2\mu$. но $\sin\mu x$ нельзя разлагать

Но если разложить $\cos\mu x \approx 1 + O(\mu^2)$, $\sin\mu x = \mu x$, то

$J_1(\theta_0 = 0) = \mu\int_{x_1}^{x_2} dx \frac{1}{ch x} = \pi\mu$, $J_2 \sim \mu^2$ пренебрежем.

$E_z(\theta_0 = 0) = -M_s HR\left[J_1(\theta_0 = 0) - J_2(\theta_0 = 0)\right] = -M_s HR\pi\mu = -M_s H\pi\lambda$,

из $\mu << 1$ разложения J_1, J_2, $\cos\mu x$, $\sin\mu x$, которое запрещено.

$E_z(\theta_0 = 0) = -M_s HR\int_{\theta_1}^{\theta_2} d\theta\, m_x(\theta) = -M_s HR\int \frac{d\theta}{ch\frac{\theta}{\mu}} = -M_s HR\cdot 2\mu\left[\frac{\pi}{2} - 2e^{-\pi/2\mu}\right] =$

$= -M_s RH\cdot\mu\pi = -M_s H\lambda\pi$ верно в рамках the DW 1D model.

$A_1(\mu) = \int_{-\frac{\pi}{2\mu}}^{+\pi/2\mu} dx \frac{\cos\mu x}{ch x} = 2\int_0^{\pi/2\mu} dx \frac{\cos\mu x}{ch x}$, $A_2(\mu) = \int_{-\frac{\pi}{2\mu}}^{\pi/2\mu} dx \sin\mu x\, th x = 2\int_0^{\pi/2\mu} dx\, th x\cdot\sin\mu x$

$A_1'(\mu) = 2\int_0^{\infty} dx \frac{\cos\mu x}{ch x} = 2\frac{\pi}{2}\cdot\frac{1}{ch\left(\frac{\mu\pi}{2}\right)} \approx \pi$ при $\mu << 1$ [Prud. P. 3.981].

$A_2'(\mu) = 2\int_0^{\infty} dx\, th x\cdot\sin\mu x$, хуже при $x \to \infty$, где $A_1(\mu)$, $th x \xrightarrow[x\to\infty]{} 1$

Видимо $A_2'(\mu)$ расходится. Надо $A_2(\mu) = 2\int_0^{\pi/2\mu} dx\, th x \sin\mu x$ расход.

$J_2(\theta_0 = 0) = \int_{-\pi/2}^{\pi/2} d\theta \sin\theta\, th\frac{\theta}{\mu}$, $th\frac{\theta}{\mu} \approx 1 - 2e^{-2\theta/\mu}$ при $\mu \to 0$, но так плохо при $\theta \approx 0$.

Но если принять $th\frac{\theta}{\mu} \approx 1$, то $J_2(\theta_0 = 0) = \int_{-\pi/2}^{\pi/2} d\theta \sin\theta = -\cos\theta\Big|_{-\pi/2}^{\pi/2} = 0$.

$E_z(\theta_0 = 0) = -M_s HR\left[J_1(\theta_0 = 0) - J_2(\theta_0)\right] = -M_s HR\left[\pi\mu - 0\right] = -M_s H\lambda\pi$.

Верное $E_z(\theta_0 = 0) = -M_s H\lambda\pi$ вычисл. напрямую из $m_z = -m_y = \frac{1}{ch\frac{\theta}{\mu}}$.

Проблема с $I_2(\mu, \theta_0)$. Может MCAD неверно I_2 считает? Да, неверно.

$J_2(\mu, \theta_0) = \int_{\theta_1}^{\theta_2} d\theta \sin\theta\, th\mu^{-1}(\theta - \theta_0)$, $th\left(\frac{\theta - \theta_0}{\mu}\right) = \begin{cases} +1, & \theta > \theta_0 \\ -1, & \theta < \theta_0 \end{cases}$ при $\mu \to 0$.

$J_2(\mu \to 0, \theta_0) = \int_{-\pi/2}^{\theta_0} d\theta \sin\theta\,(-1) + \int_{\theta_0}^{\pi/2} d\theta \sin\theta\,(+1) = \cos\theta\Big|_{-\pi/2}^{\theta_0} -$

$-\cos\theta\Big|_{\theta_0}^{\pi/2} = \cos\theta_0 - \cos(-\frac{\pi}{2}) - \left[\cos\frac{\pi}{2} - \cos\theta_0\right] = 2\cos\theta_0$, ну-ну.

$J_2(\mu \to 0, \theta_0) = 2\cos\theta_0$ сошлась с MCAD расчетом, где $J_2(\mu = 0, \theta_0 = 0) = 2$.

$E_z(\text{Bitter}) = M_s HR\left[-\pi\mu\sin\Phi + \left(1 + \frac{\pi}{2}\mu\sin\Phi\right)\theta_0^2\right]$

$E_z(\text{нас}) = -M_s HR\left[\sin\Phi\cdot J_1(\theta_0) - J_2(\theta_0)\right]$, $\mu << 1$.

Сейчас $J_1(\theta_0) = \pi\mu - \frac{\pi}{2}\mu\theta_0^2$, $J_2(\theta_0) = +\theta_0^2 \Rightarrow +2\cos\theta_0 = (+2 - \theta_0^2)$

$J_2(\mu \to 0, \theta_0) = 2\cos\theta_0 = \left[1 - \frac{\theta_0^2}{2}\right]2 = 2 - \theta_0^2$. $J_1(\mu\theta_0) = \pi\mu\cos\theta_0$ при $\mu << 1$

$J_2(\mu << 1, \theta_0) \approx 2\cos\theta_0$ MCad подтверждает approx. MCad подтверждает 2cy approx.

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y [Bilitewski'23] убеждаемся $J_2(\mu=0,\theta_0) = \text{Const} + \theta_0^2$, что даст $+q^2$-член в DW energy.
Ясно $J_2(\mu=0,\theta_0) = 2\cos\theta_0 = 2 - \theta_0^2$, что даст $(-q^2)$-член в $E_2(q)$.
$J_2 \gg J_1$, т.к. $J_{\mu\nu}$ — ферромагнетик.

$J_2 = \int_0^{\theta_2} d\theta \cos\Omega \sin\theta$, [Bilitewski'23] даёт $\frac{\partial J_2}{\partial \Omega} = -\int d\theta \sin\Omega \frac{d\Omega}{d\theta} \sin\theta$

$\Omega(q) = \Omega(\theta) = 2\arctg\exp\left[\frac{1}{\mu}(\theta-\theta_0)\right]$, $\frac{\partial}{\partial\Omega} = \pi \frac{\partial}{\partial\theta_0}$, $\theta_0 = \pi q$.

$\frac{\partial J_2}{\partial\theta_0} = -\int d\theta \sin\Omega \frac{\partial\Omega}{\partial\theta_0}\sin\theta$, $\frac{\partial\Omega}{\partial\theta_0} = -\frac{\partial\Omega}{\partial\theta} = -\frac{\partial}{\partial\theta}\left[2\arctg\exp\left(\frac{\theta-\theta_0}{\mu}\right)\right] =$

$= -\frac{\partial\Omega}{\partial\theta}\arctg(e^x)$, $x = \mu^{-1}(\theta-\theta_0)$, $\frac{\partial}{\partial\theta} = \frac{1}{\mu}\frac{\partial}{\partial x}$

$\frac{\partial\Omega}{\partial x} = \frac{\partial}{\partial x}2\arctg(e^x) = \frac{1}{1+e^{2x}}e^x = \frac{1}{2}\frac{1}{ch x} = \frac{1}{2}\sin\Omega$, $\frac{\partial\Omega}{\partial\theta_0} = -\frac{\partial\Omega}{\partial\theta} = -\frac{1}{\mu}\frac{\partial\Omega}{\partial x}$

$\frac{\partial J_2}{\partial\theta_0} = -\int d\theta \sin\Omega\left(-\frac{1}{\mu}\frac{\partial\Omega}{\partial x}\right)\sin\theta = -\int d\theta \sin\Omega\left(-\frac{1}{\mu}\right)\frac{1}{2}\sin\Omega\sin\theta =$

$= \frac{1}{2\mu}\int_{-\pi/2}^{\pi/2} d\theta \sin^2\Omega \sin\theta$, $\sin^2\Omega = \frac{1}{ch^2\left(\frac{\theta-\theta_0}{\mu}\right)}$ резкий макс при $\theta=\theta_0$

$\frac{\partial J_2}{\partial\theta_0} \propto \frac{1}{2\mu}\sin\theta_0 \int d\theta \sin^2\Omega$ + обычная замена. $\int d\theta \sin^2\Omega = 2\mu$ (?), если

Sign $J_2 \sim (\pm1)\theta_0^2$ (сделать от расчёта по координатам. $x_{1,2} = \frac{1}{\mu}\left(\pm\frac{\pi}{2} - \theta\right)$

$\frac{\partial J_2}{\partial\theta_0} = \frac{1}{2\mu}\sin\theta_0 \cdot 2\mu = 2\sin\theta_0$, $J_2(\theta_0) = -2\cos\theta_0 + C \approx \frac{1}{2}\left[4 + \frac{1}{2}\theta_0^2\right] + C$ (?)

$J_2(\theta_0) = \frac{1}{2}\theta_0^2 + C_1$ согласно [Bilitewski'23], где $J_2(\theta_0) = +\theta_0^2 + C_1$.

Откуда получается в прошлом $J_2(\theta_0) = C - \theta_0^2$ и $J_2(\theta_0) = C_1 + \theta_0^2$ [Bilitewski'23]?

(5/II) E-mails to Murakami, Alice. Skype SMS — Nora, Augustin. Home — Good.

(6/II) Имеем $J_2(\theta_0) = \int_{\theta_1}^{\theta_2} d\theta \sin\theta \cdot th\left(\frac{\theta-\theta_0}{\mu}\right)$, $\theta_{1,2} = \pm\pi/2$ при $\mu \ll 1$, $\theta_0 \ll 1$.

$\frac{\partial J_2}{\partial\theta_0} = \int_{\theta_1}^{\theta_2} d\theta \sin\theta \frac{\partial}{\partial\theta_0} th\mu^{-1}(\theta-\theta_0) = \int d\theta \sin\theta \frac{1}{ch^2\mu^{-1}(\theta-\theta_0)}\frac{\partial}{\partial\theta_0}\frac{1}{\mu}(\theta-\theta_0) =$

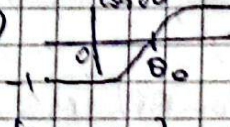
$= -\frac{1}{\mu}\int d\theta \sin\theta \frac{1}{ch^2\mu^{-1}(\theta-\theta_0)} \approx -\frac{1}{\mu}\sin\theta_0 \int d\theta \frac{1}{ch^2\mu^{-1}(\theta-\theta_0)}$

$\int_{\theta_1}^{\theta_2} d\theta \frac{1}{ch^2\mu^{-1}(\theta-\theta_0)} = \left|\frac{1}{\mu}(\theta-\theta_0) = x,\ d\theta = \mu dx\right| = \int_{\frac{1}{\mu}(\theta_1-\theta_0)}^{\frac{1}{\mu}(\theta_2-\theta_0)} \mu dx \frac{1}{ch^2 x} = \mu\, th x \Big|_{-\frac{1}{\mu}(\frac{\pi}{2}+\theta_0)}^{(\pi/2-\theta_0)/\mu} \approx$

$\approx \mu\, th x\,[{}_{-\infty}^{+\infty} = 2\mu$.

$\frac{\partial J_2}{\partial\theta_0} = -\frac{1}{\mu}\sin\theta_0 \cdot 2\mu = -2\sin\theta_0$, $J_2(\theta_0) = 2\cos\theta_0 + C = 2 + C - \theta_0^2$.

$\cos\Omega(\theta_0) = th\mu^{-1}(\theta-\theta_0)$



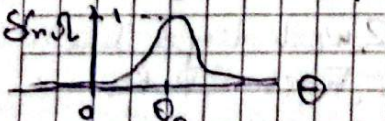
$m_z(\theta_0) = \frac{1}{ch\mu^{-1}(\theta-\theta_0)}$ [Bilitewski'23] $\vec{m}_{DW} = \sin\Omega\sin\Phi\,\hat{r} + \cos\Omega\,\hat{\theta} + \sin\Omega\cos\Phi\,\hat{z}$

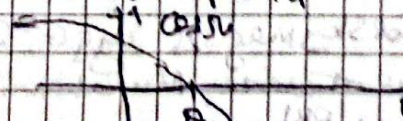
$\langle\vec{m}_{DW}\rangle_V = \langle\vec{m}_{DW}\rangle_\theta$ должно $\parallel\hat{z}$ $\forall\theta_0$ (?)

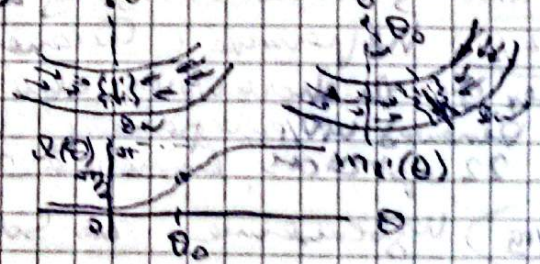
$\langle\vec{m}_{DW}\rangle_V \parallel \hat{z}$ только если самой DW

$m_z = \sin\Omega\sin\Phi$, $m_\theta = \cos\Omega$, $\hat{\theta}$, Ω, $\hat{r}$

$\Omega(\theta) = 2\arctg\left(\frac{\theta-\theta_0}{\mu}\right)$, поляризация $p = +1$







при $p=+1$, надо $\cos\Omega(\theta) = -th\mu^{-1}(\theta-\theta_0)$

$J_2(\theta_0) = \int d\theta |\cos\Omega \sin\theta|$ надо $J_2(\theta_0) = -\int d\theta \sin\theta\, th\mu^{-1}(\theta-\theta_0)$

в моем расчете, тогда $J_2(\theta_0) = -2\cos\theta_0 = -2 + \theta_0^2$ как у [Bilitewski'23] $J_2(0) = 0$, а $\frac{\partial J_2}{\partial\theta_0} \sim \sin\theta$, где θ — угол DW на наноцилиндре.

→ Вывод: расчет $E_2(\theta_0)$, $\theta_0 = \pi/R$ у [Bilitewski'23] вероятно $R = 1$.

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$\theta_0 = \frac{l}{R} \ll 1$ можемо. А $\mu = x\lambda = \frac{\lambda}{R} \ll 1$ верно ли? 3/II $\lambda(\pi/2) = \frac{le}{\sqrt{N_z}}$ DW width

$\mu = x\lambda = \sqrt{le/R} \ll 1$ при $R \simeq 10\,\mu m$, $le = 5\,nm$.

$N_z \propto -4\frac{L_z}{L}\ln\left(\frac{L_z}{w}\right)$, $\frac{L_z}{w} \sim 0.1$, $\ln\left(\frac{L_z}{w}\right) = \ln\frac{1}{10} = -\ln 10 = -2.3$.

$N_z = (-4)\frac{L_z}{wL}(-2.3) \approx 8\frac{L_z}{L}$, L_z strip thickness, L – stripe length.

$N_z \approx 8\frac{10\cdot nm}{\pi R} \approx \frac{30\cdot nm}{30\,\mu m} \approx 10^{-3}$, DW width $\lambda = \frac{5\,nm}{\sqrt{10^{-3}}} \approx 10\frac{5\,nm}{\sqrt{10}} \approx 200\,nm$

$\lambda \sim w$ (stripe width) как и должно быть. Ср. McMichael '1997.

точная DW-DW width $\Delta_0 = \pi\lambda = \pi \cdot 200\,nm \simeq 700\,nm$.

(Bretin '23) $x = \pi \cdot 10^6 \frac{1}{m}$, $L_z = 10\,nm$, $w = 20nm$ (симметр. нано).

$x\lambda \simeq \pi 10^6 \frac{1}{m} 10^2\,nm = \pi \cdot 10^8 \cdot 10^{-9} = 0.3$. $R = 10\,\mu m \to x = \frac{1}{10\,\mu m} = 0.1 \cdot 10^{-6}\frac{1}{m}$.

1D model DW width $\approx 700\,nm \gg$ DW width (xy-model) $\approx w = 100\,nm$, $\frac{L_z}{w} = 0.1$.

(1/II) Skype с Владиславом Гапоном. Обсуждение результатов – найти DW resonance in real stripe. Искать max displacement $X(\omega)$ vs. frequency. Maciej travel?

(8/II) Корректура и ответ Антону info to open Chimera-25 Conf. website. Draft of web-site готов.

(9/II) Прочел Jia et al. Twisted magnon beams". Nat. Com. 2019 и послал comments Оксане.

(10/II) Вопрос to Murakami по web-site „Chimera" и Conf. organization. Напомнил SolSkyMag-2025 на June 29 – July 3, хорошо бы за конференц (no rooms in Olavarin).

(11-12/II) Ревизия Sangar-PRA „BP DW nanowires". послал rev. MS и ответы referee Оксане.

(13/II) E-mails SolSkyMag 2025 invited о напомнил конф. на 30/VI – 4/VII.

(14/II) Вставка Introduction PhD thesis Javier (overleaf).

(15/II) Damping D_{xx} of BP nanowire – 14/I 2023. Hopf index – Oct. 2023 [S.B. Gudnason]

D_{zz} – BP nanowire – 11/I 2024. D_{xx}, D_{zz} – Feldkeller ansatz.

Feldkeller ansatz BP-DW in Cyl. wire $m_z = \frac{pz}{(s,t)\sqrt{s^2+z^2}}$ x орган ч/ш $\frac{L}{R} = p >> 1$ и small wire radius.

(7/II) E-mails to Smalyukh, Yamamoto.

(18/II) Revision RRL – Hopfion MS. Hopfion RRL – MS.

(19/II) В Университет, инет не работает.

Планы: 1) Fig. 1 – RRL, 2) update Santander, Polska home address.
3) Orden Ministerio project '25 2024; 4) Розлив Fadkella' Phys. Scr.
5) Contact Murakami. 6) JEMS abstract.

(20/II) Revised and re-submitted Hopfion RRL – MS.

(21/II) Рец. paper Fadkella Sytnyiens Sh. Scripta – reject. Вне – 30€ нового.

(22/II) Анонсы invited SolSkyMag '2025. Received 3 abstracts to date.

(23-28/II) Переписка с Оксаной по DW resonance – bent stripes (Del Mass) и Vortex GM in short wires (colab. paper с Тейо). E-mails to Murakami „Chimera" Conf. Renewed USV/Eur parking. Приглашен в Англию SolSkyMag '2025.

(1/III) 22 abstracts, из которых 3 invited SolSkyMag.

(3-4/III) Изменение дат SolSkyMag '25 будет 2 work. days, June 30, July 1.

(5/III) Mercadona с Л. переписка Tatara, Murakami, Fischer, Ruben.

(6/III) Вычитка proofs „Hopfions" RRL.

(7/III) Sent RRL proofs. С Л. „Aldi". SolSkyMag abstracts – 29 = 10 inv. + 19 reg.

(6-12/III) Info Chimera" Conf. website, переписка с SolSkyMag '25 invited speakers С Л. Carrefour Olivos 12/III. Дис. sent videos GM vortex, Klein sent preprints.

(13/III) USV/Eur – инет не работает. Нос. Клеб Л.Д. и Лена 22 кв. 63 = €.

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(14/III) Damping $D_{\varphi\varphi}, D_{zz}$ que BP DW nanowires a cruzan 14/I 2023 a 11/IX 2024 within Feldkeller ansatz ($\delta/R \gg 1$). Que Thiaville ansatz D_{ip} no cruzan.

Cruz. 14/I 2023: $D_{\varphi\varphi} = -2\pi\alpha\left(\frac{M_s}{\gamma}\right) J_\varphi$, $J_\varphi = \int dz \frac{dS}{S} m_\varphi^2$. DW BP damp

$J_\varphi(t,\beta) = \frac{1}{L}\int_{-1/2}^{1/2} dz \int_S \frac{dS}{S} m_\varphi^2(S,z,t,\beta)$, $t = \frac{\delta}{R}$, BP size. $m_\varphi^2 = 1 - m_z^2$

$m_z = p\,\frac{\beta z}{\sqrt{\beta^2 z^2 + r^2}}\,\frac{t^2}{t^2 + r^2} = m_z(S,z,t,\beta)$ – Thiaville ansatz.

Math Cad BP DW damping.mcd" nos ha dado que $\frac{1}{L} J_\varphi(t \gg 1, \beta \gg 1) = \frac{\pi}{\beta}$ – bueno



$p = 20, 30$ $J_\varphi(t, \beta = \text{const})$ bueno hasta $t = \frac{\delta}{R}$ aprox., saturating a/a $t > 5$.

$D_{zz} = -2\pi\alpha\left(\frac{M_s}{\gamma}\right) J_z$, $J_z(t,\beta) = \frac{1}{L}\int_0^1 dS \int_{-1/2}^{1/2} dz \frac{1}{m_\varphi^2}\left(\frac{\partial m_z}{\partial z}\right)^2 \frac{1}{L^2}$

$J_z(t,\beta) = \frac{R^2}{L}\int_0^1 dS \int_{-1/2}^{1/2} dz \frac{1}{m_\varphi^2}\left(\frac{\partial m_z}{\partial z}\right)^2$, $m_{\varphi,z} = m_\varphi(S, z, t, \beta)$. $S/R = r^2$, $t = \frac{\delta}{R}$.

$J_\varphi = L\frac{\pi}{\beta} = \pi R$, $J_z = \frac{1}{2}\pi R$ a/a $t \gg 1$, $\beta \gg 1$ (11/IX 2024).



Note $J_z(t,\beta)$ crece poco a poco a t, saturating a/a $t \gtrsim 1$ con $\beta = 20, 30$. $D_{zz} \sim J_z(t,\beta)$.

Notemos que $D_{zz}(t,\beta) \simeq D_{zz}(\infty,\beta)$ mismo δ para Feld. ansatz.

$D_{zz}(t,\beta) = -2\pi\alpha\left(\frac{M_s}{\gamma}\right)\frac{R}{\beta}\int_0^1 dr \int_{-1/2}^{1/2} dz \frac{1}{m_\varphi^2}\left(\frac{\partial m}{\partial z}\right)^2$. $J_z(t,\beta) = \int_0^1 dr \int_{-1/2}^{1/2} dz \frac{1}{m_\varphi^2}\left(\frac{\partial m_z}{\partial z}\right)^2$

$J_z(t \gg 1, \beta \gg 1) = \frac{\pi}{2}\beta\frac{2}{\beta} = \frac{\pi}{2}$, a/a $J_z(t \gg 1, \beta \gg 1) = \frac{1}{2}\pi R$.

$J_z(t,\beta) = \frac{R}{\beta} J_z'(t,\beta)$, $J_z'(t,\beta)$ calculado β Math Cad.

Bulog: que rígid BP DW model $D_{\varphi\varphi}(\text{Thiav}) \gg D_{\varphi\varphi}(\text{Feldkeller})$, a/a $\beta/R \sim 1$.
$D_{zz}(\text{Thiav}) \simeq D_{zz}(\text{Feld})$

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(23/IV) Перемика с Booking о комис. за ИБ. Потвър. Booking reward 277€.
(24/IV) Потвър. contract (имоти о дзиело) Oct. 1, 2024 – Dec. 30, 2027.
Мои Вх.нар Дне не.изрежим 54 k€/4 = 13,5 k€ за. Задачи из
Chimera OPUS project #1, 3, 5, 6.
(#1) (paid) Ground state and dynamics in AFM, ferri M curvilinear shape
(#3) Magnetodipolar interaction in shells and planar curvilin. stripes
(#5) SK-dynamics in inhom. electric field $\vec{E}$ (∇E) in magnetoelectric curvilinear systems
(#6) SW in ferromagnetic curvilinear films (SW)
(25/IV) C.N. restaurant „Kashaburi" San Sebastián u Mercadona. (8€+2€)
u/имейл JEMS – hopfion talk as „oral" → ticket go 30/IV, fee 320 € до 30/IV.
(26-27/IV) Нагласяване appl на MicrowaveMagnetics conf.
(28/IV) Напускане в UPV/EHU. Blackout (2-4 часа).
(29/IV) Перемика с Patricia de la Presa (IEEE Magn.) u Chychyk.
(30/IV-1/V) Submitted appl. to Conduct Microwave Magn. Conference.
(1/V) Reminders to Chimera inv. Speaker candidates. Invite JEMS
(2/V) E-mails to Arcady (JEMS), Ruben.
(7/V) E-mails to Arcady, Maciej (OPUS), Ruben (Tatara, thesis).
(5-6/V) Перемика с Oncancii. DW resonance beat stripe. No simulations sent
(6/V) Изпращам MS Sangar „BP DW dynamics" to Ph. Rev. Appl. Accepted
Изпратена бъзбрат 4.792,36 € income tax за 2024. Нарежен
(7/V) Update Chimera invited to Murakami, Ivan. пер. Аетнат, Nat. Com.
(8/V) E-mails to Nena (stripe m.e. energy) u Maciej (Sample – отказ от OPUS support
(9/V) SC ring editing, comments (Jaroslaw, Julia). TMag, Bari
(10/V) C.N. Carrefour Olaberria. Javier thesis.
(11/V) C.N. Donostia Ondarreta u Nam.
(12/V) Reg. TMag (invoice 570 € + 100 € за Пауу), request to incl. Chimera-link to TMag web-site.
Перемах OPUS travel money (Bari, TMag) to Lorz (Vienna, 3D magn.)
Magnetoelastic coupling comments to Nena.
(13/V) Javier thesis editing.
(14/V) Corrected / shortened SC-ring MS (Julia, Jaroslaw).
C.N. Ondarreta, обед Nam. UPV/EHU office.
(15/V) Вишнев u OK skyrmion MS с Zelant. Javier thesis.
(16-19/V) Javier thesis, chapt. 3. Elasticity. Landau, v. 7.
(20/V) R. Macedo (oral, SolSkyM-25) ос.мени oral talk, u/регионал to (present) substitute
oral to M. Smith (originally is poster)
(21/V) Sample invoice conf. fee TMag, Bari (570 €) u нанудване factura proforma
(22/V) Invitation let. Zelant. E-mail Arcady – my UPV/EHU affiliation.
Sample expreg Bari – 570 €. Документи Кнеб: M.D. 12 kg, Nano 6,3 kg
Thesis Javier. – 64 € за обе
(23/V) E-mail to Murakami, Chimera.
(26/V) PhD thesis comments to Javier, Ruben. E-mails. Murakami. Otani. Eli.
(27/V) Convenio Tourbureau. Invit. Bauer. TMag fee перемах (570 €).
Donostia
(28/V) перемика SolSkyMag. update. Program. Pers. Herrel, Hopfions, JAP.
Gallardo-JAP gaer $\vec{m}(\vec{r})$ когерое опит нагласване u hopfion с $Q_H = 1$.
$\vec{r} \in$ torus finite sizes large radius R (50) small radius r.
$\vec{m}[\vec{r}] = \vec{m}(\vartheta, \theta, \varphi)$, где $(\vartheta\theta\varphi)$ are not cylindrical coordinates. Zno $\neq 0$?

Escaneado con CamScanner

$r_{min}=r$, $r_{max}=R$. In xoy plane $z=0$, $\varphi=0$, $\theta=\arctg\left(\frac{y}{x}\right)$, $\rho=\sqrt{x^2+y^2}-R$.

$m_x=\sin\left(\pi\frac{\rho}{r}\right)\cos\theta$, $m_y=\sin\left(\pi\frac{\rho}{r}\right)\sin\theta$, $m_z=m_0\cos\left(\pi\frac{\rho}{r}\right)$, $|m_0|=1$.

B cilindrical cyl. coordinates $\theta\to\varphi$, $x^2+y^2=\rho^2$:

$m_x=\sin\left[\frac{\pi}{r}(\rho-R)\right]\cos\varphi$, $m_y=\sin\left[\frac{\pi}{r}(\rho-R)\right]\sin\varphi$, $m_z=\cos\left[\frac{\pi}{r}(\rho-R)\right]$

$R-r\le\rho\le R+r$. In plane $\vec m=m_x\hat x+m_y\hat y=\sin\left[\frac{\pi}{r}(\rho-R)\right]\hat\rho$, $\hat\rho=\hat x\cos\varphi+\hat y\sin\varphi$

Jto noroma ha Fig 2B Neel hopfion [Cus. Magnetism, 2024].

Wpu $\alpha=\frac{\pi}{2}$, $\cos(\varphi+\pi/2)=-\sin\varphi$, $\sin(\varphi+\frac{\pi}{2})=\cos\varphi$ → Vortex/Bloch hopfion.

$|\vec m_{xy}|=\sin\left[\frac{\pi}{r}(\rho-R)\right]=0$ wpu $\rho=R$, no $|\vec m_{xy}|\neq0$ wpu $\rho=0$

$m_z(\rho=R)=+1$, $m_z(\rho=0)=\cos\left(\pi\frac{R}{r}\right)\neq1$. T.e. Eqs (6-8) ha anecdotal toroidal hopfion $\leftarrow Q_H=1$. // Kroy ruc $|\rho-R|<r$ limited.

Dna toroidal hopfion $|\vec m_{xy}|=\frac{4a\rho(\rho^2-a^2)}{(\rho^2+a^2)^2}$ occur. or $|\vec m_{xy}|=\sin\left[\frac{\pi}{r}(\rho-R)\right]$ Gallard.

R unec cmotca the hopfion radius a'.

(29/V) E-mails - Sol Shyma u PID2022 report.

(30/V) Online meeting PID2022 project (nothing o wort regen).
Next meeting in Oviedo, Oct. 25 (?)

(31/V) Proofs of Sangar BP. DW dynamics paper PRA.

(1/VI) Reminders to inv. speakers "Chimera" to send abstracts.

(2/VI) Wpobepum en field $\vec B$ Gallard-Hertel JAP subm. Hertel's hopfion

$m_\rho=m_x\cos\varphi+m_y\sin\varphi$, $m_\varphi=-m_x\sin\varphi+m_y\cos\varphi$

B $z=0$ plane $m_x=\sin\left[\frac{\pi}{r}\rho'\right]\cos(\varphi+\alpha)$, $m_y=\sin\left(\frac{\pi}{r}\rho'\right)\sin(\varphi+\alpha)$, $m_z=\cos\left(\frac{\pi}{r}\rho'\right)$, $\rho'=\rho-R$.

$m_\rho=\sin\left(\frac{\pi}{r}\rho'\right)\cos\alpha$, $m_\varphi=\sin\left(\frac{\pi}{r}\rho'\right)\sin\alpha$, $m_z=\cos\left(\frac{\pi}{r}\rho'\right)$.

$B_i=\frac{1}{2}\varepsilon_{ijk}\vec m\cdot(\partial_j\vec m\times\partial_k\vec m)$, $B_z=\frac{1}{2}\varepsilon_{z\rho\varphi}\vec m\cdot(\partial_\rho\vec m\times\partial_\varphi\vec m)+$

$+\frac{1}{2}\varepsilon_{z\varphi\rho}\vec m\cdot(\partial_\varphi\vec m\times\partial_\rho\vec m)=\vec m\cdot(\partial_\rho\vec m\times\partial_\varphi\vec m)$

Хорошо нокаут $(\vec B\cdot\vec n)_S=0$ не вписывается на поверх. граф 2.

B_z · $B_z\neq0$ на верхней поверхности и $B_\rho\neq0$ на бок. поверхности ($z=0$).

Gallard $\rho'=\sqrt{z^2+(\rho-R)^2}$ $\Theta=\arctg\frac{y}{x}=\varphi$, $\Phi=\arctan\left(\frac{z}{\rho-R}\right)$

$v=1$, $w=1$, $m_x=\sin\left(\frac{\pi}{r}\rho'\right)\cos(\varphi-\Phi+\alpha)$, $m_y=\sin\left(\frac{\pi}{r}\rho'\right)\sin(\varphi-\Phi+\alpha)$, $m_z=\cos\left(\frac{\pi}{r}\rho'\right)$.

$m_\rho=\sin\left(\frac{\pi}{r}\rho'\right)\cos\alpha$ – совпадение ли wpu $z\neq0$?

- $m_\rho=\sin\left(\frac{\pi}{r}\rho'\right)\left[\cos(\varphi-\Phi+\alpha)\cos\varphi+\sin(\varphi-\Phi+\alpha)\sin\varphi\right]=\sin\left(\frac{\pi}{r}\rho'\right)\cos(\alpha-\Phi)$
- $m_\varphi=\sin\left(\frac{\pi}{r}\rho'\right)\left[-\sin\varphi\cos(\varphi-\Phi+\alpha)+\cos\varphi\sin(\varphi-\Phi+\alpha)\right]=\sin\left(\frac{\pi}{r}\rho'\right)\sin(\alpha-\Phi)$

$\rho'=\rho'(\rho,z)$, $\Phi=\Phi(\rho,z)$.

$B_z=\vec m\cdot(\partial_\rho\vec m\times\partial_\varphi\vec m)=\begin{vmatrix} m_\rho & m_\varphi & m_z\\ \partial_\rho m_\rho & \partial_\rho m_\varphi & \partial_\rho m_z\\ \partial_\varphi m_\rho & \partial_\varphi m_\varphi & \partial_\varphi m_z\end{vmatrix}$, $\partial_\varphi=\frac{1}{\rho}\frac{\partial}{\partial\varphi}$.

$\frac{\partial\vec m}{\partial\rho}=\frac{\partial}{\partial\rho}\left[m_\rho\hat\rho+m_\varphi\hat\varphi+m_z\hat z\right]$

Aug. 7, 1001. $\frac{\partial}{\partial x}=\cos\varphi\frac{\partial}{\partial\rho}-\sin\varphi\frac{1}{\rho}\frac{\partial}{\partial\varphi}$, $\frac{\partial}{\partial y}=\sin\varphi\frac{\partial}{\partial\rho}+\cos\varphi\frac{1}{\rho}\frac{\partial}{\partial\varphi}$

$\frac{\partial\rho}{\partial\rho}=0$ $\frac{\partial\hat\rho}{\partial\rho}=0$ $\frac{\partial\hat z}{\partial\rho}=0$, $\frac{\partial\hat\rho}{\partial\varphi}=\hat\varphi$, $\frac{\partial\hat\varphi}{\partial\varphi}=-\hat\rho$ [Arfken]

$\frac{\partial\vec m}{\partial\rho}=\hat\rho\frac{\partial m_\rho}{\partial\rho}+\hat\varphi\frac{\partial m_\varphi}{\partial\rho}+\hat z\frac{\partial m_z}{\partial\rho}$, $\frac{\partial\vec m}{\partial\varphi}=\frac{\partial}{\partial\varphi}\left[m_\rho\hat\rho+m_\varphi\hat\varphi+m_z\hat z\right]=$

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$= \hat{\rho}\frac{\partial m_\rho}{\partial \rho} + m_\rho \frac{\partial \hat{\rho}}{\partial \rho} + \hat{\varphi}\frac{\partial m_\varphi}{\partial \rho} + m_\varphi \frac{\partial \hat{\varphi}}{\partial \rho} + \hat{z}\frac{\partial m_z}{\partial \rho} = \hat{\rho}\frac{\partial m_\rho}{\partial \rho} + m_\rho \hat{\varphi} + \hat{\varphi}\frac{\partial m_\varphi}{\partial \rho} + m_\varphi(-\hat{\rho}) +$

$+ \hat{z}\frac{\partial m_z}{\partial \rho} = \hat{\rho}\left(\frac{\partial m_\rho}{\partial \rho} - m_\varphi\right) + \hat{\varphi}\left(m_\rho + \frac{\partial m_\varphi}{\partial \rho}\right) + \hat{z}\frac{\partial m_z}{\partial \rho}$

$\frac{1}{s}\left(\frac{\partial \vec{m}}{\partial \varphi} \times \frac{\partial \vec{m}}{\partial z}\right) = \frac{1}{s}\left[\left(\hat{\rho}\frac{\partial m_\rho}{\partial \varphi} + \hat{\varphi}\frac{\partial m_\varphi}{\partial \varphi} + \hat{z}\frac{\partial m_z}{\partial \varphi}\right) \times \left(\hat{\rho}\left(\frac{\partial m_\rho}{\partial z} - m_\varphi\right) + \right.\right.$

$\left.\left. + \hat{\varphi}\left(\frac{\partial m_\varphi}{\partial z} + m_\rho\right) + \hat{z}\frac{\partial m_z}{\partial z}\right)\right]$, $\hat{\rho}\times\hat{\rho} = \hat{\varphi}\times\hat{\varphi} = \hat{z}\times\hat{z} = 0$, $\hat{\rho}\times\hat{\varphi} = \hat{z}$, $\hat{\rho}\times\hat{z} = -\hat{\varphi}$, $\hat{\varphi}\times\hat{z} = \hat{\rho}$

$\frac{\partial \vec{m}}{\partial \rho} \times \frac{\partial \vec{m}}{\partial z} = \frac{\partial m_\rho}{\partial \rho}\left(\frac{\partial m_\varphi}{\partial z} + m_\rho\right)\hat{\rho}\times\hat{\varphi} + \frac{\partial m_\rho}{\partial \rho}\frac{\partial m_z}{\partial z}\hat{\rho}\times\hat{z} + \frac{\partial m_\varphi}{\partial \rho}\left(\frac{\partial m_\rho}{\partial z} - m_\varphi\right)\hat{\varphi}\times\hat{\rho} +$

$+ \frac{\partial m_\varphi}{\partial \rho}\frac{\partial m_z}{\partial z}\hat{\varphi}\times\hat{z} + \frac{\partial m_z}{\partial \rho}\left(\frac{\partial m_\rho}{\partial z} - m_\varphi\right)\hat{z}\times\hat{\rho} + \frac{\partial m_z}{\partial \rho}\left(\frac{\partial m_\varphi}{\partial z} + m_\rho\right)\hat{z}\times\hat{\varphi}$

$\rightarrow \vec{m}\cdot\left(\frac{\partial \vec{m}}{\partial \rho} \times \frac{\partial \vec{m}}{\partial z}\right) = m_\rho\left[\frac{\partial m_\varphi}{\partial \rho}\frac{\partial m_z}{\partial z} - \frac{\partial m_z}{\partial \rho}\left(\frac{\partial m_\varphi}{\partial z} + m_\rho\right)\right] + m_\varphi\left[-\frac{\partial m_\rho}{\partial \rho}\frac{\partial m_z}{\partial z} + \right.$

$\left. + \frac{\partial m_z}{\partial \rho}\left(\frac{\partial m_\rho}{\partial z} - m_\varphi\right)\right] + m_z\left[\frac{\partial m_\rho}{\partial \rho}\left(\frac{\partial m_\varphi}{\partial z} + m_\rho\right) - \frac{\partial m_\varphi}{\partial \rho}\left(\frac{\partial m_\rho}{\partial z} - m_\varphi\right)\right]$

Для Gallard-SAP ansatz: $m_\rho = m_\rho(\rho, t)$, $m_\varphi = m_\varphi(\rho, z)$, $m_z = m_z(\rho, z)$

и $\vec{m}\cdot\left(\frac{\partial \vec{m}}{\partial \rho} \times \frac{\partial \vec{m}}{\partial z}\right) = m_\rho\left[-\frac{\partial m_\varphi}{\partial z}m_\rho\right] + m_\varphi\left[-m_\varphi\frac{\partial m_z}{\partial \rho}\right] + m_z\left[m_\rho\frac{\partial m_\rho}{\partial \rho} + m_\varphi\frac{\partial m_\varphi}{\partial \rho}\right] =$

$= -\frac{\partial m_z}{\partial \rho}\left(m_\rho^2 + m_\varphi^2\right) + m_z\left[m_\rho\frac{\partial m_\rho}{\partial \rho} + m_\varphi\frac{\partial m_\varphi}{\partial \rho}\right]$, $\frac{1}{2}\frac{\partial \vec{m}^2}{\partial \rho} = \vec{m}\cdot\frac{\partial \vec{m}}{\partial \rho} \neq$

$\neq B_z = \frac{1}{s}\vec{m}\cdot\left(\frac{\partial \vec{m}}{\partial \rho} \times \frac{\partial \vec{m}}{\partial \varphi}\right)$ - emergent mag. field. $B_z = B_z(\rho, z)$.

При $z = \pm\pi$ $(\pm\pi)$, $\rho = R$ B_z $\Phi(\rho, z) = \pi/2$, $\rho' = z = \pi$

$m_\rho = 0$, $m_\varphi = 0$ из-за $\sin\left(\frac{\pi}{\pi}\rho'\right) = 0$ и $B_z = 0$ (!)

(3/VI) Проверим B_ρ на $\rightarrow B_\rho$ при $z = 0$, $\rho = R + \pi$.

$B_\rho = \frac{1}{2}\varepsilon_{\rho\varphi z}\vec{m}\cdot\left(\partial_\varphi\vec{m} \times \partial_z\vec{m}\right) + \frac{1}{2}\varepsilon_{\rho z\varphi}\vec{m}\cdot\left(\partial_z\vec{m} \times \partial_\varphi\vec{m}\right) = \vec{m}\cdot\left(\partial_\varphi\vec{m} \times \partial_z\vec{m}\right)$

$\partial_\varphi\vec{m} \times \partial_z\vec{m} = \left[\hat{\rho}\left(\frac{\partial m_\rho}{\partial \varphi} - m_\varphi\right) + \hat{\varphi}\left(\frac{\partial m_\varphi}{\partial \varphi} + m_\rho\right) + \hat{z}\frac{\partial m_z}{\partial \varphi}\right] \times \left[\hat{\rho}\frac{\partial m_\rho}{\partial z} + \hat{\varphi}\frac{\partial m_\varphi}{\partial z} + \hat{z}\frac{\partial m_z}{\partial z}\right] =$

$= \left[-m_\varphi\hat{\rho} + \hat{\varphi}m_\rho\right] \times \left[\hat{\rho}\frac{\partial m_\rho}{\partial z} + \hat{\varphi}\frac{\partial m_\varphi}{\partial z} + \hat{z}\frac{\partial m_z}{\partial z}\right] = -m_\varphi\frac{\partial m_\varphi}{\partial z}\hat{\rho}\times\hat{\varphi} - m_\varphi\frac{\partial m_z}{\partial z}\hat{\rho}\times\hat{z} +$

$+ m_\rho\frac{\partial m_\rho}{\partial z}\hat{\varphi}\times\hat{\rho} + m_\rho\frac{\partial m_z}{\partial z}\hat{\varphi}\times\hat{z} = -m_\varphi\frac{\partial m_\varphi}{\partial z}\hat{z} + m_\varphi\frac{\partial m_z}{\partial z}\hat{\varphi} - m_\rho\frac{\partial m_\rho}{\partial z}\hat{z} + m_\rho\frac{\partial m_z}{\partial z}\hat{\rho}$

$B_\rho = \vec{m}\cdot\left(\partial_\varphi\vec{m} \times \partial_z\vec{m}\right) = \frac{1}{\rho}\left(m_\rho\hat{\rho} + m_\varphi\hat{\varphi} + m_z\hat{z}\right)\left[m_\rho\frac{\partial m_z}{\partial z}\hat{\rho} + m_\varphi\frac{\partial m_z}{\partial z}\hat{\varphi} - \hat{z}\left(m_\rho\frac{\partial m_\rho}{\partial z} + m_\varphi\frac{\partial m_\varphi}{\partial z}\right)\right] =$

$= \frac{1}{\rho}\left[m_\rho^2\frac{\partial m_z}{\partial z} + m_\varphi^2\frac{\partial m_z}{\partial z} - m_z\left(m_\rho\frac{\partial m_\rho}{\partial z} + m_\varphi\frac{\partial m_\varphi}{\partial z}\right)\right]$

$0 = \frac{\partial}{\partial z}\left(\vec{m}^2 = 1\right) = 2\vec{m}\cdot\frac{\partial \vec{m}}{\partial z} = 2\left[m_\rho\frac{\partial m_\rho}{\partial z} + m_\varphi\frac{\partial m_\varphi}{\partial z} + m_z\frac{\partial m_z}{\partial z}\right]$, $m_\rho\frac{\partial m_\rho}{\partial z} + m_\varphi\frac{\partial m_\varphi}{\partial z} = -2m_z\frac{\partial m_z}{\partial z}$

$B_\rho = \frac{1}{\rho}\left(m_\rho^2 + m_\varphi^2 + 2m_z^2\right)\frac{\partial m_z}{\partial z} = \frac{1}{\rho}\left(1 + m_z^2\right)\frac{\partial m_z}{\partial z}$. Note $B_\rho \sim \frac{\partial m_z}{\partial z}$.

Gallard-SAP ansatz $m_z(\rho, z) = \cos\left(\frac{\pi}{\pi}\rho'\right)$, $\rho' = \sqrt{z^2 + (\rho - R)^2}$.

$\frac{\partial m_z}{\partial z} = -\sin\left(\frac{\pi}{\pi}\rho'\right)\frac{\pi}{\pi}\frac{\partial \rho'}{\partial z}$, $\rho'(z=0, \rho = R+\pi) = \pi$ и $\sin\left(\frac{\pi}{\pi}\cdot\pi\right) = \sin(\pi) = 0$

$\frac{\partial m_z}{\partial z}(z=0, \rho = R+\pi) = 0$ и $B_\rho(z=0, \rho = R+\pi) = 0$ (!) $\left(\vec{B}\cdot\vec{n}\right) = \left(\vec{B}\cdot\hat{\rho}\right) = 0$

$\rightarrow$ Вывод: $\left(\vec{B}\cdot\vec{n}\right)_S = 0$ в граничных точках, лежащих на S (сфера $(z=\pi, \rho=R)$ и $(z=0, \rho=R+\pi)$).

и Gallard ansatz $\vec{m}(\vec{r})$ может быть с gauge-invariant Hopf index.

(4/VI) 21/VII Теорема о покрытии (Hopf theorem) (Wiki)

in Bounded domain $V \subset \mathbb{R}^3$ с гладкой границей S из $\vec{B} = \nabla\times\vec{A}$ следует

$\vec{A}(\vec{r}) = -\frac{1}{4\pi}\int_V \vec{B}'\cdot\frac{\vec{R}\times\vec{B}'}{R^3}$, $\vec{R} = \vec{r} - \vec{r}'$, $\vec{B} = \vec{B}(\vec{r}')$. С учетом $\nabla_{\vec{r}}\times\left(\frac{\vec{B}'}{R}\right) = -\frac{1}{R^3}(\vec{r} - \vec{r}')\times\vec{B}'$

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Обычно следует $\vec{A}(\vec{r}) = \frac{1}{4\pi} \nabla_r \times \int dV' \frac{\vec{B}(\vec{r}')}{R}$, это использовал Hertel JAP submitted.

Но $\operatorname{div}\vec{A} = 0$ это выбор gauge. Это vector potential и этот поток (flow) divergence-free.

Для gauge-invariant надо либо infinite sample, либо $(\vec{B}\cdot\vec{k})$ finite в complex.

Bio-Savart (Law Wiki) $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int dV' \vec{J}(\vec{r}') \times \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = \frac{\mu_0}{4\pi} \int dV' \vec{J} \times \frac{\vec{R}}{R^3}$

или $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \nabla_r \times \int dV' \frac{\vec{J}(\vec{r}')}{R}$ $\operatorname{div}\vec{B} = 0$.

$\nabla_r \times \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \nabla_r \times \nabla_r \times \int dV' \frac{\vec{J}(\vec{r}')}{R} = \frac{\mu_0}{4\pi} \nabla_r \times \nabla_r \times \vec{X} = \frac{\mu_0}{4\pi} \left[\nabla_r (\nabla_r \cdot \vec{X}) - \nabla_r^2 \vec{X}\right]$

$\nabla_r \times \vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int dV' \nabla_r (\nabla_r \cdot \vec{J}') - \frac{\mu_0}{4\pi} \nabla_r^2 \int dV' \frac{\vec{J}(\vec{r}')}{R} \cdot \nabla^2_r \left(\frac{1}{R}\right) = -4\pi\delta(\vec{r}-\vec{r}')$

$\int dV' \nabla_r \left(\nabla_r \cdot \frac{\vec{J}'}{R}\right) = \int dV' \vec{J}' \nabla_r \left(\nabla_r \cdot \frac{1}{R}\right)$, $\nabla_r \cdot \frac{\vec{J}'}{R} = \operatorname{div}\left(\frac{\vec{J}'}{R}\right) = \frac{\vec{R}\cdot\operatorname{div}\vec{J}'}{R^2} - \nabla_r\left(\frac{1}{R}\right)\vec{J}'$ (0)

$= -\nabla_r(R)\cdot\vec{J}'\frac{1}{R^2}$, $\nabla_r(R) = \frac{\vec{R}}{R}$

$\int dV' \nabla_r \left(\nabla_r \cdot \frac{\vec{J}'}{R}\right) = \int dV' \nabla_r \left[-\frac{\vec{R}}{R^3}\cdot\vec{J}'\right]$ это $= 0$ и $\nabla_r \times \vec{B}(\vec{r}) = \operatorname{rot}\vec{B} = \mu_0\vec{J}$ Maxwell eq

Gallay: $\Delta\vec{A} = \nabla(\nabla\cdot\vec{A}) - \nabla\times\nabla\times\vec{A} = -\nabla\times\vec{B}$

$\nabla\cdot(\vec{e}_i\times\vec{B}) = (\nabla\times\vec{e}_i)\cdot\vec{B} - \vec{e}_i\cdot\operatorname{rot}\vec{B} = -\vec{e}_i\cdot\operatorname{rot}\vec{B} = -\vec{E}_i\cdot\vec{A} = -A_i$.

$\Delta\vec{A}\cdot\vec{e}_i = -(\nabla\times\vec{B})\cdot\vec{e}_i = -A_i = \nabla\cdot(\vec{e}_i\times\vec{B}) = \Delta A_i$, eq (6) Gallard Appendix

Above is checking of Eqs. из MS Gallard - Hertel, submitted JAP.

(5-12/VII) Переписка с соавт. программ Chimera'25. Переписка Salsky mag.

(13/VII) С А. в UPV/EHU, начинка internet.

(14/VI) Соавтор и коллеги Oksana Short (8 pages) CV.

(15-16/VII) Переписка, program Chimera'2025.

(17/VI) Запрос в САИ на настройку internet и phone.

(18/VII) Meet. с Ion/САИ, настройки inet и phone. Passw. для UPV-vpn.

(19/VII) Chimera program - Murakami.

(20/VII - 29/VII) Переписка SolSkyMag'25 и Chimera'25.

(30/VII - 1/VIII) Проведение SolSkyMag'25, Olatain. Рес. „Kaskazuri" Zoom Smalyuk

2/VII Чтение Ch. 5-6 PhD thesis Javier. Подбор Tribunal members.

(21/VII) Переписка Chimera'25. Посылка 1.0 - 11 kg (32€). Переписка с Ruben, Javier, Oksana. Padre. IVA и проблемы CONSTA/VTO.

(21/VII) Slavin, ICNM'26 будет в San-Sebastian.

(22/VII) DWS project. Мой вклад по Task 3: dipolar interaction in (shells) и planar curvilinear stripes. DW resonance Stripe

Curv. $\varkappa = 1/R$, λ - DW width. Res. frequency bent stripe

энергия. $E_H \sim \frac{1}{R}$, $\omega_0 = \frac{\gamma}{2\pi M_s}\sqrt{K_1 K_2}$, $K_1 \sim H$, K_2 зависит от A, Ms (magnetostatic) man

и Zeeman H. $E_H(\varkappa)$ минимум Bitzenhorst 24 если $\varkappa\lambda \ll 1$.

$E_{ex}(\varkappa)$, но E_{dip} взято в прост. форме и $\sigma \sim \varkappa = 1/R$ не работает.

Проверить E_{dip} bent stripe с DW (Task 3).

Вихрь может Vortex resonance in bent stripe (Parkin, Nat. 2008)

(23/VIII) Vortex G.M - Bent Stripe radius R. Assume Vortex Core eq. three

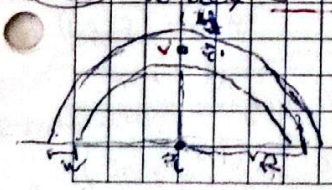
is $\vec{r}'(0,0,z_1)$, $\vec{r}' = 0$ at vortex center, $\vec{r}' = \vec{r} - \vec{R}$

$\vec{r} = \vec{s}$ center cyl. coordinates, $z_1 = L/2$ мин - не важно.

$\vec{m}(\vec{r}',t) = \vec{m}(\vec{r}', \vec{X}(t)) = \vec{m}(\vec{r}' - \vec{X}(t))$ - rigid model.

$|\vec{X}|$ - is small. GM oscillation around $\vec{r}' = 0$ point.

$L/w \ll 1$, $\vec{s} \to \vec{r}$, $\vec{s}' \to \vec{r}'$, z - coord. небольшая.



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ур-е: $\vec{G}\times\dot{\vec{X}} = -\frac{\partial W(\vec{X})}{\partial \vec{X}}$. Наклон shifted vortex energy for bent stripe geometry.

$W(\vec{X})$

$W(\vec{X}) = \frac{1}{2}\kappa_x x^2 + \frac{1}{2}\kappa_y y^2$, no cyl. symmetry anis. $\vec{\tau}=\vec{s}'=0$.

Линейн. член $\sim x'xy$ нулев.

$\vec{G}\times\dot{\vec{X}} = (0,0,G)\times(\dot{x},\dot{y},0) = \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & G \\ \dot{x} & \dot{y} & 0 \end{pmatrix} = \hat{y}G\dot{x} - \hat{x}G\dot{y}$, $(x,y)\to(s,\varphi)$?

$\left\{\begin{array}{l} -G\dot{y} = -\kappa_x x - x'y \\ G\dot{x} = -\kappa_y y - x'x \end{array}\right\}$ $\dot{x} = x_0 e^{i\omega t}$, $\dot{y} = y_0 e^{i\omega t}$

$\left\{\begin{array}{l} -G\,i\omega\, y_0 e^{i\omega t} = -\kappa_x x_0 e^{i\omega t} - x' y_0 e^{i\omega t} \\ G\, x_0\, i\omega\, e^{i\omega t} = -\kappa_y y_0 e^{i\omega t} - x' x_0 e^{i\omega t} \end{array}\right\}$

$\left\{\begin{array}{l} -i\omega G y_0 = -\kappa_x x_0 - x' y_0 \\ i\omega G x_0 = -\kappa_y y_0 - x' x_0 \end{array}\right\}$ $\left\{\begin{array}{l} -\kappa_x x_0 + (i\omega G - x') y_0 = 0 \\ -\kappa_y y_0 + (-x' + i\omega G) x_0 = 0 \end{array}\right\}$

$\det\begin{pmatrix} -\kappa_x & (i\omega G - x') \\ -(x' + i\omega G) & -\kappa_y \end{pmatrix} = 0$, $\kappa_x\kappa_y + (i\omega G - x')(x' + i\omega G) = 0$

$\kappa_x\kappa_y - |x' + i\omega G|^2 = 0$. $\kappa_x\kappa_y - x'^2 - \omega^2 G^2 = 0$

GM frequency $\omega_G^2 = \frac{1}{G^2}(\kappa_x\kappa_y - x'^2)$ – magnetostatic limit.

Ybm, $\kappa_x\kappa_y$ / x' – нелокальность на аналитическ. функцию (Math Cad).

(24 / 25/VIII) Paper нанося на изобразить CONSTANTO.

Readers и tribunal members PhD Javier.

(26/VIII) Bent stripe resonance: Vortex GM не бифуркация, нарушение ~ TW resonance [Bittencourt '23].

E_d (Bit) непонят, это E_{dip} for uniformly magnetized stripe, у на (DW) есть Transverse DW head/head or tail/tail – глобальное явление.

Если shift DW $q \ll L$ и DW width $\Delta_w = \pi\lambda \ll L$, то получим E_2 - Eq.(3)?

DW $\vec{m}(\vec{r}) = \sin\Omega\sin\Phi\,\hat{x} + \cos\Omega\,\hat{\theta} + \sin\Omega\cos\Phi\,\hat{z}$, см 3-6/II.

1D DW: $\Omega(\theta) = 2\arctg[e^{(S-q)/\lambda}]$, $S = R\theta$ coordinate along stripe

$E_2 = +M_S R H \int_{\theta_1}^{\theta_2} d\theta\,[\sin\Omega\cos\theta\sin\Phi - \cos\Omega\sin\theta]$ q - DW center displacement from $q=0$

$J_1(\theta_0) = \int_{\theta_1}^{\theta_2} d\theta\,\sin\Omega\cos\theta$, $J_2(\theta_0) = \int_{\theta_1}^{\theta_2} d\theta\,\cos\Omega\sin\theta$

$x = \frac{S-q}{\lambda} = \frac{R\theta - R\theta_0}{\lambda} = \frac{\theta-\theta_0}{\varkappa\lambda}$ θ - angle of current DW position

$\varkappa = 1/R$

$\Omega = \Omega\left(\frac{\theta-\theta_0}{\varkappa\lambda}\right) = \Omega\left(\frac{\theta-\theta_0}{\mu}\right)$. Вычисление 3-6/II $\to$ то же, что и $\mu\to 0$

$\cos\Omega(\theta)$ – step function при $\theta=\theta_0$, $\sin\Omega(\theta)$ sharp max при $\theta=\theta_0$.

(27/VII) Bitten $E_2 \to 0$ limit $\mu = \lambda/R \to 0$, large ring Radius R.

Вывод: E_2 - exact < $E_2(\mu)$ - Bitten, но отличие мал, 2-3% при $\mu = 0.1$ – 0.2.

„Zeeman energy - stripe - mc4" 1D model of transverse DW.

E_{dip}?. Bittencourt $E_{dip} = 4\pi\lambda M_S^2(N_2\sin^2\Phi + N_2\cos^2\Phi)$, ~~$\Phi = \frac{\Delta}{N_2}$~~ нет

$E_{dip}(B)$ one. uniformly magnetized strip, а у нас two domains. $\frac{1+2\pi}{2\pi}$

Bitten: Φ angle между $\hat{z}$ и projection $\vec{m}$ на $\hat{z}\cdot\hat{z}$

Inconsistency. PRL – если analytic $\vec{m}(x,y)$ для TW & exchange approxim. Plane (Fig.1)

E_d^{surf} и E_d^{vol}. $\Phi = \pi/2$ $\vec{m} = \vec{m}(\theta) = \sin\Omega(\theta)\hat{z} + \cos\Omega(\theta)\hat{\theta}$, θ угол вдоль bent stripe

$m_z = \sin\Omega$, $m_\theta = \cos\Omega$. $\operatorname{div}\vec{m} = \frac{1}{r^2}\frac{\partial(r^2 m_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(m_\theta\sin\theta)$

Для 1D DW model $\vec{m}$: $m_z = \sin\Omega$, $m_\theta = \cos\Omega$. $\Phi = \frac{\pi}{2}$ для 1D model, $m_z = 0$

Rotation in $\hat{\theta}$-$\hat{z}$ - plane

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$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 m_r) = \frac{2}{r} m_r + \frac{\partial m_r}{\partial r} = \frac{2}{r} m_r \;\Rightarrow\; \frac{\partial m_r}{\partial r} = \frac{\partial}{\partial r}\sin\Omega(\theta) = 0$.

$\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\cos\Omega\sin\theta) = \frac{1}{r\sin\theta}(+1)\sin\Omega\sin\theta + \frac{1}{r\sin\theta}\cos\Omega(-1)\cos\theta =$

$= -\frac{1}{r}\sin\Omega - \frac{1}{r}\operatorname{ctg}\theta\cos\Omega$

$\operatorname{div}\vec{m}(\theta) = \frac{2}{r}\sin\Omega - \frac{1}{r}\sin\Omega - \frac{1}{r}\operatorname{ctg}\theta\cos\Omega = \frac{1}{r}\sin\Omega - \frac{1}{r}\operatorname{ctg}\theta\cos\Omega$.

$\operatorname{div}\vec{m}(\theta')$ надо $\theta \to \theta'$, $r \to R$ для 1D model.

$E_{dip}^{vol} = \frac{1}{2}M_s^2 \int dV\,dV' \frac{\operatorname{div}\vec{m}(\theta)\cdot\operatorname{div}\vec{m}(\theta')}{|\vec{r}-\vec{r}'|}$. Note $\operatorname{div}\vec{m} \sim \frac{1}{R}$

$E_d^{vol} \sim \frac{1}{R^2} \sim \alpha^2$. $|\vec{r}-\vec{r}'|$ в сферич. coordinates? Учесть два домена

$\vec{r}(r,\theta,\varphi)$, $\vec{r}'(r',\theta',\varphi')$, $|\vec{r}-\vec{r}'|^2 = r^2+r'^2-2rr'(\sin\theta\sin\theta'\cos(\varphi-\varphi') + \cos\theta\cos\theta')$

$J = \int dV\,dV' \frac{\operatorname{div}\vec{m}\,\operatorname{div}\vec{m}(\vec{r}')}{|\vec{r}-\vec{r}'|} = \int_0^{R_0} dr \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta' r^2\sin\theta' \int_0^{2\pi} d\varphi' \times \frac{\operatorname{div}(\cdot)\,d\vec{r}(\cdot)}{|\vec{r}-\vec{r}'|}$

$\operatorname{div}\vec{m}(\vec{r}) = f(r,\theta)$. Можно $\int d\varphi d\varphi' \to 2\pi\int d\varphi$.

$|\vec{r}-\vec{r}'|^2 = r^2 + r'^2 - 2rr'\cos(\theta-\theta') - 2rr'(1 - \sin\theta\sin\theta'\cos\varphi)$

$J(R,w) \int_{R-w}^{R+w} dr(\cdot)$, где ширина thickness $\sim |\sin\theta\sin\theta'[\cos\varphi - 1]$.

L вводится при интегрировании по φ? См. Fig. 1 Bitter 23.

(29/VIII – 7/IX) Отдых дома. Отказ от conf. Bari, Italy.

(8/VIII) Визит к дантисту Inna → Aneta → Tolosa. Опасный диагноз.

(15/VIII) Платное K-RT (130 €) в Clinica Miramon. Рак полости? Emergency. Помещен в Hospital. Отделение "Urología".

(21/VIII) Вторая лазерная полость. Рак не подтвердился.

(28/VIII) Перевели в отд. "Oncología". Четвертая операция.

(31/IX) Отменили диагноз и сам левой ноги. Ампутация. Неоперабельно. Прописали "Cabometix", дорогое и сильное лекарство.

(4/IX) Начали принимать таблетки. Переливание крови. Hemo 6.4.

(4–18/IX) Лежал в Hospital "Donostia" со всеми процедурами и таблет.

10 сеансов радиотерапии. Переписка с Javier, Ph.D. thesis.

(18/IX) Перевели домой. Принимаю новые таблетки.

(29/IX – 2/X) Chimera-25 Conf. Германия. Отказ.

Escaneado con CamScanner

3/Σ Научный проект „Qmus"

Task 3. Study of the influence of magnetic dipolar int...

Две частицы с Bloch points с осью OZ. Радиус R

Center-to-center distance $2R+D$. (Fig. 1 RRB2023)

$|\vec{r}_1 - \vec{r}_2| = |\vec{R} - (\vec{r}-\vec{r}')|$, $\vec{r}, \vec{r}'$ – local coordinates within R

1-ви и вторая частица.

$d = \frac{R}{R} = \frac{2R+D}{R} = 2 + \frac{D}{R} = 2 + \delta \geq 2$

Multipole moments single particle $Q_\ell^m = \left(\frac{4\pi}{2\ell+1}\right)^{1/2} \int d^3\vec{r}\, \rho(\vec{r})\, r^\ell Y_{\ell m}(\vec{r})$

$\rho(\vec{r}) = -\mathrm{div}\vec{m}(\vec{r})$

Rosensatz (Feldkeller): $\Theta(\vec{r}) = \rho\alpha + \frac{\pi}{2}(1-\rho)$, $\Phi(\vec{r}) = \varphi + f(\vec{r})$,

$\vec{r} = (r, \varphi, \theta)$ сферични координати, сферен радиус R.

$\vec{m}$ components: $m_r = \sin^2\theta\cos f + \rho\cos^2\theta$, $m_\theta = \sin\theta\cos\theta(\cos f - \rho)$

$\vec{m}(r, \theta, \varphi)$. $m_\varphi = \sin\theta \sin f$.

Verifying $f(r) = f(r, \theta)$.

$\mathrm{div}\vec{m} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 m_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(m_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial m_\varphi}{\partial\varphi}$

$\vec{m} = \vec{m}(r, \theta)\vec{r}$ – няма φ, azimuthal symmetry. $\rho(\vec{r}) = -\mathrm{div}\vec{m}(\vec{r})$

$\int d^3\vec{r}\, \mathrm{div}\vec{m}\, r^\ell Y_\ell^m(\vec{r})$. $Y_\ell^m(\vec{r}) = Y_\ell^m(\theta, \varphi) = \sqrt{\frac{2\ell+1}{4\pi}}\sqrt{\frac{(\ell-m)!}{(\ell+m)!}} P_\ell^m(\cos\theta) e^{im\varphi}$

$Q_\ell^m \sim \int d^3\vec{r}\, \mathrm{div}\vec{m}(r, \theta)\, r^\ell\, a_\ell^m P_\ell^m(\cos\theta) e^{im\varphi} = \int dr r^2 \sin\theta\, \mathrm{div}\vec{m}\, r^\ell\, a_\ell^m P_\ell^m(\cos\theta) e^{im\varphi} =$

$= \left| \int_0^{2\pi} d\varphi\, e^{im\varphi} = 2\pi\delta_{m0} \right| = \int dr r^2 \sin\theta\, d\theta\, \mathrm{div}\vec{m}\, r^\ell\, a_\ell^0 P_\ell^0(\cos\theta) \cdot 2\pi =$

$= 2\pi\sqrt{\frac{2\ell+1}{4\pi}} \int dr r^2 \sin\theta\, \mathrm{div}\vec{m}\, r^\ell P_\ell^0(\cos\theta)\, d\theta.$

$Q_\ell^m = \left(\frac{4\pi}{2\ell+1}\right)^{1/2} \int d^3\vec{r}\, \mathrm{div}\vec{m}\, r^\ell Y_\ell^{m*} = \left(\frac{4\pi}{2\ell+1}\right)^{1/2} 2\pi\sqrt{\frac{2\ell+1}{4\pi}} \int dr r^2 \sin\theta\, \mathrm{div}\vec{m}\, r^\ell P_\ell^0(\cos\theta)$

$Q_\ell^m = 2\pi \int dr r^2 d\theta \sin\theta\, \mathrm{div}\vec{m}\, r^\ell P_\ell^0(\cos\theta)$, $m=0$ Q_ℓ^0 остава.

$\mathrm{div}\vec{m}(r, \theta) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 m_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(m_\theta \sin\theta) = (1) + (2)$

$Q_\ell^0 = (1) = 2\pi \int dr r^2 d\theta \sin\theta \cdot \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 m_r) r^\ell P_\ell^0(\cos\theta) =$

$= 2\pi \int dr\, d\theta \sin\theta \frac{\partial}{\partial r}(r^2 m_r) r^\ell P_\ell^0(\cos\theta)$ от (1).

$Q_0^0 = 2\pi \int dr\, d\theta \sin\theta \frac{\partial}{\partial r}(r^2 m_r) P_0^0(\cos\theta) = 2\pi r^2 m_r \Big|_{r=0}^{r=R} \int d\theta \sin\theta P_0^0(\cos\theta)$

$P_\ell^0 = P_\ell$. $P_0 = 1$, $P_1(\cos\theta) = \cos\theta$, $P_2(\cos\theta) = \frac{1}{2}(3\cos^2\theta - 1)$

(1) $Q_0^0 = 2\pi r^2 m_r \Big|_0^R \int d\theta \sin\theta = (*) \int d(\cos\theta)(-1) = (*)(-1)\cos\theta \Big|_0^{2\pi} = 0$. $\langle \vec{m} \rangle = 0$. ?

$Q_0^0(1)$

$Q_1^0 = 2\pi \int dr\, d\theta \sin\theta \frac{\partial}{\partial r}(r^2 m_r) r^1 \cos\theta\, dr$

$Q_0^0(2) = 2\pi \int dr r^2 d\theta \sin\theta \frac{1}{2}(\sin\theta)^{-1}\frac{\partial}{\partial\theta}(m_\theta \sin\theta) = \int dr r\, d\theta \frac{\partial}{\partial\theta}(m_\theta \sin\theta) \cdot 2\pi$

$m_\theta \sin\theta = \sin^2\theta\cos\theta(\cos f - \rho)$ и как $Q_0^0(2) = 0$?

(4/Σ) $\int dV\, \mathrm{div}\vec{m} = \int ds\, \vec{n}\cdot\vec{m} = |\vec{n} = \hat{r}| = \int dS\, m_r = \int d\theta d\varphi r^2 \sin\theta\, m_r =$

$= 2\pi R^2 \int d\theta \sin\theta \left[\sin^2\theta\cos f + \rho\cos^2\theta\right]$, $r = R$

$\int d\theta \sin\theta \cos^2\theta = -\int_0^\pi d(\cos\theta)\cos^2\theta = -\frac{1}{3}\cos^3\theta \Big|_0^\pi = -\frac{1}{3}[(-1) - (+1)] = \frac{2}{3}$

$\int dV\, \mathrm{div}\vec{m} = 2\pi R^2 \left[\int_0^\pi d\theta \sin^3\theta \cos f(R, \theta) + \rho\frac{2}{3}\right]$, $Q_0^0 = 2\pi \int dr r^2 d\theta \sin\theta\, \mathrm{div}\vec{m}$

$Q_0^0 = \int dV\, \mathrm{div}\vec{m}$, защото $Q_0^0 \neq 0$ за Bloch point. → ест. dip-dip term $\sim \frac{1}{d^3}$.

Ест. това $Q_0^0(1)$, $Q_0^0(2)$, … за $k=0, m=0$, $U_{int} = \sum_{\ell\ell' mm'} T_{\ell\ell' mm'} Q_\ell^m(1) Q_{\ell'}^{m'}(2)$, 1–2 частици

$U_{int} = T_{0000} Q_0^0(1) Q_0^0(2)$

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Magnetostatic interaction 2 by x spherical particles ϵ Bloch plane inside.

$\mathcal{H}_{int} = \begin{pmatrix} 0 \\ \vec{0} \end{pmatrix} T_0^0(\vec{R}) = \sqrt{4\pi}\, \frac{Y_0^0(\hat{R})}{R} = \sqrt{4\pi}\frac{1}{R}\frac{1}{\sqrt{4\pi}} = \frac{1}{R}$, $T_{0000} = \frac{1}{R}$.

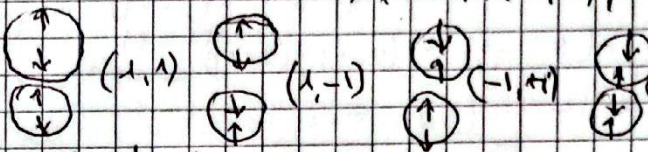
$\rightarrow U_{int} = \frac{1}{R} Q_0^0(1) Q_0^0(2) = \frac{1}{R}\int_{(1)} dV\, div\vec{m}(\vec{r}) \cdot \int_{(2)} dV'\, div\vec{m}(\vec{r}')$, Разлиш 3 вида $\theta(\gamma)$

$\gamma(1) = \gamma$, $\gamma(2) = -\gamma$. Но $\int dV\, div\vec{m} = F(\cos\gamma)$ и не зависит от sign γ. (!)

Если есть зависимость $U_{int}(\gamma)$, то она в сред. очень разложима по $\frac{1}{d}$, $\sim 1/d^3$. $m_z(\rho) = \sin^2\theta\cos\gamma + \rho\cos^2\theta$ зависит от polarity, ρ.

$\int dV\, div\vec{m} = 2\pi R^2 \int_0^{\pi} d\theta \sin\theta \left[\sin^2\theta\cos\gamma + \rho\cos^2\theta\right] = F(\rho)$.  $\rho = -1$.

$U_{int} = \frac{1}{R}\int dV\, div\vec{m}(\vec{r},\rho_1)\int dV\, div\vec{m}(\vec{r}',\rho_2)$. $U_{int} = U_{int}(\rho_1,\rho_2)$

U_{int} в сильном приближении $U_{int}(\rho_1\rho_2)$, $U_{int}(1,-1)$, $U_{int}(1,1)$, $U_{int}(-1,1)$ и $U_{int}(-1,+1)$ разные. 

$U_{int}(1,-1) = U_{int}(-1,1)$ $\downarrow\uparrow$ $\uparrow\downarrow$. Но head-to head $\downarrow\uparrow$ и tail-to-tail $\uparrow\downarrow$ разные.

$U_{int}(\rho_1,\rho_2) = \frac{1}{R}\int dV\, div\vec{m}(\vec{r},\rho_1) div\vec{m}(\vec{r}',\rho_2) dV'$, $\mathcal{R} = 2R + D$, $\frac{\mathcal{R}}{R} = 2 + d$, $d = D/R$,

$\rightarrow U_{int}(\rho_1,\rho_2) = \frac{1}{R}\frac{1}{(2+d)}\int dV\, div\vec{m}(\vec{r},\rho_1)\cdot\int dV' div\vec{m}(\vec{r}',\rho_2)$. $\frac{1}{\mathcal{R}} = \frac{1}{2R+D} = \frac{1}{R}\frac{1}{2+d}$.

У Наити $\int dV\, div\vec{m}(\vec{r},\rho) = 2\pi R^2 \int d\theta \sin\theta \left[\sin^2\theta\cos\gamma + \rho\cos^2\theta\right]$,

через случай $\gamma_1 = \gamma_2 = \gamma_0 = const$. $\int dV\, div\vec{m}(\vec{r},\rho) = 2\pi R^2 \int d\theta \sin\theta \left[\sin^2\theta\cos\gamma_0 + \rho\cos^2\theta\right]$

$\int_0^{\pi} d\theta \sin^3\theta = \int d\theta(\cdot) = \int d(\cos\theta)(-1)(1-\cos^2\theta) = (-1)\left[\cos\theta - \frac{1}{3}\cos^3\theta\right]_0^{\pi} =$

$= (-1)\left[(-1) - \frac{1}{3}(-1) - 1 + \frac{1}{3}\right] = (-1)\left[-2 + \frac{2}{3}\right] = \frac{4}{3}$.

$\int dV\, div\vec{m}(\vec{r},\rho) = 2\pi R^2\left[\cos\gamma_0\cdot\frac{4}{3} + \rho\frac{2}{3}\right] = 2\pi R^2\cdot\frac{2}{3}\left[2\cos\gamma_0 + \rho\right] = \frac{4\pi}{3}R^2\left[2\cos\gamma_0 + \rho\right]$

$U_{int} = \frac{1}{R}\frac{1}{(2+d)}\left(\frac{4\pi}{3}\right)^2 R^4\left[2\cos\gamma_0 + \rho_1\right]\left[2\cos\gamma_0 + \rho_2\right] = \left(\frac{4\pi}{3}\right)^2\frac{R^3}{(2+d)}\left(2\cos\gamma_0 + \rho_1\right)\left(2\cos\gamma_0 + \rho_2\right)$

$U_{int}(\rho_1,\rho_2) = \left(\frac{4\pi}{3}\right)^2\frac{R^3}{(2+d)}\left(2\cos\gamma_0 + \rho_1\right)\left(2\cos\gamma_0 + \rho_2\right)$, $d = D/R$ – edge-to-edge distance.

Sphere volume $V_0 = \frac{4}{3}\pi R^3$, $U_{int}(\rho_1\rho_2) = \frac{4\pi}{3}\frac{V_0}{(2+d)}\left(2\cos\gamma_0 + \rho_1\right)\left(2\cos\gamma_0 + \rho_2\right)$

Тело '202': $\gamma(\tau,\theta\theta)$ меняется от от 70° до 110° для разных R [see Fig. 3].

$\overline{\gamma(\tau,\theta\theta)} \approx \pi/2$ (90°) $\rightarrow$ $U_{int}(\rho_1\rho_2) = \frac{4\pi}{3}\frac{V_0}{(2+d)}\rho_1\rho_2$, если $\gamma_0 \approx \pi/2$.

$\rho_1\rho_2 = (-1)$ min $U_{int}(\rho_1\rho_2)$, $\rho_1\rho_2 = +1$ дает max $U_{int}(\rho_1\rho_2) = \frac{4\pi}{3}\frac{V_0}{(2+d)}$ 

Splitting due magn. interaction $U_{int}(\rho_1\rho_2 = +1) - U_{int}(\rho_1\rho_2 = -1) = \frac{8\pi}{3}\frac{V_0}{(2+d)}$

Оценим $W_{int}(\rho_1\rho_2)$ при $\gamma_0 = \pi/2$. $W_{int} = \left(\frac{4\pi}{3}\right)^2\frac{R^3}{(2+d)}\rho_1\rho_2$.

$|W_{int}| = |d=0| = \left(\frac{4\pi}{3}\right)^2\frac{R^3}{2} = \frac{8\pi^2}{9}R^3$. $R = 50$ nm. $W_{int} = \frac{8\pi}{9}^2(50\cdot10^{-7}\text{cm})^3 =$

$= \frac{8\pi^2}{9}5^3\cdot10^{-18}\text{cm}^3$. Abs. $W_{int} = \frac{1}{9}8\pi^2\cdot125\,(98\cdot10^6\text{G})^2\cdot10^{-18}\text{cm}^3 = \frac{1}{9}8\pi^2\cdot125\cdot0{,}64\cdot10^{-6}\text{cm}^3 =$

$= 702\cdot10^{-6}\text{erg} = 0{,}7\cdot10^{-4}\text{erg}$. Всего-то! Тело $W_{Bol(m)} \approx 1\cdot10^{-17}\text{J} = 10^{-10}\text{erg}$. !

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(12/X) The weakest BP interaction should $W_{int} \sim 10^{-4}$ erg. a [Feb '21] $W_m \sim 10^{-10}$ erg.
Magnetic analogy. Two point charges q_1, q_2, int. force CGS $F = \frac{q_1 q_2}{r^2}$ um
CGS - $W_{int} = +\frac{q_1 q_2}{r}$, $F = -\frac{\partial W_{int}}{\partial r}$.
Jackson: multipole expansion ch.4, magnetic charge $\rho_m(\vec{r}) = -\operatorname{div}\vec{M}$ (2)
Interaction energy $W_{int} = \frac{1}{R} Q_0^0(1) Q_0^0(2)$, $Q_0^0 = -\int dV \operatorname{div}\vec{m}(\vec{r}) = \int dV \rho_m(\vec{r}) = Q_m$
$Q_0^0 = -\frac{4\pi}{3} R^2 [2\cos\beta_0 + P]$, $W_{int} = \left(\frac{4\pi}{3}\right)^2 \frac{R^4}{R} (2\cos\beta_0 + P_1)(2\cos\beta_0 + P_2)$
$R = 2R + D$, $d = D/R$. Min d=0, $W_{int} = \left(\frac{4\pi}{3}\right)^2 \frac{1}{2} R^3 (2\cos\beta_0 + P_1)(2\cos\beta_0 + P_2)$
$W_{int} = \frac{8\pi^2}{9} R^3 M_s^2 = \frac{8\pi^2}{9} (50 \cdot 10^{-7}\,\text{cm})^3 (0{,}8 \cdot 10^3)^2 = \frac{2\pi^2}{9} 125 \cdot 10^{-18} (0{,}64) \approx 10^6$ erg =
$= \frac{8\pi^2}{9} \cdot 125 \cdot 0{,}64 \cdot 10^{-12}$ erg $= 702 \cdot 10^{-12}$ erg $= 7{,}02 \cdot 10^{-10}$ erg. typ. $W_m \sim 1 \cdot 10^{-10}$ erg.

(13/X) $W_{int} = \frac{8\pi^2}{9} \frac{R^4}{R(d=0)} (2\cos\beta_0 + P_1)(2\cos\beta_0 + P_2)$, $\cos\beta_0 = c$
$(2c + P_1)(2c + P_2)$. Razmyka $(P_2 = -1)$ a $(P_1 P_2 = +1)$ energii BP ale
ecb $\sim (2c+1)(2c-1) - (2c+1)(2c+1)^{P_2=+1} = 4c^2 - 1 - [4c^2 + 4c + 1] = -4c$ (?)
$\Delta W_{int}(P_1, P_2) \sim (-1)c$. $(P_1 P_2 = -1)$ cos. menší než energie, než $(P_1 P_2 = +1)$
Dle R=50 nm $\beta(P,v) = \beta_0 = \frac{1}{3}(92° + 85° + 75°) = 84°$, $\cos 84° = 0{,}105$
$\Delta W_{int} = \frac{8\pi^2}{9} R^3 (-4) \cdot 0{,}105$, $W_{int}(P_1 P_2) = \frac{8\pi^2}{9} R^3 (0{,}105 + P_1)(0{,}105 + P_2)$
$\Delta W_{int} = W_{int}(P_1 = +1, P_2 = -1) - W_{int}(P_1 P_2 = +1) \sim (2c+1)(2c-1) - (2c+1)^2 = 4c^2 - 1 -$
$-(4c^2 + 4c + 1) = -2 - 4c = -2(1 + 2c) = -2(1 + 2\cos\beta_0)$
Min d=0 $\Delta W_{int} = \frac{8\pi^2}{9} R^3 (-2)(1 + 2\cos\beta_0)$. Fe splitting min pro $\beta_0 = \pi/2$.

(14/X) Zakomeno u kolegů René a Macieje report Task 3 EPOS project „magnetostatic interaction..."

(3/XI) Saldo dosh: 2050 + 1.480 + 1.250 + 4150 = 8930 € [4 kamb.)
+1000 € n. = 9930 €
-30 € nope.

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